Extending seismic bandwidth using the continuous wavelet transform

Michael Smith¹, Gary Perry¹, Jaime Stein², Alexandre Bertrand³ and Gary Yu² describe the realization of a Bandwidth Extension technique with synthetic and real examples.

Resolution is the ability to identify individual features or details in a given image. By the 3D nature of seismic data, seismic resolution involves both vertical (temporal) and horizontal (spatial) resolution. Temporal resolution is a function of the frequency content of a given signal. Achieving optimal thin bed resolution requires a broadband spectrum. Since most seismic is very band-limited it is desirable to extend the bandwidth of the data in a manner that is both verifiable and consistent with the geology.

In seismic processing, many methods are employed to extend the bandwidth of the signal while maintaining an acceptable signal-to-noise ratio (SNR). Many deconvolution processes, such as spiking, attempt to sharpen the wavelet. Spectral whitening is also often employed to boost frequencies, most often on the high end (but sometimes on the low end as well). Generally speaking, the two problems most often encountered with any of these methods are that they tend to increase the noise level more than the signal and the amount of improvement expected is usually substantially less than an octave.

However, new methods have been introduced that produce results that compare favourably to both well log synthetic seismograms and to geologic conditions (e.g., high frequency imaging or HFI, Hamarbatan et al., 2006). Most of these methods have concentrated on extending the upper end of the spectrum but, especially when one is going to invert the seismic data, extending the lower end is also very desirable. The low frequency earth model should, ideally, extend no further than to about 4-5 Hz. Any information lower than this point tends to improperly influence the resulting inversion between control points.

With these criteria in mind a proprietary method called BE (Bandwidth Extension) was developed utilizing the continuous wavelet transform (CWT) to extend the signal frequencies both upward and downward. We first introduce the background theory on resolution issues. Then we introduce the BE technique, which is finally applied to both synthetic and real examples.

Theoretical background on resolution
Extending the bandwidth of a signal, both in the high end and the low end of the spectrum, has been a goal of seismic data acquisition and processing for a long time. Spectral broadening, primarily high frequency extension, has been a controversial subject because of the belief that, due to the earth absorption of preferentially high frequencies, these are lost and it is impossible to restore them to the spectrum. Many techniques, like Q-compensation (e.g., Wang, 2006) have proven this assumption incorrect.

Given a temporal sampling rate $\Delta t$, the maximum theoretical frequency, called the Nyquist frequency $F_{ny}$, at which the data will remained un-aliased can be defined as:

$$F_{ny} = \frac{1}{2\Delta t}$$  \[1\]

This limit represents the maximum frequency at which the signal contains energy still compatible with the sampling theorem. The classical resolution limit $\lambda_R$ derived from the Rayleigh criteria states that top and bottom reflectivities of a thin bed can be distinguished for thickness of up to a quarter of the dominant wavelength $\lambda_D$ ($\lambda_R \sim 1/4 \lambda_D$). However, this does not mean that it is impossible to detect thinner beds than this: Widess (1973) proved conclusively that the amplitude of a reflection coming from such a thin bed varies with the thickness of the bed. He showed that although completely resolving top and base of a thin bed may be difficult, this thickness still has a measurable effect on the signal. His work pushed the limit of resolution to a new minimum of about half the Rayleigh criterion. The new thickness of resolution $\lambda_W$ is given by $\lambda_W \sim 1/2 \lambda_R \sim 1/8 \lambda_D$.

This evidence of reflectivity below resolution implies that the seismic wavelet contains information about reflectors beyond the dominant frequency of the seismic wavelet and that high frequencies could potentially be restored through digital processing – the motivation of the new bandwidth extension development.

Bandwidth extension (BE) using CWT
The traditional method of analyzing a seismic trace in the frequency domain uses the Fourier transform or the fast Fourier transform (FFT). The problem with using a FFT is that it

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transforms local information into global information. The solution of using a short Fourier transform alleviates some of the problem in that it preserves some local information. However, the short window restricts the frequencies that can be analyzed and the extent of localization is limited by the single window length chosen. The wavelet transform does not have these same limitations and permits the analysis of both local information and global information simultaneously.

