TRACK SEEKING CONTROL OF HARD DISK DRIVES BASED ON NEW TWO-DEGREE-OF-FREEDOM CONTROL SCHEME WITH VIBRATION MINIMIZED Trajectories

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Abstract: This paper proposes a new TDOF control scheme to minimize the tracking error caused by the neglected vibration modes of the plant. The design parameter is optimized from experimental data. The effectiveness is shown by simulation using a precise model of hard disk drive. Copyright © 2005 IFAC.

Keywords: hard disk drives, track seeking control, two-degree-of-freedom control, vibration minimized trajectories, residual stiffness

1. INTRODUCTION

In track-seeking control of hard disk drives (HDD), a read/write head must be driven from one track to another as fast as possible. Since the system has resonance modes at high frequencies, vibration minimized trajectories can reduce residual vibration to improve the seeking time. Thus, the design methods of these trajectories such as the minimum jerk input have been proposed (Y. Mizoshita et al. 1996, Hara et al. 2000), and our research group has also been developed a simple method to calculate them for sampled-data system by using final-state control (FSC) method (Hirata et al. 2002).

These trajectories should be used with feedback controller because actual system has various disturbances, and two-degree-of-freedom (TDOF) control scheme is effective (Hirata et al. 1992, Ishikawa et al. 1996, Yi and Tomizuka 1999). However, in (Hirata and Nonami 2004), it has been shown that TDOF controller may increase the positioning error at the target position under some conditions. For these reasons, a new TDOF control system has been proposed to improve the positioning error at the end of seeking by introducing a simple scheme into the conventional TDOF control system (Hirata and Nonami 2004). The method requires the model of high frequency resonance modes. However, in general, it might be difficult to obtain an accurate model of these resonance modes, and it will be fluctuated by piece-to-piece variations of the products.

This paper proposes a practical method to optimize the design parameter of the new TDOF control system from experimental data. It is shown that the tracking error is an affine function of the design parameter under an simple approximation, and the parameter is optimized from a measured tracking error profile based on the least-square method. The case that the gain of the plant has a perturbation is also considered. The effectiveness of the proposed method is shown by simulation using a precise model of HDDs.

2. HARD DISK DRIVES AND PLANT MODEL

Fig. 1 shows a typical hard disk drive, and it has read/write heads, a voice coil motor (VCM) and magnetic disks which are driven by a spindle motor. The control input u is a voltage [V] to the current amplifier for VCM and the measurement output y is a head position [track]. In this study, a 2.5inch hard disk drive is used and the frequency response, which was obtained by a servo analyzer, is shown in Fig. 2 by dotted line. The transfer function including high frequency vibration modes is assumed to be given as:

\[ P(s) = k_p \left( \frac{1}{s^2} + \sum_{i=1}^{\ell} \frac{k_i \omega_i^2}{s^2 + 2\zeta_i \omega_i s + \omega_i^2} \right), \] (1)

where \( \ell \) is the number of vibration modes. By fitting the parameters of (1) to the measured frequency re-
response shown in Fig. 2 by dotted line, the 12th order model with five vibration modes is obtained, e.g., \( L = 5 \) in (1). On the other hand, for simplicity, the nominal model of \( P(s) \) is defined as a double-integrator plant as follows:

\[
P_n(s) = \frac{k_p}{s^2}.
\]

The nominal model (2) can be used to design a vibration minimized trajectory and a feedback controller. The frequency responses of \( P(s) \) and \( P_n(s) \) are shown in Fig. 2 by solid line and dashed line respectively. The discrete-time model of the plant is obtained by the zero-order hold method. The sampling frequency of the plant is 10kHz and the Nyquist frequency is 5kHz. Therefore almost all the resonance modes reside above the Nyquist frequency. From this point of view, it might also be appropriate to define the nominal model as the double-integrator plant.

3. VIBRATION MINIMIZED TRAJECTORY AND TDOF CONTROL

3.1 Vibration minimized trajectory

In this paper, two kinds of vibration minimized trajectory are used as the input to the new TDOF control system.\n
**FSC input** In (Y.Mizoshita et al. 1996), a vibration minimized control input was obtained so that the square integral of the differentiated acceleration (jerk) is minimum. This input can be easily obtained by using final-state control in discrete-time domain.

**FFSC input** If the frequency components of FSC input can be minimized at a certain frequency region, the residual vibration is reduced more. In this case, a frequency shaped final-state control (FFSC) method is effective (Hirata et al. 2002). This method can calculate a feedforward input so that the gain of the Fourier transform of the control input, which is generated though zero-order hold, is minimum at desired frequency points. The frequency shaped final state control input will be referred to as **FFSC input** in this paper.

