Cooperative Cognitive Radio for Multiple Primary and Secondary Users

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Abstract

We propose a distributed spectrum access algorithm for cognitive radio relay networks with multiple primary users (PU) and multiple secondary users (SU). The key idea behind the proposed algorithm is that the PUs negotiate with the SUs on both the amount of monetary compensation, and the amount of time the SUs are either (i) allowed spectrum access, or (ii) cooperatively relaying the PU’s data, such that both the PUs’ and the SUs’ minimum rate requirement are satisfied. The proposed algorithm is shown to be flexible in prioritizing either the primary or the secondary users. We prove that the proposed algorithm will result in the best possible stable matching and is weak Pareto optimal. Numerical analysis also reveal that the distributed algorithm can achieve a performance comparable to an optimal centralized solution, but with significantly less overhead and complexity.

Index Terms

Cognitive radio, stable matchings, cooperative relaying, overlay model.

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I. Introduction

Cognitive radio has been proposed as a promising technology to improve the spectral efficiency of wireless networks. This is achieved by allowing unlicensed secondary users (SU) to coexist with licensed primary users (PU) in the same spectrum. This coexistence is facilitated by spectrum access techniques, such as those involving an agreement between the PUs and SUs on an acceptable spectrum access strategy. The key idea is that the PUs are motivated to lease spectrum bands to the SUs in exchange for some form of compensation.

Monetary compensation have been well studied (see e.g., [1–8]), with the predominant approach for spectrum access and performance analysis involving the use of tools from game theory. For these monetary payment schemes, the PUs are assumed to have sufficient spectrum for leasing to the SUs, such that their own performance requirements are not affected. In practice, however, the PUs may desire higher data rates than what its current spectrum can provide.

To allow for higher data rates, the use of cooperative relaying has emerged as a powerful technique due to its ability to exploit user diversity and provide high reliability and capacity in wireless networks [9]. This is achieved by the use of intermediate relay nodes to aid transmission between the source and destination nodes. The use of cooperative relaying is particularly advantageous when the direct link between the source and destination is weak, due to, for example, high shadowing.

In this paper, we consider a model where the SUs act as cooperative relays to assist the PUs’ transmission in exchange for both spectrum access and monetary compensation, and thus the SUs are effectively providing both a monetary and performance compensation to the PUs. When only performance compensation is considered, this is commonly referred to as the overlay model, and various schemes have been proposed which have focused on this model [10–12]. However, these schemes considered the scenario where the increase in PU’s performance does not necessarily translate into a satisfactory performance for the SUs, and in some cases, the SUs have limited spectrum access opportunities if the PUs have regular data to transmit [10]. This issue was addressed in [13, 14], where a scheme was proposed which increased the PU’s performance while simultaneously satisfying the SU’s requirements. However, these papers [10–14] considered the simplified scenario with only one PU, and did not consider monetary compensation.
In this paper, we propose a spectrum access strategy for a general cognitive radio relay network with multiple PUs and multiple SUs under the overlay model, which guarantees a minimum rate requirement for all matched PUs and SUs. In [15], multiple PUs and SUs were also considered, but for a multiple-access PU and SU network. In contrast, we consider an ad-hoc PU and SU network where each user comprises of a transmitter-receiver pair. We define a PU and SU as matched if the SU cooperatively relays the PU’s data, in exchange for spectrum access and monetary compensation. Determining an appropriate spectrum access strategy is not trivial, due to the complex interactions and varying performance requirements of the multiple PUs and multiple SUs.

We note that current game theory techniques in cognitive radio literature focus predominantly on a framework which facilitates a competitive strategy among the users of the same type, i.e. either the PUs or the SUs compete for access to the spectrum resources [16]. In contrast, we consider a framework which facilitates a joint competitive strategy between the PUs and the SUs. This is achieved by utilizing concepts from auction theory [17] and matching theory [18], [19], which provides a convenient framework for algorithm development and performance analysis.

In particular, we propose a distributed matching algorithm which determines the matched pairings between PUs and SUs, such that the SU will provide monetary compensation, and relay its paired PU’s data in exchange for spectrum access. The key idea behind the algorithm1 is that the PUs negotiate with the SUs on the amount of monetary compensation, in addition to the time the SUs are either (i) allowed access to the spectrum, or (ii) cooperatively relaying the PU’s data, such that both the PUs’ and the SUs’ minimum rate requirement are satisfied.

Again by using concepts from auction theory and matching theory, we then analyze the performance of the proposed algorithm, showing that it results in the best possible stable matching, and is weak Pareto optimal. We introduce a utility function, which incorporates both the rate and monetary factors. We demonstrate through numerical analysis that the algorithm can achieve utilities (i) comparable to the utilities achieved by an optimal centralized algorithm, and (ii) significantly greater than the utilities achieved by a random matching algorithm, while also being able to achieve a high number of matchings with low overhead and complexity. When only the rate is important, and monetary compensation is not a priority,

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1Our algorithm can be viewed as a variant of a dynamic multi-item auction [17]. This auction is considered as a distributed game with imperfect information [17].
we show that our algorithm is flexible in terms of prioritizing either the PUs or SUs, by a simple manipulation of global parameter values. This is in contrast to the conference version of this paper [20], which did not consider monetary compensation in the utility function, and thus design flexibility was not achievable. Finally, we show that the PUs which utilize the SUs for cooperative relaying achieves a rate greater than what it would achieve without cooperative relaying, i.e., direct transmission, and thus motivates their participation in the proposed algorithm.

This paper is organized as follows. In Section II, we first describe our system model. We then formulate the optimization problem we are trying to solve in Section III, and present a distributed solution to this problem in Section IV. Finally, we analyze the performance and the implementation aspects of our proposed algorithm in Section V. For convenience, Table I provides a description of the parameter values we will be utilizing in this paper.

II. SYSTEM MODEL

We consider an overlay cognitive radio wireless network, comprising of \( L_{PU} \) PU transmitter \( \{PT_i\}_{i=1}^{L_{PU}} \)–PU receiver \( \{PR_i\}_{i=1}^{L_{PU}} \) pairs, with the \( \ell \)th pair having a rate requirement of \( R_{PU_{\ell,req}} \), and with each pair occupying a unique spectrum band of constant size. In the same network, there are \( L_{SU} \) SU transmitter \( \{ST_i\}_{i=1}^{L_{SU}} \)–SU receiver \( \{SR_i\}_{i=1}^{L_{SU}} \) pairs, with the \( q \)th pair having a rate requirement of \( R_{SU_{q,req}} \), and seeking to obtain access to one spectrum band occupied by a \((PT, PR)\) pair. We assume that there are \( T \) time-slots per transmission frame, and each \((ST, SR)\) pair has access to a monetary value \( C \).

Each PT attempts to grant spectrum access to a unique \((ST, SR)\) pair, as determined by the various matching algorithms, described in Sections III and IV, in exchange for (i) the ST cooperatively relaying the PT’s data to the corresponding PR, and (ii) monetary compensation. In particular, without loss of generality (w.l.o.g), let us consider \((PT_\ell, PR_\ell)\), whose transmission is relayed by \( ST_q \) during a fraction \( \beta_{\ell,q} \) \((0 \leq \beta_{\ell,q} \leq 1)\) of \( T \), whilst also receiving a fraction \( \zeta_{\ell,q} \) \((0 \leq \zeta_{\ell,q} \leq 1)\) of \( C \) from \( ST_q \), as depicted in Fig. 1. We will refer to \( \zeta_{\ell,q} \) and \( \beta_{\ell,q} \) as the price and time-slot allocation numbers respectively, whose exact values will be determined by the matching algorithms described in Sections III and IV.

During the cooperative relaying stage in the initial \( \beta_{\ell,q}T \) time-slots, a fraction \( \tau_{\ell,q} \) \((0 < \tau_{\ell,q} < 1)\) is first allocated for \( PT_\ell \) to broadcast its signal to \( ST_q \) and \( PR_\ell \), thus occurring in the first \( \beta_{\ell,q}\tau_{\ell,q}T \) time-slots. In the subsequent \( \beta_{\ell,q}(1 - \tau_{\ell,q})T \) time-slots, \( ST_q \) cooperatively
relays the signal from PT to PR, PR then applies maximum ratio combining (MRC) to the signal received from PT in the first $\beta_{\ell,q}\tau_{\ell,q}T$ time-slots, and the signal received from ST in the subsequent $\beta_{\ell,q}(1 - \tau_{\ell,q})T$ time-slots. After this cooperative relaying stage, PT ceases transmission, allowing ST to transmit to SR over the spectrum occupied by (PT, PR) in the final $(1 - \beta_{\ell,q})T$ time-slots.

In this paper, we consider the amplify-and-forward (AF) relaying protocol, due to its simple and practical operation, and thus set $\tau_{\ell,q} = \frac{1}{2}$. We note, however, that the proposed algorithm is applicable to any relaying protocol, such as the decode-and-forward or compress-and-forward protocol. The AF gain at ST is chosen such that its instantaneous transmission power is constrained to $P_{SU,q}$.

