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NETWORK RESOURCE REALLOCATION STRATEGY BASED ON AN IMPROVED CAPACITY-LOAD MODEL

STRATEGIA REALOKACJI ZASOBÓW SIECIOWYCH OPARTA O UDOSKONALONY MODEL PRZEPUSTOWOŚCI-OBCIĄŻENIA

Network resource reallocation is a common way to help restore performance of network systems subject to cascading failures. Majority of current network resource allocation strategies either give little regard to or make impractical assumptions about the relationship between capacity and load of network nodes, despite this relationship is closely related to the propagation of network failures. In this work we present and verify an improved nonlinear network capacity-load model based on the actual relation between network capacity and load. According to the verified model and realistic dynamic characteristics of network loads, we propose a new network resource reallocation strategy for networks under attacks from the perspective of maintenance. The strategy aims to effectively reallocate new capacity to network nodes after cascading failures occur. Both theoretical analysis and empirical studies are performed on three typical types of complex networks. Results show that the proposed network resource reallocation strategy is more efficient in mitigating devastating impact of cascading failures on network performance, in comparison to other three existing network resource reallocation strategies.

Keywords: *network reliability, maintenance, resource reallocation, cascading failures, capacity-load relation-ship.*

Realokacja zasobów sieci jest powszechnym sposobem, stosowanym w celu przywrócenia działania systemów sieciowych ob-jętych awariami kaskadowymi. Większość współczesnych strategii alokacji zasobów sieciowych kładzie mały nacisk lub czyni niepraktyczne założenia dotyczące zależności między przepustowością i obciążeniem węzłów sieci, choć zależność ta jest ściśle związana z rozchodzeniem się awarii sieci. W niniejszej pracy przedstawiono i zweryfikowano udoskonalony nieliniowy model przepustowości-obciążenia sieci na podstawie rzeczywistej relacji między przepustowością sieci i jej obciążeniem. Na podstawie zweryfikowanych modelu i realistycznych cech dynamicznych obciążeń sieciowych, proponujemy nową strategię realokacji zaso-bów dla sieci poddawanych atakom z perspektywy utrzymania ruchu. Celem strategii jest skuteczna realokacja nowej przepusto-wości węzłom sieci po wystąpieniu kaskadowych awarii. Przeprowadzono zarówno teoretyczne analizy, jak i badania empiryczne na trzech typowych rodzajach sieci złożonych. Wyniki pokazują, że proponowana strategia realokacji zasobów sieci jest bardziej skuteczna w zwalczaniu niszczącego wpływu kaskadowych awarii na przepustowość sieci w porównaniu do pozostałych trzech wykorzystywanych strategii realokacji zasobów sieciowych.

Słowa kluczowe: *niezawodnośćsieci, utrzymanie ruchu, realokacja zasobów, awarie kaskadowe, zależność przepustowość/obciążenie.*

1. Introduction

Cascading failure is a common phenomenon in complex network systems, especially in some real-world infrastructure networks, such as transportation systems, power grids and the Internet. It could lead to a sharp degradation of network performance and even collapse the whole network. For example, an initial disturbance happening on Au-gust 14, 2003 in Ohio, USA eventually brought about a disastrous blackout which affected millions of people for up to 15 hours [12]. A similar case occurred in Buenos Aires on November 7, 2012, in which the crippled critical power grids triggered large negative effects over one million households, resulting in prolonged chaos in many impor-tant departments [15]. Massive cascading failures also take place in communication, social and economic network systems [7, 33]. Due to the devastating effects of cascading failures over the entire networks,

it is of great significance to study characteristics of cascades as well as relevant mitigation strategies aiming to protect real-world infra-structure networks.

Optimal resource allocation has been used as a usual way to im-prove the network robustness to cascading failures [10, 22, 31]. Spe-cifically, overloads (or congestion) on network nodes (edges) are one of the main reasons for triggering cascading failures in networks in reality. For instance, overloads (currents or voltages) on substations or buses in power grids often happen. Hereby, the optimal allocation of limited network capacity resource for reducing the occurrence of overload failures has been one of the hot topics in the complex net-work study [11, 21, 29, 32, 35, 37]. Lee and Hui [21] proposed a dynamic reallocation scheme based on nodes' willingness to share resources, upon which networks could achieve a better performance. Xia and Hill [35] presented an algorithm that adjusts flow rate and

capacity distribution to maximize the system utility and the utilization ratio of capacity. Wang et al. [32] introduced an idea of cost-based attacks on complex networks and investigated the problem of optimal limited network defence resource distribution for minimizing attacks' damage to infrastructure networks. However, most of current resource allocation strategies are only attempt to optimally allocate resource for initial network construction, ignoring reallocating the subsequent resource after networks are established, and do not take the occurrence of cascading failures under dynamics of network loads into account. In addition, most of prior resource reallocation strategies are still lack of considering essential dynamic characteristics of real-world infrastructure networks which have important impacts on network reliability. Particularly, the practical relationship between capacity and load of infrastructure network nodes is closely related to overload failures in networks.