Here, we use the CWT to perform a time-series analysis of a seismic trace that decomposes the trace into its respective amplitude and phase components in both frequency and time domains. The CWT is defined as the convolution of a time series \( f(t) \) with a scaled \((s)\) and translated \((\tau)\) wavelet \( \psi(t) \).

\[
W(\tau,s) = \int f(t) \frac{1}{\sqrt{|s|}} \psi^* \left( \frac{t-\tau}{s} \right) dt
\]  

[2]

where \( (\ast) \) indicates the complex conjugate.

The scaled wavelets are called daughter wavelets as they are scaled from the mother wavelet \( \psi \).

The CWT ‘is in theory infinitely redundant - the extreme redundancy is less of a problem than one might imagine; a number of researchers have found ways of rapidly extracting the essential information from these redundant transforms’ (Hubbard, 1998). Because the implementation of the CWT is a discrete operator and not a truly continuous operator, a choice needs to be made as to how many daughter wavelets will be used, thus how much redundancy (infinite redundancy is just not practical). A minimum of 10 scales (voices) per octave is sufficient to recreate the input time series from the transform by computing its reconstruction. Furthermore, the mother wavelet \( \psi \) must also meet the admissibility condition (Qian, 2002) as the analyzing wavelet is going to be used to reconstruct the original time series after bandwidth is extended.

We chose to use the Morlet wavelet as the mother wavelet. The Morlet wavelet is a complex function representing a plane wave modulated by a Gaussian function. The complex nature of the wavelet allows the calculation of amplitude and phase for each scale at distinct times. The choice of this wavelet was driven by the nature of the problem we are analyzing. In our case, we are analyzing a seismic trace, and a Gaussian modulated plane wave would be a good match for seismic time series analysis. The Morlet wavelet is given by:

\[
\psi_0(x) = \pi^{-\frac{1}{4}} e^{ix^2} e^{\frac{-x^2}{2}}
\]  

[3]

The CWT ‘provides a very redundant, but also a very finely detailed description of a signal in terms of both time and frequency’ (Walker, 1999). These properties are utilized to predict the harmonics and sub-harmonics used for bandwidth extension. Because the CWT operates in the time-frequency domain, it is limited by the uncertainty principle that states we cannot know time and frequency with the same accuracy simultaneously. This obviously limits the amount of information usable to predict the higher and lower order harmonics at any given time. This is why the redundant nature of the CWT was chosen as our transform in contrast with the discrete wavelet transform, (DWT) which uses an orthonormal basis and thus has no redundancy. The fuzzy nature of the CWT allows us to compute harmonics and sub-harmonics in a fuzzy way, that is, with incomplete information in both the time and frequency domains.

The time and frequency resolutions vary with scale(s) by the standard deviations of the mother wavelet in the time domain \( \Delta_t \) and the frequency domain \( \Delta_\omega \) as \( s \Delta_t \) and \( \Delta_\omega / s \). So the time resolution increases and the frequency resolution decreases with smaller scales (higher frequencies), and the time resolution decreases and the frequency resolution increases with larger scales (lower frequencies). In other words, in the low end of the spectrum frequency resolution is more important than time resolution, and at the high end of the spectrum just the opposite is true. The result is that the product of the standard deviations of time and frequency are constant for the entire frequency spectrum analyzed (Qian, 2002). This gives us the most optimal picture of the time-frequency domain.