A. Utility functions

To evaluate the performance of each (PT, PR) and (ST, SR) pair, we consider the utility function, which comprises of both rate and monetary factors. Specifically, for (PT, PR), the achievable instantaneous rate is given by [9]

$$R_{PU_{\ell,q}}(\beta_{\ell,q}) = \frac{\beta_{\ell,q}T}{2} \log_2 \left( 1 + \frac{\gamma_{PT}\vert h_{PT,\ell,PR}\vert^2 \left\{ \frac{\gamma_{ST}\vert h_{ST,\ell,ST}\vert^2 \vert h_{ST,\ell,PR}\vert^2}{\frac{\delta_{PT,\ell,ST}}{d_{PT,\ell,ST}} + \frac{\delta_{ST,\ell,PR}}{d_{ST,\ell,PR}} + 1} \right\}}{\frac{\gamma_{PT}\vert h_{PT,\ell,ST}\vert^2 \vert h_{ST,\ell,PR}\vert^2}{\frac{\delta_{PT,\ell,ST}}{d_{PT,\ell,ST}} + \frac{\delta_{ST,\ell,PR}}{d_{ST,\ell,PR}} + 1} + 1} \right)$$

(1)

where $\gamma_{PT} = \frac{P_{PT}}{\sigma^2}$ is the transmit signal-to-noise-ratio (SNR) for PT, $\gamma_{ST} = \frac{P_{ST}}{\sigma^2}$ is the transmit SNR for ST, $\sigma^2$ is the noise variance, $P_{PT,\ell}$ is the transmission power at PT, $h_{PT,\ell,ST}$ and $d_{PT,\ell,ST}$ are respectively the channel and distance from PT to ST, $h_{ST,\ell,PR}$ and $d_{ST,\ell,PR}$ are respectively the channel and distance from ST to PR, and $\alpha$ is the path loss exponent.

To allow for both the rate and monetary value to be combined into one utility function, we introduce a variable $\bar{c} \in \mathbb{R}^+$, with unit defined as: rate per unit monetary value. We can thus express the utility for (PT, PR) as

$$U_{PU_{\ell,q}}(\beta_{\ell,q}, \zeta_{\ell,q}) = R_{PU_{\ell,q}}(\beta_{\ell,q}) + \bar{c}\zeta_{\ell,q}C.$$  

(2)

For (ST, SR), the achievable instantaneous rate is given by

$$R_{SU_{\ell,q}}(\beta_{\ell,q}) = (1 - \beta_{\ell,q})T \log_2 \left( 1 + \frac{\gamma_{ST}\vert h_{ST,\ell,SR}\vert^2}{\frac{\delta_{ST,\ell,SR}}{d_{ST,\ell,SR}} + 1} \right)$$

(3)

where $h_{ST,\ell,SR}$ is the channel coefficient from ST to SR in the spectrum band occupied by (PT, PR) and $d_{ST,\ell,SR}$ is the distance from ST to SR. The utility for (ST, SR) is
thus given by
\[ U_{SU_{q,\ell}}(\beta_{\ell,q}, \zeta_{\ell,q}) = R_{SU_{q,\ell}}(\beta_{\ell,q}) - \bar{k}_q \zeta_{\ell,q} C \]
(4)

where \(\bar{k} \in \mathbb{R}^+\) is a variable which is defined to allow for both the rate and monetary value to be combined into one utility function, with unit defined as: rate per unit monetary value. We consider a general case where \(\bar{c}\) and \(\bar{k}\) have the same units, but their values may be different. This is to allow for flexibility in the algorithm design, as will be discussed in Section V.

III. Problem Formulation

In this section, we describe the optimization problem we aim to address. To proceed, we introduce some notation. We first define the primary and secondary user sets respectively as \(P = \{PU_\ell = (PT_\ell, PR_\ell)\}_{\ell=1}^{L_{PU}}\) and \(S = \{SU_q = (ST_q, SR_q)\}_{q=1}^{L_{SU}}\). Moreover, we define a \(L_{PU} \times L_{SU}\) matching matrix \(M\), with \(m_{\ell,q} = 1\) if \(PU_\ell\) is matched with \(SU_q\), and \(m_{\ell,q} = 0\) otherwise, where the notation \(x_{i,j}\) denotes the \((i,j)\)th entry of matrix \(X\). From this matrix, we introduce an injective function \(\mu : (P \cup S) \rightarrow (P \cup S \cup \{\emptyset\})\), such that (a) \(\mu(PU_\ell) \in (S \cup \{\emptyset\})\), (b) \(\mu(SU_q) \in (P \cup \{\emptyset\})\), and (c) \(\mu(SU_q) = PU_\ell\) and \(\mu(PU_\ell) = SU_q\) if \(m_{\ell,q} = 1\), for \(\ell = 1, \ldots, L_{PU}\) and \(q = 1, \ldots, L_{SU}\), (d) \(\mu(SU_q) = \emptyset\) if \(m_{\ell,q} = 0\), for \(\ell = 1, \ldots, L_{PU}\), and (e) \(\mu(PU_\ell) = \emptyset\) if \(m_{\ell,q} = 0\), for \(q = 1, \ldots, L_{SU}\).

We also define an \(L_{PU} \times L_{SU}\) price allocation matrix \(G\) with \(g_{i,j} = \zeta_{i,j}\), and an \(L_{PU} \times L_{SU}\) time-slot allocation matrix \(B\) with \(b_{i,j} = \beta_{i,j}\), and where \(g_{i,j} = b_{i,j} = 0\) if \(m_{i,j} = 0\). We denote the price and time-slot allocation matrices with continuous elements as \(G^{\text{cont}}\) and \(B^{\text{cont}}\) respectively. Mathematically, this implies that the elements of \(G^{\text{cont}}\) and \(B^{\text{cont}}\) respectively take values from the sets \(\{g_{i,j}^{\text{cont}} = \zeta_{i,j} \in \mathbb{R} : 0 \leq \zeta_{i,j} \leq 1\}\) and \(\{b_{i,j}^{\text{cont}} = \beta_{i,j} \in \mathbb{R} : 0 \leq \beta_{i,j} \leq 1\}\).

Now the main goal for each primary and secondary user is to ensure their minimum rate requirements are satisfied. When this is achieved, the secondary goal is to maximize their utility functions. Note that the secondary goals for the primary and secondary users cannot be achieved simultaneously, as a higher utility for the primary user will result in a lower utility for the matched secondary user, and vice-versa. It is natural, and often considered in literature (see e.g. [21]), to give preference to the primary users, i.e. focus on maximizing the primary users’ utility. As such, we now present the optimization problem:
\[
\{ \mathbf{M}^{\text{opt}}, \mathbf{B}^{\text{opt}}, \mathbf{G}^{\text{opt}} \} = \arg \max_{\{ \mathbf{M}, \mathbf{B}, \mathbf{G} \}} \sum_{\ell=1}^{L_{\text{PU}}} \sum_{q=1}^{L_{\text{SU}}} m_{\ell,q} U_{\ell,q}(\zeta_{\ell,q}, \beta_{\ell,q})
\]

s.t.:

\begin{align*}
(a) & \quad R_{\ell, p_{\mu - \text{ind}(\mu)}}(\beta_{\ell, p_{\mu - \text{ind}(\mu)}}) \geq R_{\ell, \text{req}}, \forall \ell \in \{1, 2, \ldots, L_{\text{PU}}\} \\
(b) & \quad R_{\mu, q_{\mu - \text{ind}(q)}}(\beta_{\mu, q_{\mu - \text{ind}(q)}}) \geq R_{\mu, \text{req}}, \forall q \in \{1, 2, \ldots, L_{\text{SU}}\} \\
(c) & \quad R_{\mu, q_{\mu - \text{ind}(q)}}(\beta_{\mu, q_{\mu - \text{ind}(q)}}) - \zeta_{\mu - \text{ind}(q), \bar{k}} \geq 0, \forall q \in \{1, 2, \ldots, L_{\text{SU}}\} \\
(d) & \quad \sum_{\ell=1}^{L_{\text{PU}}} m_{\ell,q} \leq 1, \forall q \in \{1, 2, \ldots, L_{\text{SU}}\} \\
(e) & \quad \sum_{q=1}^{L_{\text{SU}}} m_{\ell,q} \leq 1, \forall \ell \in \{1, 2, \ldots, L_{\text{PU}}\} \\
(f) & \quad 0 \leq \zeta_{\ell,q} \leq 1, \forall q \in \{1, 2, \ldots, L_{\text{SU}}\}, \forall \ell \in \{1, 2, \ldots, L_{\text{PU}}\} \\
(g) & \quad 0 \leq \beta_{\ell,q} \leq 1, \forall q \in \{1, 2, \ldots, L_{\text{SU}}\}, \forall \ell \in \{1, 2, \ldots, L_{\text{PU}}\}
\end{align*}

where \( \mu - \text{ind}(p_\ell) = q \) if \( \mu(p_\ell) = s_q \), and \( \mu - \text{ind}(s_q) = \ell \) if \( \mu(s_q) = p_\ell \).

Conditions (a) and (b) ensure that the minimum required rate for the PUs and SUs are satisfied\(^2\), respectively. Condition (c) ensures that the SUs always receive a positive utility. Conditions (d) and (e) respectively ensure that each PU will only be matched with one SU, and vice-versa. Finally, conditions (f) and (g) respectively ensure that the price and time-slot allocation values are kept within their bounds.

In practice, a centralized controller is required to solve the optimization problem in (5). However, there are three key issues regarding this approach:

- **Overhead:** The centralized controller will require the feedback on channel conditions and minimum rate requirements from each primary and secondary user. Moreover, after the optimization problem is solved, the resultant matching between PUs and SUs and price and time-slot allocation numbers will then have to be transmitted to the corresponding users. The amount of overhead required for this increases with the number of users, and can be quite high, rendering it impractical.

- **Complexity:** The optimization problem is non-linear, and requires an exhaustive search over all possible matching, price and time-slot allocation combinations. Such a problem

\(^2\)From the output of the optimization problem, it can be shown that the final time slot allocation numbers are chosen such that the achievable rate for the matched SUs is equivalent to their minimum rate requirement, and the price-time slot allocation numbers are chosen such that the utility for each matched SU is zero. We can thus present an alternate formulation accordingly, however, we leave the formulation as described to make it clear that the SU’s rate and utility requirements are satisfied.
is known to be NP-hard [5].

