Intrawell and interwell magnetoexcitons in $In_xGa_{1-x}As$/GaAs coupled double quantum wells

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We theoretically study the ground and many of the excited optically active s states of spatially direct (intrawell) and indirect (interwell) quasi-two-dimensional excitons in $In_xGa_{1-x}As$/GaAs coupled double quantum wells (DQW's) in quantizing magnetic fields $B>2$ T. The evolution of the eigenstates and oscillator strengths with the perpendicular magnetic B and electric $\mathcal{E}$ fields (when, in particular, the direct-indirect crossover and a number of anticrossings occur in the spectra) is studied. The results of the theory are in good agreement with the recently obtained experimental data [L. V. Butov, A. Zrenner, G. Abstreiter, A. V. Petinova, and K. Eberl, Phys. Rev. B **52**, 12 153 (1995)]. The physical origin of the excitonic symmetric-antisymmetric splittings Δ_X in DQW's is discussed. It is shown that, in the wide-barrier regime, Δ_X are determined by the two-particle tunneling through the barrier and suppressed by the excitonic effects.

I. INTRODUCTION

Excitons in symmetric coupled double quantum wells (DQW's) have been the subject of a number of experimental[1–10] and theoretical[11–15] studies during the past decade. In particular, the interest in these structures is connected with a possibility to rearrange the exciton ground state by applying external electric fields $\mathcal{E}$: it is the indirect (interwell) exciton that becomes the ground state at sufficiently high $\mathcal{E}$. Since the overlap between the electron (e) and hole (h) from the different wells is diminished, radiative lifetimes of indirect excitons are large (and can be controlled by $\mathcal{E}$, see, e.g., Ref. 7). This opens an interesting possibility to achieve critical conditions for excitonic condensation for spatially separated electrons and holes[16,6] (see, also, recent experimental work on type-II GaAs/AlAs coupled quantum wells[17,18] in high magnetic fields).

The important parameter that determines many of the properties of excitons in DQW's (in particular, the details of the crossover from the direct to indirect regimes in $\mathcal{E}$), is the tunnel barrier width L_b, which regulates the coupling between the wells. For a symmetric DQW (the two wells are identical), without the excitonic effects, each pair of (exactly degenerate, in the absence of coupling) states splits to the symmetric and antisymmetric states: $\zeta_{s(a)}(z_e) = \pm\zeta_{s(a)}(-z_e)$ for e and $\xi_{s(a)}(z_h) = \pm\xi_{s(a)}(-z_h)$ for h. The single-particle symmetric-antisymmetric splittings $\Delta_\alpha = E_a^{(\alpha)} - E_s^{(\alpha)}$ $[\alpha = e,h]$ decrease exponentially with the barrier width L_b. Thus, for sufficiently large L_b, Δ_α are much smaller than the difference in the Coulomb binding energies of the direct, E_D, and the indirect, E_I, excitons: $\Delta_e, \Delta_h \ll \delta E_{\rm DI} = E_D - E_I$ ($\delta E_{\rm DI} \to E_D = {\rm const}$, when $L_b \to \infty$). In this limit, the excitons are indeed predominantly direct (D) or indirect (I) in nature. The exact symmetry of total Hamiltonian [which includes the e-h interaction term $U_{\rm eh}(|\mathbf{r}_e - \mathbf{r}_h|)$] implies that also the excitonic states should be either symmetric, S, or antisymmetric, A, under a *simultaneous* inversion ($z_e \to -z_e, z_h \to -z_h$). Thus, in order of increasing energy, e.g., for the 1s exciton, the four excitonic states appear: the symmetric direct SD, the antisymmetric direct AD, the antisymmetric indirect AI, and the symmetric indirect SI; only symmetric excitonic states are optically active.[13,15] Numerical results[13,15] show that the two-particle *excitonic* symmetric-antisymmetric splittings Δ_X are very small; typically, $\Delta_X \ll \Delta_e, \Delta_h$. Also, Δ_X *decrease* with increasing the excitonic effects. As far as we know, the physical origin of this effect has never been discussed so far. Below we will give an explanation of such a behavior, which makes the underlying physics transparent.

For sufficiently thin barriers, the other limit is realized $\Delta_e, \Delta_h \gg \delta E_{\rm DI}$ in which the excitons no longer have predominantly either direct or indirect character, but rather are strongly mixed. Many of the features of this situation can be understood in the single-particle approximation neglecting excitonic effects (see, e.g., Refs. 2,5).

In this paper, we theoretically study the effect of quantizing magnetic fields B on excitons in strained symmetric (or nearly symmetric) $In_xGa_{1-x}As$/GaAs DQW's. Such structures have a simple valence band (the light-hole subband is split off, due to the strain). This allows unambiguous identification of virtually all spectral lines in the region below crossings with the light-hole terms. This should be contrasted with the complicated magneto-optical spectra of GaAs/Ga_xAl_{1-x}As DQWs,[8] where valence band mixing plays an essential role. A detailed experimental magneto-optical study of $In_{x_1}Ga_{1-x_1}As$/GaAs/$In_{x_2}Ga_{1-x_2}$As DQW's, with the nominal values $x_1 = x_2 \simeq 0.2$, $L_1 = L_2 = L_b = 60$ Å, has been performed recently.[10] There are several interesting features, which $In_xGa_{1-x}As$/GaAs DQW's exhibit in B. For $In_xGa_{1-x}As$/GaAs structures, the wells are more shallow (in comparison with GaAs/Ga_xAl_{1-x}As DQW's), and for barrier widths $L_b \simeq 60$ Å at $B=0$, the intermediate regime

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$\delta E_{\mathrm{DI}} \sim \Delta_e$ is realized (while $\Delta_h \ll \Delta_e$). With increasing magnetic field, the Coulomb e-h attraction is enhanced. As a result, the regime which is realized for the $\mathrm{In}_x\mathrm{Ga}_{1-x}\mathrm{As/GaAs}$ DQW is gradually changed in B toward that characteristic for a wide-barrier DQW. This leads to the redistribution of the oscillator strengths from the indirect to direct exciton lines.

