

**DISORIENTED CHIRAL CONDENSATE
FORMATION IN
HEAVY ION COLLISIONS? ***

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Dedicated to André Krzywicki

Abstract

One of the main aims of present and upcoming high energy heavy ion collision experiments is to study new phases of matter at extreme temperature and density. It is expected that a nontrivial classical pion field configuration can occasionally form during the out-of-equilibrium chiral phase transition. We have recently shown that, contrarily to what has been assumed so far, this configuration is not identical to the so-called disoriented chiral condensate (DCC), proposed in the early 1990's. A detailed analysis reveals that a more realistic picture is that of an "unpolarized" DCC, where the Fourier modes of the field have completely independent orientations in isospin space instead of being aligned with each other as in the original DCC. This has important implications concerning the possible detection of the phenomenon. In particular, the main expected signature of the original DCC, which is used in most experimental searches, is absent in the unpolarized case. We point out that the fact that no evidence of DCC formation has been reported so far in nuclear collisions actually agrees with our present theoretical understanding. New experimental strategies should be designed to look for the unpolarized DCC in existing data from SPS as well as in future searches at RHIC and LHC.

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1 The Disoriented Chiral Condensate

Large number of pions with energies below or of the order of a few hundred MeV are commonly produced in high energy hadronic or nuclear collisions experiments. Very soon, it has been proposed to interpret this phenomenon as resulting from classical radiation, in analogy with the classical nature of electromagnetic waves corresponding to a large number of photons. This idea has received only a marginal attention until the early 1990's where it has been rediscovered and further developed, on solid theoretical grounds [1, 2, 3]. In Ref. [2], Blaizot and Krzywicki studied the dynamics of a classical pion field configuration described by the non-linear sigma model, in the context of a simple idealized model of a high energy collision. They found a solution where the classical pion field oscillates coherently in a given direction in isospin space. In parallel, Bjorken proposed that, in such collisions, the chiral quark condensate could be momentarily misaligned from its vacuum orientation in isospin space, resulting in a non-trivial long wavelength configuration of the pion field [3]. This intuitive picture corresponds to Blaizot and Krzywicki's solution. The "disoriented chiral condensate" (DCC) subsequently decays toward ordinary vacuum by coherent (classical) radiation of soft pions. The emitted pions can be thought of as being in a coherent state [4], schematically¹:

$$|\text{DCC}\rangle \sim \exp\left(\sum_{i=1}^3 \int d^3k J_a(\mathbf{k}) a_a^\dagger(\mathbf{k})\right) |0\rangle, \quad (1)$$

where $a_a^\dagger(\mathbf{k})$ is the creation operator of a free pion with isospin $a = 1, 2, 3$ and momentum \mathbf{k} and where $J_a(\mathbf{k})$ is the classical source emitting pions (summation over repeated indices is implied). The latter has the following form:

$$J_a(\mathbf{k}) = J(\mathbf{k}) e_a, \quad (2)$$

that is, its orientation in isospin space is *momentum independent*. This reflects the fact that the disorientation of the pion field in isospin space is assumed to be constant in space (in the volume occupied by the DCC). The classical pion field configuration corresponding to this state is therefore a

¹Strictly speaking, the following state contains components of arbitrary charge. Generalized coherent or squeezed states of definite charge can be constructed [5]. This is, however, of no relevance for the present discussion.

superposition of waves, which oscillate in the very same direction in isospin space. One can speak of a (linearly) polarized field configuration [6]. This property is at the origin of the most striking signature of the DCC, namely the anomalously large event-by-event fluctuations of the neutral fraction of the *total* number of radiated pions

$$f = \frac{N_{\pi^0}}{N_{\pi^0} + N_{\pi^\pm}} \equiv \frac{N_{\pi^3}}{N_{\pi^1} + N_{\pi^2} + N_{\pi^3}},$$

where $N_{\pi^a} \equiv \int d^3k |J_a(\mathbf{k})|^2$ is the number of pions of type a . Indeed, exploiting isospin symmetry of strong interactions, it is an easy exercise to show that the probability distribution of the above ratio is given by the inverse square root law (see e.g. [2, 3]):

$$P(f) = \frac{1}{2\sqrt{f}}. \quad (3)$$

This is a direct manifestation of the coherence of the DCC state and is to be contrasted with the sharply peaked binomial distribution one obtains in the case of incoherent pion production. In particular, the probability that less than 10% of the total number of radiated pions² be neutral is expected to be as large as 30%. This striking property is at the basis of all present strategies for experimental searches (see e.g. [7] and references therein)³.