Fig. 3 shows the time responses and the Fourier transform (FT) of FSC and FFSC inputs of 50 tracks seeking with step number \( N = 40 \). FFSC input is obtained so that the frequency component of the control input is reduced from 2kHz to 10kHz. Fig. 4 shows the simulation results of the open-loop response when FSC and FFSC inputs are applied to the high order model \( P(s) \). It shows that FFSC input achieves lower residual vibration after track seeking than FSC input, and the seeking time might be shorten.

3.2 Two-degree-of-freedom control

FSC and FFSC inputs should be implemented with feedback controller because actual systems have various disturbances and plant uncertainties. In this case, a two-degree-of-freedom (TDOF) control is effective and commonly used. The block diagram of TDOF control system is shown in Fig. 5, where \( P, P_n \) and \( K \) are an actual plant, a nominal model and a feedback controller respectively, and \( K \) works to minimize the tracking error \( y_m - y \). If \( P_n = P \) and the tracking error is zero, the transfer function from \( u_m \) to \( y \) becomes \( P_n \).
A possible way is to use a high order model for the transfer function of the neglected vibration modes as follows (Nagamatsu 1985):

In this paper, it is assumed that the feedback controller has been designed in advance by using the discrete-time $H_{\infty}$ control theory to have a good disturbance rejection and a robustness to the unmodeled dynamics of the plant (Takiguchi et al. 2002, Hirata et al. 2003). Fig. 6 shows the result when FSC and FFSC inputs are applied to the conventional TDOF control system. It shows that the FFSC input can improve the residual vibration after track seeking. However, the tracking error at $t = 2[\text{ms}]$ is degraded in both cases compared with the result of the open-loop control as shown in Fig. 4. This is a problem because the TDOF control is introduced to improve the performance but it fails. If the tracking error $y_m - y$ is small enough, the TDOF control system acts as feedforward control and the seeking error at $t = 2[\text{ms}]$ will be improved. One possible way is to use a high order model for the nominal model $P_n(s)$. However, it might be difficult to obtain a precise model of high frequency resonance modes with a good accuracy. It is also undesirable to have vibration modes in the nominal model because the reference input $y_m$ might have oscillation.

If the frequencies of resonance modes of the plant are higher than that of control bandwidth, these vibration modes can be approximated by a static gain, which is referred to as residual stiffness (Nagamatsu 1985). Since vibration minimized trajectories do not excite high frequency vibration modes, the introduction of the static gain in the nominal model is effective to include the property of high frequency resonance modes in the model. Thus a new TDOF control system with a static gain $\alpha$ as shown in Fig. 7 has been proposed to improve the tracking error $y_m - y$ (Hirata and Nonami 2004).

Remarks: From the structure of Fig. 7, the feedforward input $u_m[k]$ must satisfy $u_m[N] = 0$ so that $y[k] = y_m[k]$ holds at $k = N$ for a nonzero $\alpha$.

It is straightforward way to determine $\alpha$ from the residual stiffness itself (Hirata and Nonami 2004). The residual stiffness is defined by substituting $s = 0$ into the transfer function of the neglected vibration modes as follows (Nagamatsu 1985):

$$y_m = k_p \sum_{i=1}^N k_i.$$  \hspace{1cm} (3)

$k_{res}$ corresponds to the gain of the neglected vibration modes at low frequency, and it was obtained as $k_{res} = -0.244$ using the high order model $P(s)$. Fig. 8 shows the simulation results of the new TDOF control with $\alpha = k_{res}$. Comparing with Fig. 6, the error at the end of seeking is improved in both FSC and FFSC inputs. Especially for the result with FFSC inputs, a good seeking performance without residual vibration is achieved.

4. PROPOSED METHODS TO DETERMINE $\alpha$ FROM EXPERIMENTAL DATA

In the previous section, it has been shown that the parameter $\alpha$ can be obtained from the residual stiffness, which is the sum of the residue of each vibration mode. However, in actual systems, it is difficult to identify the correct value of residue from the measured frequency response of the plant. Thus, in this section, methods to obtain the optimum $\alpha$ from the measured tracking error profile during track seeking are proposed.

4.1 Method A

This section shows a basic idea to obtain the optimum $\alpha$ corresponding to the residual stiffness from experimental data. It is assumed that the high frequency vibration modes of the actual plant can be modeled as a constant gain because vibration minimized trajectory does not excite these vibration modes. Thus, the actual plant can be represented by the nominal model with an additive gain $\alpha$ corresponding to the residual stiffness.
αmeasured tracking error signal, the relation between as shown in Fig. 9(a). To identify the parameter
Fig. 10. ODOF control system with feedforward inputs

In Fig. 9(a), the tracking error $e$ becomes a nonlinear function of $\alpha$ for a given feedforward input $u_m$ because the feedback property is affected by the change of $\alpha$. However, the tracking error is small, e.g., smaller than 1track as shown in Fig. 6, and the compensation input $u_k$ generated by the feedback controller is also very small compared with the feedforward input $u_m$. Therefore Fig. 9(a) can be approximated by Fig. 9(b). Moreover Fig. 9(b) is equivalent to an one-degree-of-freedom (ODOF) control system with feedforward inputs as shown in Fig. 10.

