On the effective bandwidth estimation in communication network

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ABSTRACT

In this paper we apply regenerative method to estimate the required effective bandwidth (EB) of a station in an acyclic communication network. Earlier this problem for a tandem network has been addresses in [1]. We give a brief introduction to the EB theory and then discuss basic statistical methods for the EB estimation. For a queueing network described by a regenerative process we construct the estimator which gives an upper bound of predetermined QoS requirement. Simulation results show that an overestimation of a QoS parameter obtained by this method is acceptable to calculate workload-based EB for the nodes being components of highly-reliable communication systems.

INTRODUCTION

In this paper, we investigate the accuracy of the regenerative EB estimator of a node (station) being a component of a communication (or computer) network. It is assumed that the basic process describing the network is *regenerative* or *quasi-regenerative*. Similar problem for a tandem queueing network has been earlier addressed in [1]. The analysis of regenerative input to a node is natural because the regenerative input, passing a network with no *bottleneck* (saturated) stations, remains regenerative [9]. By this reason, each station of the network can be analyzed as an isolated station with regenerative input. Simulation a tandem network presented in [1] has demonstrated that regeneration-based EB estimation can be effectively applied in highly reliable systems, because, in the most of cases, obtained overestimation of turns out to be insignificant. In this paper, we generalize this approach to study more complicated network. We show that, in this setting, regeneration-based estimator of the EB (that is server capacity C) also ensures QoS requirement while keeping a moderate overestimation of C. In the modern complex networks, simulation based on classical regeneration (occurring when an arrival meets an empty system) seems difficult to be realized by the rarity (or impossibility) of such regenerations. As a result, simulation time to estimate required parameter (for instance, C) with a given accuracy becomes unacceptable. In this regard, an approximation, called quasi-regeneration, can be used in which case (quasi)regenerations of the basic process happen much more often. Although (quasi)regenerations allow a dependence between regeneration cycles, but this dependence turns out to be negligible in practice. A key aspect of this research is to reveal a suitable regenerative structure of the basic process. In a classic setting, capacity C of a system (the average work served in unit of time) is assumed to be known and equal 1. However, in practice an inverse problem appears: calculate the capacity C which assures a given QoS requirement. Namely this required capacity C, as a rule, is called *effective bandwidth* (EB). (The paper [7] can be recommended as the best introduction to the EB theory.) Stationary waiting time W, stationary loss probability, stationary overflow probability, etc., can be considered as QoS parameters. In this paper, the overflow probability of the stationary workload is the QoS parameter. We show that the regeneration-based simulation is applicable to estimate, in a highly-reliable acyclic network, the EB of a node with a regenerative input, and it is an important contribution of the paper. Another contribution of the work is that, to calculate corresponding estimates, we apply the so-called *quasi-regenerations* (instead of more rare classical regenerations). It allows to speed-up simulation.

The paper is organized as follows. First we give a brief introduction to the EB theory. Then two basic methods, batch means and regenerative, are discussed. In the following Section, regenerative structure of an acyclic network is described with focus of construction of quasi-regenerations. Finally, in the last Section, we present simulation results which illustrate the quantity of the overestimation caused by this method. We note that the networks under consideration can be treated as fragments of a general acyclic communication network.

WORKLOAD-BASED EFFECTIVE BANDWIDTH

We show in brief the EB calculation and estimation in the simplest case, when the input process has independent

