LDPC Code Performance and Optimum Code Rate for Contention Resolution Diversity ALOHA

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Abstract—In this paper we investigate several central aspects of using the unslotted Contention Resolution Diversity ALOHA (CRA) scheme with Successive Interference Cancellation (SIC). A model for the co-user-interference under consideration of time-, phase- and frequency-offsets is presented. With this model, the throughput and Packet Loss Ratio (PLR) performance of CRA is investigated when using Low Density Parity Check (LDPC) codes, showing that CRA can achieve remarkable performance also in these conditions. Moreover, the throughput and PLR gain of CRA in presence of power unbalance is studied, showing that significant gains can be expected. Following this, the central question is answered which channel rates optimize the goodput of CRA, showing that for CRA with few replicas a low, robust channel rate achieves optimum while for high numbers of replicas a high channel rate performs optimum. Finally an investigation of the interference distribution of an asynchronous TDMA scheme with replicas and the SIC operations in it are shown.

I. INTRODUCTION

After being used in communication systems for decades now, Random Access (RA) systems attract new attention, going far beyond providing a channel access for log-on, or signalling. In the last couple years, significant enhancements of the well known ALOHA [1] and Slotted ALOHA (SA) [2] schemes were developed, being based on Successive Interference Cancellation (SIC). Contention Resolution Diversity Slotted ALOHA (CRDSA) [3] and its extensions CRDSA++ [4], Irregular Repetition Slotted ALOHA (IRSA) [5] and coded slotted ALOHA [6] pave the way for new fields of application, exploiting the presence of a RA channel also for transmission of user data. Since these new and highly efficient schemes have been introduced, research has spread into several different directions, some focussing on the efficient integration of Demand Assigned Multiple Access (DAMA) with RA channels [7], others on the further extension of the efficiency, e.g. by applying variable rate coding to the SIC [6]. The stability of SIC based RA schemes is investigated e.g. in [8] for channels with retransmissions while [9] investigates the coded slotted ALOHA technique, where reliability is achieved without retransmissions.

Also recent standards, such as DVB-RCS2, have adopted CRDSA for transmission of signalling as well as user data. The usage of RA channels is particularly appealing for traffic profiles where small messages have to be delivered within a short time interval. Especially in Geostationary Orbit (GEO) satellite scenarios, where the procedure of requesting and assignment of capacity on a DAMA channel requires additional signalling (for the request and later for the assignment message) and generates additional delay (1 Round Trip Time (RTT) ≈ 500 ms is required for the request and assignment message exchange), the user data in a RA channel can in best case be transmitted directly, saving signalling overhead and delay. It was shown in [7] that the efficient integration of a CRDSA RA channel with a DAMA channel can improve the overall delay for this reason.

While the aforementioned applications are based on slotted RA schemes, where users need to be synchronized to time slots and can only transmit a fixed amount of data in each slot, an asynchronous TDMA scheme exploiting SIC, named Contention Resolution Diversity ALOHA (CRA), was proposed in [10]. There, also a first performance estimation showing the potential of this scheme was presented, assuming the decoding threshold resulting from the Shannon capacity limit. It was shown there that the CRA scheme can benefit from strong Forward Error Correction (FEC) and the presence of partial interference, which naturally occurs in unslotted schemes\(^1\), achieving theoretical throughputs far beyond SA and reaching even the basic version of CRDSA with 2 replicas. At the same time the asynchronous scheme can achieve very low PLR for low to moderate offered traffic loads \(G\), even below the one of slotted SIC schemes in a basic configuration, such as CRDSA with 2 replicas. This makes CRA a very appealing protocol in scenarios where a large number of user terminals shall access the medium in order to save the synchronization and fragmentation overhead, or where small and bursty data is generated, but needs to arrive very reliably.