By using the time-series analysis of the CWT, we can compute harmonics and sub-harmonics from the available bandwidth in the seismic wavelet. Harmonics are an integer multiple of a fundamental frequency, and sub-harmonics a ratio of one over an integer multiple (e.g., \( \frac{1}{2} \)). Since we are using a complex wavelet, amplitude and phase information is available for this computation. By convolving harmonic and sub-harmonic frequencies onto the seismic trace, we can recover this information and better reveal reflectivity that otherwise is difficult to detect. This unmasking of reflectivity is accomplished by increasing the bandwidth of the seismic wavelet with harmonic and/or sub-harmonic frequencies added to the seismic trace. Since this is a convolution-like process, harmonic or sub-harmonic frequencies, which correspond to reflectivity in the seismic trace, will be added to the seismic wavelet, but harmonics that do not correspond to reflectivity will effectively fall out of the transform.

The final step is reconstruction of the modified time series from the CWT domain of time and scale to a function of time only, our new seismic trace with extended bandwidth. The reconstruction formula is given by:

\[
f(t) = \frac{1}{C_\psi} \int \int W(\tau,s) \frac{1}{\sqrt{s}} \psi^* \left( \frac{t-\tau}{s} \right) d\tau ds
\]  

[4]

where \( C_\psi \) is given by the admissibility condition.

In implementation, the seismic trace is transformed by the CWT. The fundamental frequencies are chosen based on
user input as to the frequency range to use for computing harmonics and sub-harmonics. The frequency points representing the range of frequencies to use are called the pivot frequencies. Each pivot frequency is chosen from analyzing the seismic spectrum, and an octave below this pivot is used for computing harmonic frequencies or scales. For sub-harmonics, a different pivot frequency is chosen from the seismic spectrum, and an octave above this pivot is used for computing sub-harmonic frequencies or scales. The harmonics and/or sub-harmonics can extend upward/downward 1-2 octaves, of course honouring Nyquist and 0 Hz boundaries.

Energy density adjustments are made so that the harmonic and sub-harmonic scales produce a better shaped amplitude spectrum. These adjustments are made prior to adding back the harmonic scales to the spectrum of the seismic wavelet. The transformed seismic trace, with added harmonics and/or sub-harmonics, is then reconstructed to give the new bandwidth extended seismic trace.

**Wedge model example**

We have tested the high frequency extension of the BE algorithm on a simple wedge model to analyze its performance.

Figure 1 Bandwidth Extension applied to the wedge model. (a) Low-frequency synthetic (input to BE). (b) High-frequency extension of (a) (output of BE). (c) High frequency synthetic to compare to BE results. Blue and green arrows indicate the limit of resolution of the top and bottom reflectors of the wedge respectively.

Figure 2 Input data (left) and bandwidth-extended data (right) from an onshore 3D survey. Red traces are synthetic seismograms produced from well log reflectivity using an extracted wavelet from the respective data. Horizontal timelines are separated by 100 ms.
in a classic resolution problem. The model is made of a flat top with positive reflectivity and a dipping bottom event of equal positive reflectivity to the top. The bottom reflector has a positive dip of 0.3 ms/trace, which means that the wedge is thickening with increasing trace numbers towards the right of the section. Realistic frequency content (7-55 Hz at half points) is used to model the low frequency response of the wedge (Figure 1a). At this frequency range the bed is fully resolved, according to Rayleigh’s criteria, at trace 40 (thickness = 11.7 ms) where both top and bottom reflectors can be distinguished.

The new BE approach is used in an attempt to improve the resolution of the wedge. After BE (Figure 1b), the top and bottom reflectors of the wedge can be distinguished at trace 28, which correspond to the same resolution as the same wedge modeled using a frequency range of 7-85 Hz (Figure 1c).

Figure 3 Amplitude and phase spectra of the extracted wavelets (Input on the left and BE on the right). These are very diagnostic of the process as the input data is shown to have a good fit with the well at 17-55 Hz (less than two octaves) with near zero phase while the bandwidth extended data match the well from 10-120 Hz (3.5 octaves), still maintaining the near-zero phase fit.

