Two-Conductor HF Transmission-Line Solution for Symmetrical Indoor Triple-Pole Cables

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Abstract

Solution of the transmission-line equations is demonstrated in the HF band, for sheathed tricels with three-phase structural symmetry. The rationale commences from the diagonal form of second-order equations' matrix and the resulting distributed parameters, already derived in literature by virtue of cables' symmetrical configuration. Supposing transmission to take place between phase and neutral, voltage and current are deduced identical as over an isolated two-conductor line. Further use of cable's symmetry is hereupon made, as well as of the fact that ground is neither excited, nor terminated to either phase or neutral. Relevant hints of practical value are also provided.

1. Introduction

Impending utilisation of the High Frequency (HF) band seems highly promising towards Power-Line Communication (PLC) systems. The applicable ETSI standards specify hereupon occupation of the 1.6-9.4MHz zone over the access network, whereas the 11-30MHz zone is reserved for in-house systems [1]. Mainly single-phase networks are found in both domestic and premises installations, and PLC channels exhibit intensively fading properties [2,3]. Modelling of indoor LV cabling in the HF band is thus essential in order to develop efficient channel-simulating tools.

The cable type considered, cross section of which is depicted in Fig. 1, is widely used in Europe for in-house electrical plants. Such a configuration comprises the phase conductor, the neutral and the ground one, all of equal radii. Each conductor lies inside a dielectric jacket, and an additional insulating sheath encloses all three wires. Flexible cables are assembled of litz wires, wherein the metallic conducting strands are self-twisted too as a whole; for permanent though installations, one-ply conductors are used, and the wires lie parallel inside the cable, not twisted. Several cable types per VDE that are widely used in Europe for permanent installations in buildings fall into the structure studied herein – e.g. NYM and NYY.

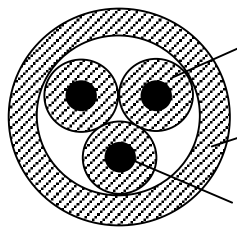

Fig. 1. Cable's cross-section.

Certain two-conductor modelling attempts of the cable type in question comprising experimental verification have already been presented [4-6]. Most of related articles involving estimation of the response – transfer function, power delay profile – of PLC channels in terms of the vicinal grid topology, assume cable compliance with the two-conductor Transmission Line (TL) model [7-11] too. The latter proves in fact particularly convenient for the treatment of signal propagation along TL networks, requiring no tedious multi-mode study that would be essential in case of a multi-conductor TL approach [12,13].

In [14], diagonalisation of the second-order TL equations' matrix is demonstrated for the cable structure in question, regarded as a three-conductor line with reference taken upon neutral, and respective distributed parameters are extracted. Regarding the phase-neutral transmission circuit, any arbitrary voltage or current excitation may thereby be considered a propagation mode of which the modal and physical values moreover coincide. Solution of the first order TL equations is the scope of this paper, interrelating the line voltage and current quantities on the pure isolated two-conductor basis. The analysis developed applies to tricels bearing the so-called three-phase symmetry, defined by cross-sectional rotation of 120° around the central longitudinal axis. The work presented in [14] is hence complemented, and sound establishment of the isolated two-conductor TL model is provided, applicable to single-phase HF signalling transmitted over the cable type considered.

As far as paper's structure is concerned, the formulation basis is set out in Section 2, both with an outline of the results extracted in [14]. The novel theoretical derivations are conducted in Section 3, and a concluding summation closes the paper in Section 4.

2. Case Establishment

2.1. Three-Conductor Reference Model

The cable is approached by the three-conductor uniform TL model of Fig. 2, regarded and formulated as in [13]. Subscript "G" is thus assigned to the Generator conductor, "R" to the Receptor conductor, and "0" denotes the common Reference conductor, whereas "m" indicates distributed parameters of mutual coupling between generator and receptor. By the term "uniform", longitudinal invariability of geometrical and material properties is denoted. The fundamental Transverse-Electro-Magnetic (TEM) wave propagation concept is presumed to allow definition of the TL formulism. Strictly, existence of a TEM field structure demands all three wires to be perfect conductors, lying inside homogeneous dielectric parallel to each other. All the same, for common good conductors, electrically small cross-sectional dimensions (conductor separations), and frequencies below the GHz range, the TEM assumption is widely supported in literature as reasonably accurate below the microwave spectral area [13,15]. As, besides, far as indoor installations are concerned, the wires are indeed parallel inside the cable – not twisted.