CRA allows the transmission of variable packet lengths without the need of fragmentation into slots and thus avoids the fragment header signalling and padding overheads. Furthermore the timing requirements can be relaxed compared to slotted schemes. While slotted schemes have to comply

\(^1\)Compared to slotted schemes where interference is always affecting an entire slot.
with guard times among different slots, for framed CRA it is sufficient to comply with guard times at the beginning and end of a frame, which can be significantly larger. When considering transmission of the CRA replicas in individual sliding windows (in contrast to a common frame) as proposed in [11], the synchronization requirements can be even further relaxed, as will be explained in the following section.

In this paper we investigate several practical aspects of CRA which so far have not been investigated. They include the achievable throughput and PLR performance when using LDPC codes instead of assuming the Shannon bound as in [10], a model for Co-User-Interference (CUI) considering frequency-, time- and phase- offsets among users, the impact of power unbalance \(^2\) among users and finally the optimum channel rate \(R\) which achieves the maximum useful data rate after coding. In this context we also investigate the interference distribution for varying traffic loads and the path in this surface which results out of the SIC process.

II. REVIEW OF CRDSA AND CRA

The fundamental concept of CRDSA exploits the idea of sending multiple identical burst instances for each packet, where bursts carry information about the location of their replicas. Once one of the bursts can be successfully decoded, the channel is estimated from it and its waveform contribution can be reconstructed from the knowledge of the burst content, modulation index, code rate and channel estimation. Together with the information of the replica locations, the interfering waveform contributions of all replica instances can be removed which may allow the decoding of other bursts afterwards. This process is then iterated until either all bursts have been successfully decoded or until a maximum number of iterations has been reached. Fig. 1 a) illustrates this principle in a simple example where every packet consists of two burst instances (denoted degree \(d = 2\)). In the shown example, the collisions among the users prevent the decoding of any transmission.

Fig. 1 b) shows the corresponding CRA scheme with \(d = 2\) for asynchronous TDMA channels [10]. Here partial interference among bursts can occur. If the interference is not exceeding the error correction capability of the FEC code, packets can be decoded despite being partially interfered and their waveform contribution can be removed from the overall received signal. It should be noted that the CRA replica locations can be easily signalled by a simple seed, which allows the receiver to reconstruct the replica locations by means of a pseudo-random number generator. Furthermore, the start times in CRA can be also quantized, while the burst durations are variable. This way the need to fragment large packets into many chunks can be avoided and thus the fragmentation overhead can be saved. The signalling of the start indices can then also be done via a seed. While the example in Fig. 1 b) shows a fixed frame, the CRA principle can also be applied in a frameless structure. Fig. 2 illustrates this principle.

In the first case, framed CRA is shown. Here an arrival within a frame has to await the start of the next frame before the transmission can commence. Additionally, the terminal needs to be at least coarsely synchronized to the frame beginning. While this is still less demanding than being synchronized to a slot as in slotted schemes, operating CRA in a frameless manner as shown in the lower part of Fig. 2 can further reduce the synchronization requirements. Here the burst instances are sent within a defined time interval, denoted as transmission window (Tx Window). This transmission window however is not bound to a common frame structure as before but starts immediately whenever a packet is generated. The receiver stores all received waveform samples in a receive buffer and decodes by means of a sliding window which is moved over the receive buffer [11].

III. CRA IN PRESENCE OF REAL CODES

In [10] a first approximation of the possible performance of framed CRA was presented by using the Shannon bound as decoding threshold for the simulations. The minimum Signal to Noise and Interference Ratio (SNIR), \(SNIR_{min}\), which is required for a successful decoding of a packet is then:

\[
SNIR_{min} = 2^R - 1, \tag{1}
\]
where $R = R_c \cdot \log_2(M)$ denotes the overall rate of a transmission with coding rate $R_c$ and modulation index $M$. In [10], the average interference contribution was computed per burst and the burst considered lost if below the demodulation threshold $SNR_{\text{th}}$. While this approach provides a good information theoretic insight into the potential of CRA, the assumed Shannon bound is in practice not achievable by existing coding schemes. Furthermore, real interference will occur concentrated in time instead of equally distributed over the entire packet. In this section, we thus investigate the CRA performance in a scenario closer to reality where a LDPC code is used and where the interference contribution to the waveform is simulated by symbol instead of averaging the interference power over an entire burst.