2 A dynamical scenario: the out-of-equilibrium chiral phase transition

Although it had been understood that the DCC field configuration is a solution of the low energy dynamics of strong interactions, a microscopic understanding of its possible formation was still lacking. An important progress has been made in this respect in 1993, when K. Rajagopal and F. Wilczek realized that a classical long wavelength pion field configuration could indeed be formed during the out of equilibrium chiral phase transition in heavy ion collisions: the rapid expansion of the system results in a sudden suppression of initial fluctuations (quenching) which in turn triggers a dramatic amplification of soft pion modes [9].

²Of course, this only concerns the pions emitted from the DCC.

³Various other possible signatures have also been discussed [8].

2.1 The quench scenario

To model the dynamics of the far from equilibrium chiral phase transition, Rajagopal and Wilczek considered the classical $O(4)$ linear sigma model, with action

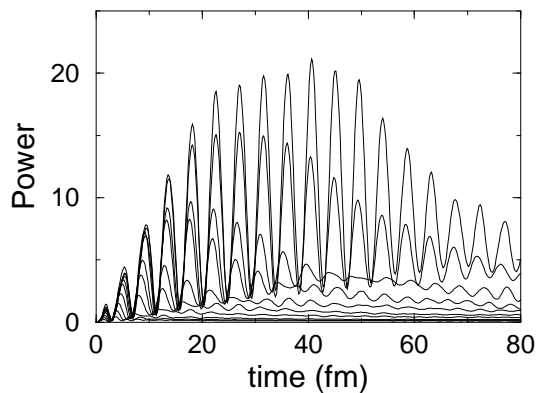
$$\mathcal{S} = \int d^4x \left\{ \frac{1}{2} \partial_\mu \phi_a \partial^\mu \phi_a - \frac{\lambda}{4} (\phi_a \phi_a - v^2)^2 + H\sigma \right\}. \quad (4)$$

The parameters v , λ and H are related to physical quantities via:

$$m_\pi^2 = m_\sigma^2 - 2\lambda f_\pi^2 = \lambda (f_\pi^2 - v^2), \text{ and } H = f_\pi m_\pi^2.$$

and are chosen so that $m_\pi = 135$ MeV, $m_\sigma = 600$ MeV and $f_\pi = 92.5$ MeV. The corresponding equations of motion are evolved numerically on a lattice for a given initial field configuration. The dramatic effect of expansion is modeled by assuming an instantaneous quench from above to below the critical temperature at initial time. In practice, the values of the field ϕ_a and its first time derivative $\dot{\phi}_a$ are chosen as independent random variables at each site of a lattice⁴. They are sampled from Gaussian distributions centered at $\phi_a = \dot{\phi}_a = 0$, corresponding to a high temperature symmetric phase, and with variances (in lattice units) $\langle \phi_a^2 \rangle = v^2/16$ and $\langle \dot{\phi}_a^2 \rangle = v^2/4$, which are *smaller* than what they would be in the high temperature phase, thereby describing quenched fluctuations.

Figure 1: *Time evolution of the power in the Fourier modes of the pion field π_3 for quenched initial conditions, as in Ref. [9]. Low momentum modes experience dramatic amplification, the softer the mode the larger the amplification, and exhibit coherent oscillations with period $\sim 2\pi/m_\pi$.*



The main result of Ref. [9] is reproduced in Fig. 1, which shows the squared amplitude of different field modes as a function of time: due to the

⁴The lattice spacing has therefore the physical meaning of the correlation length in the high temperature phase.

far from equilibrium initial condition, one observes a dramatic amplification of low momentum modes of the pion field at intermediate times. This phenomenon is analogous to that of domain formation after quenching a ferromagnet, or to the growth of quantum fluctuation during the preheating of the early universe after a period of inflation. It can be qualitatively understood in a mean-field approximation [9, 10]: at early times, the local curvature of the effective potential (effective mass squared) is negative and soft modes, for which the associated effective frequency is imaginary, experience amplification. This is the so-called spinodal instability, well known in condensed matter or nuclear physics. At intermediate times, the (quasi-)periodic oscillations of field averaged value around the minimum of its effective potential trigger a further amplification via the mechanism of parametric resonance [10, 11].

In terms of the underlying quantum dynamics, the amplification of Fourier modes amplitudes is to be interpreted as the formation of a classical configuration of the pion field. Because of expansion, which eventually causes interactions to freeze out, the system may be left in such a configuration and subsequently decay through coherent pion emission.