Considerable research efforts have been carried out to investigate the underlying mechanism of cascading failures in complex networks and network reliability [3, 5, 8, 14, 16, 38, 39]. Crucitti et al. [5] proposed a dynamic flow redistribution scheme for analyzing the cause and process of cascading failures in different complex networks. While most of previous research assumed constant [14, 24], random [17, 33], or linear [5, 20] relationship between capacity and load of nodes in network cascading simulation, Kim and Motter [18] found that there is a nonlinear capacity-load relationship actually exists in different complex infrastructure networks such as air transportation network, highway network, power-grid network and Internet router network. Consequently, prior research on complex infrastructure networks based on the three assumptions about capacity-load relationship mentioned above are inconsistent with real situations to some extent. Dou et al. [8] further researched the nonlinear relationship and presented a capacity-load model (hereinafter referred to as the C-L model) against cascading failures. However, the model has some limitations. For example, it did not identify reasonable ranges of model parameters or give an adequate explanation about the meaning of those parameters. More recently, Fang et al. [10] tackled the issue of capacity-load relationship by introducing the optimization of link capacity allocation. They found that cascade-resilient network systems tend to have a nonlinear capacity-load relation. But they just focused on the link capacity resource used for initial network construction without considering the optimal allocation of subsequent node capacity resource.

To overcome these limitations, in this paper we propose an improved nonlinear C-L model based on the capacity-load relationship of real-world infrastructure networks. Building upon this model and from the perspective of maintenance, we then put forward a new network capacity resource reallocation strategy which takes the subsequently-added network capacity as well as dynamics of network load into consideration. By conducting comparative experiments with three existing capacity resource reallocation strategies, we demonstrate the validity and feasibility of our proposed strategy in mitigating severe influence of cascading failures caused by intentional attacks on complex infrastructure networks.

The rest of this paper is organized as follows: in section 2, the improved nonlinear C-L model is proposed; in section 3, the novel resource reallocation strategy is introduced with theoretical analysis; in section 4, the process of simulating cascading failures in complex networks is described; in section 5, the improved C-L model is verified on two general networks which are subjected to different kinds of attacks, and the simulation results are analyzed; verification of performance of the novel resource reallocation strategy, and comparison with other three existing strategies are presented in section 6. Finally, conclusions as well as directions for future research are given in section 7.

2. The improved C-L model

We assume that the betweenness centrality [13] of network node i at time t represents its load $L_i(t)$ which can be calculated using the algorithm presented in [30]. Node capacity signifies the maximum load that a node could handle without congestion. Crucitti, et al. [5] assumed that because of the limited cost in practice, capacity C_i of node i is linearly proportional to its initial load $L_i(0)$ in real-life networks, as shown in the following widely-used linear C-L model (1):

$$C_i = \alpha \times L_i(0), \ i = 1, 2 \ldots N, \alpha \geq 1 \tag{1}$$

where α is the tolerance parameter, and N is the total number of network nodes.

As mentioned above, nevertheless, different real-world infrastructure networks possess a rather similar nonlinear capacity-load relationship supported by empirical data, which could be simply described as the formula: $C - L \sim L^\alpha$, where C represents capacity and L represents load [18]. Accordingly, we propose an improved nonlinear C-L model through curve fitting, intending to match the real data curves. Under our improved C-L model, the description of relationship between network node's capacity and load matches better the actual situations in infrastructure networks which are shown in [18]: heavily loaded network nodes have smaller unoccupied portions of capacity, whereas lightly loaded nodes present larger unoccupied portions of capacity. Our improved C-L model is illustrated in Eq.(2).

$$C_i = \alpha \times \left(L_i(0) + L_i(0)^{1-\mu} \right), \quad i = 1, 2 \ldots N, \alpha \geq 1, 0 < \mu < 1 \tag{2}$$

The item $\alpha \times L_i(0)^{1-\mu}$ in the improved C-L model (2) is set to describe the nonlinear characteristic of node capacity and load. α is the tolerance parameter of networks, indicating that the network node capacity is within a realistic cost constraint. Besides, it could also reflect the robustness level of different networks. μ is the nonlinear coefficient, the range of which is set with the consideration of specific situations in real-life networks. By adjusting its value, the corresponding nonlinear relationship between network node's capacity and load could be flexibly adjusted in simulation. In practice, it is more reasonable for us to reference to the data collected from real-world network systems to set these parameters.

Fig. 1 shows an example of the capacity (C)-load (L) relation of network nodes under normal condition in a log-binned scale. The results presented are obtained by applying C-L model (1) and C-L model (2), respectively, in which $\alpha = 1.1$, $\mu = 0.8$. The dashed line, assuming $C = L$ of each network node, is shown in the figure for comparison.

As shown in Fig. 1, the blue curve with symbol '+' (represents the results under the improved C-L model (2)) illuminates the nonlinear capacity-load relationship where the proportion of unoccupied node capacity is larger for nodes with smaller capacity. The smaller a node's capacity, the higher unoccupied proportion of that node's capacity. This nonlinear characteristic is closer to real-world situations in infrastructure networks [20], in contrast with the simulation results under the linear C-L model (1) (represented by the red curve in Fig. 1). In brevity, the improved nonlinear C-L model (2) could better describe the practical flow behavior of real-life infrastructure networks, upon which further analysis of network reliability could be carried out.
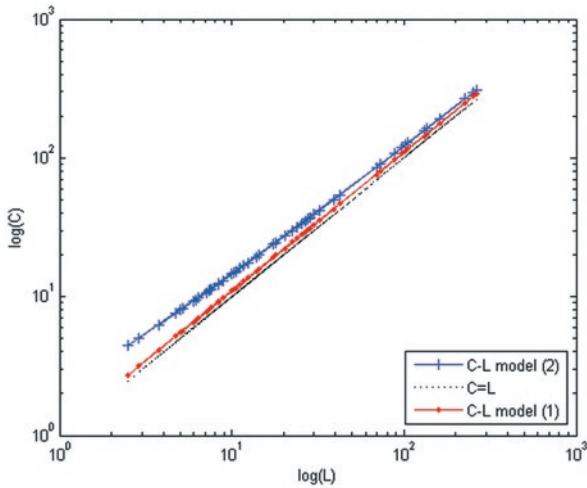
Fig. 1. A comparison between two C-L models' simulation results