### A. CRA Interference Model

For the simulation of CRA with real codes, a model for the interference power distribution in CRA is derived in the following. We consider one user as the reference (denoted as the useful user), and $N_u - 1$ users as interfering users. Every user generates a burst with length $T_s$ symbols composed of $c^{(i)}$ Quadrature Amplitude Modulated (QAM) symbols, where each symbol has duration $T_s$. The duration of the overall burst is thus $T_B = N_s \cdot T_s$. With the pulse shape $h(t) = \mathcal{F}^{-1} \left\{ \sqrt{H(f)} \right\}$, resulting from the frequency response $H(f)$ of a root-raised-cosine filter, the complex signal transmitted by user $i$ becomes:

$$x^{(i)}(t) = \sum_{k=1}^{N_s} c_k^{(i)} \cdot h(t - kT_s).$$

(2)

Every interfering user generates a burst with uniformly distributed start times in the interval $t \in [0, 2 \cdot T_B]$. The useful user generates its burst at $t = T_B$. The useful user has no time-, frequency- and phase-offset since it is considered as the reference. The waveform contributions of the other $N_u - 1$ interfering users are received with a random time-, frequency- and phase-offset, where the frequency offset $f_i$ is uniformly distributed within the bounds given by a max. frequency offset $f_{\text{max}}$, such that $f_i \sim \mathcal{U}[-f_{\text{max}}, f_{\text{max}}]$. The transmissions take place with a random time offset $\delta_i$ following a uniform distribution $\delta_i \sim \mathcal{U}[0, T_s]$ and having a phase offset $\Phi_i$, following $\Phi_i \sim \mathcal{U}[0, 2\pi]$. The received sum signal then becomes:

$$r(t) \simeq \sum_{i=1}^{N_u} g^{(i)}(t - \delta_i) \cdot e^{j(2\pi f_i t + \Phi_i)} + n(t),$$

(3)

where $g^{(i)}(t)$ is the response of the matched filter at the receiver to the signal contribution $x^{(i)}(t)$ of user $i$ and $n(t)$ is the Gaussian noise contribution of the AWGN channel. Knowing the original waveform of the useful user, we can now easily compute the interference signal contribution of the $N_u - 1$ interfering users and determine its distribution. As it turns out, the variance $\sigma^2$ of the interference distribution is proportional to the number of interfering users $N_u - 1$. If it is furthermore considered that every user transmits with a normalized power 1 (due to the QAM modulation) and that on average 50% of the reference burst are interfered due to the uniform arrival within $[0, 2 \cdot T_B]$, the variance can then be estimated from the number of interferers by:

$$\sigma^2 = \frac{N_u - 1}{2}.$$  

(4)

Fig. 3 shows the probability density function (PDF) of the interference amplitudes for $N_u = 7$ users and $N_u = 10$ users in simulations together with the idealized Gaussian distributions $\mathcal{N}(0, \sigma^2)$. As it can be seen, the distributions match very well, except for a small divergence in the area of the mean, which decreases with increasing number of interferers. The reason for this is that the arrival distribution is only approximately uniform for low number of interferers, resulting in a slightly increased probability of having interference free symbols. For increasing number of interferers the arrival distribution converges closer to the uniform distribution and the deviation diminishes.

As a quantitative measure of the fit between the simulated interference distribution $P$ and the Gaussian distribution $Q$ used in the model (following $\mathcal{N}(0, \sigma^2)$), the Kullback-Leibler divergence $D(P||Q) = \sum_{x \in X} P(x) \cdot \log_2 \frac{P(x)}{Q(x)}$ was computed for varying $N_u$, which is shown in Fig. 4 together with the measured variance $\sigma^2$.

![Fig. 3. Interference amplitude distribution for $N_u = 7$ and $N_u = 10$, simulated and ideal Gaussian with $\mathcal{N}(0, \sigma^2)$.](image)

Fig. 3. Interference amplitude distribution for $N_u = 7$ and $N_u = 10$, simulated and ideal Gaussian with $\mathcal{N}(0, \sigma^2)$.