The discrete-time state-space realization of $P_n$ and $K$ are given as follows:

$P_n[z] := \{A_n, B_n, C_n, 0\}$,
$K[z] := \{A_k, B_k, C_k, D_k\}$.

Thus the closed-loop system shown in Fig. 10 can be represented as:

$x[k+1] = Ax[k] + B_n u_m[k] + B_r (y_m[k] - \alpha u_m[k])$,
$y[k] = C x[k] + \alpha u_m[k]$.

where

$A := \begin{bmatrix} A_n - B_n D_n C_n & B_n C_n \\ -B_n C_k & A_k \end{bmatrix}$,
$B_n := \begin{bmatrix} B_n \\ 0 \end{bmatrix}$, $B_r := \begin{bmatrix} B_r D_k \\ B_k \end{bmatrix}$, $C := \begin{bmatrix} C_n & 0 \end{bmatrix}$.

For simple notation, define

$U_m := [u_m[0], \ldots, u_m[N-1]]^T$, \hspace{1cm} (5)
$E := [e[0], \ldots, e[N-1]]^T$. \hspace{1cm} (6)

Here introduce the following notation for Toeplitz matrix:

$T_p(A, B, C, D, N)$

\begin{bmatrix}
D & O & \cdots & O \\
T_{21} & D & \cdots & \vdots \\
\vdots & \vdots & \ddots & \vdots \\
T_{N1} & T_{N2} & \cdots & D \\
\end{bmatrix} \in \mathbb{R}^{N \times N}, \hspace{1cm} (7)

where $t_{ij} := CA^{i-j-1}B$ \hspace{0.5cm} $(i > j > 0)$.

Lemma 1. Assume that the initial value of the feedback system (4) is zero, and $|u_m| \gg |u_k|$ holds. The tracking error $E$ in Fig. 7 can be approximated by a linear function of $\alpha$ as:

$E = \alpha(\Omega_r - I)U_m$, \hspace{1cm} (8)

where

$\Omega_r = T_p(A, B, C, 0, N) \in \mathbb{R}^{N \times N}$.

The proof is omitted because it is straightforward. \hspace{1cm} $\square$

Here it is assumed that the tracking error $e[i]$ during track seeking can be measured, and the measured signal is defined as $\bar{e}[i]$. Using $\bar{e}[i]$, let us consider to obtain $\alpha$ so that the least square error of $e[i] - \bar{e}[i]$ for $i = 0, \ldots, N - 1$ is minimum.

Theorem 1. (Least square solution). The optimum $\alpha$ which minimizes the square error $\Sigma (e[i] - \bar{e}[i])^2$ for a given $U_m$ can be calculated as follows:

$\alpha_{opt} = \frac{U_m^T(\Omega_r - I)^T}{U_m^T(\Omega_r - I)U_m} E$, \hspace{1cm} (9)

where $E$ is defined as

$E := [e[0], \ldots, e[N-1]]^T$.

Design procedure (Method A)

Step1 Using the conventional TDOF control as shown in Fig. 5, measure the tracking error $e[i]$ during track seeking.
Step 2 From the measured data $\bar{e}[i]$ and (9), calculate the optimum $\alpha_{opt}$.

Step 3 Execute a track seeking by using the proposed TDOF control system shown in Fig. 7 with $\alpha_{opt}$.

Remarks: For actual HDD products, track seeking cannot be executed two times for one track seeking. One possible solution is to execute the identification process of step 1 and step 2 before HDD products are shipped, and store the optimum value in NVRAM. If the seeking error is increased because of aging, the identification process might be executed again to update $\alpha_{opt}$.

Method A is applied to FSC and FFSC input. Using the tracking error profile shown in Fig. 6, the optimum value of $\alpha$ is obtained. The result is $\alpha_{opt} = -0.237$ for FSC input and $\alpha_{opt} = -0.241$ for FFSC input. Fig. 11 shows the track seeking results obtained by Fig. 7 with these optimum values. It shows that the tracking errors can be improved by Method A, and the results shown in Fig. 8 and Fig. 11 have almost the same performance.

