Fast Stochastic Simulation of Silicon Waveguide with Non-Gaussian Correlated Process Variations

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Abstract: In this paper, we develop an efficient statistical simulation technique based on stochastic collocation for silicon photonics process variations with non-Gaussian correlated random parameters. Our algorithm has achieved 57-times speedup compared with standard Monte-Carlo simulation.

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1. Introduction

Silicon photonics has attracted lots of attention in recent years thanks to its ability to achieve higher bandwidth, lower power dissipation compared to electrical interconnects and its compatibility and easy integration with CMOS process [1]. However, because of the high contrast of the refractive index between silicon and silica, silicon-based optical devices are very sensitive to manufacturing process variations; for example, the geometric variations in silicon width and thickness can result in a fluctuation of the effective phase index (n_{eff}), leading not only to degraded performance in devices such as directional couplers and ring resonators but also to serious failures at the system level [2–4].

Despite some results for nanometer integrated circuits (IC) [5,6], there is still a lack of efficient uncertainty quantification techniques for silicon photonics. Monte Carlo (MC) [7] has been the mainstream statistical simulation technique in commercial design software. However, it suffers from a slow convergence rate and long simulation time. Recently, fast stochastic spectral methods have been developed based on generalized polynomial chaos expansions [8] to efficiently approximate a stochastic solution. However, one of the major assumptions in the existing publications [9,10] is that the input parameters describing the process variations are mutually independent, which is not always necessarily true. In this paper, we aim at quantifying the effects of silicon photonic process variations using a stochastic collocation (SC) scheme [11] . The geometric parameters of the silicon waveguide are assumed to be non-Gaussian correlated, which is reasonable because of the wafer thickness variations and imperfect lithography in practice. Since the silicon width depend on the wafer thickness, they are actually correlated. Based on the orthogonal basis construction [12], in Section 2 we show that the quantity of interest, such as n_{eff}, can be efficiently estimated. The simulation results of a silicon waveguide under process variations are shown in Section 3.

2. Non-Gaussian Correlated parameters

In this section, we first use a Gaussian mixture to model the parameters of the process variations, and then we demonstrate the orthogonal basis construction flow. Lastly, the idea of computing a stochastic solution dependent on the correlated non-Gaussian random variables is explained.

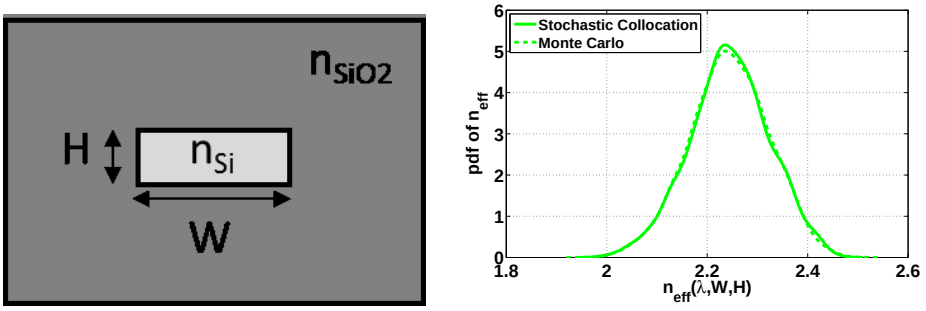
2.1. Gaussian Mixture

A Gaussian mixture for N variables with M mixed terms is a weighted sum of M multivariate Gaussian densities. For the simplicity of mathematical derivation, we set $N = M = 2$, but it is straightforward to extend our technique to the general cases where N does not necessarily equal M. With a Gaussian mixture, the process variations can be described by the following distribution

$$\begin{bmatrix}\xi_1\\ \xi_2\end{bmatrix} \sim a\cdot N(\vec{\mu}_A,\Sigma_A)+b\cdot N(\vec{\mu}_B,\Sigma_B) \quad \text{with} \quad a+b=1, \quad 0\le a,b\le 1 \tag{1}$$

and

$$\mu_A=\begin{bmatrix}\mu_{A_1}\\ \mu_{A_2}\end{bmatrix}, \quad \Sigma_A=\begin{bmatrix}\sigma_{A_1}^2 & \rho_A\sigma_{A_1}\sigma_{A_2}\\ \rho_A\sigma_{A_1}\sigma_{A_2} & \sigma_{A_2}^2\end{bmatrix}, \quad \mu_B=\begin{bmatrix}\mu_{B_1}\\ \mu_{B_2}\end{bmatrix}, \quad \Sigma_B=\begin{bmatrix}\sigma_{B_1}^2 & \rho_B\sigma_{B_1}\sigma_{B_2}\\ \rho_B\sigma_{B_1}\sigma_{B_2} & \sigma_{B_2}^2\end{bmatrix}$$



(a) (b)

Fig. 1. (a) The cross section of a Silicon waveguide with width W, height H, and refractive indices $n_{Si} = 3.48, n_{SiO_2} = 1.445$. (b) The simulated pdf of $n_{\text{eff}}(\lambda, W, H)$ with $\lambda = 1.55\ \mu$m. The solid line is the stochastic collocation result, whereas Monte Carlo result is dash line.

where $\vec{\xi} = [\xi_1; \xi_2]$ can be geometric parameters such as the width, height, and length of a silicon device.

