

EXOTIC CARBON SYSTEMS IN TWO-NEUTRON HALO THREE-BODY MODELS

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The general properties of exotic carbon systems, considered as a core with a two-neutron ($n - n$) halo, are described within a renormalized zero-range three-body model. In particular, it is addressed the cases with a core of ^{18}C and ^{20}C . In such a three-body framework, ^{20}C has a bound subsystem (^{19}C), whereas ^{22}C has a Borromean structure with all subsystems unbound. ^{22}C is also known as the heaviest carbon halo nucleus discovered. The spatial distributions of such weakly-bound three-body systems are studied in terms of a universal scaling function, which depends on the mass ratio of the particles, as well as on the nature of the subsystems.

Keywords: Halo nuclei; Binding Energies; Faddeev Equation; Three-body Systems.

1. Introduction

An unexpected exotic type of nucleus was discovered at Lawrence Berkeley National Laboratory in 1985, as reported by Tanihata et al.¹ The experiments have shown that the most neutron-rich unstable isotope of lithium, the ^{11}Li , has an unusually large size similar to a heavy nucleus like gold, with 197 nucleons. This observation have challenged the common believe that the radius of a nucleus depends only on the total number of nucleons. It was found that this lithium system has two extremely weakly bound neutrons with large probability of being located at distances very far from the ^9Li core.

Halos can occur also for other nuclear systems, as found in different experiments. They can occur when nuclei tend to have few nucleons weakly bound located in the outermost orbits. Therefore, the halo can be formed with one or more nucleons. Most of these findings have been confined to relatively light nuclei. But, it remains an open question the existence of giant multi-nucleon haloes, not only with light-nuclei core but also in heavy-nuclei.

A motivation for studying halo-nuclei systems in laboratories relies in the fact that these nuclei, called unstable nuclei (because they decay naturally), have large asymmetries in the number of protons and neutrons. They are extreme cases for bound nucleus and define the neutron or proton drip line. This opens the possibility to go beyond the nuclei on earth and look into what exists in a supernova explosion, or in other cosmic objects like novae and X-ray bursters. The creation of the majority of elements on earth took place in such explosive stellar environments, involving unstable nuclei.² Exploring and understanding unstable nuclei is relevant to study the principles and forces that bind the nucleons together to form the large variety of nuclei in our universe.

By considering exotic nuclei with neutron excess, one can also challenge the common understanding of the traditional nuclear physics, such as the single-particle character and the associated shell structure. Recent observations reported in Refs. 3, 4 suggest that large neutron excess can change the sequence of single-particle states by altering the mean-field potential experienced by a single-particle in the nucleus. Studies of nuclei far from the β - stability line have begun to indicate that the familiar shell gaps do not persist in exotic systems.

Here we report some studies on the general properties of exotic carbon systems, within a renormalized s -wave zero-ranged three-body model, which describes a core (C) plus a two-neutron ($n-n$) halo.⁵ This approach is suitable for large two-body scattering lengths, or weakly bound subsystems.

The low-energy properties of large three-body systems are universal and dominated by the Efimov effect,⁶ where the details of the interaction are less important. The Efimov effect is the prediction of an infinite series of excited three-body energy levels when the two-body state is exactly at the dissociation threshold. It is the counterpart of the Thomas collapse⁷ of the three-body ground-state energy, when the range of the two-body interaction goes to zero. Both effects are related

by a scaling transformation, such that they can be described by the same dimensionless non-relativistic quantum-mechanical three-body equation.⁸ This effect has been studied long-time ago in the context of nuclear physics, such that the Thomas collapse was crucial in determining the range of the nuclear forces, as pointed out by Bethe and Bacher.⁹

The three-body properties are set by low-energy parameters (scales), as the binding energy of the halo to the core, which is assumed to be structureless, and the corresponding scattering lengths of the two-body subsystems. The condition for the appearance of one excited state in the maximally symmetric configuration of a three-body halo system in the Efimov limit (infinite scattering lengths) defines a universal two-dimensional parametric space, where the axis are given by the energies of the subsystems in units of the three-body ground-state energy.⁵

