Adaptive Observer-Based Fault Diagnosis for a Class of MIMO Nonlinear Uncertain Systems

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Abstract — In this paper, a high-gain nonlinear observer based fault diagnosis approach is proposed for a general class of nonlinear uncertain systems. The nonlinear system under consideration contains parameter uncertainties as well as Lipschitz-like nonlinearities and may be harmed by time-varying fault. The fault diagnosis algorithm is designed based on a new adaptive estimation method for estimation of the parameters related to faults. The main result is given in a constructive manner by developing a novel nonlinear adaptive observer, without resort to any linearization. The design of the proposed observer does not necessitate the resolution of any dynamics systems and its expression is explicitly given. Its global exponential convergence is ensured, which does not rely on solving any kind of dynamic Riccati equation. A simulation example is given to illustrate the efficiency of the proposed fault diagnosis method.

I. INTRODUCTION

The increased complexity of plants and the development of sophisticated control systems have necessitated the parallel development of efficient Fault Detection (FD) systems. In these years, fault diagnosis problem is becoming strategically important for its various implications (e.g., avoiding major plant breakdowns and catastrophes; safety problems; fast and appropriate responses to emergency situations). The main function of an FD scheme is to detect a fault when it happens, which may then be acted on by sending alarm signals, taking protection measures, or reconfiguring a running control scheme.

Classical strategies for fault diagnosis are mainly applicable for linear systems (Willsky, 1976; Frank, 1990; Basseville & Nikiforov, 1993; Gertler, 1998; Chen & Patan, 1999), whereas in many practical situations, nonlinear characteristics of the control systems cannot be ignored in dealing with fault diagnosis problem. Therefore, nonlinear system fault diagnosis problem has become an active research topic recently. Several researchers have developed control techniques for fault diagnosis in restricted classes of nonlinear systems. Watanabe and Himmelblau (1982, 1983) used separate observer based techniques for detection of instrument faults and asymptotic identification of parametric faults. Wünnenberg (1990) developed unknown input observers that are valid for systems where the nonlinearities can be expressed as an explicit function of the input and transformed versions of the output. The resulting observer is nonlinear, but uses a linear transformation of the states. Seliger and Frank (1991) extended this approach to a wider class of nonlinear systems using a nonlinear transformation of the states. However, the resulting observer design is not transparent. A class of nonlinear systems, namely models consisting of a set of polynomial differential-algebraic equations has been considered by Frisk (2001). Garg and Hedrick (1995) proposed extending the detection filter concept for linear systems to systems with a nonlinear Lipschitz drift component. Yu, Shields, and Daley (1996) proposed a bilinear fault detection system for bilinear systems. The fundamental problem of residual generation was formulated for nonlinear systems and some conditions for the solvability of the same were given by Hammouri, Kinnaert, and Yaagoubi (1999). Persis and Isidori (2001) presented a differential geometric approach to this problem for systems that are linear in the input and faults and derived a necessary condition for the problem to be solvable in terms of an observability distribution. Flies, Join, and Mounier (2005) and Join, Sira-Ramírez, and Flies (2005) used differential algebra techniques for nonlinear fault diagnosis. Xu and Zhang (2004), Jiang and Chowdhury (2004), Wang and Shen (2005), Wang and Zhang (2006) and others used an adaptive observer for fault detection and estimation in certain classes of nonlinear systems. Zhang, Polycarpou, and Parisini (2002) used a bank of observers with adaptive residual threshold to detect and isolate faults in a certain class of nonlinear systems. Narasimhan, Vachhani, and Rengaswamy (2008) applied a diagonal observer to detect and identify faults in a certain class of nonlinear systems. Tan, Crusc, and Aldaen (2008) developed an extension to nonlinear fault detection observers based on sliding mode theory. Li and Zhou (2009) gave the optimal solutions to several robust fault detection problems such as $H_\infty/H_\infty$, $H_2/H_\infty$, and $H_\infty/H_\infty$ problems for linear time-varying systems in a time domain.

Motivated by the above works, in this paper we attempt to develop a fault diagnosis scheme based on a new adaptive observer technique. The significance of this paper is shown as follows: 1) We try to deal with the fault detection problem for a wider practical class of systems, compared with the ones in the existed literatures. The systems under consideration are a class of nonlinear multiple-input-multiple-output (MIMO) systems which contain parameter uncertainties as well as Lipschitz-like nonlinearities. 2) The advantage of the newly proposed adaptive observer for fault diagnosis scheme lie in its constructive manner and its validity for a class of genuine nonlinear systems without resort to any linearization. 3) The design method does not rely on numerical optimization tool but on Lyapunov-type stability results. Different from the most existing literatures for fault diagnosis problem, the gain of the proposed filter does not necessitate the resolution of any dynamics systems and it expression is explicitly given, i.e., its calibration is achieved through the choice of a single parameter. 4) The global exponential convergence of the proposed observer is ensured, which does not rely on solving any kind of dynamic Riccati equation.