- **Selfish Users:** If each of the primary and secondary users are selfish, which means their goal is to always maximize their own utilities, then the outcome of the optimization problem may not be in the best interests of at least one of these users. There are also privacy issues for which a centralized approach may not be ideal.

To address these issues, we propose a distributed low-complexity algorithm which accounts for selfish users. As we will demonstrate in Section V, our algorithm can achieve a performance close to the solution of the optimization problem in (5) for practical system parameters.

## IV. PROPOSED DISTRIBUTED MATCHING ALGORITHM

In this section, we describe the proposed algorithm which determines spectrum access for each (PT, PR) and (ST, SR) pair.

### A. Received SNR Assumptions

We first describe two scenarios we will be considering in the proposed algorithm, characterized by different assumptions on the received SNR at the transmitters and receivers.

1) **Complete Received SNR:** In the first scenario, $PT_\ell$ has perfect knowledge of the instantaneous received SNRs in \( \left\{ \frac{\gamma_{PT_\ell}|h_{PT_\ell,PR_\ell}|^2}{d_{PT_\ell,PR_\ell}}, \frac{\gamma_{PT_\ell}|h_{PT_\ell,ST_q}|^2}{d_{PT_\ell,ST_q}} \right\}_{q=1}^{L_{SU}} \). Moreover, $ST_q$ has perfect knowledge of the instantaneous received SNRs in the terms \( \left\{ \frac{\gamma_{ST_q}|h_{ST_q,SR_q,\ell}|^2}{d_{ST_q,SR_q}} \right\}_{\ell=1}^{L_{PU}} \). As such, $PT_\ell$ and $ST_q$ are able to respectively calculate their instantaneous rate in (1) and (3).

2) **Partial Received SNR:** In the second scenario, $PT_\ell$ has knowledge of the average received SNRs in the term \( \left\{ \frac{\gamma_{ST_q}}{d_{ST_q,PR_\ell}} \right\}_{q=1}^{L_{SU}} \) and the instantaneous received SNRs in the terms \( \left\{ \frac{\gamma_{PT_\ell}|h_{PT_\ell,PR_\ell}|^2}{d_{PT_\ell,PR_\ell}}, \frac{\gamma_{PT_\ell}|h_{PT_\ell,ST_q}|^2}{d_{PT_\ell,ST_q}} \right\}_{q=1}^{L_{SU}} \). Moreover, $ST_q$ has perfect knowledge of the instantaneous received SNRs in the term \( \left\{ \frac{\gamma_{ST_q}|h_{ST_q,SR_q,\ell}|^2}{d_{ST_q,SR_q}} \right\}_{\ell=1}^{L_{PU}} \). As such, $PT_\ell$ is able to calculate its instantaneous conditional rate, given by the expectation of the rate in (1) with respect to \( \left\{ h_{PT_\ell,ST_q} \right\}_{q=1}^{L_{SU}} \), while $ST_q$ is able to calculate its instantaneous rate in (3).

For both complete and partial received SNR scenarios, note that each PT and ST does not have knowledge of the instantaneous received SNRs corresponding respectively to the other.

\(^3\)Selfish users are a common assumption in cognitive radio literature [22].
PTs and STs. Moreover, the instantaneous received SNRs can be obtained through standard channel estimation techniques.

B. Users Preference Lists

Each PT has a preference list of STs which can cooperatively relay the PT’s message such that it obtains a rate greater than its minimum rate requirement. In particular, the preference list for PT_\ell is given by

\[
\text{PULIST}_\ell = \{(ST_{\phi_\ell(j)}, SR_{\phi_\ell(j)})\}_{j=1}^{K_\ell}
\]

(6)

where \(\phi_\ell(\cdot)\) is a function \(\phi_\ell : \{1, \ldots, K_\ell\} \rightarrow \{1, \ldots, L_{SU}\}\) satisfying

\[
\left\{ R_{PU_\ell,\phi_\ell(q)}(\zeta_\ell,\phi_\ell(q), \beta_\ell,\phi_\ell(q)) \geq R_{PU_\ell,\text{req}} \right\}_{q=1}^{K_\ell}
\]

(7)

for the complete received SNR scenario, and the conditions

\[
\left\{ E_{h_{ST,\phi_\ell(q) \cdot P_{R_\phi_\ell(q)}}} \left[ R_{PU_\ell,\phi_\ell(q)}(\zeta_\ell,\phi_\ell(q), \beta_\ell,\phi_\ell(q)) \right] \geq R_{PU_\ell,\text{req}} \right\}_{q=1}^{K_\ell}
\]

(8)

for the partial received SNR scenario. The function \(\phi_\ell(\cdot)\) also satisfies the ordering

\[
U_{PU_\ell,\phi_\ell(1)}(\zeta_\ell,\phi_\ell(1), \beta_\ell,\phi_\ell(1)) > \ldots > U_{PU_\ell,\phi_\ell(K_\ell)}(\zeta_\ell,\phi_\ell(K_\ell), \beta_\ell,\phi_\ell(K_\ell)),
\]

implying that the first ST in the list provides the largest utility. Moreover, \(K_\ell\) is the number of (ST, SR) pairs satisfying these conditions.

Similarly, each ST has a preference list of PTs which, if it transmits in the spectrum band occupied by the (PT, PR) pair in the list, obtains a rate greater than its minimum rate requirement and a utility greater or equal to zero. In particular, the preference list for ST_q is given by

\[
\text{SULIST}_q = \{(PT_{\psi_q(\ell)}, PR_{\psi_q(\ell)})\}_{\ell=1}^{V_q}
\]

(9)

where \(\psi_q(\cdot)\) is a function \(\psi_q : \{1, \ldots, V_q\} \rightarrow \{1, \ldots, L_{PU}\}\) satisfying the conditions

\[
\left\{ R_{SU_q,\psi_q(\ell)}(\beta_q,\psi_q(\ell)) \geq R_{SU_q,\text{req}} \right\}_{\ell=1}^{V_q}
\]

(10)

and

\[
\left\{ U_{SU_q,\psi_q(\ell)}(\zeta_q,\psi_q(V_q), \beta_q,\psi_q(\ell)) \geq 0 \right\}_{\ell=1}^{V_q}
\]

(11)

with the ordering \(U_{SU_q,\psi_q(V_q)}(\zeta_q,\psi_q(V_q), \beta_q,\psi_q(V_q)) > \ldots > U_{SU_q,\psi_q(V_q)}(\zeta_q,\psi_q(V_q), \beta_q,\psi_q(V_q))\). The ordering thus implies that the first PT in the list provides the largest utility. Moreover, \(V_\ell\) is
the number of (PT, PR) pairs satisfying these conditions.

C. Proposed algorithm to determine the matching, price and time-slot allocation matrices

The key idea of the proposed algorithm is that each (PT, PR) pair which provides the highest utility, through both cooperative relaying and monetary payment. This trading will be done by negotiating on the price and time-slot allocation numbers \( \{\zeta_{\ell,q}, \beta_{\ell,q}\}_{\ell=1}^{L_{PUU}}_{q=1}^{L_{SU}} \). We say \( PT_\ell \) makes an offer of \( (\zeta_{\ell,q}, \beta_{\ell,q}) \) to \( ST_q \) to imply that \( PT_\ell \) is willing to allow \( ST_q \) to transmit, in exchange for \( ST_q \) (i) cooperatively relaying \( PT_\ell \)'s message with time slot allocation number \( \beta_{\ell,q} \) and, (ii) providing a monetary payment with price allocation number \( \zeta_{\ell,q} \).

The specific details of the main algorithm are given in Table II. Note that the main algorithm calls upon the function ‘Proposal Update Unit (PUU)’, denoted as PUU(·, ·, ·), and detailed in Table III. To summarize the main algorithm (MA), each PT will first make an offer to the ST which is first in its preference list (MA-Step 2-A). The ST will then check if the offering PT is in it’s preference list (MA-Step 2-B-1). If it is, and the ST is already matched with another PT, the ST has two choices: (a) if the offering PT can provide a better utility than the ST’s current matching, then the ST will reject its current matching in favor of the new matching (MA-Step 2-B-1-a-i), or (b) if the offering PT can not provide a better utility than the ST’s current matching, the ST will reject the PT’s offer (MA-Step 2-B-1-a-ii). If the ST is not matched, then the ST will be matched with the offering PT (MA-Step 2-B-1-b). If the offering PT is not in the ST’s preference list, the ST will reject the offering PT (MA-Step 2-B-2). The algorithm will then repeat this procedure with each PT until no more matchings are possible.

We observe that the proposed algorithm:

- Extends the classic Gale-Shapely algorithm [18] through the exchange and updating of the price and time-slot allocation numbers.
- Applies to both complete and partial received SNR scenarios through the different
construction of the preference list \( \{ \text{PULIST}_\ell \}_{\ell=1}^{K} \).

- Produces a matching, price allocation, and time-slot allocation matrix. Note that the price and time-slot allocation matrix have non-zero entries, which take values from a discrete set, in contrast to the continuous set considered in the optimization problem in (5). This is due to the update procedure, where the price and time-slot allocation numbers change according to the price and time-slot step numbers \( \delta \) and \( \epsilon \). We denote the price and time-slot allocation matrices with discrete elements corresponding to the particular \( \zeta_{\text{init}}, \beta_{\text{init}}, \delta \) and \( \epsilon \) values as \( G_{\text{disc}}(\zeta_{\text{init}}, \delta) \) and \( B_{\text{disc}}(\beta_{\text{init}}, \epsilon) \) respectively. Mathematically, the elements of \( G_{\text{disc}}(\zeta_{\text{init}}, \delta) \) and \( B_{\text{disc}}(\beta_{\text{init}}, \epsilon) \) respectively take values from the sets \( \{ g_{i,j}^{\text{disc}} = \zeta_{i,j} \in \zeta_{\text{init}} - m \delta : m = 1, \ldots, \lfloor \frac{\zeta_{\text{init}}}{\delta} \rfloor \} \) and \( \{ b_{i,j}^{\text{disc}} = \beta_{i,j} \in \beta_{\text{init}} - m \epsilon : m = 1, \ldots, \lfloor \frac{\beta_{\text{init}}}{\epsilon} \rfloor \} \).