3. The proposed resource reallocation strategy

When an infrastructure network system broke down and its performance dropped, depending on the available budget and cost, people usually reallocate some new network resource to a part of important network nodes to maintain the performance of this network (i.e., upgrade those nodes, such as enlarging the capacity of substations in power grids and routers in the Internet). By this way, people intend to restore network performance to some extent to meet the requirements about certain network services. For some real-life infrastructure networks, e.g., power grids, network components upgrade as discussed here is more declined to be taken as a strategic action to maintain network systems, which could work in the long run. While for some other network applications, e.g., the Internet, this action could be adopted as a response to address emergencies as we do in this paper.

In this section, based on the proposed nonlinear C-L model (2), we propose a new capacity resource reallocation strategy (*Cnlinr*) for efficiently reallocating the subsequently-added network capacity against cascading failures. The core part of this new reallocation strategy is described in Eq.(3), which decides the amount of new capacity that every node in the network is reallocated:

$$C_{\Delta i} = \theta \times \left(\frac{L_i(0)}{C_i} \right)^{\eta} \times \left(\frac{L_i(T)}{C_i} \right)^{\gamma} \times |L_i(T) - C_i| \qquad (3)$$

where $0 < \theta \leq 1, \eta \geq 1, 0 < \gamma \leq 1$, $C_{\Delta i}$ is the subsequent capacity added to arbitrary node i, C_i is the initial capacity of node i. $L_i(0)$ is the initial load of node i, and $L_i(T)$ is its final load at time T when network G stays stable again after cascades occurred. θ is the scale parameter, and η, γ are nonlinear coefficients. These three arguments, giving a certain degree of flexibility, are used to modify the reallocation proportion of subsequent capacities adding to each network node. By comparing simulation results, we could find a proper combination of the three parameters θ, η and γ for a better capacity reallocation effect in improving performance of damaged networks. The total capacities C_{iall} of network node i after capacity resource reallocation are: $C_{iall} = C_i + C_{\Delta i}$.

Specifically, the scale parameter θ in Eq.(3) is used to control the total amount of newly added capacities. The role played by this parameter is mainly for coordinating with the other two model parameters η and γ. Meanwhile, its size could be tuned based on the available capacity resource, signifying the limitation of cost in practice. The

second item in Eq.(3), $L_i(0)/C_i$, is the initial load to capacity ratio of node i, denoting the nonlinear C-L relationship when the network is under a normal condition, which has a profound influence on the spread of cascades. Consequently, the ratio is highly referred for reallocating new network capacities and the particular scope of parameter η emphasizes the importance of this ratio. The third item $L_i(T)/C_i$ actually reflects the following steady state of node i after cascading failures occur from its load viewpoint. Thus, the ratio should also be taken into account in determining the new capacity reallocation proportion. The range of designed parameter γ is set to control the impact of this ratio, $L_i(T)/C_i$. The last item, $|L_i(T) - C_i|$, in Eq.(3) is the difference between steady load $L_i(T)$ after cascades and corresponding initial capacity C_i of node i. This item presents the changes of node load after the dynamic redistribution of network flows resulted from cascades.

According to [18], in many real-world infrastructure networks, the larger a node's load is, the higher proportion of occupied node's capacity is. As a result, the network nodes with larger loads are prone to be overloaded (malfunctioned). Moreover, load-based intentional attacks as well as other attacks, aiming at heavily loaded nodes, would bring a severely adverse influence on network performance if those important nodes break down. Consequently, by taking the redistribution of node loads and its relationship with corresponding node capacities before and after failure propagation into account, we tend to nonlinearly reallocate more subsequent capacity resource to the nodes with larger loads, as illustrated in Eq.(3) above.

Constrained by some real-world factors, e.g., available investment cost, subsequently-added network resource that can be used for reallocation are limited. Hence, different resource reallocation strategies' effects on improving network performance might be very different. In section 'Verification of proposed resource reallocation', we compare our new proposed resource reallocation strategy with other three existing reallocation strategies by simulating cascading failures in scale-free networks, random networks as well as small-world networks. These three types of complex networks are widely-recognized topologies of typical networks nowadays, including the World-Wide-Web, the Internet, social network, the electrical power-grid network, highway network, air transportation network, etc [2, 9, 26, 34]. Simulation results illuminate that the new capacity resource reallocation strategy proposed in this paper could efficiently defend against cascades triggered by intentional attacks on network nodes.

According to related research works [1, 27, 36], some properties of real-life infrastructure networks are independent of sizes of network systems, for example, the relative size of the largest clusters in networks. Thus, for shortening simulation time, network sizes in the simulations presented in this paper are selected but still representative.