In addition to the ground direct $1s$-D and indirect $1s$-I exciton states, at high B, a number of excited optically active s-D and s-I states (associated with $n_e = n_h \equiv n = 1,2,\ldots$ Landau levels) have large oscillator strengths and are resolved experimentally;[10] hereafter, we denote these states D_n, I_n, using the high-field notations. When, in addition to B, the perpendicular electric field $\mathscr{E}$ is applied, there is a set the direct-indirect crossovers for each group of the D_n and I_n levels with a given Landau-level index n. Further, a number of anticrossings between the D_{n+1}, D_{n-1} and I_n excitons occur, which result in shifts and the redistribution of the oscillator strengths. Our main goal is to quantitatively describe these features of the excitonic spectra in the DQW's in quantizing magnetic fields and compare the results with the experimental data.[10]

II. THEORETICAL MODEL

We consider strained $\mathrm{In}_x\mathrm{Ga}_{1-x}\mathrm{As/GaAs}$ DQW's with $x_1, x_2 \simeq 0.2$ and GaAs outer barriers. We will be interested here in heavy-hole excitons and, thus, the split-off light-hole band (see, e.g., Refs. 19,20) is neglected. The Hamiltonian describing the system in the perpendicular electric $\mathscr{E} = (0,0,\mathscr{E})$ and magnetic $\mathbf{B} = (0,0,B)$ fields can be written as follows:

$$H = H_{ez} + H_{hz} + H_{2\mathrm{D}} + U_{\mathrm{eh}} \equiv H_0 + U_{\mathrm{eh}}, \tag{1}$$

where

$$H_{ez} = -\frac{\hbar^2}{2m_e}\frac{\partial^2}{\partial z_e^2} + V_e(z_e) + e\mathscr{E}z_e, \tag{2}$$

$$H_{hz} = -\frac{\hbar^2}{2m_{hz}}\frac{\partial^2}{\partial z_h^2} + V_h(z_h) - e\mathscr{E}z_h \tag{3}$$

are the parts describing the free z motion. The confining potentials $V_\alpha(z)$ $[\alpha = e,h]$ for the DQW are given by

$$V_\alpha(z) = \begin{cases} 0, & z < -L_1 - \frac{1}{2}L_b \\ -V_{\alpha 1}, & -L_1 - \frac{1}{2}L_b < z < -\frac{1}{2}L_b \\ 0, & -\frac{1}{2}L_b < z < \frac{1}{2}L_b \\ -V_{\alpha 2}, & \frac{1}{2}L_b < z < \frac{1}{2}L_b + L_2 \\ 0, & z > -L_2 + \frac{1}{2}L_b. \end{cases} \tag{4}$$

Here $L_{1(2)}$ is the left (right) well width, L_b is the GaAs barrier width, and the well depths $V_{\alpha i} = V_\alpha(x_i)$ are[19] $V_{ei} = 0.8\Delta E_g(x_i)$, $V_{hi} = 0.2\Delta E_g(x_i)$, where $\Delta E_g(x_i) = E_g(0) - E_g(x_i)$ is the band offset, $E_g(x) = 1.519 - 1.47x + 0.375x^2$ eV is the band gap of $\mathrm{In}_x\mathrm{Ga}_{1-x}\mathrm{As}$ and we adopt the following values of the effective masses: $m_e = 0.067$, $m_{hz} = 0.35$ (see, also, Appendix). Exciton energies are given relative to the GaAs bang gap $E_g(0)$.

Hamiltonian $H_{2\mathrm{D}}$, describing the in-plane two-dimensional (2D) motion of a free e-h pair in the magnetic field B, is of the form

$$H_{2\mathrm{D}} = \frac{1}{2m_e}\left(-i\hbar\nabla_{\boldsymbol{\rho}_e} + \frac{e}{c}\mathbf{A}_e\right)^2 + \frac{1}{2m_{h\|}}\left(-i\hbar\nabla_{\boldsymbol{\rho}_h} - \frac{e}{c}\mathbf{A}_h\right)^2. \tag{5}$$

$\mathbf{A} = \frac{1}{2}\mathbf{B} \times \boldsymbol{\rho}$ is the vector potential in the symmetric gauge, $\boldsymbol{\rho} = (x,y)$ and $\mathbf{r} = (\boldsymbol{\rho}, z)$ is a 3D vector. Note that, in Eqs. (2), (3), and (5), we assumed an isotropic dispersion for electrons, while taking into account that the in-plane and the perpendicular hole masses are different $m_{h\|} \neq m_{hz}$ (see Appendix for the discussion of $m_{h\|}$ nonparabolicity). In what follows, we will neglect the difference of the effective masses in the wells and in the barriers.

The part describing the Coulomb e-h attraction, when the difference in dielectric constants of GaAs ($\epsilon_1 = 12.5$) and $\mathrm{In}_{0.2}\mathrm{Ga}_{0.8}\mathrm{As}$ ($\epsilon_2 = 13$) is neglected, has the usual form

$$U_{\mathrm{eh}} = U_{\mathrm{eh}}(|\mathbf{r}_e - \mathbf{r}_h|) = -\frac{e^2}{\epsilon|\mathbf{r}_e - \mathbf{r}_h|}, \tag{6}$$

where, e.g., it can be set $\epsilon = (\epsilon_1 + \epsilon_2)/2$. When the difference between ϵ_1 and ϵ_2 is taken into account, due to the image potentials in a QW, $U_{\mathrm{eh}}(|\mathbf{r}_e - \mathbf{r}_h|) \rightarrow U_{\mathrm{eh}}(|\boldsymbol{\rho}_e - \boldsymbol{\rho}_h|; z_e, z_h)$. Below we will briefly discuss how these effects are incorporated into our model.

Our approach to finding the eigenstates of Hamiltonian (1) is to diagonalize the part describing the e-h interaction, U_{eh}, within the basis of noninteracting two-particle e-h states in the DQW *and* in the magnetic B and electric $\mathscr{E}$ fields [i.e., using the eigenstates of H_0 in (1)]. Such an approach proved useful in the problem of correlated quasi-two-dimensional two-electron D^- centers in magnetic fields.[21]

First, let us discuss the properties and symmetries of H_0, in which the 1D z motions of e and h decouple from each other and from the in-plane motion. The solutions of the 1D Schrödinger equations

$$H_{ez}\zeta_i(z_e) = E_i^{(e)}\zeta_i(z_e), \tag{7}$$

$$H_{hz}\xi_j(z_h) = E_j^{(h)}\xi_j(z_h), \tag{8}$$

corresponding to the two lowest-lying discrete states, which exist for $\mathrm{In}_{0.2}\mathrm{Ga}_{0.8}\mathrm{As/GaAs}$ DQW's, we find numerically. To avoid difficulties in dealing with the continuous spectra when $\mathscr{E} \neq 0$, at sufficiently large distances from the DQW ($200-500$ Å), we impose boundary conditions corresponding to infinite barriers. When $\mathscr{E} = 0$ and the DQW is symmetric (i.e., the two wells are identical $x_1 = x_2$, $L_1 = L_2$), the discrete indices $i,j = s,a$ label the ground symmetric, s, and the excited antisymmetric, a, single particle e and h states:

$$\zeta_{s(a)}(z_e) = \pm\zeta_{s(a)}(-z_e), \quad \xi_{s(a)}(z_h) = \pm\xi_{s(a)}(-z_h).$$