Figure 4 The input data on the left exhibits useable frequencies up to about 55 Hz while the bandwidth-extended data (right) has high signal data well past 120 Hz with good correlation with the well.
Figure 5 At the top of this figure are vertical sections through the 3D at normal bandwidth (left) and after bandwidth extension (right). At bottom are two horizon slices 20 ms below the top of the carbonate marker. The top of the carbonate reservoir is displayed as a black-dash horizon in the vertical sections and the vertical section location is marked with the bold black line.
The application of BE to the wedge model has proved successful at increasing seismic resolution according to Rayleigh’s criteria. In other words, BE can significantly improve the temporal resolution by extending optimally the original bandwidth (Figure 1a) to a broader bandwidth (Figure 1b) that matches the targeted bandwidth (Figure 1c) and ultimately increases the thin bed detectability.

Real data example 1
This first example is from an onshore 3D survey with thin siliclastic reservoirs. The comparison before-and-after BE is shown in Figure 2. The input frequency content is approximately 17-55 Hz, and a relatively good match is obtained with the synthetic (correlation of 70%). With the application of BE, harmonics and sub-harmonics are used to model one additional octave of data on both the low and high side of the spectrum. The output frequency content is 10-120 Hz. With this broader spectrum, we can observe much richer low frequencies, as well as more high frequency details. One disadvantage of most spectral broadening techniques is that they also reduce SNR. Here, the SNR stays at the same level as indicated by good match between the BE result and the high frequency synthetic (correlation of 68%).

The amplitude and phase spectra of the extracted wavelets (Input and BE) also are presented in Figure 3. They are another diagnosis of the confidence that lays in the bandwidth-extended data compared to the true Earth reflectivity measured by the well data. It shows that the match between earth and seismic is good over a much broader frequency range (from less than two octaves to 3.5 octaves). It also shows that BE produces a very stable zero-phase spectrum over the entire frequency range.

Real data example 2
This second real example is a carbonate reef play from an onshore 3D dataset. The geological objective is to identify porosity with higher resolution data. Figure 4 illustrates the before and after BE comparison of the pre-stack time migration (PSTM) section with its respective amplitude spectrum and the synthetics using extracted wavelet. The input frequency is up to 55 Hz and the BE-enhanced frequency is up to 140 Hz. The seismic and well synthetic correlations before and after BE were quite good and comparable at 70%.

To further understand the impact of broader bandwidth, a before-and-after comparison is presented using a horizon slice at the top of reservoir and shown in Figure 5. The improvement of resolution after BE for interpretation in terms of subtle geological features and trends is very evident. The colour scale was calibrated with wells and dark blue represents good porosity. With higher bandwidth, it is now much easier and more reliable to identify the porosity trend - a great time saver and productivity tool because we can see and interpret the geology quickly with good confidence.

Conclusions
Extension of bandwidth using harmonics and sub-harmonics predicted and computed with the CWT will enhance the seismic resolution allowing a more refine and detail interpretation. There are limitations: the sampling theorem limits the maximum recoverable reflectivity to Nyquist and anti-aliasing filters will set this limit below Nyquist. Also, frequencies available for harmonic prediction and limits introduced by a hard cut-off field filter and notch filter, as well as the uncertainty principle, affect the ability to extend bandwidth.

The Widess model suggests that there is seismic reflectivity available below one fourth of the dominant frequency wavelength. This information can be extracted, resulting in an increase in resolution by adding harmonic and sub-harmonic frequencies back to the data. In addition, it is well known that it is desirable to have at least one and a half octaves, if not two octaves, of bandwidth to decrease side lobes and remove the ‘ringy’ character of our seismic data. Once this is done, many features such as minor faults, on-laps, pinch outs, and other stratigraphic subtle features come to light. All of these features can have a significant impact on interpretation of seismic data and field discovery.

Extension of bandwidth to the low end of the spectrum is also possible with the use of sub-harmonics predicted from the available data. This capability is especially helpful for broadband inversion work when the input seismic is missing a large part of the low end of the spectrum.

Three examples presented here (both synthetic and real data) have proven the benefits of the new methodology. The thin bed resolution of the wedge model was significantly improved. In the real case studies not only were broader spectrums attained but the results were confirmed by well-based diagnostics.

References