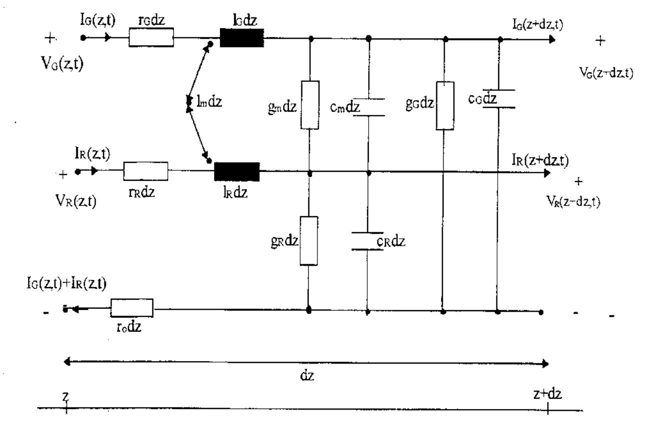

Fig. 2. Three-conductor transmission-line model.

As long as only TEM modes propagate on the line, the field structure is identical to the static (dc) case, wherein the zero-potential reference may be arbitrarily selected causing no loss of generality. Supposing transmission to take place between phase and neutral, the latter is adopted as common-return reference conductor, the former as generator, and the ground wire as receptor. Denoting the per-unit-length resistance, inductance, conductance and capacitance respectively by r, l, g, and c, the per-unit-length parameter matrices are

$$\mathbf{R} = \begin{bmatrix} r_G + r_0 & r_0 \\ r_0 & r_R + r_0 \end{bmatrix}, \tag{1a}$$

$$\mathbf{L} = \begin{bmatrix} l_G & l_m \\ l_m & l_R \end{bmatrix}, \tag{1b}$$

$$\mathbf{G} = \begin{bmatrix} g_G + g_m & -g_m \\ -g_m & g_R + g_m \end{bmatrix}, \tag{1c}$$

and

$$\mathbf{C} = \begin{bmatrix} c_G + c_m & -c_m \\ -c_m & c_R + c_m \end{bmatrix}. \tag{1d}$$

The voltage vector is

$$\mathbf{V}(z,t) = \begin{bmatrix} V_G(z,t) \\ V_R(z,t) \end{bmatrix}, \tag{2a}$$

and performing sinusoidal steady-state analysis in the frequency domain, the phasor voltage vector comes up

$$\hat{\mathbf{V}}(z) = \begin{bmatrix} \hat{V}_G(z) \\ \hat{V}_R(z) \end{bmatrix}, \tag{2b}$$

defined by

$$\mathbf{V}(z,t) = \mathrm{Re}\left\{\hat{\mathbf{V}}(z)e^{j\omega t}\right\}. \tag{2c}$$

The current vector is

$$\mathbf{I}(z,t) = \begin{bmatrix} I_G(z,t) \\ I_R(z,t) \end{bmatrix}, \tag{3a}$$

and the phasor current vector

$$\hat{\mathbf{I}}(z) = \begin{bmatrix} \hat{I}_G(z) \\ \hat{I}_R(z) \end{bmatrix}, \tag{3b}$$

defined by

$$\hat{\mathbf{I}}(z,t) = \mathrm{Re}\left\{\hat{\mathbf{I}}(z)e^{j\omega t}\right\}. \tag{3c}$$

Matrices

$$\hat{\mathbf{Z}} = \mathbf{R} + j\omega\, \mathbf{L} \tag{4a}$$

and

$$\hat{\mathbf{Y}} = \mathbf{G} + j\omega\,\mathbf{C} \tag{4b}$$

are respectively designated the per-unit-length impedance and admittance matrix.