The symmetric-antisymmetric single-particle splittings, $\Delta_\alpha = E_a^{(\alpha)} - E_s^{(\alpha)}$, are physically related with the single-particle quantum-mechanical tunneling through the barrier: $\Delta_\alpha = E_\alpha \exp(-S_\alpha)/\pi$, where $S_\alpha = \sqrt{2m_{\alpha z}|E_\alpha|}L_b/\hbar$ is the (imaginary) action associated with the tunneling and E_α is the energy level in a single well.[22] Thus, Δ_α exponentially decrease with the barrier width L_b and strongly depend on the particle mass $m_{\alpha z}$. For the considered case of $In_{0.2}Ga_{0.8}As$/GaAs DQW, with $L_1=L_2=L_b\simeq 60$ Å, numerical calculations give $\Delta_e\simeq 4.9$ meV and $\Delta_h\simeq 0.6$ meV; $\Delta_h \ll \Delta_e$ reflects the fact that $m_{hz}=0.35>m_e=0.067$. When $\mathscr{E}\neq 0$ and/or the DQW is slightly asymmetric, the states labeled by the indices i, j can be thought of as those developed from the s and a states with increasing $\mathscr{E}$ (and with changing the parameters of the DQW from the symmetric case).

Hamiltonian H_{2D} (5), describing the motion of a strictly 2D neutral e-h pair in a uniform field B, is effectively translationally invariant, despite the presence of the coordinate-dependent vector-potentials $\mathbf{A}(\boldsymbol{\rho}_e)$, $\mathbf{A}(\boldsymbol{\rho}_h)$. In fact, there exists (a gauge-dependent) exact integral of motion[23,24]

$$\hat{\mathbf{K}}=\left(-i\hbar\nabla_{\boldsymbol{\rho}_e}+\frac{e}{c}\mathbf{A}_e\right)+\left(-i\hbar\nabla_{\boldsymbol{\rho}_h}-\frac{e}{c}\mathbf{A}_h\right)+\frac{e}{c}(\boldsymbol{\rho}_e-\boldsymbol{\rho}_h)\times\mathbf{B}, \tag{9}$$

which plays the role of the momentum of the center-of-mass motion for a neutral e-h pair in a uniform B[24] (see also Ref. 25). In what follows, we will consider only optically active $\mathbf{K}=0$ s excitons (the angular momentum projection of the relative motion $\ell_z=0$).[26] The wave function for such states can be written as the following expansion:

$$\Psi_{\mathbf{K}=0,s}(\mathbf{r}_e,\mathbf{r}_h)=\exp\left(\frac{i[\boldsymbol{\rho}\times\mathbf{R}]_z}{2\ell_B^2}\right)\Phi_s(\rho;z_e,z_h), \tag{10}$$

$$\Phi_s(\rho;z_e,z_h)=\sum_{i,j=1,2}\sum_n A_{ijn}\zeta_i(z_e)\xi_j(z_h)\phi_{nn}(\rho). \tag{11}$$

Here, $\ell_B=(\hbar c/eB)^{1/2}$ is the magnetic length, $\mathbf{R}=(m_e\boldsymbol{\rho}_e+m_{h\|}\boldsymbol{\rho}_h)/M$ is the center-of-mass and $\boldsymbol{\rho}=\boldsymbol{\rho}_e-\boldsymbol{\rho}_h$ is the relative in-plane coordinates, $M=m_e+m_{h\|}$,

$$\phi_{nn}(\rho)=\langle\boldsymbol{\rho}|nn\rangle=\frac{1}{(2\pi\ell_B^2)^{1/2}}L_n\left(\frac{\rho^2\ell_B^2}{2}\right)\exp\left(-\frac{\rho^2\ell_B^2}{4}\right)$$

is the factored oscillator wave function (see, e.g., Ref. 27), $L_n(x)$ is the Laguerre polynomial. (A similar expansion has been used[28] to study strictly 2D hydrogenic levels in B; see, also, a review[29] on optics of quasi-two-dimensional magnetoexcitons and the references therein.) Note that the e-h pair wave functions $\phi_{nn}(\rho)$ describe the states that are bound by the magnetic field [with the characteristic length $\langle nn|\rho^2|nn\rangle=2(2n+1)\ell_B^2$]. Thus, expansion (11) can be considered as that using the *exciton* wave functions. Note also that (11) allows for the subband coupling (cf. the discussion in Ref. 13).

We obtain eigenenergies E and eigenfunctions of Hamiltonian (1) by numerically solving the secular equation,

$$\mathrm{Det}\Big([E_i^{(e)}+E_j^{(h)}+\hbar\omega_{ce}(n+\tfrac{1}{2})+\hbar\omega_{ch}(n+\tfrac{1}{2})-E]\times\delta_{ii'}\delta_{jj'}\delta_{nn'}+U_{ijnn}^{i'j'n'n'}\Big)=0, \tag{12}$$

where $\omega_{ce}=eB/m_ec$, $\omega_{ch}=eB/m_{h\|}c$, the matrix elements of the e-h interaction are given by

$$U_{ijnn}^{i'j'n'n'}=\langle i'j'n'n'|U_{eh}|ijnn\rangle=\int\frac{d^2q}{(2\pi)^2}\left(-\frac{2\pi e^2}{\varepsilon q}\right)F_{ij}^{i'j'}(q)\mathscr{D}_{nn}^{n'n'}(q), \tag{13}$$

$$\mathscr{D}_{nn}^{n'n'}(q)=\frac{\min(n,n')!}{\max(n,n')!}\left(\frac{q^2\ell_B^2}{2}\right)^{|n-n'|}\times\left[L_{\min(n,n')}^{|n-n'|}\left(\frac{q^2\ell_B^2}{2}\right)\right]^2\exp\left(-\frac{q^2\ell_B^2}{2}\right), \tag{14}$$