- Reduces to the algorithm we proposed in [20], when \( c = k = 0 \). For this specific scenario, the utility function is equivalent to the rate.

V. PERFORMANCE AND IMPLEMENTATION ANALYSIS

We now analyze the performance of the proposed algorithm, and consider related implementation issues. We first present some assumptions we will be considering in the analysis. To demonstrate that the (PT, PR) pairs are motivated to participate in the proposed algorithm, we set the minimum rate requirement of each (PT, PR) pair to be the rate of the direct PT to PR link. This is given for (PT\( \ell \), PR\( \ell \)) by \( R_{\text{PT}_\ell, \text{PR}_\ell} = T \log_2 \left( 1 + \frac{\gamma_{\text{PT}_\ell, \text{PR}_\ell} |h_{\text{PT}_\ell, \text{PR}_\ell}|^2}{d_{\text{PT}_\ell, \text{PR}_\ell}} \right) \) where \( h_{\text{PT}_\ell, \text{PR}_\ell} \) and \( d_{\text{PT}_\ell, \text{PR}_\ell} \) denote respectively the channel coefficient and distance from PT\( \ell \) to PR\( \ell \). In this paper, we thus set \( R_{\text{PU}_\ell, \text{req}} = R_{\text{PT}_\ell, \text{PR}_\ell} \).

We also assume all channels experience Rayleigh fading, distributed as \( \sim \mathcal{CN}(0, 1) \), and are constant during the duration of the proposed algorithm and subsequent \( T \) transmission time-slots. Moreover, the PTs and PRs are located on opposite sides of a square of length two, and thus \( d_{\text{PT}_\ell, \text{PR}_\ell} = 2 \), while the STs and SRs are randomly located in an internal square of length one, located within the square of length two. Moreover, the minimum rate requirements for each (ST, SR) pair is given by \( R_{\text{SU}_{\ell, \text{req}}} = \{ R_{\text{SU}_{\ell_{q, \text{req}}}} \}_{q=1}^{L_{\text{SU}}} = 0.1 \) and all curves are generated based on averaging over 20,000 instances of the algorithm.

Finally, to demonstrate the benefits of the proposed algorithm, we also consider, in addition to the centralized algorithm in (5), a random matching algorithm where the (PT, PR) pairs are randomly matched with the (ST, SR) pairs, and denoted by \( \mu_{\text{rand}} \). When \( L_{\text{PU}} > L_{\text{SU}} \), \( L_{\text{PU}} - L_{\text{SU}} \) (PT, PR) pairs will be randomly omitted from the matching, and when \( L_{\text{SU}} > L_{\text{PU}} \),
\( L_{SU} - L_{PU} \) (ST, SR) pairs will be randomly omitted from the matching. Once the matchings are established, the (PT, PR) pairs will choose their price and time-slot allocation numbers such that

\[
(B^{\text{rand}}, G^{\text{rand}}) = \arg \max_{B^{\text{cont}}, G^{\text{cont}}} \mathbb{E} \left[ \sum_{\ell=1}^{L_{PU}} U_{\text{PU}_{\ell,\mu-\text{ind}}(\ell)}(\zeta_{\ell,\mu-\text{ind}}(\ell), \beta_{\ell,\mu-\text{ind}}(\ell)) \right]
\]

where the expectation is w.r.t. the channels in (1). We observe that such a random matching requires a centralized approach, and is an upper bound to a completely distributed random matching with no overhead. However, this is sufficient for comparison purposes with the proposed algorithm. Note that the centralized and random matching algorithm represent the two extremes in the amount of overhead and complexity required for any algorithm.

For all figures, unless indicated otherwise the ‘Centralized’ curves are generated using (5), the ‘Proposed (complete SNR)’ curves are generated as described in Section IV-A1, the ‘Proposed (partial SNR)’ curves are generated as described in Section IV-A2, and the ‘Random’ curves are generated by using (12).

A. Selfish Users

A common and realistic assumption in cognitive radio networks is that the primary and secondary users are selfish, (see e.g., [1]), with each user competing with each other to maximize their individual utility function. To motivate the primary and secondary users to adopt the proposed algorithm, it is desirable for the algorithm to thus take into account the selfish nature of the users. We will show this to be the case, by demonstrating that our algorithm produces a stable matching, while simultaneously addressing privacy issues.

1) Stable Matchings: To define a stable matching\(^4\), we will first define a matching which is blocked by an individual, and a matching which is blocked by an individual. To do this, we consider \( p_\ell \in P \) and \( s_\ell \in S \), where \( \mu(p_\ell) \neq s_\ell \) and \( \mu(s_\ell) \neq p_\ell \). A matching \( \mu \) is blocked by an individual \( p_\ell \) (\( s_\ell \)) if \( p_\ell \) (\( s_\ell \)) prefers not to be matched, than being matched with its current partner under \( \mu \). Mathematically, for \( p_\ell \), this implies that \( R_{\text{PU}_{\ell,\mu-\text{ind}}(p_\ell)}(\beta_{\ell,\mu-\text{ind}}(p_\ell)) < R_{\text{PU}_{\ell,\text{req}}} \), while for \( s_\ell \), this implies that \( R_{\text{SU}_{\ell,\mu-\text{ind}}(s_\ell)}(\beta_{\mu-\text{ind}}(s_\ell), q) < R_{\text{SU}_{\ell,\text{req}}} \) or \( U_{\text{SU}_{\ell,\mu-\text{ind}}(s_\ell)}(\zeta_{\mu-\text{ind}}(s_\ell), q, \beta_{\mu-\text{ind}}(s_\ell), q) < 0 \).

A matching \( \mu \) is blocked by pair \( (p_\ell, s_\ell) \) if (i) it is not blocked by individual \( p_\ell \) and \( s_\ell \), and (ii) there exists a \( \zeta_{\ell, q} \) and \( \beta_{\ell, q} \) such that \( p_\ell \) and \( s_\ell \) can both achieve a higher utility if they

\(^4\)We extend the common definition of stable matchings [18] to incorporate the price and time-slot allocation numbers.
were matched together, as opposed to their current matching under $\mu$. The latter condition, this mathematically implies that

$$U_{P\mu}(\zeta_{\mu-\text{ind}(p)}, \beta_{\mu-\text{ind}(p)}) < U_{P\mu}(\zeta_{\mu}, \beta_{\mu})$$

$$U_{S\mu}(\zeta_{\mu-\text{ind}(s_q)}, \beta_{\mu-\text{ind}(s_q)}) < U_{S\mu}(\zeta_{\mu}, \beta_{\mu}).$$

(13)

A matching $\mu$ is then defined as stable, under the price and time-slot allocation matrices $G$ and $B$, if it is not blocked by any individual or any pair. Given this definition, we present the following theorem.

**Theorem 1:** The proposed algorithm in Section IV-C produces a stable matching, under $G^{\text{disc}}(\zeta_{\text{init}}, \delta)$ and $B^{\text{disc}}(\beta_{\text{init}}, \epsilon)$.

**Proof:** See Appendix A.

**Theorem 1** is important because it states that the proposed algorithm results in a matching where any matched $(PT, PR)$ and $(ST, SR)$ pair will not both achieve a higher utility than if they were to respectively partner with any other $(ST, SR)$ and $(PT, PR)$ pair.

2) Privacy and False Information: The optimization problem in (5) requires each primary and secondary user to transmit their channel information and minimum rate requirements to a centralized controller. However, such information can be intercepted by other users, resulting in (i) privacy issues, and (ii) the potential for other users to use this information to transmit false information to the centralized controller to obtain a better spectrum allocation. In contrast, our proposed distributed algorithm doesn’t require the channel information and minimum rate requirements of each user to be known by the other users.

For the proposed algorithm, none of the users will have any incentive to provide false information in the negotiation process. This is because the use of false information to obtain a better rate, e.g., through giving false information about a higher rate requirement than the actual rate requirement, with no knowledge of other user’s channel information and minimum rate requirements, will lead to a decrease in the size of the users’ preference list. This will subsequently increase the likelihood that the minimum rate requirements are not satisfied. As the main goal for the primary and secondary users is to satisfy their minimum rate requirements, they are thus motivated to be truthful during the negotiation process in the algorithm.

---

5In general, $G$ and $A$ can have continuous elements as considered in Section III, or discrete elements as considered in Section IV. Any analysis involving the matching $\mu$ thus has to also consider the domain of the price and time-slot allocation matrices in order to be meaningful.
B. Utility Performance

We now investigate the utility performance of the proposed algorithm. To do this, we first present the following lemma:

**Lemma 1**: The utility for every \((PT, PR)\) pair in the stable matching produced by the proposed algorithm in Section IV-C is greater than or equal to the utility obtained through a stable matching produced by any algorithm, under \(G^{\text{disc}}(\zeta_{\text{init}}, \delta)\) and \(B^{\text{disc}}(\beta_{\text{init}}, \epsilon)\).

**Proof**: See Appendix B.

Lemma 1 thus indicates that our algorithm produces the best stable matching, out of every possible stable matchings, under \(G^{\text{disc}}(\zeta_{\text{init}}, \delta)\) and \(B^{\text{disc}}(\beta_{\text{init}}, \epsilon)\).