2.2 Further developments

This proposal has attracted much attention and the quench scenario has been extensively studied and further developed. In particular, quantum effects in the mean field approximation have been included (see e.g. [12, 13]) and the drastic quench approximation has been abandoned in favor of a description taking explicitly expansion into account [13, 14]. In this context, the importance of initial conditions has been pointed out [14]. It has been realized that, in most cases, initial fluctuations are not suppressed efficiently enough and expansion hardly drive the system into a region of instability. This observation led to the conclusion that the formation of a strong pion field is a rare phenomenon. Exploiting the assumption of local equilibrium at initial time, it has been possible to estimate the probability that a potentially observable signal is produced in this scenario [15] (see also Ref. [6]). For central Pb-Pb collisions at CERN SPS energies, this analysis yields the upper bound

$$\text{Proba}(\text{“observable” DCC}) \lesssim 10^{-3}. \quad (5)$$

where “observable” means that the number of produced DCC pions is required to be greater than a typical multiplicity fluctuation [15]. Note that

this estimate is obtained within the quench scenario and, therefore, corresponds to a conditional probability, which assumes that the conditions leading to a quench are met. Thus the DCC is expected to be a fairly rare phenomenon, a fact which has important implications. For instance, it is very unlikely that more than one “observable” DCC domain be formed in a single collision⁵. Moreover, it is clear that the phenomenon should be looked for on an event-by-event basis (see also [16]).

The quench scenario has been widely accepted as a microscopic description of DCC formation in heavy ion collisions and, as such, has been extensively used for DCC phenomenology [17]. Paradoxically, the expected large event-by-event fluctuations of the neutral ratio (cf. Eq. (3)) have never been observed in actual simulations of the out-of-equilibrium phase transition [18]. Of course, deviations from the perfect law (3) were expected on the basis of a microscopic model underlying the original picture of a perfectly polarized DCC, cf. Eqs. (1) and (2). However, the complete absence of large fluctuations in the quench scenario suggests that this idealized picture is far from being realized in the context of the far-from-equilibrium chiral phase transition [6].

3 A more realistic picture: the unpolarized DCC

Whether the original picture is realized or not in a realistic model is a question of great phenomenological importance and requires a careful investigation. If the rapid suppression of initial fluctuations provides a robust mechanism leading to the formation of a strong coherent pion field, it is not clear whether it can explain the hypothetical polarization (2) originally proposed. The key point is to realize that the usual assumption of a (locally) thermalized initial state implies that the field modes are completely uncorrelated at initial time: they are independent (Gaussian) random numbers. Therefore, in order to generate a DCC configuration, the microscopic mechanism at work needs not only to be efficient in amplifying the modes amplitudes, but it should also build correlations between amplified modes.

To study this question, we have performed a detailed statistical analysis of generic pion field configurations produced after a quench, with particular emphasis on their isospin structure [6]. For this purpose, it is sufficient to

⁵This does not exclude the possibility that many DCCs with small number of pions be formed. However, these do not correspond to classical field configurations.

consider the original model of Rajagopal and Wilczek [9], which exhibits all the relevant physical features. In particular, the classical field approximation allows one to take into account the full nonlinearities, which are crucial for the study of correlations. We sample initial field configurations in the Gaussian statistical ensemble described in Sec. 2.1, and we follow the exact time evolution of each configuration by solving numerically the equations of motion corresponding to the classical action (4). In this way, we can study the produced configurations on an event-by-event basis. To analyze the isospin orientations of distinct field modes, a sensitive observable is the neutral fraction of pions in each mode \mathbf{k} , defined as⁶

$$f(\mathbf{k}) = \frac{n_3(\mathbf{k})}{n_1(\mathbf{k}) + n_2(\mathbf{k}) + n_3(\mathbf{k})}, \quad (6)$$

where $n_{a=1,2,3}(\mathbf{k})$ are the averaged occupation numbers corresponding to the classical field configuration at the time t_f of measurement:

$$n_a(\mathbf{k}) = \left| \frac{i\dot{\varphi}_a(\mathbf{k}, t_f) + \omega_k \varphi_a(\mathbf{k}, t_f)}{\sqrt{2\omega_k}} \right|^2. \quad (7)$$

Here φ_a and $\dot{\varphi}_a$ denote the Fourier component of the field and its time derivative respectively and $\omega_k = \sqrt{k^2 + m_\pi^2}$ is the free pion dispersion relation.

In the quench approximation, where the initial fluctuations are suppressed by hand, the system is initially unstable and all initial configurations undergo a dramatic amplification, as in Fig. 1. This procedure therefore selects the interesting events. To distinguish between different mechanisms responsible for amplification, we analyze the properties of the statistical ensemble at various final times t_f . The plots presented here are obtained for $t_f = 10$ fm, corresponding to the end of the spinodal instability, but the results to be discussed below are found to be insensitive to this choice.