The Telegrapher's equations for the phasor voltage and current vectors are

$$\frac{d\hat{\mathbf{V}}(z)}{dz} = -\hat{\mathbf{Z}}\hat{\mathbf{I}}(z) \tag{5a}$$

and

$$\frac{d\hat{\mathbf{I}}(z)}{dz} = -\hat{\mathbf{Y}}\hat{\mathbf{V}}(z)\,, \tag{5b}$$

wherefrom the second-order TL equations

$$\frac{d^2\hat{\mathbf{V}}(z)}{dz^2} = \hat{\mathbf{Z}}\,\hat{\mathbf{Y}}\,\hat{\mathbf{V}}(z) \tag{6a}$$

and

$$\frac{d^2\hat{\mathbf{I}}(z)}{dz^2} = \hat{\mathbf{Y}}\,\hat{\mathbf{Z}}\,\hat{\mathbf{I}}(z) \tag{6b}$$

are obtained.

All quantities and relations appearing in (1)-(6) are well known, already established in literature [12,13]. The above synoptic quotation is provided as a sound definition of parameters and dominant equations indispensable for the foundation of the work to be presented, as well as a basis for the reasoning to begin.

2.2. Parameter Properties

In [14], the per-unit-length impedance and admittance matrices are extracted as

$$\hat{\mathbf{Z}} = (r + j\omega l)\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \tag{7a}$$

and

$$\hat{\mathbf{Y}} = (g + j\omega c)\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} \tag{7b}$$

respectively, so that

$$\hat{\mathbf{Z}}\hat{\mathbf{Y}} = \hat{\mathbf{Y}}\hat{\mathbf{Z}} = \begin{bmatrix} \gamma_{\mathrm{III}}^2 & 0 \\ 0 & \gamma_{\mathrm{III}}^2 \end{bmatrix}\,, \tag{8a}$$

$$\frac{d^2\hat{\mathbf{V}}(z)}{dz^2} = \begin{bmatrix} \gamma_{\mathrm{III}}^2 & 0 \\ 0 & \gamma_{\mathrm{III}}^2 \end{bmatrix}\hat{\mathbf{V}}(z)\,, \tag{8b}$$

and

$$\frac{d^2\hat{\mathbf{I}}(z)}{dz^2} = \begin{bmatrix} \gamma_{\mathrm{III}}^2 & 0 \\ 0 & \gamma_{\mathrm{III}}^2 \end{bmatrix}\hat{\mathbf{I}}(z) \tag{8c}$$

are obtained, where

$$\gamma_{\mathrm{III}}{}^2 = 3(r + j\omega\, l)(g + j\omega\, c)\,. \tag{8d}$$

That is, having diagonal second-order TL equations' matrix with a single, degenerate eigenvalue, any arbitrary voltage or current excitation may be considered a propagation mode with transmission constant γ_{III}, the modal and physical values of which moreover coincide. The relation between voltage and current quantities is obtained from (5) through suitable differentiation.

Being conductors imperfect, i.e. $r \neq 0$ and $l^{in} \neq 0$, yields a small longitudinal electric-field component, so that the line field is no more of a pure TEM kind. The TEM assumption is of course indispensable for the line voltages and currents to be uniquely defined; deviation from the pure TEM field structure is though normally small for common good conductors, typical cross-sectional dimensions and frequencies below the GHz range [13]. It is the so-called "quasi-TEM mode" case, wherein the field structure is considered a TEM one, but the effects of conductors' losses upon the modal propagation constants are taken into account, reflected into the r and l^{in} parameters.