Technically, we firstly use a mild (nonlinear) coordinates transformation to formulate the considered system into a special form for furthermore disposal. (The class of systems are truly nonlinear in the sense that they cannot be linearized by coordinate change and output injection). Then in a new coordinates a high-gain adaptive observer is constructed under some special regularity assumptions.
and an excitation condition, which has exponential convergence in errors and the estimation of unknown parameters related to possible faults. Finally, the fault diagnosis problem is solved in the original coordinates through a nonlinear transformation.

The paper is organized as follows. In Section II, the considered system model and the form of parameterized fault are given. In Section III, the proposed approach to fault diagnosis through adaptive estimation is presented, and the main result is proved. Then a numerical example is presented in Section IV before the paper is concluded in Section V.

II. PROBLEM STATEMENT

The considered deterministic nonlinear systems subject to faults is assumed to have the form

\[
\begin{aligned}
\dot{x} &= F(s, x)x + G(u, s, x) + \Psi_f(t)\vartheta_f, \\
y &= \tilde{C}x,
\end{aligned}
\]  

(1)

where \( x = [x^1, x^2, \ldots, x^q]^T \in \mathbb{R}^n \) is the state of the system, with \( x^k \in \mathbb{R}^{n_k}, k = 1, \ldots, q \) and \( q = n_1 \geq n_2 \geq \cdots \geq n_q \), \( \sum_{k=1}^q n_k = n; u \in U \) is the input of the system, where \( U \) is a compact subset of \( \mathbb{R}^m; y \in \mathbb{R}^p \) is the output of the system; \( s(t) \) is a known signal, which presents the parameter uncertainties. \( F(\cdot) \in \mathbb{R}^{n \times n} \), \( G(\cdot) \in \mathbb{R}^n \) are nonlinear vector functions which have the following special structures:

\[
\begin{bmatrix}
G^1(u, s, x^1) \\
G^2(u, s, x^1, x^2) \\
\vdots \\
G^n(u, s, x)
\end{bmatrix}
\]

\[
F(s, x) =
\begin{bmatrix}
0 & F_1(s, x^1) & 0 & \cdots & 0 \\
0 & 0 & F_2(s, x^1, x^2) & \cdots & \vdots \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & 0 & F_{q-1}(s, x^1, \ldots, x^{q-1}) \\
0 & 0 & \cdots & 0 & 0
\end{bmatrix}
\]

is a block matrix with each \( F_k, k = 1, \ldots, q - 1 \), denoting a \( n_k \times n_{k+1} \) rectangular matrix.

\[
\tilde{C} = [I_n, 0_{n_1 \times n_{n_2}}, 0_{n_1 \times n_{n_3}}, \ldots, 0_{n_1 \times n_q}],
\]

where \( I_n \) is the \( n \times n \) identity matrix and \( 0_{n \times n_k} \) is the \( n \times n_k \) null matrix, \( k = 2, \ldots, q \).

Remark 2.1. This fault diagnosis scheme design is proposed to a general class of nonlinear systems where parameter uncertainties are presented differently to the existing ones. In fact, all physical models presented in these papers (Farza et al., 1998), and Busanwon et al., 1998 are under the fault-free editions of form (1) and can be seen as practical examples of the systems (1).

The term \( \Psi_f(t)\vartheta_f \) represents possible faults with a time-varying matrix \( \Psi_f(t) \) and an unknown parameter vector \( \vartheta_f \). Let \( \Psi^j_f(t) \) be the \( j \)th column of the matrix \( \Psi_f(t) \) and \( \vartheta^j_f \) the corresponding component of \( \vartheta_f \), then

\[
\Psi_f(t)\vartheta_f = \Psi^1_f(t)\vartheta^1_f + \Psi^2_f(t)\vartheta^2_f + \cdots + \Psi^n_f(t)\vartheta^n_f.
\]

(2)

Each term \( \Psi^j_f\vartheta^j_f \) represents a different type of faults. The online estimation can provide a direct approach to fault diagnosis with an estimation of the magnitude of each possible fault. With the above formulation, the diagnosis of the faults modeled by the term \( \Psi_f(t)\vartheta_f \) amounts to the isolation of nonzero components of the parameter vector \( \vartheta_f \).

The main objective of this paper is to design a recursive fault diagnosis algorithm for joint estimation of the state vector \( x(t) \) and the parameter vector \( \vartheta_f \), and the estimation method is based on the techniques of high gain observer and of a nonlinear adaptive observer. We focus on building an exponential observer for the fault diagnosis problem in uncertain MIMO nonlinear systems (1).

Before the synthesis of the proposed adaptive observer-based fault diagnosis scheme, some assumptions which will be stated in due course. At this step, we assume the following:

(A1) The matrix \( \Psi_f(t) \) and unknown constant vector \( \vartheta_f \) are bounded.

(A2) The unknown signal \( s(t) \) and its time derivative \( \dot{s}(t) \) are bounded.