![Fig. 4. Kullback-Leibler divergence $D(P||Q)$ for simulated interference distribution $P$ and Gauss distribution $Q$.](image)

Fig. 4. Kullback-Leibler divergence $D(P||Q)$ for simulated interference distribution $P$ and Gauss distribution $Q$. 
As it can be seen, the distribution $P$ measured in the simulations converges very rapidly towards the Gaussian distribution $Q$. Also the measured variances $\sigma^2$, shown in Fig. 4, match very well with the expected $\bar{\sigma}^2$ from Eq. (4). It should be noted though that for low number of interferers the assumption of Gaussian distributed interference power is getting increasingly coarse, as can be seen in Fig. 4.

### B. Numerical Results in Equal Power Conditions

Based on this model, the performance of CRA is hereafter investigated with a real LDPC code and considering interference symbol-wise. In the following, we consider a scenario where the system is populated by $N_u$ users, each generating one packet per CRA frame with uniformly distributed random data inside the packet. Prior to transmission, the packets are encoded by a $R_c = 1/2$ LDPC encoder. A random interleaver is applied on the codewords to counteract bursty interference. The packet is then replicated into $d$ transmission bursts and after QPSK modulation sent over an AWGN channel. At the receiver side the number of interfering users is computed for every symbol and a Gaussian distributed interference signal following $\mathcal{N}(0, \bar{\sigma}^2) = \mathcal{N}(0, \frac{\bar{\sigma}^2}{\sqrt{c}})$ is added on top. It is then attempted to decode every burst. The integrity of the decoded bursts is then checked, e.g. by using a Cyclic Redundancy Check (CRC). If a burst has been successfully decoded, the originally transmitted waveform is reconstructed from it by re-encoding, random interleaving and re-modulation of the packet. It is assumed that the channel estimation for time-, phase- and frequency-offset is ideal among the different burst instances of a packet, i.e. the waveform contributions of all burst instances of this packet can be ideally removed from the overall signal. This decoding process is successively iterated for at max. $I_{\text{max}} = 20$ times or stopped as soon as all bursts in the frame have been decoded. Fig. 5 shows the results for the simulations of CRA when using a $R_c = 1/2$ LDPC code for different repetition rates $d = 2, 3$ and 4, a packet size of $L = 1024$ bits for $SNR = 10 \, \text{dB}$. For comparison, the CRA results for averaging the interference power per transmission burst and attempting to decode with the Shannon bound threshold (denotes as $SH$) are shown as well.

The simulation results show that the maximum throughput for decoding with the Shannon bound thresholds cannot be reached when using an LDPC code, what could be expected. While a maximum throughput $S_{SH} = 0.94$ was reached for 2 burst instances per packet for the Shannon-bound decoder, the maximum throughput of the LDPC code $S_{LDPC} = 0.60$ is reached for 3 burst instances per packet. Though the degradation w.r.t. the Shannon-bound decoder is significant, the CRA performance is anyway very good and similar to the one of CRDSA with $d = 2$ instances per packet ($S_{CRDSA-2} = 0.55$), but avoiding the necessity of slotted schemes to have fixed burst sizes and thus eliminating losses due to fragment burst overheads and slot padding.

As the PLRs in Fig. 6 show, the CRA PLRs for the LDPC coding reach very low values for low to moderate load conditions. Though the PLRs with the LDPC coding cannot reach the ones of the Shannon-decoding threshold, the scheme remains anyway very appealing for use in systems where only moderate load is generated but data needs to be delivered with a high reliability, requiring a very low PLR.