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identically distributed (i.i.d.) increments. The QoS requirement can be formulated as follows: find server capacity C such that the stationary *overflow probability* P_B satisfies inequality

$$\mathsf{P}_B := \mathsf{P}(W > B) \le \Gamma, \qquad (1)$$

where Γ is a given QoS requirement, W is the stationary workload and B is a given constant (level of the workload or the buffer size). Thus, (1) means that the probability the stationary workload W in a finite buffer system exceeds given threshold B is upper bounded by a constant Γ. We will consider a slotted time scale, and let v_i be a workload arrived in the system in time interval $[i, i-1)$, $i = 0, 1, \ldots$ Denote

$$\Lambda_n(\theta) = \frac{1}{n} \ln \mathsf{E} e^{\theta \sum_{i=0}^{n-1} v_i}, \; n \ge 1, \qquad (2)$$

where free parameter $\theta > 0$. Moreover we assume that there exists $\theta_0 > 0$ such that, for all $\theta \in (0, \theta_0)$, there exists a finite limit

$$\lim_{n\to\infty} \Lambda_n(\theta) =: \Lambda(\theta). \qquad (3)$$

This limit $\Lambda(\theta)$ is called (scaling) *limit logarithmic moment generating function* of the input process [7], [8]. In general, the sequence $\{v_i\}$ is assumed to be stationary and satisfying some mixing conditions. But in the i.i.d. setting, it is enough to assume solely [2], [8] that

$$\mathsf{E} e^{\theta v} < \infty, \qquad (4)$$

where v denotes a generic element of the i.i.d. sequence $\{v_i\}$. (Throughout the paper we suppress serial index to denote a generic element of an i.i.d. sequence.) The Large Deviation Principle (LDP) for the stationary workload W states that

$$\lim_{x\to\infty} \frac{1}{x} \ln \mathsf{P}(W > x) = -\theta^*. \qquad (5)$$

(Exact conditions for the LDP to be hold can be found in [5], [6].) Then relations (5) and (1) give the following expression for the desired parameter θ^*,

$$\theta^* = -\ln \Gamma / b > 0. \qquad (6)$$

Finally, EB C can be found as follows (see, for instance, [8]):

$$C := \frac{\Lambda(\theta^*)}{\theta^*}. \qquad (7)$$

ESTIMATION METHODS

For the i.i.d. $\{v_i\}$, function $\Lambda(\theta) = \ln \mathsf{E} e^{\theta v}$ can be found analytically for some distributions of v. However, in a general case, simulation is required to estimate $\Lambda(\theta)$. To this end, the standard sample mean estimator is constructed,

$$\Lambda_k(\theta) := \ln \frac{1}{k} \sum_{i=0}^{k-1} e^{\theta v_i}, \qquad (8)$$

which is strongly consistent, that is, with probability 1 (w.p.1), $\Lambda_k(\theta) \to \Lambda(\theta)$, $k \to \infty$. If $\{v_i\}$ are dependent, then the *batch-mean* method is often used to estimate $\Lambda(\theta)$, see [12], [13]. To apply this method, for a fixed B, the following *blocks*

$$X_j = \sum_{i=(j-1)B}^{jB-1} v_i, \quad j \ge 1,$$

are constructed, which (for B large enough) are assumed to be i.i.d. Then, by Strong Law of Large Numbers, w.p.1,

$$\lim_{k\to\infty} \frac{1}{k} \sum_{i=1}^{k} e^{\theta X_i} = \mathsf{E} e^{\theta X},$$

and thus, as $k \to \infty$,

$$\Lambda_k(\theta, B) := \frac{1}{B} \ln \frac{1}{k} \sum_{i=1}^{k} e^{\theta X_i} \to \frac{\ln \mathsf{E} e^{\theta X}}{B} := \Lambda(\theta, B),$$

where X denotes generic block. Thus $\Lambda_k(\theta, B)$ is strongly consistent estimate of $\Lambda(\theta, B)$, and the estimate of the EB based on k blocks is then defined as

$$C_k(B) = \frac{\Lambda_k(\theta^*, B)}{\theta^*} \quad (\theta^* = -\ln \Gamma / b). \qquad (9)$$

However, as experiments show [11], this method underestimates the EB value, and can not guarantee the basic QoS requirement (1). This is unacceptable if we deal with highly-reliable communication systems.