(A3) There exist two positive constants \( \mu \) and \( \nu \) such that for every \( k \in 1, \ldots, q - 1, \forall x \in \mathbb{R}^n, \forall t \geq 0 \),

\[
0 < \mu^2 I_{n_{k+1}} \leq F_k(s(t), x)^TF_k(s(t), x) \leq \nu^2 I_{n_{k+1}},
\]

where \( I_{n_{k+1}} \) is the \((n_{k+1}) \times (n_{k+1})\) identity matrix.

For every \( \xi \in \mathbb{R}^n, t \geq 0 \), let \( A(s(t), \xi) \) be the block diagonal matrix by

\[
A(s(t), \xi) = \text{diag} \left[ I_{n_1}, F_1(s(t), \xi), F_1(s(t), \xi)F_2(s(t), \xi), \ldots, \prod_{i=1}^{q-1} F_i(s(t), \xi) \right]
\]

By Assumption (A3), each \( F_k, k = 1, \ldots, q - 1 \) is left-invertible and as a result so is \( A(s(t), \xi) \). In the sequel, we shall denote its left inverse by \( A^+(s(t), \xi) \).

In the next section, we shall synthesis the desired adaptive nonlinear observer and the fault diagnosis algorithm for nonlinear uncertain MIMO system (1) subject to the faults (2).

III. FAULT DIAGNOSIS BASED ON ADAPTIVE ESTIMATION

In this section, a newly proposed form of the proposed adaptive observer will be given firstly. For the proposed observer structure, we shall present the fault diagnosis scheme and guarantee the convergence of the proposed observer under some excitation condition. The fault diagnosis observer design problem will then be taken into account. We shall choose its parameter (gain) of the presented filter explicitly.

In order to synthesize the fault diagnosis observer, we shall apply an mild (nonlinear) change of coordinates, which puts system (1) into a special form, then we will propose an adaptive observer. After the construction of the desired fault diagnosis scheme, the equations of the latter will be given in the original coordinates.

Consider the following change of coordinates: \( \Phi : \mathbb{R}^n \rightarrow \mathbb{R}^{n_1} \),

\[
x = \begin{bmatrix}
x^1 \\
x^2 \\
\vdots \\
x^n
\end{bmatrix}
\rightarrow
z = \begin{bmatrix}
z^1 \\
z^2 \\
\vdots \\
z^n
\end{bmatrix} = \Phi(s, x) = \Lambda(s, x)x
\]

\[
\begin{bmatrix}
F_1(s, x^1)x^2 \\
F_1(s, x^1)F_2(s, x^2, x^3)x^3 \\
\vdots \\
(\prod_{i=1}^{q-1} F_i(s, x^1, \ldots, x^{q-1}))x^q
\end{bmatrix}
\]

where \( z^k \in \mathbb{R}^{n_k}, k = 1, \ldots, q \). It is easy to see that \( \Phi \) is one to
Let $\Phi^c$ denote its converse. One can easily show that
\[
\dot{z} = \frac{\partial \Phi}{\partial x}(s, x) \dot{x}(t) + \frac{\partial \Phi}{\partial s}(s, x) \dot{s}(t)
\]
\[
= \Lambda(s, x) F(s, x)x + \left( \frac{\partial \Phi}{\partial x}(s, x) - \Lambda(s, x) \right) F(s, x)x
\]
\[
+ \frac{\partial \Phi}{\partial x}(s, x) \left( G(u, s) + \Psi_f(t) \vartheta_f \right) + \frac{\partial \Phi}{\partial s}(s, x) \dot{s}(t)
\]
\[
= A z + \left( \frac{\partial \Phi}{\partial x}(s, x) \Lambda^+(s, x) - \Lambda(s, x) \Lambda^+(s, x) \right) A z
\]
\[
\frac{\partial \Phi}{\partial x}(s, x) \left( G(u, s) + \Psi_f(t) \vartheta_f \right) + \frac{\partial \Phi}{\partial s}(s, x) \dot{s}(t),
\] (3)
where
\[
A = \begin{bmatrix}
0 & I_{n_1} & 0 & 0 \\
& 0 & I_{n_1} & 0 \\
& & \vdots & \ddots & \ddots \\
& & 0 & \cdots & I_{n_1} \\
0 & \cdots & 0 & 0 & 0
\end{bmatrix}
\] (4)
is $n_1 \times n_1 q$ square matrix. Note that equality (3) comes from the fact that
\[
\Lambda(s, x) F(s, x) = A \Lambda \quad \text{or equivalently} \quad F(s, x) = \Lambda^+(s, x) A \Lambda(s, x).
\]
For the clarity of statement, we shall use the following notation in the sequel:
\[
\Theta(s, z) \triangleq \left( \frac{\partial \Phi}{\partial x}(s, \Phi^c(z)) \Lambda^+(s, \Phi^c(z)) \right)
\]
\[
- \Lambda(s, \Phi^c(z)) \Lambda^+(s, \Phi^c(z)) \right)
\]
\[
\psi(s, z) \triangleq \Theta(s, z) A z,
\]
\[
\varphi(s, u, z) \triangleq \frac{\partial \Phi}{\partial x}(s, \Phi^c(z)) G(u, s, \Phi^c(z)),
\]
\[
\varphi_f(s, u, z, t) \triangleq \frac{\partial \Phi}{\partial s}(s, \Phi^c(z)) \Psi_f(t).
\]
Notice that the matrix $\Theta(s, z)$ is lower triangular with zeros on its main diagonal. As a result, the function
\[
\psi(s, z) = \begin{bmatrix}
\psi^1(s, z^1) \\
\psi^2(s, z^1, z^2) \\
\vdots \\
\psi^k(s, z^1, \ldots, z^k)
\end{bmatrix},
\]
where $\psi^k(s, z^1, \ldots, z^k) \in \mathbb{R}^{n_1}$ has a triangular structure. Similarly, since $\{\partial \Phi/\partial x\}(s, x)$ and $G(u, s, x)$ assume triangular structures, so does $\varphi(s, u, z)$. Then system (1) can be written in the new coordinates $z$ as follows:
\[
\dot{z} = A z + \psi(s, z) + \varphi(s, u, z) + \varphi_f(s, u, z, t) \vartheta_f
\]
\[
+ \frac{\partial \Phi}{\partial s}(s, \Phi^c(z)) \dot{s}(t),
\]
\[
y = C z = z^1,
\] (5)
\[
where C = [I_{n_1}, 0_{n_1}, \ldots, 0_{n_1}] \text{ is } n_1 \times n_1 q \text{ matrix with } 0_{n_1}
\text{ denoting the } n_1 \times n_1 \text{ null matrix.}
\]