In section 2, the general framework for studying the configuration space properties of three-body halo nuclei systems is presented, which is applied to some well-known rich-neutron halo nuclei in the following sections. Of particular interest, is the case of ^{20}C , modeled as $n - n - ^{18}\text{C}$ system, where $n - ^{18}\text{C}$ is bound.^{10,11} In section 3, the recently discovered ^{22}C system is studied within the three-body model $n - n - ^{20}\text{C}$, with all subsystems unbound.¹² (Note that, the properties of the $n - ^{20}\text{C}$ subsystem are not well established experimentally, but there are strong indications that such system is not bound, implying that ^{22}C is a Borromean system.). A minimal number of physical inputs are considered in the model, which are directly related to observables: the two-neutron separation energy $S(2n) = -E3$, the neutron-neutron and neutron-core s -wave scattering lengths (or the corresponding virtual or bound state energies). For the corresponding scattering formalism, details are given in Ref. 11, where the neutron- ^{19}C scattering is analyzed. The conclusions with perspectives are given in the summary section.

2. Structure of Three-body Halo-nuclei Systems

The structure of the $n - n - C$ system is described by using Jacobi coordinates, where \vec{q}_i is the relative momentum of the particle i to the center-of-mass (CM) of the pair jk and \vec{p}_i is the relative momentum of the pair jk . \vec{R}_i and \vec{r}_i are, respectively, the positions canonically conjugated to the momenta. In the equations we consider $j \equiv k \equiv n$ and $i \equiv C$. The root-mean-square (rms) distances between the particles j and k are calculated from the Fourier transform of form factors corresponding to the probability density functions. Similarly, one can obtain the corresponding rms distances between the other pair of particles.

By using the form factor $F_{nn}(Q^2)$ at zero momentum Q , one can obtain the mean-square distance between the particles j and k as

$$\langle r_{nn}^2 \rangle = -6 \frac{dF_{nn}(Q^2)}{dQ^2} \Big|_{Q^2=0}. \quad (1)$$

The form factor $F_{nn}(Q^2)$ is given in terms of the wave functions Ψ in the momentum

space representation, as follows:

$$F_{nn}(Q^2) = \int d^3 q_i d^3 p_i \langle \vec{q}_i, \vec{p}_i + \vec{Q}/2 | \Psi \rangle \langle \vec{q}_i, \vec{p}_i - \vec{Q}/2 | \Psi \rangle. \quad (2)$$

From the three-body Schrödinger equation with separable two-body potentials $v_\alpha = \lambda_\alpha |\chi_\alpha\rangle \langle \chi_\alpha|$ [$\alpha \equiv (jk), (ki), (ij)$], with H_0 the free-Hamiltonian, we have

$$(E - H_0)|\Psi\rangle = \sum_{\alpha=i,j,k} \lambda_\alpha |\chi_\alpha\rangle \langle \chi_\alpha | \Psi \rangle. \quad (3)$$

The three-body wave function in the base $|\vec{q}_i, \vec{p}_i\rangle$, is written in terms of the spectator functions $f_\alpha(\vec{q}_\alpha) = \lambda_\alpha \langle \vec{q}_\alpha | \chi_\alpha | \Psi \rangle$ by using the Dirac- δ interaction $\langle \vec{p}_\alpha | \chi \rangle = 1$, as:

$$\langle \vec{q}_i, \vec{p}_i | \Psi \rangle = \left[f_i(|\vec{q}_i|) + f_j(|\vec{p}_i - \frac{\vec{Q}}{2}|) + f_k(|\vec{p}_i + \frac{\vec{Q}}{2}|) \right] (|E_3| + H_0)^{-1}, \quad (4)$$

where f_i, f_j e f_k are, respectively, the spectator functions for the particles i, j and k ; and E_3 is the three-body binding energy. From (4) and (2) we obtain