where L_n^m are generalized Laguerre polynomials and

$$F_{ij}^{i'j'}(q)=\int_{-\infty}^{\infty}dz_e\int_{-\infty}^{\infty}dz_h\,\exp(-q|z_e-z_h|)\times\zeta_i(z_e)\zeta_{i'}(z_e)\xi_j(z_h)\xi_{j'}(z_h) \tag{15}$$

are the form factors corresponding to the wave functions of the 1D z motion. Quadratures in Eq. (15) and then in Eq. (13) are performed numerically; in our calculations we include from ten Landau levels at $B=12$ T up to 36 Landau levels at $B=2$ T. It is interesting to note that the approximate eigenvalues, which we obtain with a truncated basis $N\times N$, are upper bounds to the exact first N s-exciton levels and *all* (not only the ground one) would deepen with an increasing number of states N included in a secular equation (12). This is similar to the treatment of D^- states and is due to the generalized variational principle for the first N levels (see Ref. 21 and the references therein).

So far, we neglected nonparabolicity of the heavy holes and the effects related with the image forces. To take, approximately, into account the former effects, we replace in the secular equation (12) the energies of free heavy-hole Landau levels in a parabolic band $\hbar\omega_{ch}[n+\frac{1}{2}]$ by those for a nonparabolic band, $\varepsilon_{ch}(n)$. These are obtained using the heavy-hole density of states mass and the experimentally determined parameters (see Appendix).

To take into account the potentials due to the image forces, the factor $(-2\pi e^2/\varepsilon q)F_{ij}^{i'j'}(q)$ in Eq. (13) is replaced by

$$\int_{-\infty}^{\infty}dz_e\int_{-\infty}^{\infty}dz_hU(q;z_e,z_h)\zeta_i(z_e)\zeta_{i'}(z_e)\xi_j(z_h)\xi_{j'}(z_h), \tag{16}$$

where $U(q;z_e,z_h)=\int d^2\rho\,\exp(-i\mathbf{q}\cdot\boldsymbol{\rho})U(|\boldsymbol{\rho}|;z_e,z_h)$ is the 2D Fourier transform of the full two-particle e-h potential of interaction $U(|\boldsymbol{\rho}|;z_e,z_h)$, which includes the potentials due to the image forces. The explicit form of $U(q;z_e,z_h)$ for a DQW [which can be found analogously to the case of a single QW (Refs. 30–32) or a semiconductor superlattice[33]] will be presented elsewhere. Note that, for a symmetric DQW, $U(|\boldsymbol{\rho}|;-z_e,-z_h)=U(|\boldsymbol{\rho}|;z_e,z_h)$ and the inversion symmetry is conserved. For the $In_xGa_{1-x}As$/GaAs DQW, since the difference in dielectric constants is small (we adopt the values $\epsilon=12.5$ for GaAs and $\epsilon=13$ for $In_{0.2}Ga_{0.8}As$), the net resulting effect is also very small. For, e.g., the energy splitting between the direct, D_0, and indirect, I_0 excitons belonging to zero Landau levels, the inclusion of the image potentials gives at $B=10$ T a rather small (positive) contribution of 0.2 meV.

The matrix elements for the interband optical transitions are proportional to the probability amplitude of finding the electron and the hole at the same site,

$$|d|^2 \sim p_{cv}^2 \left| \int d^3 r \Psi_{\mathbf{K}=0,s}(\mathbf{r},\mathbf{r}) \right|^2, \tag{17}$$

where $p_{cv}^2 = |\langle c|\mathbf{p}|v\rangle \cdot \hat{e}|^2$ is the interband momentum matrix element. From (17), we have

$$|d|^2 \sim \left| \sum_{i,j=1,2} \sum_n A_{ijn} \left[\int_{-\infty}^{\infty} dz \zeta_i(z) \xi_j(z) \right] \phi_{nn}(0) \right|^2. \tag{18}$$

Note that $\phi_{nn}^2(0) = (2\pi \ell_B^2)^{-1} \sim B$ does not depend on the Landau-level index n and determines a strong dependence of oscillator strengths on B. Also, for a symmetric DQW, only states with the same inversion symmetry $[\zeta_s(z)\xi_s(z), \zeta_a(z)\xi_a(z)]$ give contribution to (18). In terms of excitons this means, as is known, that only symmetric excitons are optically active [which is also evident from (17)].

III. NUMERICAL RESULTS AND DISCUSSION

A. Exciton binding energies

We define the magnetoexciton binding energy as the absolute difference in energy between a given excitonic state and that of a noninteracting e-h pair from which it evolves [i.e., with the same Landau-level quantum numbers ($n_e = n_h = n$ for s excitons) and the same spatial symmetry]. The binding energies of the direct (indirect) excitons belonging to zero Landau levels, $E_{D(I)}^{(0)}$, for the symmetric $In_xGa_{1-x}As/GaAs$ DQW with $L_1 = L_2 = L_b = 60$ Å, $x_1 = x_2 = 0.194$, are shown in Fig. 1. The energy splittings between the symmetric and antisymmetric direct excitons, $\Delta_{\rm XD}^{(0)} = E_{\rm AD}^{(0)} - E_{\rm SD}^{(0)}$, and indirect excitons, $\Delta_{\rm XI}^{(0)} = E_{\rm SI}^{(0)} - E_{\rm AI}^{(0)}$ are also shown. It is seen that $\Delta_{\rm XI}^{(0)} \simeq \Delta_{\rm XD}^{(0)} \ll E_D^{(0)}, E_I^{(0)}$ (thus, $E_{{\rm SD}(I)}^{(0)} \simeq E_{{\rm AD}(I)}^{(0)}$ and in notations we do not discriminate between them).

The increase of $E_D^{(0)}$ with B is much steeper than that of $E_I^{(0)}$. It can be easily understood, since in the limit of high magnetic fields, when $\ell_B \ll \epsilon \hbar^2 / m_\alpha e^2$, the asymptotic behaviors are $E_D^{(0)} \sim e^2/\epsilon \ell_B \sim B^{1/2}$, while $E_I^{(0)} \sim e^2/\epsilon \sqrt{\ell_B^2 + [L_b + L_1]^2} \rightarrow e^2/\epsilon |L_b + L_1|$ (for simplicity we put here $L_1 = L_2$).