**Remark 3.1** In addition to the existing fault diagnosis techniques to deal with nonlinear uncertain systems, we first use the mild coordinates change proposed in Farza et al., (2004) which can put the nonlinear stochastic systems into a special form for which we will propose an adaptive observer under some special assumptions. It can be seen that the transformation is not resort to linearization like Persis and Isidori (2001). In the new coordinates, the construction of the observer for fault diagnosis can be developed in the ideal of high-gain observer method. To do this, we will present a new observer method to the nonlinear form of (5) with possible faults (2), which can not be dealt with by the techniques in the existing literatures.

We need the following assumption similar to many works related to high-gain observer synthesis (see e.g. Bornard & Hammouri, 1991; Gauthier et al., 1992; Farza et al., 1998):

(A4) The function $\psi(s, z)$, $\varphi(s, u, z)$, and $\varphi_f(s, u, z, t)$ are globally Lipschitz with respect to $z$ uniformly in $s$ and $u$.

Before giving the candidate observer, we introduce the following notations:

- Let $\Delta_\theta$ be the block diagonal matrix defined by
\[
\Delta_\theta = \text{diag} \left[ \frac{1}{\theta} I_{n_1}, \frac{1}{\theta^2} I_{n_1}, \ldots, \frac{1}{\theta^q} I_{n_1} \right],
\]
where $\theta > 0$ is real number.
- Let $S$ be the unique solution of the algebraic Lyapunov equation
\[
S + A^T C S + S A - C^T C = 0
\] (7)
where $A$ and $C$ are, respectively, given by Eqs. (4) and (6). It can be seen that the solution of (7) is symmetric positive definite and that
\[
S^{-1} C^T = \begin{bmatrix}
C_{1q} I_{n_1} \\
C_{2q} I_{n_1} \\
\vdots \\
C_{qq} I_{n_1}
\end{bmatrix}
\]
A candidate observer for system (5) is
\[
\dot{\hat{z}} = A \hat{z} + \psi(\hat{s}, \hat{z}) + \varphi(\hat{s}, u, \hat{z}) + \varphi_f(\hat{s}, u, \hat{z}, t) \vartheta_f
\]
\[
+ \left[ \Delta_\theta^{-1} S^{-1} C^T + \Delta_\theta^{-1} \Psi_f(t) \Gamma_f \right] (C \hat{z} - y)
\]
\[
+ \frac{\partial \Phi}{\partial s}(s, \Phi^c(z)) \dot{s}(t),
\] (8)
where $S, C, \Delta_\theta$, and $\Lambda^+(s, \Phi^c(z))$ are given above, $\Gamma \in \mathbb{R}^{1 \times k}$ is $\hat{z} = [\hat{z}^1, \hat{z}^2, \ldots, \hat{z}^k]^T \in \mathbb{R}^n$ with $z^1 \in \mathbb{R}^{n_1}, k = 1, \ldots, q$; $u$ and $y$ are, respectively, the input and the output of system (5).