Assume the input sequence $\{v_i\}$ is regenerative with regeneration points $\{\beta_n\}$, that is the random elements

$$(v_0, \ldots, v_{\beta_1 - 1}), \ldots, (v_{\beta_{k-1}}, \ldots, v_{\beta_k - 1}), \ldots \qquad (10)$$

are i.i.d. (with generic *regeneration period* length β). Hence,

$$X_k := \sum_{i=\beta_k}^{\beta_{k+1}-1} v_i, \; k \ge 0, \; \beta_0 := 0,$$

are i.i.d. blocks (with generic element X). It allows to construct the following *regenerative* estimator of function $\Lambda(\theta)$ [3], [14]:

$$\Lambda_k(\theta^*) := \frac{k}{\beta_k} \ln \frac{1}{k} \sum_{i=1}^{k} e^{\theta^* X_i}, \; k \ge 1. \qquad (11)$$

The main moment assumptions are: $\mathsf{E}\beta^2 < \infty$ and there exists $\theta_0 > 0$ such that $\mathsf{E} e^{\theta X} < \infty$, $\theta \in (0, \theta_0)$. Since, by renewal theory, $k/\beta_k \to 1/\mathsf{E}\beta$, then it follows that w.p.1,

$$\Lambda_k(\theta^*) \to \frac{\ln \mathsf{E} e^{\theta^* X}}{\mathsf{E}\beta} =: \Lambda(\theta^*), k \to \infty. \qquad (12)$$

It is expected that the desired capacity C can be obtained as the limit

$$C_k(\theta^*) := \frac{\Lambda_k(\theta^*)}{\theta^*} \to \frac{\ln \mathsf{E} e^{\theta^* X}}{\theta^* \, \mathsf{E}\beta}. \qquad (13)$$

Indeed, as simulation shows, the term at the right of (13) is a lower bound of the unknown C, that is

$$\frac{\ln \mathsf{E} e^{\theta^* X}}{\theta^* \, \mathsf{E}\beta} := U \ge C. \qquad (14)$$

The inequality (14) is a serious advantage if we evaluate the EB of the components of highly-reliable systems. We suggest that the obtained overestimation of C is caused by random length blocks. Now can illustrate the latter statement assuming i.i.d. variables $\{v_i\}$ which are independent of the period length β. In this case, by the property of conditional expectation,

$$\ln \mathsf{E} e^{\theta^* X} = \ln \mathsf{E}\Big(\mathsf{E}(e^{\theta^* \sum_{i=1}^{\beta} v_i} | \beta)\Big) = \ln \mathsf{E}[\mathsf{E} e^{\theta^* v}]^{\beta},$$

and then, by the Jensen's inequality,

$$U = \frac{\ln \mathsf{E}[\mathsf{E} e^{\theta^* v}]^{\beta}}{\theta^* \, \mathsf{E}\beta} \geq \frac{\mathsf{E} \ln[\mathsf{E} e^{\theta^* v}]^{\beta}}{\theta^* \, \mathsf{E}\beta} = \frac{\ln \mathsf{E} e^{\theta^* v}}{\theta^*} = C. \quad (15)$$

From a practical point of view, implementation of the proposed regenerative estimator crucially depends on the magnitude of the overestimation. We study this problem in the last section.

QUASI-REGENERATION

To use estimator (12), we need to construct regenerations $\{\beta_n\}$. For a general acyclic network it can done as in [10]. Note, however, that it is a rather complicated construction.