$$F_{nn}(Q^2) = \int d^3 q_i d^3 p_i \left[f_i(|\vec{q}_i|) + f_j \left(\left| \vec{p}_i + \frac{\vec{Q}}{2} - \frac{\vec{q}_i}{2} \right| \right) + f_k \left(\left| \vec{p}_i + \frac{\vec{Q}}{2} + \frac{\vec{q}_i}{2} \right| \right) \right] \\ \times \frac{\left[f_i(|\vec{q}_i|) + f_j(|\vec{p}_i - \frac{\vec{Q}}{2} - \frac{\vec{q}_i}{2}|) + f_k(|\vec{p}_i - \frac{\vec{Q}}{2} + \frac{\vec{q}_i}{2}|) \right]}{(|E_3| + H_0)(|E_3| + H'_0)}, \quad (5)$$

where

$$H_0 \equiv \frac{|\vec{p}_i + \frac{\vec{Q}}{2}|^2}{2m_{nn}} + \frac{q_i^2}{2m_{nn,C}}, \quad H'_0 \equiv \frac{|\vec{p}_i - \frac{\vec{Q}}{2}|^2}{2m_{nn}} + \frac{q_i^2}{2m_{nn,C}}, \quad (6)$$

with $m_{nn} = m_n/2$ and $m_{nn,C} = 2m_C m_n / (2m_n + m_C)$. For particles n and C , the mean-square distances are obtained in an analogous way, following Eqs. (1) and (2):

$$\langle r_{nC}^2 \rangle = -6 \frac{d}{dQ^2} \left\{ \int d^3 q_k d^3 p_k \langle \vec{q}_k, \vec{p}_k + \frac{\vec{Q}}{2} | \Psi \rangle \langle \vec{q}_k, \vec{p}_k - \frac{\vec{Q}}{2} | \Psi \rangle \right\} \Big|_{Q^2=0}. \quad (7)$$

In this case, the wave function in the base $|\vec{q}_k, \vec{p}_k\rangle$ is given by:

$$\langle \vec{q}_k, \vec{p}_k | \Psi \rangle = \frac{f_i(|\vec{p}_k - \frac{\mathcal{A}}{\mathcal{A}+1} \vec{q}_k|) + f_j(|\vec{p}_k + \frac{1}{\mathcal{A}+1} \vec{q}_k|) + f_k(|\vec{q}_k|)}{|E_3| + p_k^2 \frac{\mathcal{A}+1}{2\mathcal{A}} + q_k^2 \frac{\mathcal{A}+2}{2(\mathcal{A}+1)}}, \quad (8)$$

where $\mathcal{A} = m_C/m_n$ is the mass ratio of the particles n and C .

The mean-square distances of n and C to the three-body CM are calculated by using the respective form factors, as follows:

$$\langle r_n^2 \rangle = -6 \left(\frac{m_n + m_C}{2m_n + m_C} \right)^2 \frac{d}{dQ^2} \left\{ \int d^3 q d^3 p \langle \vec{q}_k + \frac{\vec{Q}}{2}, \vec{p}_k | \Psi \rangle \langle \vec{q}_k - \frac{\vec{Q}}{2}, \vec{p}_k | \Psi \rangle \right\} \Big|_{Q^2=0}, \quad (9)$$

and

$$\langle r_C^2 \rangle = -6 \left(\frac{2m_n}{2m_n + m_C} \right)^2 \frac{d}{dQ^2} \left\{ \int d^3q_i d^3p_i \langle \vec{q}_i + \frac{\vec{Q}}{2}, \vec{p}_i | \Psi \rangle \langle \vec{q}_i - \frac{\vec{Q}}{2}, \vec{p}_i | \Psi \rangle \right\} \Big|_{Q^2=0}, \quad (10)$$