A natural question now arises as to how our algorithm performs when compared with non-stable matchings. To answer this, we first show that our algorithm produces a weak Pareto optimal matching, denoted by \(\mu_{\text{Pareto}}\). A weak Pareto optimal matching is defined as a matching where there exists at least one matched \((PT, PR)\) pair under \(\mu_{\text{Pareto}}\) which obtains a utility at least greater than any other matching, stable or non-stable. Given this definition, we present the following theorem:

**Theorem 2**: The proposed algorithm in Section IV-C is weak Pareto optimal, under \(G^{\text{disc}}(\zeta_{\text{init}}, \delta)\) and \(B^{\text{disc}}(\beta_{\text{init}}, \epsilon)\).

**Proof**: See Appendix C.

Theorem 2 indicates there exists at least one matched \((PT, PR)\) pair under the proposed algorithm that achieves a utility at least greater than the utility achieved under a non-stable centralized optimal algorithm, under \(G^{\text{disc}}(\zeta_{\text{init}}, \delta)\) and \(B^{\text{disc}}(\beta_{\text{init}}, \epsilon)\). This optimal algorithm is similar to the algorithm in (5), but under \(G^{\text{disc}}(\zeta_{\text{init}}, \delta)\) and \(B^{\text{disc}}(\beta_{\text{init}}, \epsilon)\), not \(G^{\text{cont}}(\zeta_{\text{init}}, \delta)\) and \(B^{\text{cont}}(\beta_{\text{init}}, \epsilon)\).

In fact, the proposed algorithm can achieve a utility for every matched \((PT, PR)\) pair very close to the centralized optimal algorithm in (5), even under \(G^{\text{cont}}(\zeta_{\text{init}}, \delta)\) and \(B^{\text{cont}}(\beta_{\text{init}}, \epsilon)\). This can be observed in Fig. 2, which plots the average sum-utility of all matched \((PT, PR)\) pairs vs. time-slot step number \(\epsilon\) for the proposed algorithm, the centralized algorithm in (5), and the random algorithm. Note that the average sum-utility corresponds to the sum over all utilities achieved by the matched \((PT, PR)\) pairs, averaged over the channel realizations, and given by \(U_{\text{PU; } \mu} = \sum_{\ell \in P_{\mu}} E \left[ U_{\ell, \mu - \text{ind}(\ell)}(\zeta_{\ell, \mu - \text{ind}(\ell)}, \beta_{\ell, \mu - \text{ind}(\ell)}) \right]\), where \(P_{\mu}\) corresponds to all the \((PT, PR)\) pairs matched under \(\mu\).
We first observe in Fig. 2 that for the proposed algorithm, the complete and partial received SNR scenarios achieve very similar performance, despite the different channel assumptions. We next observe that the proposed algorithm (i) achieves a sum-utility comparable with the sum-utility of the centralized algorithm for sufficiently small $\epsilon$, and (ii) performs significantly better than the random matching algorithm. For example, when the time-slot step number $\epsilon = 0.1$ and $L_{SU} = 10$, we observe that the proposed algorithm with complete and partial received SNR achieves respectively (i) $\approx 97\%$ of the sum-utility of the centralized algorithm, and (ii) $\approx 165\%$ sum-utility increase compared to the random matching algorithm.

C. Rate Performance

As observed in Section IV-C and Fig. 2, the proposed algorithm’s primary focus is on maximizing the PU’s sum-utility, while simultaneously satisfying both the PU’s and SU’s minimum rate requirement. However, some networks may require that (i) only the rate is important and monetary factors are not a consideration, and (ii) the SUs also prefer a higher rate, greater than their minimum data rate requirement. For such networks, the proposed algorithm is flexible in adapting to these design needs.

In particular, observe from (2) that increasing $\bar{c}$ has the effect of increasing the contribution of the monetary value in the $(PT, PR)$’s utility function. As observed in the Proposal Update Unit, this has the effect of the PUs choosing to offer a lower time-slot allocation number, rather than a lower price allocation number, to the SUs. This will subsequently result in a higher rate for the SUs, at the expense of a lower rate for the PUs. Thus the parameter $\bar{c}$ can be used to control the rates of the PUs and SUs, in accordance with specific system requirements. When monetary factors are not of primary concern in the system, the total monetary value per SU $C$ can thus be interpreted as ‘virtual money’ whose purpose is a design tool to provide the SUs with more negotiating power in order to achieve a higher rate.

This can be observed in Figs. 3 and 4, which respectively plots the average sum-rate of all matched $(PT, PR)$ pairs vs. $\bar{c}$, and the average sum-rate of all matched $(ST, SR)$ pairs vs. $\bar{c}$, for the complete received SNR scenario. Note that the average sum-rate of all matched $(PT, PR)$ pairs corresponds to the sum over all rates achieved by the matched $(PT, PR)$ pairs, averaged over the channel realizations, and is given by $R_{PU_{\Sigma, \mu}} = \sum_{\ell \in P_{\mu}} E \left[ R_{\ell, \mu \leftarrow \text{ind}(\ell)} (\beta_{\ell, \mu \leftarrow \text{ind}}) \right]$, where $P_{\mu}$ corresponds to all the $(PT, PR)$ pairs matched under $\mu$. A similar definition can be made for the sum-rate of the matched $(ST, SR)$ pairs, denoted as $R_{SU_{\Sigma, \mu}}$. For comparison, we also
plot the sum-rate achieved by a centralized optimal algorithm. In particular, in Fig. 3, the centralized algorithm produces a sum-rate which maximizes the total sum-rate of all \((PT, PR)\) pairs, and given by substituting \(\bar{c} = 0\) into the optimal algorithm produced from (5). In Fig. 4, the centralized algorithm produces a sum-rate which maximizes the sum-rate of all \((ST, SR)\) pairs, and formulated similar to (5), but interchanging the \((PT, PR)\) pairs with the \((ST, SR)\) pairs in the optimization equation. This results in the optimization

\[
\arg \max_{\{M,B^{\text{cont}},G^{\text{cont}}\}} \sum_{\ell=1}^{L_{PU}} \sum_{q=1}^{L_{SU}} m_{\ell,q} R_{SU,q,\ell}(\zeta_{\ell,q}, \beta_{\ell,q})
\]

subject to the conditions in (5).

We observe in Figs. 3 and 4 that \(R_{PU}\) decreases with \(\bar{c}\), while \(R_{SU}\) increases with \(\bar{c}\), as expected. This shows the flexibility of our algorithm in adapting to different primary and secondary user priority levels. When \(\bar{c}\) is low, we observe in Fig. 3 that the sum-rate of the matched \((PT, PR)\) pairs of our proposed algorithm achieves a high percentage of the optimal algorithm and significantly greater than the random matching algorithm. Similarly, when \(\bar{c}\) is high, we observe in Fig. 4 that the sum-rate of the matched \((ST, SR)\) pairs of our proposed algorithm achieves a high percentage of the optimal algorithm and significantly greater than the random matching algorithm.

\textbf{D. Total number of matchings}

The total number of matched \((PT, PR)\) and \((ST, SR)\) pairs is also an important consideration of any matching algorithm, and is proportional to the total number of users which can achieve their minimum rate requirements. Fig. 5 shows the percentage of matched \((PT, PR)\) pairs vs. the SU’s SNR \(\gamma_{ST} = \gamma_{ST_1} = \gamma_{ST_2}\), for different number of \((ST, SR)\) pairs \(L_{SU}\). We observe that the percentage of matched \((PT, PR)\) pairs increases with \(\gamma_{SU}\) and \(L_{SU}\), due to the higher achievable rates that the matched \((PT, PR)\) pairs can achieve through cooperative relaying. Remarkably, we observe that the proposed scheme can achieve a very high matching percentage at high SNR even when \(L_{SU} = 2\), i.e., \(\geq 80\%\) when \(\gamma_{SU} \geq 20\) dB. Moreover, the proposed algorithm delivers a percentage of matched users comparable with the centralized algorithm, and significantly greater than the random algorithm. Note that as the minimum rate requirement for the PUs is equal to the rate of their corresponding direct link transmission without cooperative relaying, Fig. 5 thus indicates that the PUs are well motivated to participate in the trading framework with the SUs.
In practice, the unmatched PTs will transmit directly to their corresponding PRs and thus \((PT_\ell, PR_\ell)\) will achieve the rate \(R_{PT_\ell, PR_\ell}\). However, the unmatched STs will not be able to transmit at all. To remedy this, various modifications to the proposed algorithm can be made, such as integrating a fairness mechanism into the algorithm so each ST has a turn transmitting, though at different times.

**E. Overhead and Complexity**

The proposed algorithm is distributed, and thus incurs significantly less overhead and complexity compared to centralized algorithms. It can be observed from the proposed algorithm that the total number of times the PTs communicates with the STs scales as

\[
N \sim L_{PU} L_{SU} \left( a \left\lceil \frac{\zeta_{init}}{\delta} \right\rceil + b \left\lceil \frac{\beta_{init}}{\epsilon} \right\rceil \right)
\]

(14)

where \(a, b \in \mathbb{R}^+\). It can be shown from Table II that (14) also serves as an upper bound to the number of iterations required for convergence of the proposed algorithm. We observe in (14) that the amount of overhead, and thus the number of iterations, decreases with \(\epsilon\) and \(\delta\). This is confirmed in Fig. 6, which plots the total number of communication packets exchanged between each PT and all the STs it communicates with, vs. time-slot step number \(\epsilon\), with the same parameters used in Fig. 2. We see that the total number of communication packets converge to a constant at sufficiently high \(\epsilon\). This is because if \(\epsilon\) is sufficiently large, the time-slot allocation numbers are updated in the algorithm in such a way that the preference lists for each \((PT, PR)\) and \((ST, SR)\) pair remain unchanged.