Let the uniform two-conductor TL model of Fig. 3 be considered, consisting of the generator and return conductors, with R, L, G, and C the per-unit-length series resistance, series inductance, shunt conductance and shunt capacitance respectively, representing the configuration of two parallel, identical circular conductors inside homogeneous dielectric. Solution of the scalar TL equations yields the voltage and current phasors

$$\hat{V}(z) = \hat{V}_m^+ e^{-\gamma_{\mathrm{II}} z} + \hat{V}_m^- e^{\gamma_{\mathrm{II}} z} \tag{9a}$$

and

$$\hat{I}(z) = \frac{\hat{V}_m^+}{\hat{Z}_C^{\mathrm{II}}} e^{-\gamma_{\mathrm{II}} z} - \frac{\hat{V}_m^-}{\hat{Z}_C^{\mathrm{II}}} e^{\gamma_{\mathrm{II}} z} \tag{9b}$$

respectively, where transmission constant is given by

$$\gamma_{\mathrm{II}}^2 = (R + j\omega\, L)(G + j\omega C) \tag{10}$$

and characteristic impedance is

$$\hat{Z}_C^{\mathrm{II}} = \sqrt{\frac{R + j\omega\, L}{G + j\omega\, C}}\,. \tag{11}$$

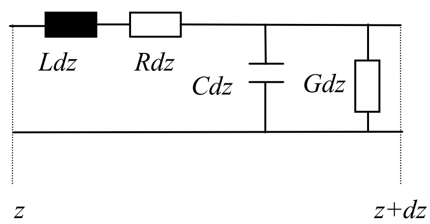

Fig. 3. Two-conductor transmission line model.

The reasoning developed in [14] results in

$$r=\frac{R}{2}\ , \tag{12a}$$

$$l=\frac{L}{2}\ , \tag{12b}$$

$$g=\frac{2G}{3}\ , \tag{12c}$$

and

$$c=\frac{2C}{3}, \tag{12d}$$

giving

$$\gamma_{\mathrm{III}}=\gamma_{\mathrm{II}}\ , \tag{13}$$

i.e. propagation constant of the generator-reference transmission circuit is identical as if the two conductors formed an isolated uniform transmission line at equal distance in between.

3. Two-Conductor Model Concurrence

Continuing the formulism on a voltage-excitation basis, as such is normally applied at transmission and captured at reception, the general solution of (8b) is

$$\hat{V}_G(z)=\hat{V}^{+}_{mG}\,e^{-\gamma_{\mathrm{III}}z}+\hat{V}^{-}_{mG}\,e^{\gamma_{\mathrm{III}}z}\ , \tag{14a}$$

$$\hat{V}_R(z)=\hat{V}^{+}_{mR}\,e^{-\gamma_{\mathrm{III}}z}+\hat{V}^{-}_{mR}\,e^{\gamma_{\mathrm{III}}z}\ , \tag{14b}$$

which substituted in (5a) give

$$\hat{I}_G(z)=\sqrt{\frac{y}{3\zeta}}\left[\left(2\hat{V}^{+}_{mG}-\hat{V}^{+}_{mR}\right)e^{-\gamma_{\mathrm{III}}z}-\left(2\hat{V}^{-}_{mG}-\hat{V}^{-}_{mR}\right)e^{\gamma_{\mathrm{III}}z}\right], \tag{15a}$$

$$\hat{I}_R(z)=\sqrt{\frac{y}{3\zeta}}\left[\left(2\hat{V}^{+}_{mR}-\hat{V}^{+}_{mG}\right)e^{-\gamma_{\mathrm{III}}z}-\left(2\hat{V}^{-}_{mR}-\hat{V}^{-}_{mG}\right)e^{\gamma_{\mathrm{III}}z}\right], \tag{15b}$$

where

$$\zeta=r+j\omega\,l\ , \tag{16a}$$

$$y=g+j\omega\,c\ . \tag{16b}$$

Considering the three-wire set, excitation is applied to generator that represents phase and may be connected to reference via load termination, whereas receptor is neither excited, nor terminated to either reference or generator, as it corresponds to ground. Due thus to the three-phase symmetry, receptor's voltage equals half the voltage of generator, i.e.