Next, the coupled integral equations for the spectator functions, in dimensionless units are presented. Our units are such that $\hbar = 1$ and $m_n = 1$. In these units, the three-body energy is given by ϵ , with the corresponding energies for the two-body subsystem nn and nC (assumed to be bound or virtual) given, respectively, by ϵ_{nn} and ϵ_{nC} . The equations are regularized by a kernel subtraction, as follows:

$$\begin{aligned} f_j(\vec{q}) = & \left(\frac{\mathcal{A} + 1}{2\mathcal{A}} \right)^{3/2} \frac{1}{\pi} \left(\sqrt{|\epsilon| + \frac{q^2(\mathcal{A} + 2)}{2(\mathcal{A} + 1)} \mp \sqrt{\epsilon_{nC}}} \right)^{-1} \int_0^\infty k^2 dk \int_{-1}^1 dy \\ & \times \left\{ \left[\frac{1}{|\epsilon| + q^2 + \frac{\mathcal{A} + 1}{2\mathcal{A}} k^2 + kqy} - \frac{1}{1 + q^2 + \frac{\mathcal{A} + 1}{2\mathcal{A}} k^2 + kqy} \right] f_i(\vec{k}) \right. \\ & \left. - \left[\frac{1}{|\epsilon| + \frac{(\mathcal{A} + 1)}{2\mathcal{A}} q^2 + \frac{(\mathcal{A} + 1)}{2\mathcal{A}} k^2 + \frac{kqy}{\mathcal{A}}} - \frac{1}{1 + \frac{(\mathcal{A} + 1)}{2\mathcal{A}} q^2 + \frac{(\mathcal{A} + 1)}{2\mathcal{A}} k^2 + \frac{kqy}{\mathcal{A}}} \right] f_j(\vec{k}) \right\}. \quad (11) \end{aligned}$$

$$\begin{aligned} f_i(\vec{q}) = & \frac{2}{\pi} \left(\sqrt{|\epsilon| + \frac{(\mathcal{A} + 2)}{2\mathcal{A}} q^2 \mp \sqrt{\epsilon_{nn}}} \right)^{-1} \int_0^\infty k^2 dk \\ & \times \int_{-1}^1 dy \left[\frac{1}{|\epsilon| + \frac{(\mathcal{A} + 1)}{2\mathcal{A}} q^2 + k^2 + kqy} - \frac{1}{1 + \frac{(\mathcal{A} + 1)}{2\mathcal{A}} q^2 + k^2 + kqy} \right] f_j(\vec{k}). \quad (12) \end{aligned}$$

In the regularization procedure an energy-subtraction point $\mu_{(3)}^2$ is introduced in the kernel. Next, by considering all the energies in units of $\mu_{(3)}^2$, with the momenta q and k in units of $\mu_{(3)}$, the above coupled equations are found.¹³ Note that the spectator functions f_j and f_k are identical. In front of the energy square-root, the minus (-) sign refers to a bound state subsystem and the plus sign (+) to a virtual state subsystem. Four types of nnC systems are possible according to the two-body interactions, such as: *Borromean* configuration,¹⁴ when all the two-body subsystems are unbound; *tango* configuration,¹⁵ when we have two unbound and one bound subsystems; *samba* configuration,¹³ when just one of the two-body subsystems is unbound; and *all-bound* configuration, when there is no unbound subsystems. The differences in the intensity of the attraction in the kernel for the above configurations, with a fixed three-body energy, creates the following size sequence: *all-bound* > *samba* > *tango* > *Borromean*. In the case of a more attractive kernel, the particles can be more separated in comparison to the case of a less attractive kernel, where the particles should stay closer to allow the same three-body bound state energy.

In the three-body *scaling limit*,¹⁶ with binding energy E_3 , a given observable \mathcal{O} , with dimension of energy to the power η , at a particular energy E , can be written as a function of the ratios between the two and three-body energies: $\mathcal{O}(E, E_3, E_{nn}, E_{nC}, \dots) = (|E_3|)^\eta \mathcal{F}\left(\sqrt{\frac{E}{E_3}}, \pm\sqrt{\frac{E_{nn}}{E_3}}, \pm\sqrt{\frac{E_{nC}}{E_3}}, \mathcal{A}\right)$. The two-body energies $E_{n\gamma}$ ($\gamma = n, C$), are negative quantities, corresponding to bound ($\sqrt{|E_{n\gamma}|}$ positive) or virtual ($\sqrt{|E_{n\gamma}|}$ negative) states. The different rms distances in the bound $n - n - C$ system are functions defined from this relation with $E = E_3$, which depend on the mass ratio \mathcal{A} , the ratios of the two and three-body energies and the subsystem type (bound or virtual).