For some chosen symmetric positive-definite matrix $\Gamma \in \mathbb{R}^{1 \times k}$, then the estimation $\hat{\vartheta}_f$ of $\vartheta_f$ in the system (8) is generated as follows:
\[
\dot{\hat{\vartheta}}_f = \Delta_\theta [A - \Delta_\theta^{-1} S^{-1} C^T C] [\Delta_\theta^{-1} \Psi_f(t)
\]
\[
+ \Delta_\theta \frac{\partial \Phi}{\partial s}(s, \Phi^c(z)) \dot{s}(t) ,
\] (9)
\[
\dot{\hat{z}} = \Delta_\theta^{-1} \Gamma \vartheta_f \Gamma^T (C \hat{z} - y)
\] (10)

A persistent excitation condition is required to ensure the sensitivity of the observer to the possible faults, otherwise the sensitivity may still hold, but not guaranteed.

(A5) Let $\Gamma \in \mathbb{R}^{n \times n}$ be a matrix of signals generated by the ordinary differential equation (ODE) system
\[
\dot{\Gamma} = \theta [A - S^{-1} C^T C] \Gamma + \Delta_\theta \frac{\partial \Phi}{\partial x}(s, \Phi(z)) \Psi_f(t),
\]
Assume that $\Delta_\theta \frac{\partial \Phi}{\partial x}(s, \Phi(z)) \Psi_f(t)$ is persistently exciting, so that there exist positive constants $\alpha, \beta, T$ and some bounded symmetric positive-definite matrix $\Sigma(t) \in \mathbb{R}^{n_1 \times n_1}$ such that, for all $t$, the following inequality holds:
\[
\alpha I \leq \int_{t}^{t+T} \Gamma^T(t) \Psi_f(t) \Sigma(t) \Psi_f^T(t) \Gamma^T(t) d\tau \leq \beta I.
\]
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Theorem 3.2. Assume that system (5) satisfies assumptions (A1)-(A5) and for constant ϑ_f, the ODE system
\[ \dot{\hat{z}} = A\hat{z} + \varphi(s, u, \hat{z}, t)\dot{\varphi}_{f} + \left[ \Delta_{\theta}^{-1}S^{-1}C^{T}\Delta_{\theta}^{-1}\Psi f \right] C\hat{z} - y \]
\[ + \Delta_{\theta}\frac{\partial\Phi(s, \Phi(\hat{z}))}{\partial x}(t), \]
\[ Y = \Delta_{\theta}[A - \Delta_{\theta}^{-1}S^{-1}C^{T}]C\Delta_{\theta}^{-1}\Psi f \]
\[ + \Delta_{\theta}\frac{\partial\Phi(s, \Phi(\hat{z}))}{\partial x}(t)\Delta_{\theta}^{-1}, \]
\[ \dot{\varphi}_{f} = \Delta_{\theta}^{-1}Y^{T}C^{T}[C\hat{z} - y] \]
is a global adaptive exponential observer for system (5), i.e., for any initial conditions \( z(t_0), \hat{z}(t_0), \varphi_f(t_0) \) and \( \dot{\varphi}_f(t_0) \), the errors \( \tilde{z}(t) - z(t) \) and \( \tilde{\varphi}_f(t) - \varphi_f(t) \) tend to zero when \( t \to \infty \).

Proof: Let \( Y' = Y_{f}\Delta_{\theta} \), and note that \( \theta\Delta_{\theta}^{-1}A\Delta_{\theta} = A \), \( \theta C^{T}C\Delta_{\theta} = C^{T}C \). From (9), we can obtain that
\[ \frac{d}{dt}(Y_{f}\Delta_{\theta}) = \theta(A - \Delta_{\theta}^{-1}S^{-1}C^{T}) (Y_{f}\Delta_{\theta}) + \Delta_{\theta}\frac{\partial\Phi(s, \Phi(\hat{z}))}{\partial x}(t)\Psi f. \]
It can be seen that from assumption of (A5) \( Y_{f} \) generated from (9) is bounded.