### C. Numerical Results for Power Unbalance

In the scenario investigated in the previous section it was assumed that all users transmit with the same power. It was shown in [4] that the presence of power unbalance among users can further boost the performance of the SIC process in CRDSA. For this reason, we investigate in this section the impact on the throughput and PLR performance of CRA in presence of power unbalance. In real systems the received power levels of different users may vary due to manifold reasons such as different fading conditions, antenna gains, user terminal characteristics or geometrical reasons such as significantly different distances between transmitters and receivers. On the other hand power unbalance may be also intentionally introduced to benefit from the capture effect or in order to boost the SIC process. While the investigation of the optimum power unbalance distribution which maximizes the overall throughput is out of scope of this work, we focus here on slowly varying large scale signal fading, which is commonly
described by a log-normal distribution [12]. To analyze the performance of CRA in presence of log-normal distributed user power unbalance, in the following each user selects its transmission power randomly from a log-normal distribution where the expectation value of the log-normal distributed random variable is set to 1 (to ensure that the average transmission power is equal in all compared scenarios), and the variance of the log-normal distributed random variable $\sigma^2$ is variable. While the maximum transmission power in real systems would be bounded by the amplifiers, we do not consider such a limitation in the following.

Fig. 7 shows the achieved CRA-3 throughput for different variances $\sigma^2$ of the transmission power.

As can be seen, also CRA can significantly benefit from power unbalance and its throughput increases for LDPC coding in the simulated cases up to a maximum $S_{\text{max}}(\sigma^2 = 1) = 0.88$. The main reason for this significant gain is that also in situations where the interference ratio among two colliding packets is very high, the stronger of the two packets can be decoded and its signal contribution can be removed from the overall waveform. The improvement in presence of power unbalance is also evident in the PLR performance shown in Fig. 8.

It can be noted that the PLRs show an error floor which increases with the power unbalance. The explanation for this floor is that for high values of power unbalance a significant amount of packets is transmitted with low power, so that the decoder is not able to successfully decode the bursts even in interference free conditions. This is also the reason why the maximum achievable throughput shows a maximum and does not arbitrarily increase with $\sigma^2$.

IV. CRA OPTIMUM TRANSMISSION RATE

In slotted schemes (e.g. CRDSA) interference affects always an entire burst, while bursts in CRA are affected likely only by partial interference. For this reason CRA can benefit from FEC also in equal power conditions since also then interfered parts of a burst may be corrected. For the performance investigations of CRA and CRDSA in literature, always a fixed coding rate was assumed, e.g. $R_c = 1/2$ in [4] for CRDSA and in [10] for CRA. In the following we investigate which channel rate achieves the optimum useful throughput performance for CRA. Hereby it has to be considered that a robust FEC is able to correct also bursts which experience a higher level of interference, but reduces the useful rate for the user (assuming equal burst length) since the number of information bits is decreasing with the code rate. On the other hand, a burst with higher code rate can only tolerate few interference but has a higher efficiency due to a larger number of information bits per burst. For determining the optimum overall rate (resulting from code rate and modulation index) we are assuming in the following that bursts have a constant physical layer length of $N_s$ symbols. Moreover we are interested in the useful rate $\bar{S}$, i.e. the information bits which can be transmitted after decoding. While the investigations in [10] and the previous sections were focussing in the normalized throughput in terms of packets per frame, we now define the useful rate as:

$$\bar{S} = S \cdot R = S \cdot R_c \cdot \log_2(M),$$

(5)

where $S$ is the normalized throughput as in the previous sections. $\bar{S}$ now provides a measure for the amount of information bits which can be transmitted on average when transmitting packets of equal length, code rate $R_c$ and modulation index $M$. For finding the maximum useful rate, the overall rate was varied within $R \in [0.1, 3.5]$. For every rate, the CRA throughput was simulated for normalized offered traffic loads $G$ in the range $G \in [0.1, 20]$ Erl, a nominal SNR = 10 dB, a maximum number of SIC iterations of $I_{\text{max}} = 20$, and a packet length of $N_s = 1504$ symbols. This means that e.g. for QPSK, $R_c = 1/2$ the information length corresponds to an MPEG2-TS packet of 188 Bytes. For the decoding process, the Shannon bound was assumed, where the interference power is averaged over the entire burst. This was done since the Shannon bound provides decoding thresholds for arbitrary rates, while the usage of LDPC codes only allows discrete channel rates. If the average SNIR falls below the Shannon decoding
threshold the burst is considered lost and successfully decoded otherwise. As before, we assume that the channel estimation is ideal and successfully decoded bursts can be ideally removed from the waveform together with their replicas. Fig. 9 shows the results for CRA with 2, 3 and 4 burst instances per packet.