4. Cascading process

In this paper, we model a complex network as an undirected, weighted, self loop-free as well as single edge graph G with N nodes and E edges. Single edge graph means that there is only one edge possibly existing between two connected network nodes. For instance, G can represent a power grid with N stations and E transmission lines. Mathematically, it is depicted by an adjacency matrix $\{w_{ij}\}_{N \times N}$ with $w_{ij} \in [0,1]$ representing the weight (communication efficiency) associated with the edge between node i and node j. $w_{ij} = 0$ implies that there is no edge between node i and node j; $w_{ij} = 1$ implies that the edge between node i and node j works in a perfect condition. We as-

sume that the smaller the entry w_{ij} is, the less efficient communication along the edge between node i and node j is.

At the initial time of simulation $t = 0$, we set $w_{ij} = 1$ for all existing edges in the network, assuming that those transmission edges all work perfectly. Also, we assume that all network loads are only transmitted along the most efficient paths between a pair of nodes. We use e_{ij} to depict the efficiency of the most efficient path [19] between node i and node j in the network, which is defined as follows:

$$e_{ij} = \left(\sum \frac{1}{w_k} \right)^{-1} \qquad (4)$$

where w_k represents the communication efficiency of edge, as introduced above, involved in the most efficient path between node i and node j.

Furthermore, network efficiency $E(G)$ defined in Eq.(5) is used to measure the performance of network G [19]:

$$E(G) = \frac{1}{N(N-1)} \sum_{i \neq j \in G} e_{ij} \qquad (5)$$

The simulation process of cascading failures in this work is structured as follows: First set up network G, according to the assumptions above, initial node load will not exceed node capacity, and network load distribution along with the most efficient paths will not change. Based on Eq.(5), network efficiency $E(G)$ will remain stable at this stage, i.e., its value will not change, which means that network G is in a steady state. Then the removal (breakdown) of one network node gives rise to the dynamic redistribution of network loads, triggering more overloaded (congestion) nodes. In the next seciton two strategies are explained for choosing the node to be removed. One node removal will lead to alterations of some edges' transmitting efficiencies, i.e., the corresponding entries in $\{w_{ij}\}_{N \times N}$ decrease. As a consequence, the composition and efficiency e of some of the most efficient paths in the network would change, which in turn causes the redistribution of network loads. The above process repeats until all the loads of remaining nodes are below their capacity. At this time, network G stays steady on a lower performance level, which means that network efficiency $E(G)$ eventually converges to a relatively stable value in the cascading simulation. We calculate each edge's weight (transmitting efficiency w) by applying the following iterative rule (6) at every time step t in the simulation [5]:

$$w_{ij}(t+1) = \begin{cases} w_{ij}(0) \times \dfrac{C_i}{L_i(t)}, & L_i(t) > C_i \\ w_{ij}(0), & L_i(t) \leq C_i \end{cases} \qquad (6)$$

where j represents the first neighbor nodes of network node i. As seen in the Eq.(6), the matrix $\{w_{ij}\}_{N \times N}$ is recalculated based on the real-time relationship between node's capacity and load in each simulation step. Through cascading simulations, we can get the overall changing trends of network efficiency $E(G)$, upon which we could observe and further analyze the whole cascading process in the network.

The above cascading failure model based on the complex network theory is relatively comprenhensive and abstract, which has the advantage of using graph theory techniques to model cascading dynamics in real-world infrastructure systems and providing a good understanding of dynamics of cascades. Therefore, it has been recognized to offer a universal perspective and a useful way to research on cascading process on power grids [4, 6, 10].

5. Verification of improved C-L model

In this section, we execute cascading simulations on BA scale-free networks [9] and ER random networks [34]. BA network model is a widely-adopted one to describe scale-free networks, which possesse a power-law distribution of node degree. Note that $P(k)$ represents the distribution probability of network nodes with degree k, $P(k) \sim k^{-\gamma}, \gamma \approx 3$. Similarly, ER network model, proposed by Erdos and Renyi, is a typical model for constructing random networks, whose load distribution and degree distribution all follow the Poisson distribution. Two representative triggering strategies are used here: load-based intentional attack where a network node with the largest load is removed; and random failure where one node chosen at random will be removed from the network. By analyzing the cascading process and the difference generated by adopting the existing linear C-L model (1) and the improved C-L model (2), the performance and feasibility of our improved nonlinear C-L model (2) is validated.

5.1. Simulation on BA scale-free network

The simulation process of an intentional attack on a BA network is as follows: At first, initial network capacities are allocated to network nodes according to the nonlinear C-L model (2). Then the node with the largest load is removed, causing the dynamic redistribution of network loads which is accompanied with a cascade of overload failures. Meanwhile, the weight of each edge and network average efficiency $E(G)$ at every simulation time step are calculated until network G reaches a steady state again. The same process is repeated for the linear C-L model (1). Moreover, the similar cascading simulations are performed for both of the considered C-L models under a random failure where the node initially removed is chosen at random. To minimize random errors, simulation results under random failures correspond to the average of results over four failure triggers.