Substitute (10) into (8) then we get
\[ \dot{\hat{z}} = A\hat{z} + \varphi(s, \hat{z}) + \varphi(s, u, \hat{z}, t)\dot{\varphi}_{f} + \left[ \Delta_{\theta}^{-1}S^{-1}C^{T}\Delta_{\theta}^{-1}\Psi f \right] C\hat{z} - y + \Delta_{\theta}^{-1}Y_{f}\Delta_{\theta}\dot{\varphi}_{f} \]
Let \( \tilde{z}(t) = z(t) - \hat{z}(t) \), \( \tilde{\varphi}_f(t) = \dot{\varphi}_f(t) - \varphi_f(t) \), and notice that \( \dot{\varphi}_f(t) = 0 \), then
\[ \tilde{z} = (A - \Delta_{\theta}^{-1}S^{-1}C^{T}) \tilde{z} + [\varphi(s, \hat{z}) - \varphi(s, z)] \]
\[ + \left[ \Delta_{\theta}\frac{\partial\Phi(s, \Phi(\hat{z}))}{\partial x}(t) - \Delta_{\theta}\frac{\partial\Phi(s, \Phi(\hat{z}))}{\partial x}(t)\right] \]
\[ + \left[ \Delta_{\theta}^{-1}Y_{f}\Delta_{\theta}\dot{\varphi}_{f} - \varphi(s, u, \hat{z}, t)\dot{\varphi}_{f} + \Delta_{\theta}^{-1}Y_{f}\Delta_{\theta}\dot{\varphi}_{f} \right] \]
Let us definite the combination of estimation errors \( \tilde{z} \) and \( \tilde{\varphi}_f \)
\[ \eta = \tilde{z} - \Delta_{\theta}^{-1}Y_{f}\Delta_{\theta}\dot{\varphi}_{f} \]
then we get, after some algebraic manipulation
\[ \eta = \tilde{z} - \Delta_{\theta}^{-1}Y_{f}\Delta_{\theta}\dot{\varphi}_{f} - \Delta_{\theta}^{-1}Y_{f}\Delta_{\theta}\dot{\varphi}_{f} \]
\[ = (A - \Delta_{\theta}^{-1}S^{-1}C^{T}C)\eta + \frac{\partial\Phi(s, \Phi(\hat{z}))}{\partial x}(t)\Psi f + \left[ \Delta_{\theta}^{-1}S^{-1}C^{T}\Delta_{\theta}^{-1}\Psi f \right] C\hat{z} - y \]
\[ + \Delta_{\theta}\frac{\partial\Phi(s, \Phi(\hat{z}))}{\partial x}(t)\Psi f + \left[ \Delta_{\theta}^{-1}Y_{f}\Delta_{\theta}\dot{\varphi}_{f} - \varphi(s, u, \hat{z}, t)\dot{\varphi}_{f} + \Delta_{\theta}^{-1}Y_{f}\Delta_{\theta}\dot{\varphi}_{f} \right] \]
\[ \eta = (A - \Delta_{\theta}^{-1}S^{-1}C^{T}C)\eta + \left[ (A - \Delta_{\theta}^{-1}S^{-1}C^{T}C)\Delta_{\theta}^{-1}Y_{f}\Delta_{\theta} + \frac{\partial\Phi(s, \Phi(\hat{z}))}{\partial x}(t)\right] \Psi f \]
\[ + \left[ \Delta_{\theta}^{-1}Y_{f}\Delta_{\theta}\dot{\varphi}_{f} - \varphi(s, u, \hat{z}, t)\dot{\varphi}_{f} + \Delta_{\theta}^{-1}Y_{f}\Delta_{\theta}\dot{\varphi}_{f} \right] \Psi f + \left[ \Delta_{\theta}\frac{\partial\Phi(s, \Phi(\hat{z}))}{\partial x}(t)\right] \Psi f \]
\[ + \left[ \Delta_{\theta}\frac{\partial\Phi(s, \Phi(\hat{z}))}{\partial x}(t)\right] \Psi f + \left[ \Delta_{\theta}^{-1}Y_{f}\Delta_{\theta}\dot{\varphi}_{f} - \varphi(s, u, \hat{z}, t)\dot{\varphi}_{f} + \Delta_{\theta}^{-1}Y_{f}\Delta_{\theta}\dot{\varphi}_{f} \right] \Psi f \]
\[ \eta = (A - \Delta_{\theta}^{-1}S^{-1}C^{T}C)\eta + \left[ (A - \Delta_{\theta}^{-1}S^{-1}C^{T}C)\Delta_{\theta}^{-1}Y_{f}\Delta_{\theta} + \frac{\partial\Phi(s, \Phi(\hat{z}))}{\partial x}(t)\right] \Psi f \]
\[ + \left[ \Delta_{\theta}^{-1}Y_{f}\Delta_{\theta}\dot{\varphi}_{f} - \varphi(s, u, \hat{z}, t)\dot{\varphi}_{f} + \Delta_{\theta}^{-1}Y_{f}\Delta_{\theta}\dot{\varphi}_{f} \right] \Psi f \]
\[ + \left[ \Delta_{\theta}\frac{\partial\Phi(s, \Phi(\hat{z}))}{\partial x}(t)\right] \Psi f + \left[ \Delta_{\theta}\frac{\partial\Phi(s, \Phi(\hat{z}))}{\partial x}(t)\right] \Psi f \]
\[ \eta = (A - \Delta_{\theta}^{-1}S^{-1}C^{T}C)\eta + \Xi(s, u) \]
where \( \Xi(s, u, \hat{z}, t) \) is bounded, and by Assumption (A4), \( \|Y_{f}\| \) is bounded, then
\[ V = -\theta\tilde{\eta}^{2}S\tilde{\eta} + 2\kappa\left(\|S\|\|\tilde{\eta}\|^{2} + \|Y_{f}\|\|\tilde{\eta}\|\|\tilde{\varphi}_{f}\|\right) \]
\[ + 2\|\tilde{\varphi}_{f}\||\tilde{\varphi}_{f}\| - 2\theta\|Y_{f}\|\|\tilde{\varphi}_{f}\|^{2} \]
\[ \leq -\theta(2\kappa - \kappa\|Y_{f}\| - \|\tilde{\varphi}_{f}\|)\|\tilde{\varphi}_{f}\|^{2} \]
\[ \leq -\theta\|Y_{f}\||\tilde{\varphi}_{f}\| \leq \|\tilde{\varphi}_{f}\| \leq \lambda_{\max}(\Sigma)\|\tilde{\varphi}_{f}\| \]