Denoting the random variable \(Y\) as the total number of communication packets as a function of the user channels, Fig. 7 plots the cumulative distribution function (c.d.f.) of \(Y, PR (Y \leq y)\), vs. \(y\) for different number of secondary users \(L_{SU}\), where \(y\) is a realization of \(Y\). We observe that the proposed algorithm can achieve low overhead with high probability in various practical scenarios. For example, when \(L_{SU} = 6\), we observe that 90% of the time, a maximum of only 15 communication packets are exchanged between the PUs and the SUs.

Figs. 2 and 6, and (14) reveal that \(\epsilon\) can be designed to ensure an acceptable amount of overhead and achievable rate. In particular, we observe from (14) and Fig. 6 that decreasing the overhead by increasing \(\epsilon\) or \(\delta\) will result in both a lower magnitude and number of price and time-slot allocation numbers which can be offered to the \((ST, SR)\) pairs from the \((PT, PR)\) pairs. This will result in a lower utility for the \((PT, PR)\) pairs, and increase the
chance the STs will reject any offer made. This can be observed in Fig. 2, and illustrates a tradeoff between performance and overhead. A similar argument can be made for decreasing $\zeta_{\text{init}}$ and $\beta_{\text{init}}$.

We note that the packet length required for communication between the PTs and the STs is very short. In particular, assuming that $\zeta_{\text{init}}, \beta_{\text{init}}, \delta$ and $\epsilon$ are initially known to all users, each PT is only required to send one bit to the first ST in its preference list indicating an offer, and the corresponding ST only needs to send one bit back to the offering PT indicating either acceptance or rejection. As demonstrated in Fig. 6, the total number of communication packets for each PT can be designed to be reasonably small, and thus given the short packet lengths, the total running time and amount of overhead from the proposed algorithm can be quite small.

VI. Conclusion

We proposed a distributed algorithm for spectrum access which guarantees that the PUs’ and SUs’ rate requirement are satisfied. We proved that the proposed algorithm results in the best possible stable matching and is weak Pareto optimal. Numerical analysis also revealed that the distributed algorithm achieves a performance comparable to an optimal centralized algorithm, but with significantly less overhead and complexity.

APPENDIX

A. Proof of Theorem 1

We first define some notations. Without loss of generality, let \( \{(\text{PT}_\ell \text{PR}_\ell)\}_{\ell=1}^{L_{\text{prop}}} \) be the set of matched (PT, PR) pairs at the conclusion of the algorithm. We first consider $p_1 = (\text{PT}_1, \text{PR}_1)$, with the preference list at the conclusion of the algorithm denoted by $\text{PULIST}_1 = \{s_1, \ldots, s_{K_1}\}$. Associated with this preference list are the final price and time-slot allocation numbers associating $p_1$ and $\{s_q\}_{q=1}^{K_1}$, and denoted respectively by $\{\zeta_{\text{prop}}^{\text{prop}}\}_{q=1}^{K_1}$ and $\{\beta_{\text{prop}}^{\text{prop}}\}_{q=1}^{K_1}$. Note that due to the ordering of the preference list, the proposed algorithm will match $p_1$ with $s_1$.

We now prove that the proposed algorithm doesn’t produce any blocking pairs by contradiction. Assume that $p_1$ and $s_q$, for $q = 2, \ldots, K_1$, can constitute a blocking pair. This implies that there exists a $\zeta_{1,q}$ and $\beta_{1,q}$ such that $U_{\text{PU}_1,q}(\zeta_{1,q}, \beta_{1,q}) > U_{\text{PU}_{1,1}}(\zeta_{1,1}^{\text{prop}}, \beta_{1,1}^{\text{prop}})$, implying that either (i) $\zeta_{1,q} > \zeta_{1,q}^{\text{prop}}$, (ii) $\beta_{1,q} > \beta_{1,q}^{\text{prop}}$ or (iii) both $\zeta_{1,q} > \zeta_{1,q}^{\text{prop}}$ and $\beta_{1,q} > \beta_{1,q}^{\text{prop}}$. Due to the ordering of the preference list PULIST$_1$, this cannot occur as there would have been a period
during the algorithm when \( s_q \) initially rejected \( p_1 \) at this \( \beta_{1,q} \) and/or \( \zeta_{1,q} \) value, as \( s_q \) received a better offer from another (PT, PR) pair. For this scenario, \( p_1 \) and \( s_q \) thus do not form a blocking pair. A similar proof can be made for the other \( p_\ell s \), for \( \ell = 2, \ldots, L_{PU} \). The stable matching proof follows by noting that there are no blocking individuals due to the preference lists, i.e., no (PT, PR) and (ST, SR) pairs will be matched respectively with a (ST, SR) and (PT, PR) pair not on its preference list.

B. Proof of Lemma 1

The proof is by contradiction. Denote \( \mu^{prop} \) as the stable matching produced by the proposed algorithm, and \( \mu^{alt} \) as another stable matching. Then let us assume that for all (PT, PR) pairs, the utility achieved under \( \mu^{prop} \) is less than the utility achieved under \( \mu^{alt} \). W.l.o.g, consider (PT\(_\ell\), PR\(_\ell\)), and thus the previous statement mathematically implies that

\[
U_{PU,\ell,\mu^{alt} - \text{ind}(\ell)}(\zeta_{\ell,\mu^{alt} - \text{ind}(\ell)}, \beta_{\ell,\mu^{alt} - \text{ind}(\ell)}) > U_{PU,\ell,\mu^{prop} - \text{ind}(\ell)}(\zeta_{\ell,\mu^{prop} - \text{ind}(\ell)}, \beta_{\ell,\mu^{prop} - \text{ind}(\ell)}),
\]

where \((\zeta_{\ell}, \beta_{\ell})^{prop}\) and \((\zeta_{\ell}, \beta_{\ell})^{alt}\) denote the price and time-slot allocation numbers under matching \( \mu^{prop} \) and \( \mu^{alt} \) respectively. Denoting \( q = \mu^{alt} - \text{ind}(\ell) \), this means that under the proposed matching \( \mu^{prop} \), PT\(_\ell\) offered \((\zeta_{\ell,\mu^{alt} - \text{ind}(\ell)}, \beta_{\ell,\mu^{alt} - \text{ind}(\ell)})\) to ST\(_q\) but was rejected. There are two ways for this to happen. The first is if under matching \( \mu^{prop} \), PT\(_\ell\) with proposal \((\zeta_{\ell,\mu^{alt} - \text{ind}(\ell)}, \beta_{\ell,\mu^{alt} - \text{ind}(\ell)})\) was not in ST\(_q\)’s preference list SULIST\(_q\). However, this contradicts the implicit assumption that under matching \( \mu^{alt} \), PT\(_\ell\) with proposal \((\zeta_{\ell,\mu^{alt} - \text{ind}(\ell)}, \beta_{\ell,\mu^{alt} - \text{ind}(\ell)})\) is in ST\(_q\)’s preference list, and thus should also be in ST\(_q\)’s preference list under matching \( \mu^{prop} \).

The second alternative is if under matching \( \mu^{prop} \), ST\(_q\) rejected PT\(_\ell\) in favor of another primary transmitter, denoted as PT\(_\nu\). W.l.o.g., let us assume that this is the first rejection that has taken place under \( \mu^{prop} \). This implies that PT\(_\nu\) prefers ST\(_q\) to every other (ST, SR) pair, and thus the matching \( \mu^{alt} \) is blocked by pair \((\text{PT}_\nu, \text{PR}_\nu), (\text{ST}_q, \text{SR}_q)\)\), which contradicts the assumption that \( \mu^{alt} \) is a stable matching.

C. Proof of Theorem 2

We outline the proof for non-stable matchings, and note that the proof for stable matchings follows directly from Lemma 1. The proof is by contradiction.

Denote \( \mu^{prop} \) as the matching produced by the proposed algorithm, and \( \mu^{opt} \) as an arbitrary non-stable matching. Then let us assume that every matched (PT, PR) pair in \( \mu^{prop} \) achieves a utility less than its utility obtained under matching \( \mu^{opt} \). Mathematically, this is represented by
\( \{U_{\ell,\mu^{\text{prop}}-\text{ind} (1)} (\zeta_{\ell,\mu^{\text{prop}}-\text{ind} (1)}, \beta_{\ell,\mu^{\text{prop}}-\text{ind} (1)}) < U_{\ell,\mu^{\text{opt}}-\text{ind} (1)} (\zeta_{\ell,\mu^{\text{opt}}-\text{ind} (1)}, \beta_{\ell,\mu^{\text{opt}}-\text{ind} (1)}) \}_{\ell=1}^{L_{\mu^{\text{prop}}}} \), where \((\zeta_{\ell,\mu^{\text{prop}}-\text{ind} (1)}, \beta_{\ell,\mu^{\text{prop}}-\text{ind} (1)})\) and \((\zeta_{\ell,\mu^{\text{opt}}-\text{ind} (1)}, \beta_{\ell,\mu^{\text{opt}}-\text{ind} (1)})\) denote the price and time-slot allocation numbers under matching \(\mu^{\text{prop}}\) and \(\mu^{\text{opt}}\) respectively.