Fig. 2 and Fig. 3 illustrate the evolution of network efficiency $E(G)$ after intentional attack on the node with the heaviest load in a BA network ($N = 100$, $E = 500$) under C-L models (2) and (1), respectively. Parameters used are $\mu = 0.3$ and $\alpha = 1.01, 1.05$. Apparently, the changing trends of network efficiency $E(G)$ under two C-L models are quite different. The collapse factor Cp shown in the figures means the rate of decline in network efficiency caused by cascading failures. Cp is the ratio of the difference between initial efficiency $E(0)$ and steady efficiency $E(T)$ after cascades spread to initial network efficiency $E(0)$, as the following equation (7) shows. This factor could clearly illuminate the degradation of network efficiency caused by cascading failures:

$$Cp = (E(0) - E(T)) / E(0) \qquad (7)$$

We define that a range of ± 2 % on the steady value of network efficiency $E(G)$ is the error band of network efficiency in this paper. Transition time T_r is the time when network efficiency remains stable again within the error band after cascades take place, which manifests that the network reaches the stationary state. Values of T_r obtained in the cascading simulations on BA networks subject to intentional attacks are shown in Table 1. The unit of T_r is the simulation time step.

Table 2 shows experimental results of T_r and decrease extent of collapse in network efficiency (ΔEd) in the simulations on the BA network ($N = 100$, $E = 500$) which is subjected to random failures. ΔEd is calculated according to the following equation (8):

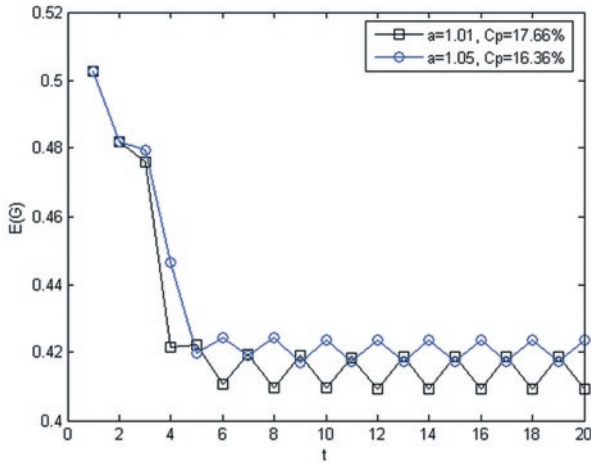$$\Delta Ed = (E_{\Delta 1} - E_{\Delta 2}) / E_{\Delta 1} \qquad (8)$$

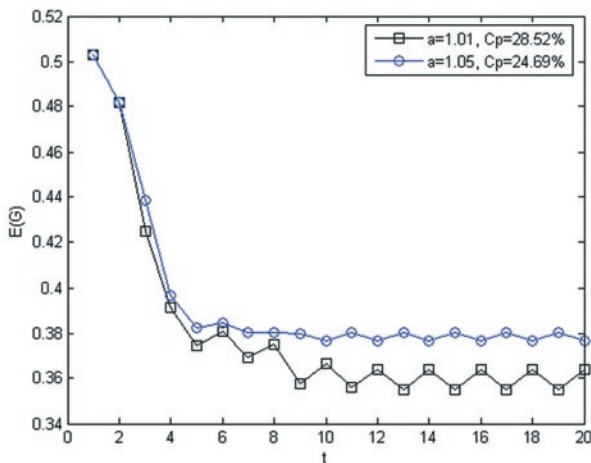Fig. 2. Changing curves of BA network efficiency under C-L model (2)

Fig. 3. Changing curves of BA network efficiency under C-L model (1)

Table 1. Transition time T_r of BA networks under two C-L models

	α	μ	Transition time T_r	
			Network 1	Network 2
Linear C-L model (1)	1.01 (1.05)	*	9 (5)	5 (7)
Nonlinear C-L model (2)	1.01 (1.05)	0.3	4 (5)	5 (5)
	1.01 (1.05)	0.5	4 (4)	4 (5)

Network 1: N = 100, E = 500. Network 2: N = 200, E = 1000.

Table 2. Decrease extent ΔEd and T_r under two C-L models

	α	μ	T_r	ΔEd
Linear C-L model (1)	1.01 (1.05)	*	6 (6)	33.76%
Nonlinear C-L model (2)	1.01 (1.05)	0.5	5 (6)	5.33%
	1.01 (1.05)	0.8	5 (5)	12.22%

where $E_{\Delta 1}$ is the decline rate of network efficiency caused by cascading failures when α = 1.01, and $E_{\Delta 2}$ is the drop of network efficiency when α = 1.05.

Table 3. Decrease extent ΔEd and T_r on ER networks

	α	μ	Network 1		Network 2	
			T_r	ΔEd	T_r	ΔEd
Linear C-L model (1)	1.01 (1.05)	*	5 (6)	27.9%	7 (6)	14.54%
Nonlinear C-L model (2)	1.01 (1.05)	0.8	4 (5)	6.64%	5 (6)	10.17%

Table 4. Decrease extent ΔEd and T_r on ER networks

	α	μ	Network 1		Network 2	
			T_r	ΔEd	T_r	ΔEd
Linear C-L model (1)	1.01 (1.05)	*	7 (2)	63.25%	7 (2)	68.76%
Nonlinear C-L model (2)	1.01 (1.05)	0.5	6 (3)	35.23%	5 (2)	21.79%

Network 1: N = 100, p = 0.101, Network 2: N = 200, p = 0.0503. p is the probability that each edge is included.