![Fig. 9. Maximum achievable normalized useful rate $S_{max}$ for CRA-2, 3 and 4 instances per packet.](image)

The results show that for CRA-2 a low rate $R = 0.4$ achieves the overall highest throughput with $S_{max} = 1.2$ bit/s/Hz. For increasing number of burst instances per packet, the useful rate reaches a local maximum around $R = 1$ but the global maximum is located at high values of $R$, close to the maximum achievable Shannon rate $R = 3.45$. CRA-3 reaches here a maximum useful rate of $S_{max} \approx 1.05$ bit/s/Hz. For all number of burst instances the results show that the initial increase in useful rate is followed by a decrease for intermediate rates $R$, until it finally increases again towards high rates. This is due to the two conflicting effects mentioned before. For low rates $R$, a high amount of interference can be tolerated since the strong FEC is more robust to interferences. Since the interference is increasing with the offered traffic load $G$, the maximum useful rate is here reached for high levels of $G$, but with a low spectral efficiency $R$ per burst. For high values of $R$, only few interference can be tolerated before loosing a packet. The highest useful rates are here reached for low offered traffic loads $G$, but for a very high spectral efficiency in every transmission. An interesting observation can be made for all curves in the area around $R \approx 0.9$. The curvature of all curves changes from a concave to a convex shape. Additionally, the decrease of the CRA-2 useful rate slows down for $R = 0.9$ before then declining faster again. The explanation can be found when analyzing the interference distribution in more detail. Fig. 10 shows the probability of getting $SNIR = x$ (as a measure for the degree of interference) in dependency of the offered traffic load $G$, i.e. $Pr(SNIR = x|G)$.

The curve $E\{SNIR(G)\}$ shows the resulting average SNIR in dependence of the load when starting the SIC process. It can be seen that the average SNIR increases with decreasing $G$, which can be expected since the probability of two packets interfering each other decreases together with $G$.

![Fig. 10. Interference distribution expressed in linear SNIR for varying $G$ and CRA-2](image)

Furthermore, Fig. 10 shows that the monotonic decrease of $Pr(SNIR = x|G)$ for $SNIR \geq x^*$ with $x^* = \arg \max G \approx 0.9$ and $G \approx 0.5$, where $Pr(SNIR = x|G)$ first increases sharply before decreasing monotonically afterwards again.

Considering that for each rate $R$ there exists a minimum $SNIR_{min}$ according to Eq. (1) for which packets with $SNIR \geq SNIR_{min}$ can be decoded, the SIC decoding process can now be represented as a trajectory in this surface. Starting from an initial offered traffic load $G_0$, the corresponding initial SNIR distribution results in the average SNIR on the previously mentioned curve $E\{SNIR(G)\}$ shown in Fig. 10. Due to the SIC process, packets with a higher SNIR can be decoded and their interference contribution can be removed from the frame for the next SIC round. This next round can then be equivalently thought of as starting from a frame with lower offered traffic load $G_1 < G_0$ resulting from the packets remaining after the first round, since the correctly decoded packets have been removed. At the same time the average SNIR (considering also the decoded packets) will increase since less interferers are present and correctly decoded packets do not interfere anymore. The iterative reduction of load and increase of SNIR results then in a trajectory from high loads $G$ and low SNIRs towards low loads and high SNIR. The path of the trajectory is hereby also determined by the rate $R$. Fig. 10 shows the average trajectories from the given starting point, i.e. the average change in $G$ and $SNIR$ for each iteration step. Three examples for rates $R = 0.14, 0.25$ and $0.4$ are illustrated when starting from a high initial offered load $G_0 = 3.06$ Erl and their respective trajectories in the SNIR-G surface. For a rate $R = 0.14$, the SIC process decodes most of the packets already in the first iteration, so it moves directly towards low loads and high SNIR. When increasing the rate $R$ to $R = 0.25$, it can be seen that the SIC process can remove successfully decoded packets from the frame (strong decrease of load $G$) in the first iteration while the increase of SNIR remains limited. Only in the second iteration other packets can be decoded and removed, reducing the load $G$ further and approaching high SNIRs. Finally, for a rate $R = 0.4$ and $G_0 = 3.06$ Erl, i.e. the configuration resulting in the max. normalized throughput...
as shown in Fig. 9, a qualitatively similar behaviour can be observed. The decrease of $G$ in the first iteration leads only to a slight improvement in the average SNIR and it takes several iterations to resolve most of the collided packets in the frame, resulting in low load and high SNIR. It should be also noted that for this rate the trajectory approaches the peak of the local maximum.