2.2. *Basis Functions*

Let $u(x, \vec{\xi})$ be the quantity of interest smoothly dependent on the process variations $\vec{\xi}$. In order to investigate the process variations of a silicon waveguide, $u(x, \vec{\xi})$ can be, for example, n_{eff}; $\vec{\xi}$ can be a parameter vector containing the width and height of the waveguide; and x can be the operating wavelength that affects n_{eff}. Given a joint probability density function(pdf) of $\vec{\xi}$, which in our case is assumed to be a Gaussian mixture, we can use a set of N-dimensional orthogonal basis functions $\Psi_{\vec{\alpha}}(\vec{\xi})$ to approximate the quantity of interest [12] :

$$u(x, \vec{\xi}) \approx \sum_{\vec{\alpha}} C_{\vec{\alpha}}(x) \Psi_{\vec{\alpha}}(\xi_1, \cdots, \xi_N) \tag{2}$$

where $C_{\vec{\alpha}}(x)$ is the corresponding coefficient with a multivariable index $\vec{\alpha} = (\alpha_1, \cdots, \alpha_N) \in \mathbb{N}^N$. In order to construct such orthogonal bases $\{\Psi_{\vec{\alpha}}\}$, we first employ the three term recurrence relations [13] to construct some orthogonal polynomials with the marginal pdf of ξ_i for each parameter ξ_i. Subsequently, we use tensor product and the reshaping scheme in [12] to obtain $\{\Psi_{\vec{\alpha}}\}$ that are orthogonal for the joint pdf.

2.3. *Stochastic Collocation*

The coefficients $C_{\vec{\alpha}}(x)$ in (2) can be computed as

$$C_{\vec{\alpha}}(x) = \frac{\langle u(x, \vec{\xi}), \Psi_{\vec{\alpha}}(\vec{\xi}) \rangle_{p_{\vec{\xi}}}}{\langle \Psi_{\vec{\alpha}}(\vec{\xi}), \Psi_{\vec{\alpha}}(\vec{\xi}) \rangle_{p_{\vec{\xi}}}} \tag{3}$$

since $\{\Psi_{\vec{\alpha}}\}$ is an orthogonal set for the measure $p_{\vec{\xi}}$ (i.e., the joint pdf of $\vec{\xi}$). The denominator can be obtained as a by-product when constructing $\Psi_{\vec{\alpha}}$ in Section 2.2. The numerator of $C_{\vec{\alpha}}(x)$ can be approximated based on a SC scheme. Specifically, we first use an electromagnetic finite difference mode solver to simulate the device at a small number of Gauss quadrature points computed according to the marginal pdf of each parameter. Then, the denominator can be computed by a N-dimensional Gauss quadrature rule.

3. Silicon Waveguide example

In this section, the computational flow in Section 2 is applied to estimate n_{eff} of the silicon waveguide shown in Fig. 1(a). We consider the uncertainties of the width W and thickness H, and approximate $n_{\text{eff}}(\lambda, W, H)$ as the linear combination of several orthogonal functions $\{\Psi_{\vec{\alpha}}(W, H)\}$, where λ is the wavelength. Since W depends on H in process variation, they are actually correlated. The calculation of $C_{\vec{\alpha}}(\lambda)$ involves calling a deterministic finite difference

mode solver at some N-dimensional Gauss-Hermite quadrature points [which are given as some samples of (W,H,λ)]. In our numerical experiments, we set the nominal width W as 0.4 μm, the nominal height H as 0.22 μm. The refractive index of silicon and silica are 3.48 and 1.445, respectively. The wavelength of interest is 1.55 μm. The Gaussian mixture of W and H used in our simulation is

$$\begin{bmatrix} W \\ H \end{bmatrix} \sim 0.8 \cdot N(\vec{\mu}_A, \Sigma_A) + 0.2 \cdot N(\vec{\mu}_B, \Sigma_B)$$

with

$$\mu_A = \begin{bmatrix} 400 \\ 220 \end{bmatrix} \text{nm}, \quad \Sigma_\text{A} = \begin{bmatrix} 400 & 0.1 \\ 0.1 & 100 \end{bmatrix} \text{nm}^2, \quad \mu_\text{B} = \begin{bmatrix} 420 \\ 230 \end{bmatrix} \text{nm}, \quad \Sigma_\text{B} = \begin{bmatrix} 300 & 0.1 \\ 0.1 & 150 \end{bmatrix} \text{nm}^2$$

All simulations are performed using a core i7-4700 CPU and RAM 8GB laptop. Fig. 1(b) shows the simulated n_eff of the silicon waveguide. The solid line is the pdf of the simulated n_eff by our SC method whereas the dashed line is the standard MC with 5000 samples. Our result matches the MC result quite well. The total number of quadrature points and basis functions used in our algorithm are both 49. Therefore, our solver is about 57$\times$ faster than standard MC. In practice, a 49-quadrature-points SC is adequate to provide highly accurate results, at the CPU time of 180 seconds.

4. Conclusions

In this paper, we proposed a SC scheme to simulate non-Gaussian correlated parameters. The numerical procedures of the basis functions construction and basis coefficient computation are briefly presented. Our computational technique is applied to analyze the process variations of a silicon photonics waveguide. The simulated n_eff of our numerical method is consistent with that from MC. Compared with standard MC, our algorithm has achieved 57$\times$ speedup.

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