5.2. Simulation on ER random network

Using the similar procedures, we perform cascading simulations on ER random networks [34] which are subjected to intentional attacks as well as random failures, respectively. Simulation results of decrease extent of drops in network efficiency (ΔEd) and transition time T_r in the cascading process on ER networks under different C-L models are listed in Table 3 (under intentional attacks) and Table 4 (under random failures).

5.3. Results analysis

According to the above cascading simulations on different kinds of networks, the following observations can be made:

(1) As the tolerance parameter α increases (1.01 to 1.05), the network efficiency E(G) is reduced by 13.43% for the BA scale free network using the linear C-L model (1) (Fig. 3). Under the same condition, as shown in Fig. 2, the collapse rate of network efficiency is declined by 7.36% (for μ = 0.3) and 5.33% (for μ = 0.5) when adopting the nonlinear C-L model (2). This implies that increasing the capacity of each network node in the same proportion may not greatly enhance the network performance. It is consistent with the existing findings [23, 25], which verifies the feasibility of the proposed nonlinear C-L model (2). Simulation results on other BA networks with different scales and ER networks present the similar characteristics.

(2) As illustrated in tables 1-4, transition time T_r under the improved nonlinear C-L model (2) is shorter than that under the linear C-L model (1). It demonstrates that cascading failures triggered by intentional attacks or random failures spread rapidly on the two kinds of experimented networks. After cascades occur, network efficiency E(G) will finally stay stable at a lower level. It is in compliance with the current findings [28, 40], which further verifies the feasibility of the proposed nonlinear C-L model (2).

6. Verification of proposed resource reallocation strategy

When a network which is subjected to intentional attacks reaches a new stationary state, i.e., network efficiency stays at a lower level within error band after cascades occurred, we reallocate subsequent network capacities of the same amount to network nodes using our proposed resource reallocation strategy and other three existing reallocation strategies. By analyzing the upward trends of network efficiency as well as eventual steady-state values, we compare effectiveness of these four resource reallocation strategies in restoring network performance against cascading failures, and accordingly verify the performance of our proposed one.

6.1. Simulation method

The three existing resource reallocation strategies [11] for comparison are summarized as follows: (i) the newly added capacity of each node is linearly proportional to the node's degree (***CDlinr***); (ii) the newly added capacity of each node is linearly proportional to its initial load (***CLlinr***); (iii) each node's newly added capacity is equal (***Ceql***). The corresponding equations are as below:

$$C_{\Delta i} = b \times D_i, \quad i = 1,2...N \quad (9)$$

$$C_{\Delta i} = b \times L_i(0), \quad i = 1,2...N \quad (10)$$

$$C_{\Delta i} = c, \quad i = 1,2...N \quad (11)$$

where $C_{\Delta i}$ is the new capacity reallocated to node i, b is the scaling coefficient, D_i in Eq. (9) denotes the degree of node i, and c in Eq. (11) is a constant, signifying the new capacity added to each node.

Fig. 4 shows the averaged relationship between the newly added capacity (C') and initial load ($L(0)$) of each node in the network, which is subjected to an intentional attack, in a logarithmic scale after reallocating equal capacity resource to network nodes based on the four reallocation strategies. The dotted black line (assuming $C' = L(0)$) is the reference line.
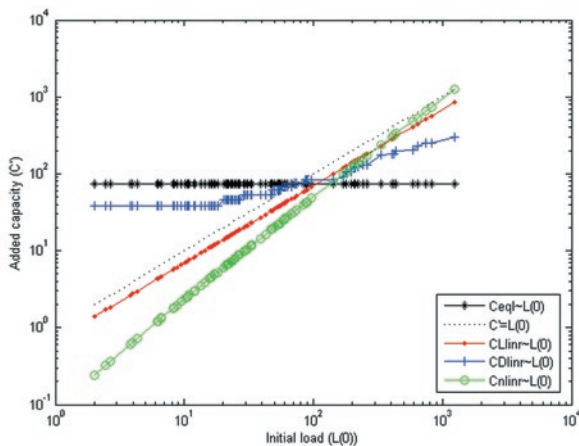

Fig. 4. The relation between added capacity and initial load of each node

As shown in Fig. 4 (the green curve with circles), the proposed new reallocation strategy (***Cnlinr***) reallocates more subsequent capacities to the nodes with larger loads than the ones with smaller loads nonlinearly. It is based on the dynamics of network load redistribution and the corresponding relations with node capacity.

Fig. 5 shows the main steps of subsequent capacity resource reallocation simulation on different networks undergoing cascades which are caused by intentional attacks.
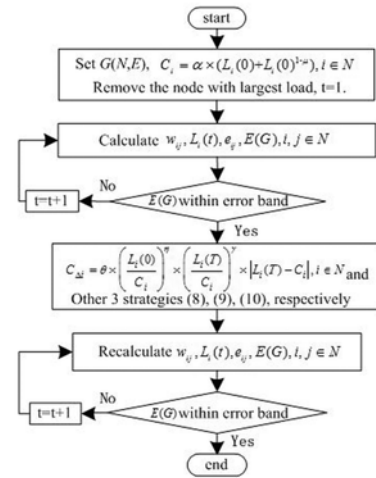

Fig. 5. The simulation steps of reallocating new capacities

Specifically we first use the method introduced in section 'Cascading process' to simulate the cascading process triggered by an intentional attack on network G, adopting the improved nonlinear C-L model (2). When network G reaches a new stationary state after cascades spread, we reallocate subsequent capacity resource of the same amount to network nodes according to the four resource reallocation strategies, respectively. Then we simulate the dynamic redistribution of network loads ensued as mentioned before, and calculate the shortest paths in the network and network efficiency $E(G)$ at every time step, until $E(G)$ ascends to a higher level and remains relatively stable again. It means that network G relaxes to a steady state. By this way, the overall changing trends of network efficiency of scale-free networks, random networks and small-world networks which were subjected to intentional attacks, after reallocating subsequent capacity resource, can be obtained.