where \( \lambda_{\max}(\cdot) \) and \( \lambda_{\min}(\cdot) \) represent the biggest and the smallest eigenvalue of the corresponding matrix, respectively. It can be seen that there exist a \( \theta \) large enough to tend \( V \) to satisfy that
\[ V \leq -\rho(\theta)V, \]
where \( \rho(\theta) \) is an increasing positive function in \( \theta \) when \( \theta \) satisfies that \( \theta > \max\{\kappa\lambda_{\max}(S) + 1\}, 2\kappa + (\kappa + \lambda_{\max}(S)^{-1})\|Y_{f}\| \).
This implies that \( \tilde{\eta} \) and \( \tilde{\varphi}_{f} \) converge exponentially to zero. And so does \( \tilde{z} = \eta + \Delta_{\theta}^{-1}Y_{f}\Delta_{\theta}\dot{\varphi}_{f} \). The exponential converge of the estimation errors \( \tilde{z} \) and \( \tilde{\varphi}_{f} \) are thus proved.

Now, we shall transform the form of the observer (5) into the one in the original coordinates in the following theorem, which has an explicit form and solve the fault diagnosis problem for the nonlinear uncertain system (1) in the end.
**Theorem 3.3.** Assume that system (5) satisfies assumptions (A1)-(A5) and for constant \( \psi_f \), the ODE system
\[
\dot{x} = F(s, \dot{x}) + G(u, s, \dot{x}) + \Psi_f(t) \dot{\theta}_f + \left( \frac{\partial \Phi}{\partial s}(s, \dot{x}) \right)^+ \\
\times \left[ \Delta_0^{-1} S^{-1} C^T \Gamma C + \Delta_0^{-1} \hat{\gamma}_f \Gamma T_c^T C \right] \dot{x} - x,
\]
\[
\dot{\theta}_f = \Delta_0^{-1} \hat{\gamma}_f \Gamma T_c^T C (\dot{x} - y),
\]
(13)
\[
\dot{y} = \Delta_0^{-1} \hat{\gamma}_f \Gamma T_c^T C (\dot{x} - y)
\]
is an exponential adaptive observer for system (1), i.e., for any initial conditions \( x(t_0), \dot{x}(t_0), \dot{\theta}_f(t_0) \) and \( \forall \theta_f \in \mathbb{R}^l \), the errors \( \dot{x}(t) - x(t) \) and \( \dot{\theta}_f - \theta_f \) tend to zero when \( t \to \infty \).

**Proof:** It suffices to show that system (8) can be written in the original coordinates under form (13). Indeed, let \( \tilde{z} \in \mathbb{R}^{n+q} \) such that
\[
\Phi(s, \dot{x}) = \dot{z}.
\]
Differentiating each term of the above equality with respect to time gives
\[
\frac{\partial \Phi}{\partial s}(s, \dot{x}) \dot{s} + \frac{\partial \Phi}{\partial x}(s, \dot{x}) \dot{x} = \dot{z}.
\]
Thus,
\[
\dot{x} = \left( \frac{\partial \Phi}{\partial s}(s, \dot{x}) \right)^+ \left( \dot{z} - \frac{\partial \Phi}{\partial x}(s, \dot{x}) \dot{x} (t) \right)
\]
\[
= \left( \frac{\partial \Phi}{\partial s}(s, \dot{x}) \right)^+ \left[ A \dot{z} + \frac{\partial \Phi}{\partial x}(s, \dot{x}) \Lambda^+ - \Lambda \Lambda^+ \right] A \dot{z}
\]
\[
+ \frac{\partial \Phi}{\partial x}(s, \dot{x}) (G(s, u, x) + \Psi_f \dot{\theta})
\]
\[
+ \left[ \Delta_0^{-1} S^{-1} C^T + \Delta_0^{-1} \hat{\gamma}_f \Gamma T_c^T C \right] (C \dot{z} - y)
\]
\[
= F(s, \dot{x}) + G(u, s, \dot{x}) + \Psi_f \dot{\theta}_f + \left( \frac{\partial \Phi}{\partial x}(s, \dot{x}) \right)^+ \\
\times \left[ \Delta_0^{-1} S^{-1} C^T + \Delta_0^{-1} \hat{\gamma}_f \Gamma T_c^T C \right] (C \dot{x} - x).
\]
This ends the proof of the theorem.