Table 1. Neutron-neutron rms distances in halo nuclei. The cores are in the first column, with the two-neutron separation energies $S(2n)$ (equal to the absolute value of the three-body ground state energies) in the second column. For bound two-body nC subsystem, $-E_{nC} = S(n)$ (one-neutron separation energy). The virtual states are indicated by (v), and the nn virtual state energy is taken as -143 keV. The experimental values (last column) are quoted in Ref. 13.

Core	$S(2n)$ (MeV)	$-E_{nC}$ (MeV)	$\sqrt{\langle r_{nn}^2 \rangle}$ (fm)	$\sqrt{\langle r_{nn}^2 \rangle}_{exp}$ (fm)
⁴ He	0.973	0	5.1	5.9±1.2
⁹ Li	0.29	0.05 (v)	8.5	6.6±1.5
¹² Be	1.337	0	4.6	5.4±1.0
¹⁸ C	3.50	0.53	3.0	—

For a generic three-body system, the above approach was applied in Ref. 13 considering some well-known halo nuclei. Table 1 presents the rms distances of two-neutrons in the halo-nuclei ⁶He, ¹¹Li, ¹⁴Be and ²⁰C. (The case of ⁶He, cannot be considered realistic in the model, because ⁵He was treated as an *s*-wave state.). In the above, we have reported a formalism and results obtained in Ref. 13 for the sizes of generic three-body systems. One quite interesting conclusion of this study it that, by considering the same three-body energy for all the configurations, the following sequence should be applied for the mean-square distances: $\langle r_{Borromean}^2 \rangle < \langle r_{tango}^2 \rangle < \langle r_{samba}^2 \rangle < \langle r_{all-bound}^2 \rangle$.

In the next, we review some recent results related to ²²C.

3. The Borromean ²²C system

The mean-square distance of a halo neutron with respect to the center-of-mass of ²²C was recently estimated by our group,¹² within the above discussed renormalized three-body model. The analysis of Tanaka et al.¹⁷ of experiments involving this carbon isotope reported a fairly large extracted value of the matter radius of 5.4±0.9 fm, which characterizes ²²C as the heaviest and largest halo nuclei discovered until now. In Ref. 17, they also quote a value of 0.42±0.94 MeV for the two-neutron separation energy, S_{2n} . These experimental results indicate that ²²C

is weakly-bound, having a very large two-neutron halo with a ^{20}C core. The corresponding observables are probably dominated by the tail of the three-body wave function in an ideal s -wave three-body model.¹⁸

With the above information on ^{22}C from Ref. 17, an energy region for experimental investigations of the unbound ^{21}C virtual state is suggested in Ref. 12. As indicated by the renormalized three-body model of Ref. 12, the two-neutron separation energy in ^{22}C is expected to be found below ~ 0.4 MeV. The $n - n - ^{20}\text{C}$ configuration of ^{22}C is Borromean, in our model of two s -wave halo neutrons bound to a structureless ^{20}C . In this Borromean configuration with the virtual-state energy of ^{21}C close to zero, would make the ^{22}C the most promising candidate to present an excited bound Efimov state or a continuum three-body resonance.