Parameters $R^{(n)} = \max(\Delta_e, \Delta_h)/\delta E_{\rm DI}^{(n)}$ (where $\delta E_{\rm DI}^{(n)} = E_D^{(n)} - E_I^{(n)}$) determine a spatial character of excitons in the z direction: the smaller $R^{(n)}$, the more strongly excitons are predominantly direct or indirect. From Fig. 1, it is clear that $R^{(0)}(B)$ is a decreasing function of B; however, only at sufficiently high magnetic fields $R^{(0)} \ll 1$. Thus, at zero and low B, the intermediate-barrier regime is realized for the DQW's under study; with increasing B, a crossover to a wide-barrier regime DQW occurs.

The binding energies of the excitons belonging to the first Landau level and the corresponding symmetric-antisymmetric excitonic splittings are shown in Fig. 2. It is seen that (1) The binding energies $E_D^{(1)}$, $E_I^{(1)}$ are smaller than these for zero Landau level. It is explained by a larger extent of the $2s$ exciton wave function in the x-y plane; for a strictly 2D magnetoexciton, in the limit of high magnetic fields $E_D^{(1)} = \frac{3}{4} E_D^{(0)}$.[25] (2) Similar to the case of $n=0$ excitons, $E_D^{(1)}$ is a more steep function of B than $E_I^{(1)}$. (3) Excitonic symmetric-antisymmetric splittings $\Delta_{\rm XI}^{(1)} \simeq \Delta_{\rm XD}^{(1)}$

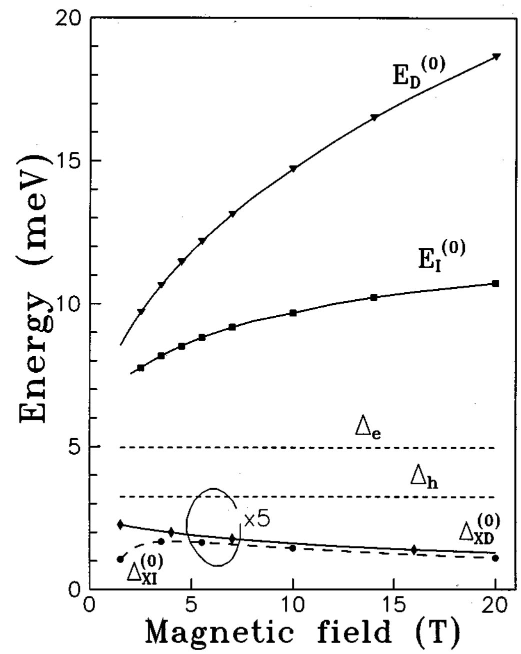

FIG. 1. The binding energies of the direct, $E_D^{(0)}$, indirect, $E_I^{(0)}$, excitons belonging to the zero Landau levels and the corresponding excitonic symmetric-antisymmetric splittings $\Delta_{\rm XD}^{(0)}$, $\Delta_{\rm XI}^{(0)}$. The single-particle electron and hole symmetric-antisymmetric splittings Δ_e, Δ_h (which do not depend on B) are also shown. For clarity, $\Delta_{\rm XD}^{(0)}$, $\Delta_{\rm XI}^{(0)}$, and Δ_h are multiplied by a factor of 5.

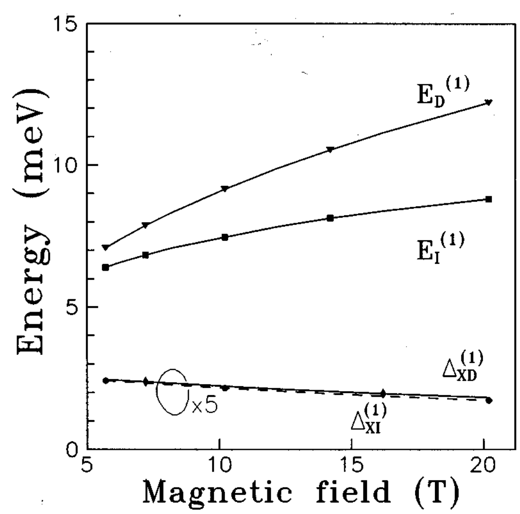

FIG. 2. The binding energies of the direct, $E_D^{(1)}$, and indirect, $E_I^{(1)}$, excitons belonging to the first Landau levels. The corresponding excitonic symmetric-antisymmetric splittings $\Delta_{\rm XD}^{(1)}$, $\Delta_{\rm XI}^{(1)}$ are also shown.

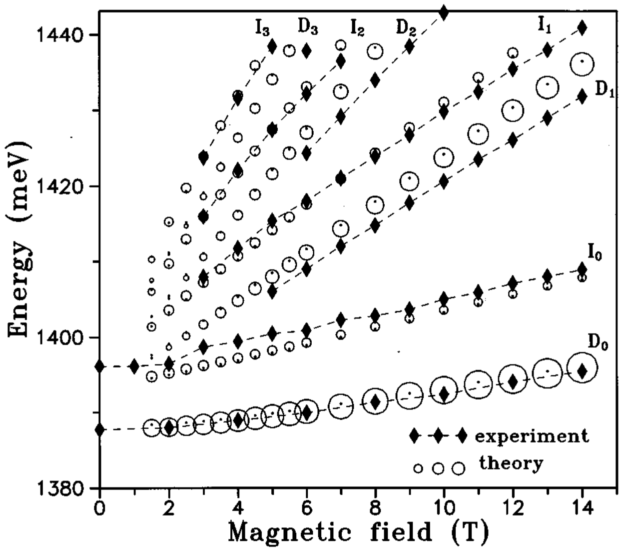

FIG. 3. The magnetic field dependence of the exciton transition energies and oscillator strengths for the symmetric $In_xGa_{1-x}As/GaAs$ DQW, with $L_1=L_2=L_b=60$ Å, $x_1=x_2=0.194$ at zero electric field $\mathcal{E}=0$. The area of open circles is proportional to the oscillator strength. Small dots show positions of antisymmetric excitons with vanishing oscillator strengths. The experimental data taken from Ref. 10 are shown by full symbols connected by dashed lines.

$>\Delta_{XI}^{(0)},\Delta_{XD}^{(0)}$. Both $n=1$ and $n=0$ excitonic splittings decrease with increasing B. An explanation of such a behavior will be given below. (4) The parameter $R^{(1)}>R^{(0)}$; this means that magnetoexcitons associated with higher Landau levels become predominantly direct or indirect at comparatively higher fields than $n=0$ magnetoexcitons.