In this research instead we focus on the so-called *quasi-regenerations*. Unlike classical regeneration, quasi-regeneration allows a dependence between adjacent regeneration cycles and, as a rule, are more frequent and easier to identify in simulation. (We note that the estimation based on quasi-regeneration gives, as a rule, a high accuracy in an acceptable simulation time [1], [3].) Being an approximation, quasi-regeneration can be defined by various ways. Below we illustrate it for an *acyclic* queueing network with M single-server stations and with renewal input to station 1, with arrival instants $\{t_n,\, n \geq 1\}$. We restrict our analysis by the networks where station M is the *last* one, see Figures 1,2. Assume that the network is initially empty, and denote $t_n^{(i)}$ the arrival instant of customer n to station i (so $t_n^{(1)} = t_n$). Also denote $z_n^{(i)}$ the instant customer n leaves station i. We define $t_n^{(i)} = \infty$, $z_n^{(i)} = 0$ if customer n does not visit station i. For a fixed n, denote

$$T_n := \max_{1 \leq i \leq M} \max_{k<n} z_k^{(i)}. \quad (16)$$

Thus, there are no customers $k < n$ in the system after instant T_n. Denote $R(n)$ the set of stations visited by customer n until he leaves the network, and denote $W_n^{(i)}$ his waiting time at station i. We call customer n *quasi-regenerative*, if

$$W_n^{(i)} = 0, \; i \in R(n) \text{ and } \min_{k>n} t_k^{(M)} > t_n^{(M)} > T_n. \quad (17)$$

For a tandem network, we need only the 1st condition in (17), which becomes $W_n^{(i)} = 0, i = 1, \ldots, M$, see also [1]. By conditions (17), regenerative customer n crosses the network (along the route $R(n)$) with no collisions with other customers. Moreover, he arrives at the (empty) target station M before any customer $k > n$ and after the instant T_n when all customers $k < n$ leave the network. (Note that some customers $k < n$ can be in the network while customer n crosses the network.) Arrival of a regenerative customer in an empty station M generates quasi-regenerative cycle of the input.

SIMULATION RESULTS

In this section we present simulation results. We have simulated tandem network (Fig.1) and a fragment of an acyclic network (Fig.4). First we construct estimate (13). Then we evaluate the overestimation when this estimate is used as the required EB of the *last* station M. We stress that an overestimation caused by regenerative estimate (13) has been established in previous works for isolated stations and tandem networks [1]. In this paper we develop these experiments to cover a wider class of the networks.

Experiment 1: tandem network

Consider tandem network with M stations, see Fig 1.

Fig. 1. Tandem network

It is assumed that service time S_j in station j is exponential with parameter $\mu_j = 1$, $j = 1, \ldots, M-1$. We consider Poisson input to station 1, with parameter $\lambda = 0.4$. For station M we take QoS parameter $\Gamma = 10^{-5}$ (in all experiments) and the threshold $B = 20$, implying $\theta^* = 0.58$ (see (6)). The following dependent data $\{v_i\}$ (at station M) are considered,

$$v_i = v_{i-1} + \eta_i, \;\; 1 \leq i \leq \beta, \quad (18)$$

where the i.i.d. variables $\{\eta_i\}$ are uniformly distributed over interval $[-v_{i-1}/2;\; v_{i-1}/2]$ (to avoid negative values), and β is the generic (quasi)regenerative period. We calculate the sample mean $\hat{D}$ of the cycle length, the cycle length variance $Var\hat{D}$ and the regenerative estimate $\hat{C}$ of the EB C. For each $\hat{C}$, we evaluate the overflow frequency ratio $\hat{\Gamma}$, and then find the overestimation Δ which is defined by the following condition: capacity $(1-\Delta)\hat{C}$ gives the overflow frequency ratio $\hat{\Gamma}_\Delta$ (estimate of the overflow probability) such that $\hat{\Gamma}_\Delta \approx \Gamma = 10^{-5}$. Table 1 presents simulation results (based on $k = 1000$ cycles) for the tandem networks with M stations. (Indeed we give tight bounds of Δ.) In particular, if, for $M = 4$, we use estimate $C_k(\theta^*) := \hat{C} = 0.15$ as the EB, then the overflow frequency ratio $\hat{\Gamma}$ equals $4.76 \cdot 10^{-6}$ and is less than $\Gamma = 10^{-5}$. This overestimation becomes negligible when capacity equals $(1-\Delta)\hat{C} = 0.85\hat{C}$. Moreover, when the number of stations M increases, then $Var\hat{D}$ increases as well (see Fig.2). Note that similar results for a tandem network with another dependence between $\{v_i\}$ have been presented in [1]. Moreover Table 2 shows for $M = 5$ stations that $\hat{D}$, $Var\hat{D}$ and Δ increase together with the input rate λ, and it is an expected result.