Let us concentrate on the amplified long wavelength modes, which are the one we are interested in. The event-by-event distribution of the neutral fraction (6) for the zero mode (the most amplified) is shown in Fig. 2. At a given time, all the amplified modes exhibit a similar distribution. In particular, we observe that nothing significant happens concerning their isospin structure: the neutral fraction distribution is essentially the same as the corresponding one in the initial ensemble. The latter can be computed exactly [6] and is

⁶For a more detailed discussion, see Ref. [6].

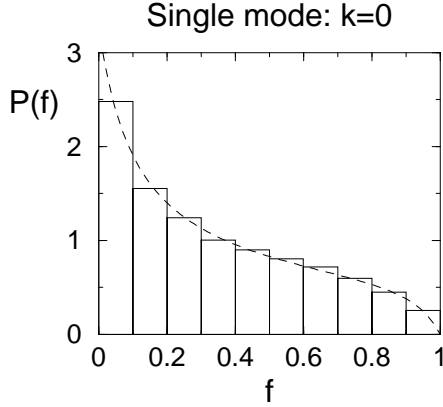


Figure 2: *Neutral fraction distribution in a single mode, here $\mathbf{k} = 0$, at initial time (dashed line) and at final time, here $t_f = 10$ fm (histogram).*

represented by the dashed line in Fig. 2. For a given mode \mathbf{k} , it reads⁷

$$P(f) = \frac{1}{2} \left[F_{\Omega}(f) + F_{-\Omega}(f) \right], \quad (8)$$

where

$$F_{\Omega}(f) = [\Omega - (1 - f)] \left(\frac{\Omega + 1}{\Omega - (1 - 2f)} \right)^{3/2},$$

and⁸

$$\Omega = \frac{4 + \omega_k^2}{4 - \omega_k^2}.$$

Although not exactly $1/\sqrt{f}$, the distribution is very broad, exhibiting large fluctuations around the mean value $\langle f \rangle = 1/3$, which is the relevant point for phenomenology. As emphasized above, these fluctuations are a direct manifestation of the classical nature of the field modes. The small deviations from the perfect law (2) come from the fact that the latter are not strictly linearly polarized [6].

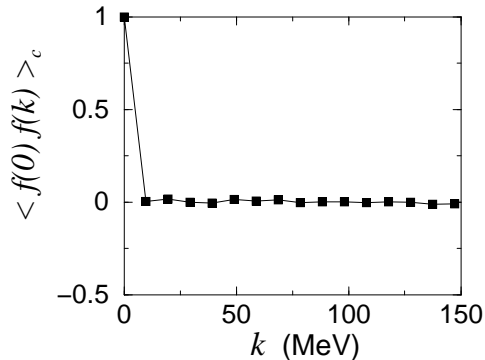
However, we find that the that distinct modes have completely independent polarizations in isospin space, as can be seen on Fig. 3, which shows the correlation between the neutral fractions in different (amplified) modes.

⁷There is a subtlety associated with the choice of boundary conditions for the field [6]. The following formula is obtained for Neumann boundary conditions, which are convenient for discussing the question of polarization.

⁸For a general Gaussian ensemble, one has $\Omega = (b^2 + \omega_k a^2)/(b^2 - \omega_k a^2)$, where $a^2 \equiv \langle \varphi_a^2 \rangle$ and $b^2 \equiv \langle \dot{\varphi}_a^2 \rangle$ are the variances of the Fourier components of the field and its time derivative respectively.

These modes oscillate coherently, but their directions of oscillation in isospin space are completely random. In other words, different modes act as independent DCCs. This has the important phenomenological consequence that the

Figure 3: *Normalized connected correlation between the neutral fraction in the zero mode and in mode k as a function of momentum.*



large event-by-event fluctuations of the neutral fraction are rapidly washed out when the contributions of several modes are added in a momentum bin – as commonly done in experimental analysis – even when one limits one’s attention to soft modes only. This is illustrated on Fig. 4, where we show the neutral fraction distribution in a bin containing 10 modes. The expected signal is considerably reduced, already for such a small bin. This explains the absence of large fluctuations reported in previous works [18], where the authors typically considered the contribution of a large number of modes⁹.

In conclusion, contrarily to what is usually assumed, the non-linear dynamics does not build the required correlation between modes: the state produced in the simplest form of the quench scenario, where no correlations are present initially, is not the originally proposed DCC. Our detailed analysis reveals that a more realistic picture is that of a superposition of waves having independent orientations in isospin space: an “unpolarized” DCC configuration, as depicted on Fig. 5.