The application of our proposed capacity resource reallocation strategy is briefly presented as follows: as we get real-time information of each network node's load and capacity (e.g., voltage and capacity of substations in power grids) in real-life infrastructure networks, a proper combination of model parameters in Eq.(3) can be determined through the above described simulations. For this reason, we can obtain a theoretical reallocation proportion of newly-added capacities for each network node, which can provide guidance in designing robust infrastructure networks, along with mitigating cascading failures in reality.

6.2. Simulation results and analysis

Fig. 6 illuminates the evolution trends of network efficiency $E(G)$ after reallocating new resource to nodes based on the four strategies for a BA network ($N = 100$, $E = 500$) subject to cascading failures triggered by intentional attacks. The first three values of each trend curve are stable values of network efficiency when networks reach steady states after cascading failures occur. The corresponding relative increments of network efficiency (ΔR) are listed in Table 5. ΔR is calculated based on equation (12), where $E(T)$ is the steady value of network efficiency after cascades spread, E_f is the final stable value of network efficiency after reallocating new capacities. Parameter values used are $\mu = 0.5$, $\gamma = 0.3$, $\theta = 0.8$, $\eta = 2$, and $\alpha = 1.01$, 1.05. ΔC is the ratio between the overall newly added capacities and initial capacities in all.

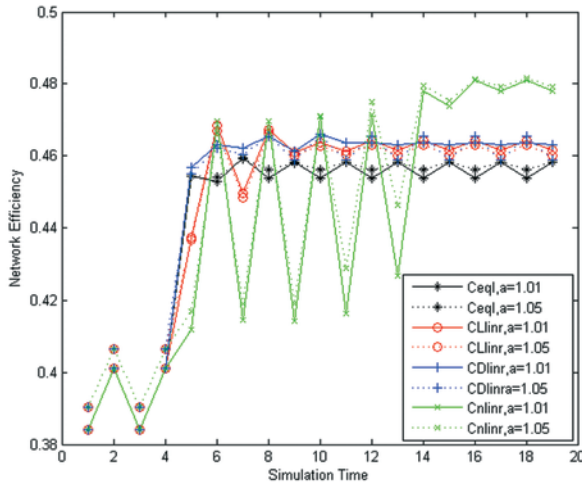$$\Delta R = (E_f - E(T)) / E(T) \qquad (12)$$

Fig. 6. Rising trends of BA network efficiency after resource reallocation

Table 5. Relative increment ΔR of network efficiency under four strategies

	Ceql	CLlinr	CDlinr	Cnlinr	ΔC	α
ΔR	18.19%	20.55%	20.68%	24.87%	43.99%	1.01
	17.13%	18.28%	18.41%	23.1%	37.07%	1.05

It can be seen from Fig. 6 that the changing curve (with green crosses) of network efficiency under the proposed resource reallocation strategy is obviously different from others. Initially, it fluctuates severely, and then does cost a longer time to reach a stable level with higher final value (see details in Table 5). We also performed simulations on BA networks with different scales and got the similar conclusion that the proposed resource reallocation strategy (**Cnlinr**) can enable the network efficiency of BA networks to be upgraded the most.

Fig. 7 illustrates the comparative results for an ER network ($N = 100$, $p = 0.101$) subject to an intentional attack on the node with the highest loads. Similar to Fig. 6, the first three values of those curves are final network efficiency when networks stay stable again after cascading failures occurred. In the cascading simulations, $\alpha = 1.05$, $\mu = 0.5$, $\gamma = 0.3$, $\theta = 0.6$, $\eta = 2.3$, and $\Delta C = 20.35\%$ is the overall increments of new capacities in each of the four resource reallocation strategies. The values of the factor 'rise' presented in the figure are equal to ΔR.
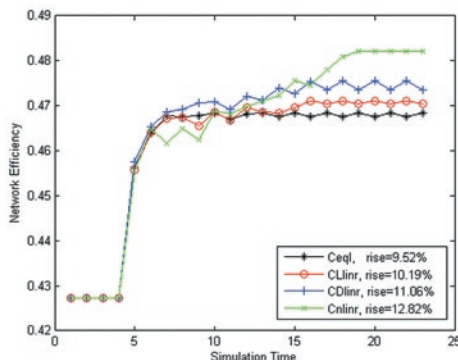

Fig. 7. Rising trends of ER network efficiency after resource reallocation

As shown in Fig. 7, aside from the longer time is taken for network efficiency to stay stable, the proposed resource reallocation strategy (the green curve with crosses) could make the ER network suffering intentional attacks get the largest improvement in the network efficiency, by comparison to results under other three reallocation strategies. Additionally, similar conclusions are obtained in simulations on other ER networks with different scales, which further testify the suitability of the proposed resource reallocation strategy.