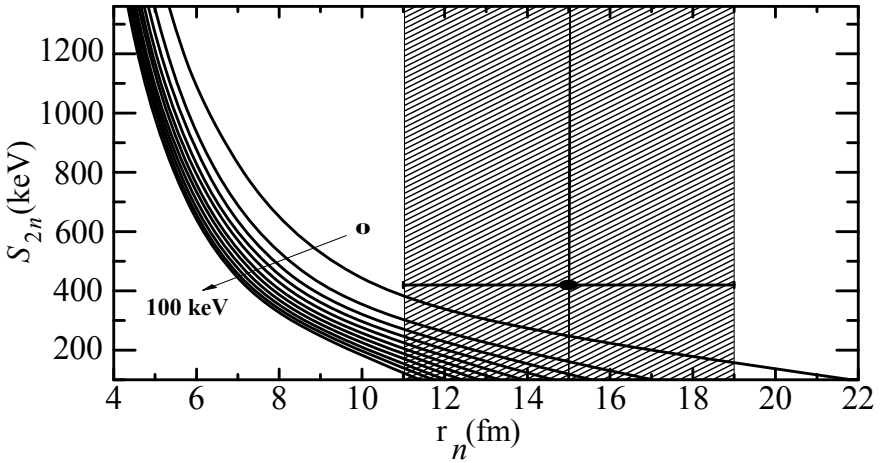


Fig. 1. Model results (solid curves) for two halo neutron separation energies S_{2n} in ^{22}C , with ^{21}C virtual-state energies varying from 0 to 100 keV. The results are given in terms of the root-mean-square distances of a halo neutron with respect to the three-body CM (r_n). The experimental data, with corresponding uncertainties, define the shaded area for S_{2n} (Extracted from Ref. 12).

The results for the two halo neutron separation energies in ^{22}C are shown in Fig.1, in terms of the rms distance of a halo neutron with respect to the three-body CM. The rms (r_n) can be estimated by using the additional information on the matter radius of the loosely bound ^{20}C , which is given in Ref. 19 ($R_M^{20\text{C}} = 2.98 \pm 0.05$ fm). Note that, in view of the large difference between the radius of ^{22}C and ^{20}C , it is a reasonable approximation to assume ^{20}C as the core of ^{22}C , such that we still can use a three-body approach. The result of our estimation is:

$$r_n \simeq \sqrt{\frac{22}{2} \left[(R_M^{22\text{C}})^2 - \frac{20}{22} (R_M^{20\text{C}})^2 \right]} \approx 15 \pm 4 \text{ fm} , \quad (13)$$

where $r_n \equiv \sqrt{\langle r_n^2 \rangle}$ and $R_M^i \equiv \sqrt{\langle (R_M^i)^2 \rangle}$, with $i = 20, 22$.

In the calculation, different values for the ^{21}C virtual-state energy were used between 0 to 100 keV, resulting in a window for the two-neutron separation energy given by $100 \text{ keV} \leq S_{2n} \leq 1360 \text{ keV}$, taking into account r_n (13).

4. Conclusions and Perspectives: Four-body Model for ^{21}C

In the present paper, we report some interesting results on the structure of halo nuclei, with particular emphasis in the rich-neutron carbon isotopes. By considering the recently discovered ^{22}C , together with available data for ^{20}C , it was also possible to infer some relevant information on ^{21}C sensible to the ^{22}C structure.

In view of the promising results obtained for exotic carbon isotopes, within a renormalized three-body model, in particular considering the carbon halo-nuclei, it is necessary to improve our understanding of the neutron halo of ^{21}C . This system was studied within the ^{22}C three-body model in Ref. 12 giving an insight on the properties of the neutron halo of ^{21}C .

One possibility to study the halo formation in ^{21}C is to consider a recent renormalized zero-range four-body approach^{20,21}, and also by using the Faddeev-Yakubovsky (FY) formalism extended to non-identical four particle systems.²² In principle, the ^{21}C system can be modeled as a core given by ^{18}C plus three neutrons. The Pauli principle applied to the halo neutrons in ^{21}C makes unnecessary the introduction of a four-body short-range scale to regularize the FY equations, in opposition to the four-boson case.^{20,21} Thus, the approach is nicely simplified and the properties of the three-neutron halo in ^{21}C , can be completely determined from the neutron-neutron scattering length and the one- and two-neutron separation energies in ^{20}C . This suggests that one can put strong constraints in the possible values of the $n - ^{20}\text{C}$ energy. In this case, it looks appealing to model ^{21}C by an inert ^{18}C plus three neutrons, with the inputs of the zero-range FY equations given by the existing available data of the low-energy observables of the subsystems, without the need of any new information.

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