In order to investigate the reason for the curve shape around $R = 0.9$ in Fig. 9, also the average trajectory for $R = 0.9$ and load $G_0 = 1.2 \text{Erl}$ is shown. As it can be seen, the average trajectory traverses the center region of the local maximum. While for the trajectories of the other rates the probability $\Pr(SNIR = x|G)$ was monotonically decreasing, the trajectory for $R = 0.9$ reaches at the local maximum a point where an increasing fraction of packets has an SNIR high enough to be decoded. Due to this, the amount of bursts which can be decoded is not decreasing as before and causes that the useful rate remains constant. As soon as the local maximum has been traversed, the fraction of packets decreases monotonically again and the throughput decreases together with it.

This finding is also reflected in the PLR curves, as shown in Fig. 11.

As can be seen, the PLR curves consist of three regions. In the first region for rates up to $R \leq 0.8$, where the trajectories are not intersecting the local maximum in Fig. 10, the PLRs have a similar slope. In the following region of transition for $0.85 \leq R < 1.0$ the trajectories are traversing the local maximum, which results in a change of PLR slope, in particular in the region for small $G$ where the local maximum occurs. Finally the third region for $R \geq 1.0$ does not intersect the local maximum anymore and the PLR curves in this region have thus all similar slopes again.

V. CONCLUSIONS

In this paper we have derived an interference model of co-user interference for SIC in CRA and shown that the CUI under consideration of frequency-, time- and phase-offsets can be appropriately modeled by a Gaussian distribution with a variance proportionally to the number of interfering users. Furthermore results on the performance of the CRA scheme have been presented for the usage of LDPC codes and considering the frequency-, time- and phase-offsets occurring in real systems. It was shown that the CRA performance is reduced when using LDPC codes compared to the Shannon decoding bounds, but reaches anyway impressive throughputs outperforming SA and getting close to CRDSA, without having the disadvantages of a time-synchronized slotted scheme. It was confirmed that the PLR performance of CRA is in particular interesting for applications which require very low PLR at low to moderate offered traffic loads. Also the throughput and PLR gain which results from the presence of power unbalance (irrespective whether natural or artificially generated) was shown, proving that CRA can benefit from the presence of power unbalance in a similar manner as CRDSA. Finally it was investigated which channel rate (combination of modulation index and code rate) is optimum to achieve the highest useful rate in a CRA system. The investigations have shown that the useful rate can be maximized for low rates $R = 0.4$ when using low-order CRA with 2 burst instances where high levels of interference can be tolerated for packets carrying few information bits. For CRA with 3 and 4 burst instances instead it was shown that efficient rates close to the maximum achievable rate maximize the throughput, i.e. the system operates optimum in the region where only few interference exists but packets have a high fraction of information bits. In this context also the distribution of SNIR versus offered load was investigated and how the SIC process of CRA transits this plane. The results have shown that the interference distribution has a non-linear region which is the cause of a non-monotonic decrease of the achievable useful rate from its maximum.

REFERENCES