We continue to validate the proposed resource reallocation strategy by conducting similar network capacity reallocation simulations on another type of networks called small-world networks. It is a type of network between regular network and random network with short average path length and high clustering coefficient. We choose WS small-world model [1] to generate small-world networks, which is a widely-accepted model with a probability parameter P. In the test network, $N = 100$, $P = 0.5$ which is the probability that each initial edge is reconnected. After reallocating subsequent resource based on four strategies separately, the dynamic redistribution of network loads follows. The corresponding four rising trends of network efficiency are obtained, as shown in Fig. 8. Similarly, the first three values of each curve in the figure are the final steady-state network efficiency after cascades spread triggered by attacks intentionally. The values of parameters used in the simulations are: $\alpha = 1.04$, $\mu = 0.3$, $\gamma = 0.33$, $\theta = 0.66$, $\eta = 1.9$, $\Delta C = 24.97\%$. The values of factor 'rise' listed in Fig. 8 present the values of ΔR as mentioned above. Other WS networks with different scales are also tested and similar findings are obtained.
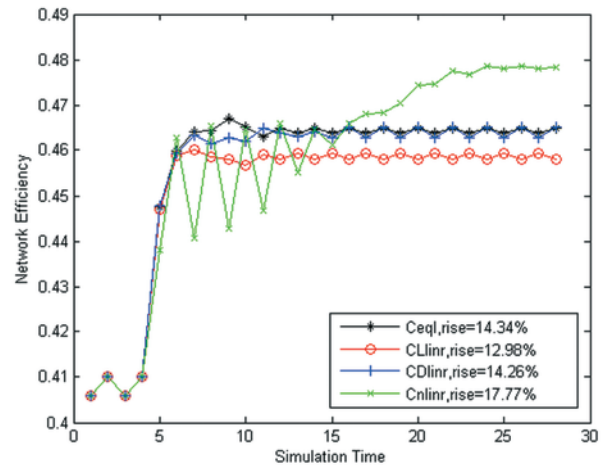

Fig. 8. Rising trends of WS network efficiency after resource reallocation

It can be seen from Fig. 8 that except for the initial fluctuation, the green curve with crosses obtained by adopting our proposed strategy has the largest final value of network efficiency among all curves with the same initial efficiency values. It strongly verifies the performance of our proposed resource reallocation strategy again.

In summary, as compared with the other three existing reallocation strategies, the changing curves of network efficiency for different networks subject to intentional attacks under the proposed new reallocation strategy may fluctuate more drastically, and reach steady states slower in the cases studied. Nevertheless, the corresponding steady-state values of network efficiency under our proposed reallocation strategy are the highest, indicating that network performance can be restored the best after being damaged by cascading failures.

As illuminated in Eq.(3), under the proposed new reallocation strategy, the new subsequent capacity resources are reallocated to network nodes based on not only initial node's capacity but also real-time features of networks, i.e., the distribution of node's load before and after failure propagation, as well as the nonlinear Capacity-Load

relationship. In comparison, the other three strategies reallocate new capacities of the same amount just according to some fixed network properties without combining certain real-time key information of networks. The simulation results demonstrate the effectiveness of the proposed new resource reallocation strategy (*Cnlinr*) in improving the network efficiency against cascading failures triggered by intentional attacks.

7. Conclusion

In this paper, we first propose an improved C-L model based on the realistic, nonlinear relationship between capacity and load of network nodes in many real-life infrastructure networks. We demonstrate its feasibility by simulating cascading failures on two typical types of networks, namely, scale-free networks and random networks, considering both intentional attacks and random failures. The proposed C-L model presents the practical flow behavior of network systems under normal condition, which could be used as a basis for analyzing network failures and their impacts on network performance. According to this improved nonlinear C-L model and some important dynamic characteristics of infrastructure networks, we further propose a network capacity resource reallocation strategy for network systems suffering intentional attacks for the purpose of maintenance. Experimental results are obtained by simulating the proposed resource reallocation strategy on three widely-recognized types of networks, in comparison to other three existing resource reallocation strategies. These comparative results show that our proposed strategy could reallocate subsequently-added capacity more efficiently by identifying

important network nodes as well as subsequently reallocating more new capacity to them. As a result, the proposed reallocation strategy could bring about a more marked improvement of network performance after it drops due to cascades. Further, our proposed capacity resource reallocation strategy could effectively reduce damages on infrastructure networks caused by cascading failures, which is of great importance to maintain real-world infrastructure networks with the consideration of limited investment cost. Findings from this research have theoretical importance and could be practically useful for improving the reliability-based design of real-life infrastructure networks, along with defense of cascading failures in them from economic perspective.

Some hypotheses have been made in the cascading simulations, e.g., we assume only one network node is initially removed from the network after it is attacked or breaks down, and the amount of new subsequent capacities is not particularly regulated in resource reallocation simulations. However, they are not necessarily the case for many infrastructure networks in practice. Therefore, one direction of our future work is to relax these assumptions. Besides, we mainly verify the performance of the proposed capacity resource reallocation strategy subject to intentional attacks. The study of its performance for random failures will be another direction of further research.

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