B. Transition energies and oscillator strengths versus B; zero electric field $\mathcal{E}=0$

Theoretical results for the transition energies and oscillator strengths of s excitons for a symmetric DQW, with $L_1=L_2=L_b=60$ Å, are shown in Fig. 3 and compared with the experiment.[10] Since the In content $x=0.2$ in the experimentally used $Ga_{1-x}In_xAs/GaAs$ DQW's is only nominally known, we choose x to ensure the coincidence of the theoretical and experimental[10] values for the D_0 line at $B=10$ T. Thus, chosen x turns out to be $x=0.194$. No other fitting parameters were used in the theory. Small dots in Fig. 3 represent antisymmetric excitons, which have vanishing oscillator strength (these are shown for clarity). It is seen from Fig. 3 that several terms develop in B forming a Landau fan; we denote these by the quantum numbers D_n, I_n ($n_e=n_h=n$) showing their predominant character in high B.

In each pair D_n, I_n, with increasing B, a redistribution of the oscillator strength f ($f\sim|d|^2$) to the direct line occurs. For example, at $B=2$ T theory gives $f_{D_0}/f_{I_0}=3.2$, while at $B=12$ T $f_{D_0}/f_{I_0}=13.1$. This is explained by the crossover to a wide-barrier regime in high B (see also Ref. 15). In experiment[10] a qualitatively similar behavior was observed which, however, is not so steep with B as in theory. [A situation is qualitatively similar, in regard to comparison of theory and experiment, for the oscillator strengths of the zero Landau-level excitons in a superlattice[34] characterized (in high electric fields $\mathcal{E}$) by the Wannier-Stark indices $p=0$ (direct excitons) and $p=-1$ (indirect excitons).]

For terms belonging to higher Landau levels, such a redistribution of the oscillator strengths $I_n\rightarrow D_n$ occurs in higher magnetic fields. This can be explained by the fact that the parameters $R^{(n)}$ are larger for larger n (see the discussion in Sec. III A).

At lower fields $B\lesssim 4$ T a behavior is complicated by a number of anticrossings that occur in the spectra. Figure 3 shows, for example, that at $B\lesssim 3$ T, the anticrossing between the D_1 and I_0 states occurs, which leads to a redistribution of the oscillator strength from a (nominally stronger) direct D_1 state to (a weaker) I_0 line. Similar behavior takes place, at slightly higher magnetic fields, for the D_{n+1} and I_n anticrossings.

Figure 3 shows that our theory gives a very good description of the D_0 behavior versus B; a good agreement with the experiment[10] is also obtained for the indirect excitonic transitions I_1, I_2, I_3. The theory, however, systematically underestimates the I_n-D_n energy splittings in comparison with the experiment; the origin of this discrepancy is not fully understood. (We believe that it can only partly be explained by a systematic underestimation of the Coulombic effects in our approach [see the discussion after Eq. (15) above].) It is also seen that the theoretical energies of the lines belonging to higher Landau levels increase with B more steeply than in the experiment. This suggests that either the electron mass used ($m_e=0.067$) is too light or the parameters describing the heavy-hole in-plane dispersion and nonparabolicity (see Appendix) are not completely adequate. It should be noted that all these values are not known experimentally with high precision.

Only symmetric excitonic states are optically active and can be directly probed by interband optical measurements. In reality, DQW's can only be nominally symmetric: the well widths can be effectively different, due to fluctuations and the wells depths can differ from each other, due to a slight difference in the composition of the semiconductor materials which occurs during the growth. Thus, it is important to study to what extent small deviations from the exactly symmetric case lead to observable consequences (see, also, Ref. 13).

Figure 4 shows the absorption spectra for a slightly asymmetric DQW with different In contents $x_1=0.194$ and $x_2=0.196$ (still we use the same widths $L_1=L_2=L_b=60$ Å). For such a difference between x_1 and x_2, the well depths for electrons (holes) differ by 2.4 meV (less than 1 meV). This difference is enough, however, for the excitons developed from the antisymmetric states to gain comparatively large oscillation strengths (Fig. 4). Broadening of lines[10] does not allow experimentalists, however, to discriminate such lines.

C. Transition energies and oscillator strengths in electric fields $\mathcal{E}$

Energies and oscillator strengths of excitonic peaks in $GaAs/Ga_xAl_{1-x}As$ DQW's, as a function of applied external voltage have been studied at $B=0$ both experimentally[2,3,5,7] and theoretically (see Ref. 13 and the references cited therein). The main qualitative feature in electric fields is that

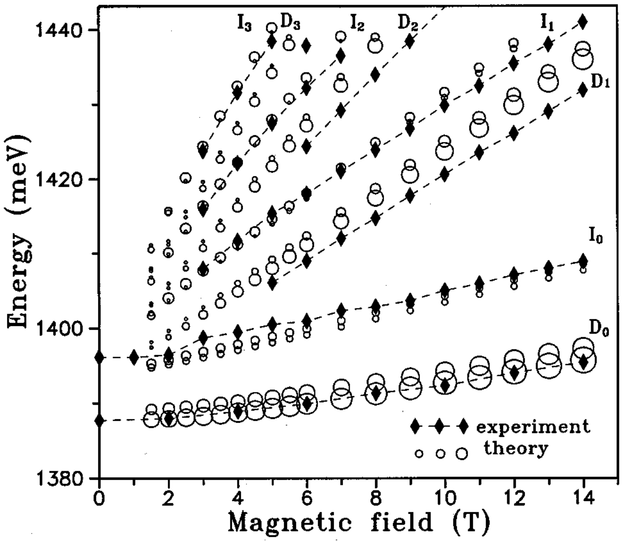

FIG. 4. Same as in Fig. 3 for a slightly asymmetric DQW with $x_1=0.194$, $x_2=0.196$ ($L_1=L_2=L_b=60$ Å). In contrast to a symmetric case, additional optically active states (evolved from antisymmetric excitons) appear in the spectrum.

from the symmetric and antisymmetric indirect excitons having zero dipole moments, two indirect excitons develop: the excited state I^+ and the lower-lying state I^-. These have large dipole moments $d_z^{\pm}\sim\mp e(L_b+L_1)$. At sufficiently high $\mathscr{E}$, after the anticrossings with the two direct excitons $D^{\pm}$ (which also experience a splitting in $\mathscr{E}$), it is the indirect I^- exciton which becomes the ground state.