We consider three scenarios, defined by the relative number of matched \((\text{ST}, \text{SR})\) pairs under \(\mu^{\text{prop}}\) compared to \(\mu^{\text{opt}}\). Specifically:

1) Same matchings: In this scenario, every matched \((\text{ST}, \text{SR})\) and \((\text{PT}, \text{PR})\) pair under \(\mu^{\text{prop}}\) is also matched under \(\mu^{\text{opt}}\). Consider the last \((\text{ST}, \text{SR})\) pair which is matched in \(\mu^{\text{prop}}\), and denote this pair as \((\text{ST}_{L_{\mu^{\text{prop}}}}, \text{SR}_{L_{\mu^{\text{prop}}}})\). Moreover, w.l.o.g. \((\text{ST}_{L_{\mu^{\text{prop}}}}, \text{SR}_{L_{\mu^{\text{prop}}}})\) is matched with (i) \((\text{PT}_{L_{\mu^{\text{prop}}}}, \text{PR}_{L_{\mu^{\text{prop}}}})\) under matching \(\mu^{\text{prop}}\), and (ii) \((\text{PT}_{1}, \text{PR}_{1})\) under matching \(\mu^{\text{opt}}\). From the initial contradiction assumption, note that under \(\mu^{\text{prop}}\), \((\text{PT}_{1}, \text{PR}_{1})\) prefers \((\text{ST}_{L_{\mu^{\text{prop}}}}, \text{SR}_{L_{\mu^{\text{prop}}}})\) with price and time slot allocation numbers \((\zeta_{1,\text{L}_{\mu^{\text{prop}}}}, \beta_{1,\text{L}_{\mu^{\text{prop}}}})\) than its current matching, which mathematically implies that

\[ U_{\text{PU}_{1},\text{L}_{\mu^{\text{prop}}}} (\zeta_{1,\text{L}_{\mu^{\text{prop}}}}, \beta_{1,\text{L}_{\mu^{\text{prop}}}}) > U_{\text{PU}_{1},\mu^{\text{opt}}-\text{ind} (1)} (\zeta_{1,\mu^{\text{opt}}-\text{ind} (1)}, \beta_{1,\mu^{\text{opt}}-\text{ind} (1)}) \].

There was thus a period during the proposed algorithm where \((\text{PT}_{1}, \text{PR}_{1})\) offered a price allocation number \(\zeta_{1,\text{L}_{\mu^{\text{prop}}}} \) and/or a time-slot allocation number \(\beta_{1,\text{L}_{\mu^{\text{prop}}}} \) to \((\text{ST}_{L_{\mu^{\text{prop}}}}, \text{SR}_{L_{\mu^{\text{prop}}}})\), and was rejected. However, as \((\text{ST}_{L_{\mu^{\text{prop}}}}, \text{SR}_{L_{\mu^{\text{prop}}}})\) is the last \((\text{ST}, \text{SR})\) pair to be matched, this is a contradiction as \((\text{ST}_{L_{\mu^{\text{prop}}}}, \text{SR}_{L_{\mu^{\text{prop}}}})\) will not reject any offer from \((\text{PT}_{1}, \text{PR}_{1})\).

2) More primary user matchings: In this scenario, there are more \((\text{PT}, \text{PR})\) pairs which are matched under \(\mu^{\text{prop}}\), then matched under \(\mu^{\text{opt}}\). The initial contradiction assumption is thus clearly violated, as the un-matched \((\text{PT}, \text{PR})\) pairs which are matched under \(\mu^{\text{prop}}\), but not under \(\mu^{\text{opt}}\), have a zero utility.

3) Less primary user matchings: In this scenario, there are less \((\text{PT}, \text{PR})\) pairs which are matched under \(\mu^{\text{prop}}\), then matched under \(\mu^{\text{opt}}\). Every \((\text{ST}, \text{SR})\) pair which is not matched under \(\mu^{\text{prop}}\) will thus have a zero utility. However, at least one of these \((\text{ST}, \text{SR})\) pairs, denoted as \((\text{ST}_{q}, \text{SR}_{q})\) will obtain a positive utility under \(\mu^{\text{opt}}\). Combined with the initial contradiction assumption, this implies that \((\text{ST}_{q}, \text{SR}_{q})\) and \((\text{PT}_{\mu^{\text{opt}}-\text{ind}(q)}, \text{PR}_{\mu^{\text{opt}}-\text{ind}(q)})\) form a blocking pair for \(\mu^{\text{prop}}\), which is a contradiction.

REFERENCES


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<tr>
<td>$L_{PU}$</td>
<td>Number of PUs</td>
</tr>
<tr>
<td>$L_{SU}$</td>
<td>Number of SUs</td>
</tr>
<tr>
<td>$R_{PU,\ell,req}$</td>
<td>PU$_\ell$'s rate requirement</td>
</tr>
<tr>
<td>$R_{SU,q,req}$</td>
<td>SU$_q$'s rate requirement</td>
</tr>
<tr>
<td>$\zeta_{\ell,q}$</td>
<td>Price allocation number</td>
</tr>
<tr>
<td>$\beta_{\ell,q}$</td>
<td>Time-slot allocation number</td>
</tr>
<tr>
<td>$T$</td>
<td>Length of transmission frame</td>
</tr>
<tr>
<td>$C$</td>
<td>Total money each SU has access to in each transmission frame</td>
</tr>
<tr>
<td>$\tau_{\ell,q}$</td>
<td>Time-slot fraction that PT$_\ell$ transmits its signal to ST$_q$</td>
</tr>
<tr>
<td>$h_{PT,\ell,PR,\ell}$</td>
<td>Channel from PT$<em>\ell$ to PR$</em>\ell$</td>
</tr>
<tr>
<td>$h_{PT,\ell,ST,q}$</td>
<td>Channel from PT$_\ell$ to ST$_q$</td>
</tr>
<tr>
<td>$h_{ST,q,PR,\ell}$</td>
<td>Channel from ST$<em>q$ to PR$</em>\ell$</td>
</tr>
<tr>
<td>$h_{ST,q,SR,\ell}$</td>
<td>Channel from ST$<em>q$ to SR$<em>q$ in the spectrum band occupied by (PT$</em>\ell$, PR$</em>\ell$)</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Path loss exponent</td>
</tr>
<tr>
<td>$R_{PU,\ell}$</td>
<td>PU$_\ell$'s sum-rate while cooperating with ST$_q$</td>
</tr>
<tr>
<td>$R_{SU,q,\ell}$</td>
<td>SU$<em>q$'s sum-rate while cooperating with PU$</em>\ell$</td>
</tr>
<tr>
<td>$R_{PT,\ell,PR,\ell}$</td>
<td>Direct rate of PT$<em>\ell$ to PR$</em>\ell$ link</td>
</tr>
<tr>
<td>$\gamma_{PT,\ell}$</td>
<td>Transmit signal-to-noise-ratio for PT$_\ell$</td>
</tr>
<tr>
<td>$\gamma_{ST,q}$</td>
<td>Transmit signal-to-noise-ratio for ST$_q$</td>
</tr>
<tr>
<td>$\bar{c}$</td>
<td>Rate per unit monetary value for PT</td>
</tr>
<tr>
<td>$k$</td>
<td>Rate per unit monetary value for ST</td>
</tr>
<tr>
<td>$U_{PU,\ell}$</td>
<td>PU$_\ell$'s utility while cooperating with ST$_q$</td>
</tr>
<tr>
<td>$U_{SU,q,\ell}$</td>
<td>SU$<em>q$'s utility while cooperating with PU$</em>\ell$</td>
</tr>
<tr>
<td>$M$</td>
<td>Matching matrix</td>
</tr>
<tr>
<td>$B$</td>
<td>Time-slot allocation matrix</td>
</tr>
<tr>
<td>$G$</td>
<td>Price allocation matrix</td>
</tr>
<tr>
<td>PULIST$_\ell$</td>
<td>PT$_\ell$'s preference list</td>
</tr>
<tr>
<td>SULIST$_q$</td>
<td>ST$_q$'s preference list</td>
</tr>
<tr>
<td>$\delta$</td>
<td>PT's price step number</td>
</tr>
<tr>
<td>$\epsilon$</td>
<td>PT's time-slot step number</td>
</tr>
</tbody>
</table>

**TABLE I**

**PARAMETER DESCRIPTIONS**
TABLE II
MAIN ALGORITHM (MA)

**Step 1: Initialization.**
- Set \((\zeta_{k,q}, \beta_{k,q}) = (\zeta_{\text{init}}, \beta_{\text{init}})\), for \(k = 1, \ldots, L_{\text{PU}}, q = 1, \ldots, L_{\text{SU}}\).
- Set the price step number \(\delta\).
- Set the time-slot step number \(\epsilon\).
- Construct \(P_{\text{ULIST}}_k\) based on \((\zeta_{\text{init}}, \beta_{\text{init}})\), for \(k = 1, \ldots, L_{\text{PU}}\).
- Construct \(S_{\text{ULIST}}_q\) based on \((\zeta_{\text{init}}, \beta_{\text{init}})\), for \(q = 1, \ldots, L_{\text{SU}}\).
- Construct the list of all PTs which are not matched, denoted by \(\text{MATCH} = \{\text{PT}_1, \ldots, \text{PT}_{L_{\text{PU}}}\}\).
- Set the indexing parameter \(\ell = 1\).

**Step 2: Find a suitable ST for each PT**

**Step 2-A:** \(\text{PT}_\ell\) makes an offer of \((\zeta_{\ell,q}, \beta_{\ell,q})\) to the first ST in its preference list \(P_{\text{ULIST}}_\ell\).
- Let us denote this ST as \(S_{\text{T}}_q\). If \((\zeta_{\ell,q}, \beta_{\ell,q}) \neq (\zeta_{\text{init}}, \beta_{\text{init}})\), then \(S_{\text{T}}_q\) will update \(S_{\text{ULIST}}_q\) based on \((\zeta_{\ell,q}, \beta_{\ell,q})\).