Of course, one cannot exclude the possibility that the required correlations between modes be indeed (partially) formed in actual nuclear or hadronic collisions by means of some other mechanism¹⁰. The resulting configuration would lie between the two extreme pictures represented on Fig.

⁹The absence of large fluctuations has also been reported in a recent work in a slightly different context [19].

¹⁰For example, the scenario proposed in Ref. [20] could give rise to correlations in the initial state.

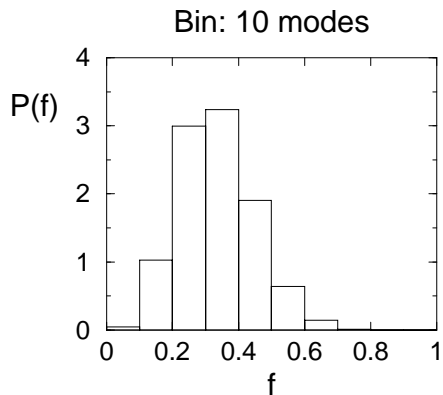


Figure 4: *Event-by-event distribution of the neutral fraction in a bin in momentum space containing 10 amplified modes. The expected DCC signal is already considerably reduced, due to the absence of correlation between modes.*

5. Our point here is that there exists, at present, no microscopic justification for the polarized DCC state which has been looked for so far in experiments [21, 22]. Instead, within our present theoretical understanding of DCC formation in heavy ion collisions, one expects an unpolarized state to be produced. It is, therefore, important to take this information into account in further theoretical as well as experimental investigations.

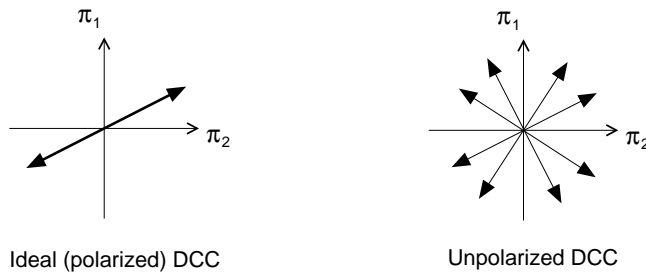


Figure 5: *Schematic representation of the ideal DCC configuration (left), where all field modes oscillate in the same direction in isospin space (see Eq. (2)), and of the unpolarized DCC formed after a quench (right), where the modes have independent directions of oscillation.*

4 What next?

Since it has been proposed in the early 1990's, the idea that a DCC might be formed in high energy collisions experiments has triggered an intense activity, both theoretical and experimental. Attempts to detect this phe-

phenomenon include the T-864 MiniMax experiment, a dedicated search in proton-antiproton collisions at the Fermilab Tevatron [21], or event-by-event analysis of Pb-Pb collisions by the WA98 and NA49 collaborations at CERN SPS [22]. No clear experimental evidence for DCC formation has been reported so far¹¹. This is a bit disappointing in view of the fact that the phenomenon seems so natural, at least in the context of heavy ion collisions. Indeed, the growth of long wavelength fluctuations after a quench is a generic phenomenon, observed in many actual situations, for example in condensed matter experiments, and which is likely to happen in heavy ion collisions as well. In particular, it has been shown to occur in various different models [25, 11]. The absence of experimental evidence may indicate that the appropriate conditions for DCC formation have not been met yet (see the interesting discussion in Ref. [26], where it is argued that semi-peripheral heavy ion collisions at RHIC energies might provide more favorable conditions for efficient quenching).

However, we would like to point out that the present experimental situation can actually be simply understood within our present theoretical understanding. Indeed, the upper bound (5) for the probability of DCC formation at SPS energies is at the edge of the current experimental limit probed by the WA98 and NA49 collaborations [22]. Moreover, our results concerning the (iso-)polarization of the DCC indicates that the phenomenon we are seeking may actually be more difficult to observe than originally thought. In particular, it would be of first interest to analyze existing data in the light of the present results and to design new strategies for future DCC searches at RHIC and LHC [23], taking into account its unpolarized nature.

Besides providing a useful tool for the experimental study of the QCD chiral phase transition [26], the detection of a DCC is of fundamental interest on its own. Indeed, the possible formation of a coherent classical pion field is to be expected on very general grounds as a direct consequence of the bosonic nature of pions. This phenomenon is analogous to that of Bose-Einstein condensation, currently observed in condensed matter experiments. The fact that such a macroscopic wave of QCD matter may be produced is a very exciting possibility. I think it is worth the effort.

¹¹DCC has, however, been proposed to be responsible for so-called Centauro events, reported in the cosmic ray literature [24, 1].

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