In quantizing magnetic fields B, a number of higher-lying excitonic states $I_n^{\pm}$ belonging to higher Landau levels are resolved experimentally in $In_xGa_{1-x}As/GaAs$ DQW's.[10] At a fixed B, with increasing $\mathscr{E}$, many anticrossings occur in the spectra between I_n^- (I_n^+) indirect excitons and direct excitons D_k, with $k\leqslant n$ ($k>n$). Figure 5 shows the results of our theory for the evolution of excitonic lines, with $\mathscr{E}$ at a fixed magnetic field $B=10$ T. In theoretical calculations, a symmetric DQW structure was considered. Theoretical results are in reasonable agreement with the experiment,[10] for the behavior and positions of the D_0 lines. Those for the energies of the higher-lying $n=1$ excitons do not coincide too well with the experiment: the theory gives too larger splittings between free Landau levels (see the discussion in Sec. III B). Thus, the particular values of the electric fields $\mathscr{E}$ at which (anti)crossings occur are affected (see, for example, the I_0^+-I_1^- anticrossing in Fig. 5). For this particular anticrossing, I_0^+-I_1^-, theory predicts extremely small interaction —the energy splitting and redistribution of oscillator strengths — between these terms. This is because the two excitons are essentially indirect in nature and have rather different spatial properties (are strongly orthogonal). Theory predicts, at a higher $\mathscr{E}$, a larger interaction between the I_0^+ and D_1 excitons, which are less orthogonal. The enhancement of the I_0^+ oscillator strength is visible in Fig. 5. This is in complete agreement with the experiment (see Fig. 3 of Ref. 10).

It is also seen from Fig. 5 that, at intermediate electric fields $\mathscr{E}=3-10$ kV/cm, the theory gives smaller splittings between the two D_0 lines than in the experiment. To check if

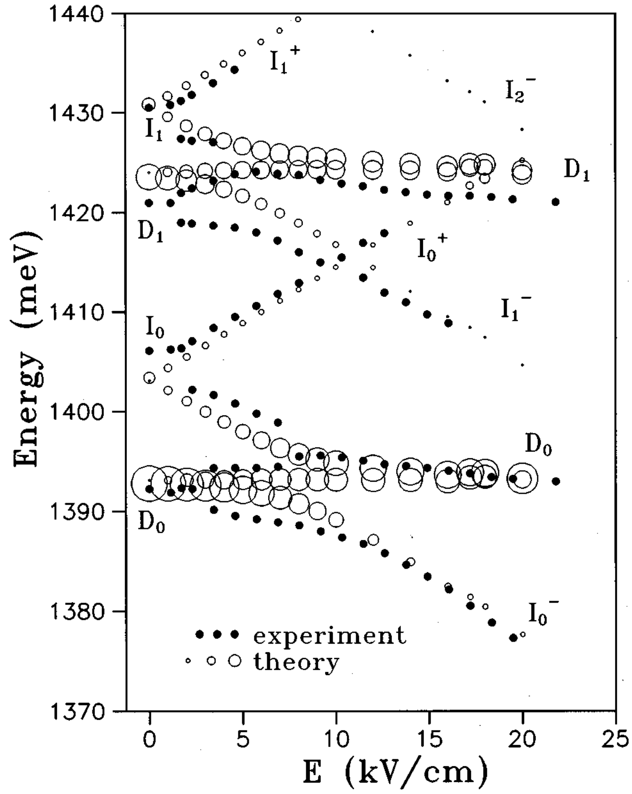

FIG. 5. The comparison of theory for the electric field dependence of the exciton transition energies at $B=10$ T, for the symmetric DQW (the parameters are the same as in Fig. 3), with the experimental data (the full symbols) taken from Ref. 10. The area of open circles is proportional to the transition oscillator strength.

small asymmetry between the two wells can account for such a discrepancy, we calculated spectra for a slightly asymmetric DQW with the same parameters considered earlier in Sec. III B ($x_1=0.194$, $x_2=0.196$). The results (Fig. 6) show a better agreement with the experiment, which probably indicates small asymmetry in the DQW structures experimentally studied in Ref. 10.

It is interesting to follow how excitonic spectra evolve with the magnetic field B at different fixed electric fields $\mathscr{E}$. Figure 7 shows such an evolution for $\mathscr{E}=17.2$ kV/cm. At this sufficiently high electric field and at low magnetic fields, the low-lying excitons are I^- excitons, then a group of the direct excitons D lies and, next, the group of the indirect excitons I^+ appears. With increasing B, Landau fan structure develops, which leads to a number of anticrossings in the spectra. Again, D-I_n^- and D-I_n^+ anticrossings lead to visible redistribution of the oscillator strengths (the enhancement of indirect lines), while virtually there is no interaction I^--I^+ between indirect excitons. The theory is in excellent agreement with the experiment for the D_0 and I_0^- transitions. There is also satisfactory agreement for the (experimentally resolved) direct lines D_n, with $n=1,2,3,4$.

Figure 8 shows the results of theory for the evolution of the spectra with B at a lower fixed electric field $\mathscr{E}=7$ kV/cm. Here, at low magnetic fields, the I_0^- exciton is the ground state, as in Fig. 7. However, with increasing B, in complete agreement with the experiment,[10] a rearrangement of the nature of the ground state takes place: it is the direct

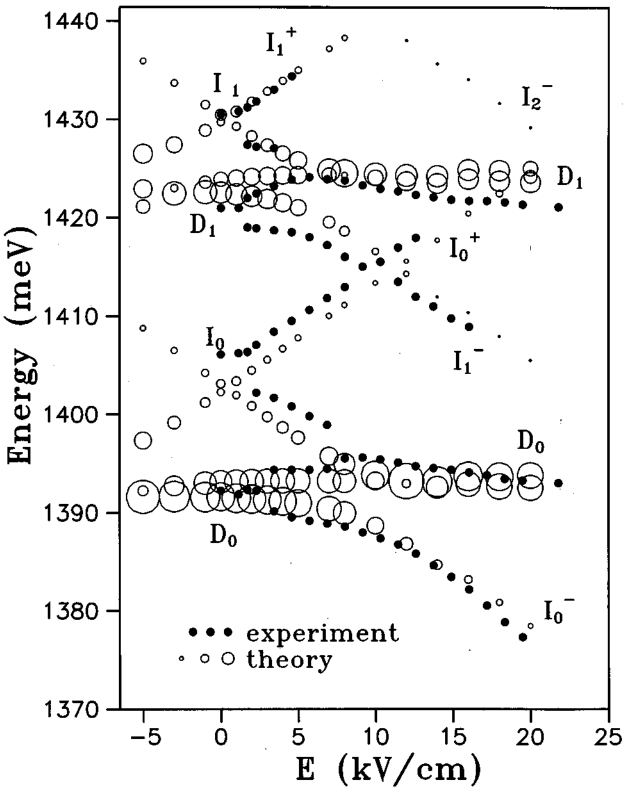

FIG. 6. Same as in Fig. 5, for slightly asymmetric DQW (the parameters are listed in caption to Fig. 4).