Table 3 demonstrates the relation between λ, M and $Var\hat{D}$. It indicates that the variance $Var\hat{D}$ can be stabilized by an appropriate choose of M and λ. However, in general, sample mean $\hat{D}$ (and Δ) increases together with M and λ.

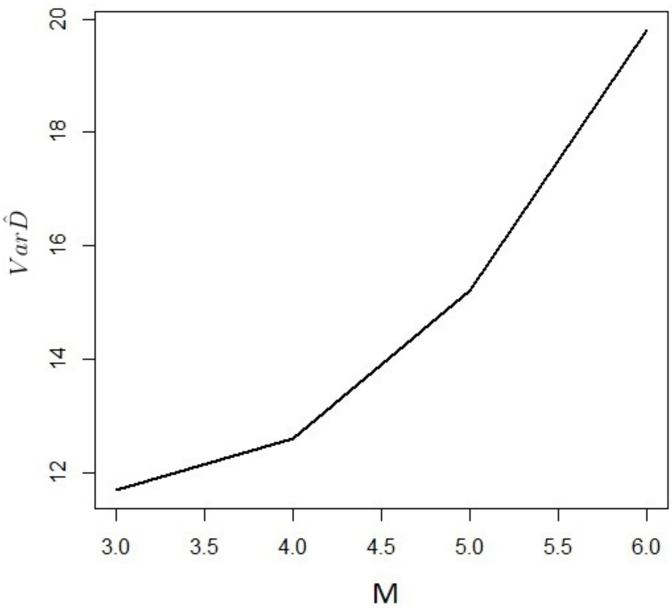

Fig. 2. M vs. $Var\hat{D}$

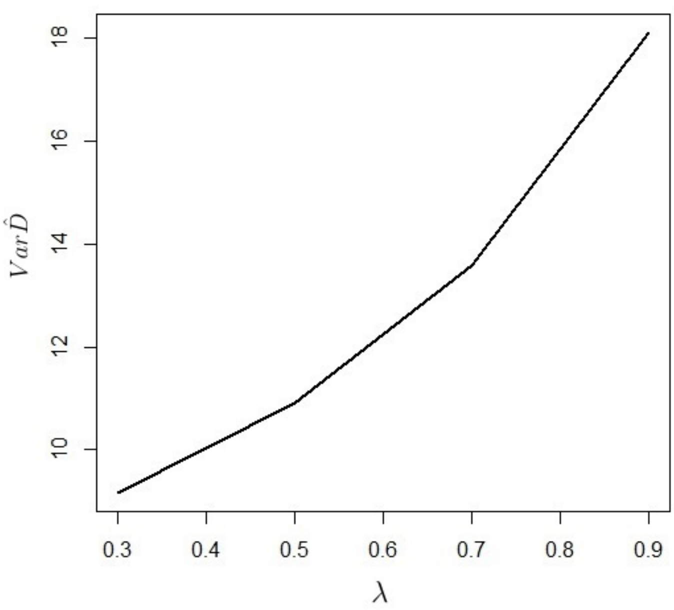

Fig. 3. λ vs. $Var\hat{D}$

TABLE I
TANDEM NETWORK, $\Gamma = 10^{-5}$

M	$Var\hat{D}$	$\hat{C}$	$\hat{\Gamma}$	Δ	$\hat{\Gamma}_\Delta$
3	11.7	0.14	$3.18 \cdot 10^{-6}$	0.06	$7.82 \cdot 10^{-6}$
3	11.7	0.14	$3.18 \cdot 10^{-6}$	0.09	$9.03 \cdot 10^{-5}$
4	12.8	0.15	$4.76 \cdot 10^{-6}$	0.12	$8.46 \cdot 10^{-6}$
4	12.8	0.15	$4.76 \cdot 10^{-6}$	0.15	$9.61 \cdot 10^{-4}$
5	15.3	0.19	$9.51 \cdot 10^{-8}$	0.13	$2.52 \cdot 10^{-6}$
5	15.3	0.19	$9.51 \cdot 10^{-8}$	0.15	$4.22 \cdot 10^{-5}$
6	19.8	0.225	$8.29 \cdot 10^{-9}$	0.15	$7.39 \cdot 10^{-6}$
6	19.8	0.225	$8.29 \cdot 10^{-9}$	0.17	$5.97 \cdot 10^{-5}$

TABLE II
TANDEM NETWORK, M=5, $\Gamma = 10^{-5}$

λ	$\hat{D}$	$Var\hat{D}$	$\hat{C}$	$\hat{\Gamma}$	Δ	$\hat{\Gamma}_\Delta$
0.3	3.93	9.17	1.80	$3.26 \cdot 10^{-6}$	0.11	$8.90 \cdot 10^{-6}$
0.3	3.93	9.17	1.80	$3.26 \cdot 10^{-6}$	0.15	$2.71 \cdot 10^{-4}$
0.5	4.27	10.80	2.03	$2.61 \cdot 10^{-6}$	0.12	$7.25 \cdot 10^{-6}$
0.5	4.27	10.80	2.03	$2.61 \cdot 10^{-6}$	0.15	$8.66 \cdot 10^{-5}$
0.7	5.06	13.47	2.49	$7.94 \cdot 10^{-7}$	0.12	$3.58 \cdot 10^{-6}$