$$\hat{V}_R(z)=\frac{\hat{V}_G(z)}{2}\ , \tag{17}$$

as illustrated in Fig. 4, where transverse admittance quantities of a Δz length of the cable are depicted. By reason of linear independency that holds between any different exponential functions, relations (14) and (17) require

$$\hat{V}^{+}_{mR}=\frac{\hat{V}^{+}_{mG}}{2} \tag{18a}$$

and

$$\hat{V}^{-}_{mR}=\frac{\hat{V}^{-}_{mG}}{2}\ , \tag{18b}$$

so that from (14b) and (15),

$$\hat{I}_G(z)=\frac{\hat{V}^{+}_{mG}\,e^{-\gamma_{\mathrm{III}}z}}{\sqrt{\frac{4\zeta}{3y}}}-\frac{\hat{V}^{-}_{mG}\,e^{\gamma_{\mathrm{III}}z}}{\sqrt{\frac{4\zeta}{3y}}}\ , \tag{19}$$

$$\hat{V}_R(z)=\frac{\hat{V}^{+}_{mG}}{2}\,e^{-\gamma_{\mathrm{III}}z}+\frac{\hat{V}^{-}_{mG}}{2}\,e^{\gamma_{\mathrm{III}}z}\ , \tag{20a}$$

and

$$\hat{I}_R(z)=0 \tag{20b}$$

are deduced.

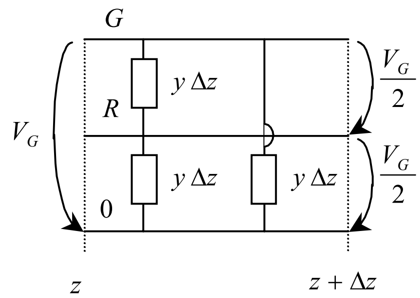

Fig. 4. Voltage allocation in the cable.

From (14a), (16), (19), and (20), HF signalling transmitted between phase and neutral propagates with transmission constant γ_{III} over characteristic impedance given by

$$\hat{Z}_C^{\mathrm{III}}=\sqrt{\frac{4(r+j\omega\,l)}{3(g+j\omega\,c)}}\overset{(12)}{=}\sqrt{\frac{R+j\omega\,L}{G+j\omega\,C}}\Rightarrow\hat{Z}_C^{\mathrm{III}}=\hat{Z}_C^{\mathrm{II}}\ , \tag{21}$$

whereas no current flows along ground, voltage of which equals at any longitudinal position half the voltage of phase at the same position, as inferred by (20).

Combining (9), (13), (14a), (19), and (21), propagation properties of the line formed of phase (generator) and neutral (reference) prove identical as if the two conductors formed an isolated uniform transmission line, at the same distance in between. Voltage and current of the phase-neutral circuit are thereby described as

$$V_z(z,t) = \mathrm{Re}\left\{\hat{V}_z(z)e^{j\omega t}\right\} \quad (22a)$$

and

$$I_z(z,t) = \mathrm{Re}\left\{\hat{I}_z(z)e^{j\omega t}\right\}, \quad (22b)$$

with voltage and current phasors given by (9), transmission constant and characteristic impedance by (10) and (11) respectively, and the z-axis supposed in the longitudinal direction. The proposed formulism is correct as long as the TEM assumption consists a valid approach of cable's field distribution, and the ground wire receives no excitation, which in fact stands. The study presented is focalised on the HF band as the only spectral area of interest with respect to broadband PLC applications in Europe [1], and covers TL properties of the cables under consideration. Any antenna-mode quantities stemming from potential electromagnetic interference penetrating the cable should of course be treated separately.

4. Conclusion

In this paper, voltage and current solutions of the TL equations have been derived, applying to indoor triple-pole cables assembled of identical adjacent cylindrical wires – phase, neutral, and ground. Propagation of HF signalling transmitted between phase and neutral ends up unaffected by the vicinal ground, and for PLC channel-simulation purposes may be unreservedly treated on the basis of the well-established two-conductor model. Symmetric tricels appear therefore strongly suggested for single-phase HF transmission, turning out preferable for in-house power electric plants. Justifying the two-conductor transmission-line model for the cable type considered and revealing consequent suitability of the latter towards PLC applications constitute the main contributions of the work presented.

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