**IV. EXAMPLE AND SIMULATION**

In this section, we will illustrate the proposed method for fault diagnosis through the following dynamical system:
\[
\dot{x}_1 = (a - x_1) x_3 + u_1 + \Psi_1 \dot{\theta}_1,
\]
\[
\dot{x}_2 = x_1 x_3 - x_2 + u_2 + \Psi_2 \dot{\theta}_2,
\]
\[
\dot{x}_3 = -k x_3,
\]
\[
y = (x_1 x_2)^T,
\]
(14)
where \( x = (x_1, x_2, x_3)^T \in \mathbb{R}^3 \), \( a > 0, k > 0 \) are constant real parameters, \( u = (u_1, u_2)^T \in \mathbb{R}^2 \) and \( y = (x_1 x_2)^T \), respectively, denote the measured inputs and outputs. It is easy to see that system (14) is under form (1) with \( q = 3 \) and \( x = (x_1 x_2)^T \), \( x^* = x_1 \). The matrix \( F_1 \) is \( F_1(x_1) = (a - x_1 x_1)^T \).

\[
F(x) = \begin{bmatrix}
0 & 0 & a - x_1 \\
0 & 0 & x_1 \\
0 & 0 & 0
\end{bmatrix},
\]
\[
G(u, s, x) = \begin{bmatrix}
u_1 \\
-u_2 + u_2 \\
-k x_3^T
\end{bmatrix},
\]
\[
z = \Phi(s, x) = \Lambda(s, x) = \begin{bmatrix}
x_1 \\
x_2 \\
x_3
\end{bmatrix} = \begin{bmatrix}
x_1 \\
x_2 \\
(a - x_1) x_3
\end{bmatrix},
\]
\[
\frac{\partial \Phi}{\partial x} = \begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
-a - x_1 & 0 & x_1
\end{bmatrix},
\]
\[
\left( \frac{\partial \Phi}{\partial x} \right)^+ = \begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & -\frac{1}{a} & \frac{1}{x_1}
\end{bmatrix},
\]

The initial values are \( x(0) = (1, 0, 0)^T \), \( \dot{x}(0) = (0, 0, 0)^T \), \( \dot{\psi}(0) = (0.5, 0.5)^T \). And the adaptive observer parameters are given as follows
\[
\theta = 8, \quad \Gamma = \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}, \quad S = \begin{bmatrix}
1 & 0 & -1 & 0 \\
0 & 1 & 0 & -1 \\
-1 & 0 & 2 & 0 \\
0 & -1 & 0 & 2
\end{bmatrix}.
\]

We choose the simulated faults as periodical effects. Square impulse signals \( \Psi_1 \) and \( \Psi_2 \) are used in the simulation.

\[
\Psi_1(t) = \begin{cases}
1, & \text{for } 0.5k \leq t < 0.5k + 0.25, \\
0, & \text{for } 0.5k + 0.25 \leq t < (k + 1)0.5,
\end{cases}
\]
\[
\Psi_2(t) = \begin{cases}
1, & \text{for } k \leq t < k + 0.5, \\
0, & \text{for } k + 0.5 \leq t < k + 1,
\end{cases}
\]

We will detect the presence of the substances \( \hat{\theta}_1 \Psi_1, \hat{\theta}_2 \Psi_2, \hat{\theta}_3 \Psi_3 \) and \( \hat{\theta}_4 \Psi_4 \) through the estimations of \( \hat{\theta}_1, \hat{\theta}_2, \hat{\theta}_3 \) and \( \hat{\theta}_4 \).

In the simulation example, initially \( \hat{\theta}_1 = 1.5, \hat{\theta}_2 = 1 \). At \( t = 10 \), \( \hat{\theta}_1 \) and \( \hat{\theta}_2 \) jump to 1.25, and then at \( t = 15 \), \( \hat{\theta}_1 \) jumps to 1.5; at \( t = 20 \), \( \hat{\theta}_2 \) jumps to 1.5 jumps to 1. The parameter estimations given by the adaptive observer are illustrated in Fig.2. It can be observed that, for each parameter change, the convergence of the parameter estimation errors are reestablished after a transient less than 5s. In order to monitor the persistent excitation, the biggest eigenvalue and smallest eigenvalue of the matrix \( \Gamma T_c^T(t) \Sigma(t) C(t) T_c^T(t) \) are plotted in Fig. 3., it can be seen that the persistent excitation condition is satisfied.

![Fig. 1. State estimation errors: \( \hat{x}_1(t), \hat{x}_2(t), \hat{x}_3(t) \)](image1)

![Fig. 2. Parameter estimations: \( \hat{\theta}_1(t) \) (solid), \( \hat{\theta}_2(t) \) (dash).](image2)
An adaptive fault diagnosis observer has been synthesized for a class of nonlinear MIMO uncertain systems. The gain of this observer does not require the resolution of any equation and is explicitly given. Its tuning is achieved through the choice of a single constant parameter. The proposed fault diagnosis method is designed in constructive manner though a novel nonlinear high-gain adaptive observer, without resort to any linearization. The exponential convergence of the estimation errors and parameters related to possible faults is guaranteed by tuning the gain of the adaptive observer. A numerical example has been presented to illustrate the performance of the proposed fault diagnosis observer.

REFERENCES