**Step 2-B:**
1) If \(\text{PT}_\ell\) is in \(S_{\text{T}}_q\)’s preference list.
   a) If \(S_{\text{T}}_q\) is already matched to a primary transmitter, denoted by \(\text{PT}_{\text{curr}}\),
      i) If \(\text{PT}_\ell\) is higher up than \(\text{PT}_{\text{curr}}\) on \(S_{\text{T}}_q\)’s preference list \(S_{\text{ULIST}}_q\):
         - SU and PU are matched. Remove PU from MATCH.
         - Update \((\zeta_{\text{curr},q}, \beta_{\text{curr},q}, \text{MATCH}, P_{\text{ULIST}}_{\text{curr}}) = \text{PUU} (\zeta_{\text{curr},q}, \beta_{\text{curr},q}, \text{MATCH}, P_{\text{ULIST}}_{\text{curr}})\).
      ii) Else
         - Update \((\zeta_{\ell,q}, \beta_{\ell,q}, \text{MATCH}, P_{\text{ULIST}}_{\ell}) = \text{PUU} (\zeta_{\ell,q}, \beta_{\ell,q}, \text{MATCH}, P_{\text{ULIST}}_{\ell})\).
   b) Else
      - SU and PU are matched. Remove PU from MATCH.

2) Else
   - Update \((\zeta_{\ell,q}, \beta_{\ell,q}, \text{MATCH}, P_{\text{ULIST}}_{\ell}) = \text{PUU} (\zeta_{\ell,q}, \beta_{\ell,q}, \text{MATCH}, P_{\text{ULIST}}_{\ell})\).

3) If \(P_{\text{ULIST}}_{\ell} = \emptyset\), remove PT from MATCH, and go to **Step 2-C**, else go to **Step 2-A**.

**Step 2-C:** If no more matchings are possible, i.e., MATCH = \(\emptyset\), then stop the algorithm. Else go to **Step 2-A** for each PT which is not matched, i.e., go to **Step 2-A** with \(\text{PT}_\ell\) being the PT corresponding to the first entry in MATCH.
TABLE III
PROPOSAL UPDATE UNIT (PUU)

Input
- Current price allocation number $\zeta_{\ell,q}^{\text{old}}$.
- Current time-slot allocation number $\beta_{\ell,q}^{\text{old}}$.
- Current MATCHLIST$^{\text{old}}$.
- Current PU$^{\text{old}}$'s preference list PULIST$^{\text{old}}$.

Output
- Updated price allocation number $\zeta_{\ell,q}^{\text{new}}$.
- Updated time-slot allocation number $\beta_{\ell,q}^{\text{new}}$.
- Updated MATCHLIST$^{\text{new}}$.
- Updated PU$^{\text{old}}$'s preference list PULIST$^{\text{new}}$.

Procedure

1) If $\zeta_{\ell,q}^{\text{old}} - \delta \leq 0$
   - $\beta_{\ell,q}^{\text{new}} = \beta_{\ell,q}^{\text{old}}$.
   - $\zeta_{\ell,q}^{\text{new}} = \zeta_{\ell,q}^{\text{old}}$.

2) Else if $\zeta_{\ell,q}^{\text{old}} - \delta > 0$ and $R_{\text{PU},\ell,q}(\beta_{\ell,q}^{\text{old}} - \epsilon, \zeta_{\ell,q}^{\text{old}}) \leq R_{\text{PU},\ell,q}$
   - $\beta_{\ell,q}^{\text{new}} = \beta_{\ell,q}^{\text{old}}$.
   - $\zeta_{\ell,q}^{\text{new}} = \zeta_{\ell,q}^{\text{old}} - \delta$.

3) Else if $U_{\text{PU},\ell,q}(\beta_{\ell,q}^{\text{old}}, \zeta_{\ell,q}^{\text{old}} - \delta) < U_{\text{PU},\ell,q}(\beta_{\ell,q}^{\text{old}} - \epsilon, \zeta_{\ell,q}^{\text{old}})$
   - $\beta_{\ell,q}^{\text{new}} = \beta_{\ell,q}^{\text{old}}$.
   - $\zeta_{\ell,q}^{\text{new}} = \zeta_{\ell,q}^{\text{old}} - \delta$.

4) Else $(U_{\text{PU},\ell,q}(\beta_{\ell,q}^{\text{old}}, \zeta_{\ell,q}^{\text{old}} - \delta) \geq U_{\text{PU},\ell,q}(\beta_{\ell,q}^{\text{old}} - \epsilon, \zeta_{\ell,q}^{\text{old}}))$
   - $\beta_{\ell,q}^{\text{new}} = \beta_{\ell,q}^{\text{old}} - \epsilon$.
   - $\zeta_{\ell,q}^{\text{new}} = \zeta_{\ell,q}^{\text{old}}$.

5) Update the old preference list PULIST$^{\text{old}}$ for (PT$^{\ell}$, PR$^{\ell}$) based on the updated $(\zeta_{\ell,q}^{\text{new}}, \beta_{\ell,q}^{\text{new}})$, to form a new preference list PULIST$^{\text{new}}$.

6) Update the old MATCHLIST$^{\text{old}}$ by putting PU$^{\ell}$ at the end of the list, to form a new MATCHLIST$^{\text{new}}$. 
Fig. 1. Secondary user and primary user spectrum-access model. The channel and price and time-slot allocation numbers are indicated for \((PT_{\ell}, PR_{\ell})\) and \((ST_q, SR_q)\).

Fig. 2. Average sum-utility of all matched (PT, PR) pairs vs. time slot step-number \(\epsilon\), with \(\zeta_{\text{init}} = 0.99, \beta_{\text{init}} = 0.99, \delta = \epsilon, \gamma_{SU_1} = \ldots \gamma_{SU_{L_{SU}}} = 25 \text{ dB}, L_{PU} = 2, \gamma_{PU_1} = \gamma_{PU_2} = 5 \text{ dB}, R_{SU,req} = 0.1, \{R_{PU_{\ell},req} = R_{PT,PR_{\ell}}\}_{\ell=1}^{L_{PU}}, \bar{c} = 1, \bar{k} = 1\) and \(\alpha = 4\).
Fig. 3. Average total sum-rate of all matched (PT, PR) pairs vs. $\bar{c}$, with $\zeta_{\text{init}} = 0.99$, $\beta_{\text{init}} = 0.99$, $\epsilon = 0.05$, $\delta = 0.05$, $\gamma_{SU_1} = \ldots \gamma_{SU_{L_{SU}}} = 25$ dB, $L_{PU} = 2$, $\gamma_{PU_1} = \gamma_{PU_2} = 5$ dB, $R_{SU,req} = 0.1$, $\{R_{PU_\ell,req} = R_{PT,PR_\ell}\}_{\ell=1}^{L_{PU}}$, $\bar{k} = 15$ and $\alpha = 4$.

Fig. 4. Average total sum-rate of all matched (ST, SR) pairs vs. $\bar{c}$, with $\zeta_{\text{init}} = 0.99$, $\beta_{\text{init}} = 0.99$, $\epsilon = 0.05$, $\delta = 0.05$, $\gamma_{SU_1} = \ldots \gamma_{SU_{L_{SU}}} = 25$ dB, $L_{PU} = 2$, $\gamma_{PU_1} = \gamma_{PU_2} = 5$ dB, $R_{SU,req} = 0.1$, $\{R_{PU_\ell,req} = R_{PT,PR_\ell}\}_{\ell=1}^{L_{PU}}$, $\bar{k} = 15$ and $\alpha = 4$. 
Fig. 5. Percentage of matched (PT, PR) pairs vs. the total number of (ST, SR) pairs $L_{SU}$, with $\alpha_{\text{init}} = 0.99$, $\beta_{\text{init}} = 0.99$, $\epsilon = 0.05$, $\delta = 0.05$, $\gamma_{\text{SU}1} = \ldots \gamma_{\text{SU}L_{SU}} = 25$ dB, $L_{PU} = 2$, $\gamma_{\text{PU}1} = \gamma_{\text{PU}2} = 5$ dB, $R_{SU,req} = 0.1$, $\{R_{PU,req} = R_{PT,PR}\}_{\ell=1}$, $\bar{c} = 1$, $\bar{k} = 2$ and $\rho = 4$.

Fig. 6. Total number of communication packets vs. $\epsilon$ for the complete instantaneous received SNR scenario, with $\alpha_{\text{init}} = 0.99$, $\beta_{\text{init}} = 0.99$, $\epsilon = 0.05$, $\delta = 0.05$, $\gamma_{\text{SU}1} = \ldots \gamma_{\text{SU}L_{SU}} = 25$ dB, $L_{PU} = 2$, $\gamma_{\text{PU}1} = \gamma_{\text{PU}2} = 5$ dB, $R_{SU,req} = 0.1$, $\{R_{PU,req} = R_{PT,PR}\}_{\ell=1}$, $\bar{c} = 1$, $\bar{k} = 1$ and $\rho = 4$. 
Fig. 7. Cumulative distribution function vs. total number of communication packets for the complete instantaneous received SNR scenario, with $\alpha_{\text{init}} = 0.99$, $\beta_{\text{init}} = 0.99$, $\epsilon = 0.05$, $\delta = 0.05$, $\gamma_{SU} = \ldots \gamma_{SU_{L \times SU}} = 25$ dB, $L_{PU} = 2$, $\gamma_{PU_1} = \gamma_{PU_2} = 5$ dB, $R_{SU_{req}} = 0.1$, $\{R_{PU_{\ell,req}} = R_{PU_{\ell,PR}}\}_{\ell=1}^{L_{PU}}$, $c = 1$, $k = 1$ and $\rho = 4$. 