0.7	5.06	13.47	2.49	$7.94 \cdot 10^{-7}$	0.15	$5.31 \cdot 10^{-5}$
0.9	6.59	18.22	2.85	$3.36 \cdot 10^{-7}$	0.15	$3.79 \cdot 10^{-6}$
0.9	6.59	18.22	2.85	$3.36 \cdot 10^{-7}$	0.18	$2.94 \cdot 10^{-4}$

Experiment 2: acyclic network

In this case we study a typical fragment of an acyclic communication network (called *acyclic network*), see Fig.4. Leaving station 1, customers join station 2 with probability

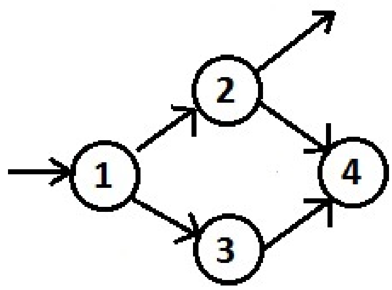

Fig. 4. Acyclic network

$p_1 = 0.6$, and station 3 with probability $q_1 := 1 - p_1 = 0.4$. Leaving station 2, customers leave the network with probability $p_2 = 0.2$, and join station 4 with probability $q_2 = 1 - p_2 = 0.8$. Values of Γ are given in Table 4. It is assumed that service times S_j are exponential with rate $\mu_j = 1$, $j = 1, 2, 3$, the input is Poisson with rate $\lambda = 0.3$, the overflow threshold $B = 20$ and variables $\{v_i\}$ (in station M) satisfy relation (18). It is required to find capacity C (EB) of station 4. Table 4 demonstrates the approximate values for overestimation Δ for this network. Note that the identification of (quasi)regenerative cycles in this network is more complicated than that in the tandem network, see conditions (17). For each $\hat{C}$, we calculate Δ and the estimate $\hat{\Gamma}_\Delta$ of the overflow probability. In particular, Table 4 shows