4.2 Method B

In track seeking control problem, the feedforward control dominates the total performance. Thus, parameter change of the plant will degrade the track seeking performance. Especially, a loop gain change of the plant is very sensitive to the performance. The upper figure of Fig. 12 shows the track seeking result when the gain of the plant changes only $-1$ percent, and the performance degradation is obvious. Using this error profile, $\alpha_{opt} = 0.132$ was obtained by Method A, and it is quite different from the value of the residual stiffness ($k_{res} = -0.244$). This value can not improve the track seeking performance as shown in the below figure of Fig. 12.

In order to cope with the gain change of the plant, another parameter $\beta$ is introduced as shown in Fig. 13(a). This parameter represents the gain change of the plant, and it is suggested to identify both parameters of $\alpha$ and $\beta$ simultaneously from the measured tracking error profile. As same as Method A, $|u_m| \gg |u_k|$ can be assumed to have an approximated system shown in Fig. 13(b). Finally, as shown in Fig. 13(c), ODOF control system with feedforward inputs is obtained.

*Lemma 2.* Assume that the initial value of the closed-loop system shown by Fig. 13(c) is zero, and $|u_m| \gg |u_k|$ holds. Then the tracking error $E$ in Fig. 13(a) can be approximated by an affine function of $\alpha$ and $\beta$ as:

$$E = Z_0 + \bar{\alpha}Z_1 + \bar{\beta}Z_2$$

where

$$\bar{\alpha} := \alpha\beta,$$

$$Z_0 := (I - \Omega_e)Y_m,$$

$$Z_1 := (\Omega_e - I)U_m, Z_2 := -\Omega_eU_m,$$

and

$$\Omega_e := T_p\{A, B_r, C, 0, N\} \in \mathbb{R}^{N \times N},$$

$$Y_m := \left[y_m[0], \ldots, y_m[N - 1]\right]^T.$$  

The proof is straightforward from Fig. 13(c).  

*Theorem 2.* (Least square solution) For given $U_m, Y_m$ and measured tracking error $\bar{e}[i]$, the optimum $\alpha$ and $\beta$ which minimize the square error $\sum (e[i] - \bar{e}[i])^2$ can be calculated as follows:

$$\alpha = \bar{\alpha}/\bar{\beta}$$

(10)

where

$$\begin{bmatrix} \bar{\alpha} \\ \bar{\beta} \end{bmatrix} = (Z^TZ)^{-1}Z^T(E - Z_3)$$

$$Z = [Z_1, Z_2]$$
Step 1 Using the conventional TDOF control as shown in Fig. 5, measure the tracking error $\bar{e}[i]$ during track seeking.

Step 2 From the measured data $\bar{e}[i]$, calculate the optimum $\alpha_{opt}$ and $\beta_{opt}$ from Theorem 2.

Step 3 Execute a track seeking by using the new TDOF control system shown in Fig. 14 with $\alpha_{opt}$ and $\beta_{opt}$.

In Fig. 14, the compensation gain $1/\beta$ is introduced only in the feedforward input $u_{\text{ff}}$ not in the feedback loop because of the following reasons:

1. In hard disk drives industry, it is very nervous to introduce an adjustment gain into the feedback loop because it may destabilize the system.

2. Since $|u_{\text{ff}}| \gg |u_k|$ holds, it is enough to compensate the feedforward input $u_{\text{ff}}$ only to improve track seeking performance.

Method B is applied to both FSC and FFSC inputs assuming a gain change of $-1$ percent. The optimized parameters of $\alpha$ and $\beta$ in step 2 are shown in Table 1. It shows that $\alpha_{opt}$ is close to the value of residual stiffness and $\beta_{opt}$ reflects the gain change of $-1$ percent correctly. Fig. 15 shows the track seeking performance obtained by using the parameters in Table 1 and the TDOF control system shown in Fig. 14. The figure shows that a good performance can be obtained in the face of the gain change of the plant by Method B.

### Table 1. $\alpha_{opt}$ and $\beta_{opt}$.

<table>
<thead>
<tr>
<th>Input</th>
<th>$\alpha_{opt}$</th>
<th>$\beta_{opt}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>FSC input</td>
<td>-0.246</td>
<td>0.990</td>
</tr>
<tr>
<td>FFSC input</td>
<td>-0.249</td>
<td>0.990</td>
</tr>
</tbody>
</table>

5. CONCLUSION

This paper has proposed a new TDOF control scheme which can improve the track error at the end of seeking, and two methods, Method A and Method B, have been proposed to optimize the performance from the measured tracking error profile. Especially for Method B, it can achieve a good performance in the face of a gain perturbation of the plant. The effectiveness of the proposed method has been shown by simulation using the high order model including five vibration modes at high frequencies. Since the proposed method does not depend on reference trajectory and feedback controller, it can apply to many other positioning systems.

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REFERENCES