TABLE III
TANDEM NETWORK, $\Gamma = 10^{-5}$

λ	M	$\hat{D}$	$Var\hat{D}$	$\hat{C}$	$\hat{\Gamma}$	Δ	$\hat{\Gamma}_\Delta$
0.2	7	3.68	8.69	1.56	$8.35 \cdot 10^{-8}$	0.15	$7.44 \cdot 10^{-6}$
0.2	7	3.68	8.69	1.56	$8.35 \cdot 10^{-8}$	0.17	$9.85 \cdot 10^{-5}$
0.25	6	3.75	8.81	1.64	$4.71 \cdot 10^{-7}$	0.13	$7.23 \cdot 10^{-6}$
0.25	6	3.75	8.81	1.64	$4.71 \cdot 10^{-7}$	0.15	$4.73 \cdot 10^{-5}$
0.3	5	3.94	9.17	1.80	$3.26 \cdot 10^{-6}$	0.14	$1.04 \cdot 10^{-5}$
0.3	5	3.94	9.17	1.80	$3.26 \cdot 10^{-6}$	0.15	$2.71 \cdot 10^{-4}$
0.35	4	4.30	9.58	2.16	$3.64 \cdot 10^{-6}$	0.10	$6.42 \cdot 10^{-6}$
0.35	4	4.30	9.58	2.16	$3.64 \cdot 10^{-6}$	0.13	$2.86 \cdot 10^{-4}$

TABLE IV
ACYCLIC NETWORK

Γ	$\hat{C}$	$\hat{\Gamma}$	Δ	$\hat{\Gamma}_\Delta$
10^{-4}	3.29	$1.93 \cdot 10^{-6}$	0.12	$4.82 \cdot 10^{-5}$
10^{-4}	3.29	$1.93 \cdot 10^{-6}$	0.17	$9.34 \cdot 10^{-4}$
10^{-5}	3.76	$4.22 \cdot 10^{-7}$	0.10	$2.26 \cdot 10^{-6}$
10^{-5}	3.76	$4.22 \cdot 10^{-7}$	0.15	$9.30 \cdot 10^{-5}$
10^{-6}	4.26	$3.81 \cdot 10^{-7}$	0.14	$7.99 \cdot 10^{-7}$
10^{-6}	4.26	$3.81 \cdot 10^{-7}$	0.20	$2.50 \cdot 10^{-5}$

that in all cases $\Delta \approx 0.15 - 0.2$.

In another experiment we consider the same acyclic network and switch (alternately) light-tailed Weibull distributions $F(x) = 1 - e^{-0.3x}$ and $F(x) = 1 - e^{-0.9x}$ (of variables $\{v_i\}$) at the beginning of each cycle. The results of estimation are given in Table 5 and on Figures 5,6. In particular, such a model is motivated by the pooling systems, where server, after an empty period, starts to serve another type of customers.

TABLE V
ACYCLIC NETWORK, WEIBULL DISTRIBUTIONS

Γ	$\hat{C}$	$\hat{\Gamma}$	Δ	$\hat{\Gamma}_\Delta$
10^{-4}	3.80	$2.01 \cdot 10^{-6}$	0.15	$5.98 \cdot 10^{-5}$
10^{-4}	3.80	$2.01 \cdot 10^{-6}$	0.17	$9.80 \cdot 10^{-4}$
10^{-5}	4.12	$7.37 \cdot 10^{-7}$	0.13	$2.31 \cdot 10^{-6}$
10^{-5}	4.12	$7.37 \cdot 10^{-7}$	0.16	$9.20 \cdot 10^{-5}$
10^{-6}	4.61	$9.24 \cdot 10^{-8}$	0.12	$3.82 \cdot 10^{-6}$
10^{-6}	4.61	$9.24 \cdot 10^{-8}$	0.17	$5.92 \cdot 10^{-5}$

As Table 5 shows, in all cases overestimation $\Delta \approx 0.16 - 0.17$. Moreover, Figures 5, 6 show, for given $\hat{\Gamma}$ and estimated $\hat{C}$, the dependence between different values $\hat{C}_i \leq \hat{C}$ and the corresponding values of the overflow probability estimations $\hat{\Gamma}_i$. It is worth mentioning that, as Figures 5,6 show, the estimate of the overflow probability grows extremely quickly as soon as EB becomes less than $(1 - \Delta)\hat{C}$. It indicates again that the overestimation caused by the regenerative estimate is 'minimal' and acceptable for highly-reliable systems.

CONCLUSIONS

In this paper, we apply regenerative estimator to calculate the EB of a station in a tandem network and in a more general acyclic communication network. Simulation confirms that this estimator overestimates the requirement QoS guarantee Γ, however in general it is acceptable for the evaluation of the

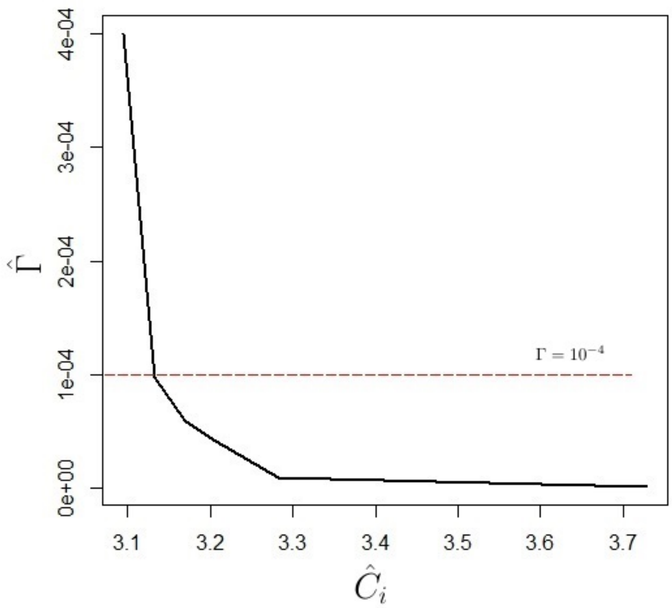

Fig. 5. Acyclic network, $\hat{C} = 3.73$: $\hat{\Gamma}$ vs. $\hat{C}_i$ with $\Gamma = 10^{-4}$.

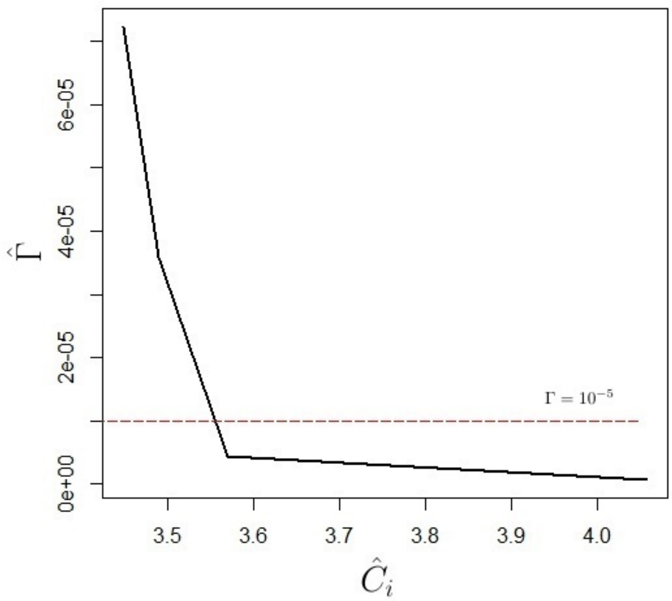

Fig. 6. Acyclic network, $\hat{C} = 4.06$: $\hat{\Gamma}$ vs. $\hat{C}_i$ with $\Gamma = 10^{-5}$.

EB of the elements of highly-reliable networks. Instead of rare classical regenerations, we apply more frequent quasi-regenerations, and it accelerates simulation time needed to estimate the EB with a given accuracy.

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