exciton D_0, which becomes the ground state at $B \gtrsim 10$ T. This is due to the fact that the negative contribution of the e-h interaction to the total exciton energy is much more enhanced for the D_0 excitons in B than for the I_0 (cf. Fig. 1). (Note that this indirect-direct crossover is much more

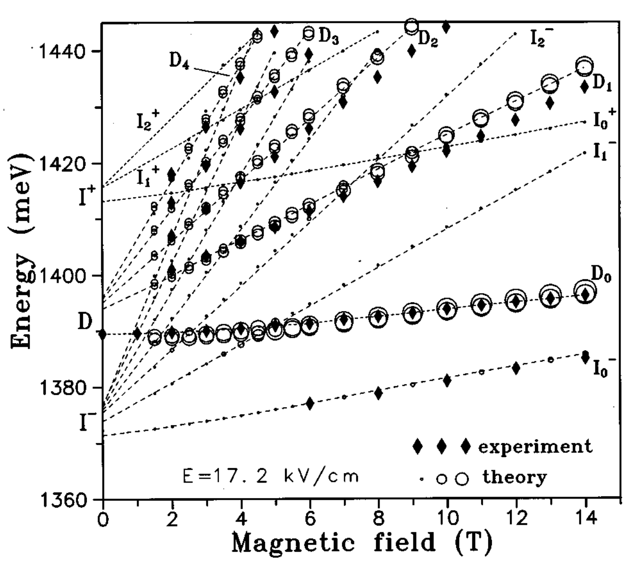

FIG. 7. The evolution of excitonic transition energies and oscillator strengths with the magnetic field B at $\mathcal{E}=17.2$ kV/cm, for the symmetric DQW, with the parameters the same as in Fig. 3. Full symbols are experimental data taken from Ref. 10. The area of open circles is proportional to the transition oscillator strength.

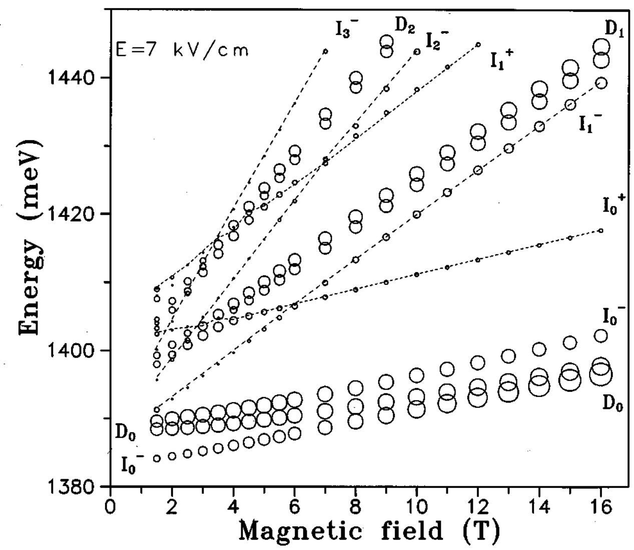

FIG. 8. The evolution of excitonic transition energies and oscillator strengths with the magnetic field B at $\mathcal{E}=7$ kV/cm, for the symmetric DQW, with the parameters the same as in Fig. 3.

smooth than that obtained[14] for a DQW, with changing parameters of an additional lateral quantization produced by a quantum dot.)

A number of other anticrossings are also evident in Fig. 8. It is of interest that here, not only the oscillator strengths are redistributed as in Fig. 7, but also a complicated behavior (splittings of lines) is present. This is explained by the fact that at lower electric fields direct and indirect states are less orthogonal (e.g., direct states D_0 contain sufficient admixture of indirect states I_0). This makes the interaction between states stronger. As an example, the theory predicts even a small I_1^--I_0^+ interaction.

D. Symmetric-antisymmetric excitonic splitting

The splittings between the symmetric and antisymmetric (both direct and indirect) excitonic states $\Delta_{\rm XD}^{(n)}$, $\Delta_{\rm XI}^{(n)}$ belonging to Landau levels $n=0,1$ are shown in Figs. 1, 2. Typically, $\Delta_{\rm XD}^{(n)} \simeq \Delta_{\rm XI}^{(n)} \simeq 0.4$ meV, i.e., are very small and much smaller than the electron single-particle symmetric-antisymmetric splitting $\Delta_e \simeq 4.9$ meV (while, $\Delta_{\rm XD(I)} \simeq \Delta_h$). It is also seen that $\Delta_{\rm XD}^{(n)}$, $\Delta_{\rm XI}^{(n)}$ (with fixed n) decrease when B is increased, and, at fixed B, $\Delta_X^{(0)} < \Delta_X^{(1)}$ [also, $\Delta_X^{(1)} < \Delta_X^{(2)}$ (not shown)]. Thus, there is a tendency that the *excitonic* symmetric-antisymmetric splittings become *smaller* with increasing the strength of the e-h interaction. Such a behavior has been also revealed by numeric studies of excitons in DQW's at $B=0$ of other authors (see, in particular, the behavior of the splittings versus lateral confinement at $\mathcal{E}=0$ in Fig. 3 of Ref. 15). Thus, generally, it can be said that the excitonic effects suppress the symmetric-antisymmetric excitonic splittings for wide-barrier DQW's (and coupled symmetric quantum dots). Below, we present a simple physical explanation of this behavior. It can be extended, with modifications due to the Pauli exclusion principle, for spatially symmetric and antisymmetric *electron* states, e.g., in coupled quantum dots.

To make underlying physics more transparent, let us consider the situation in a sufficiently strong magnetic field, so that the symmetric SD_n and antisymmetric AI_n exciton states in the DQW can be constructed out of only the nth Landau levels neglecting admixture of other Landau levels.[35] The wave functions of these states with the appropriate symmetry under inversion $(z_e \to -z_e, z_h \to -z_h)$ and with $\mathbf{K}=0$, up to inessential phase factor, can be written in the form [cf. Eq. (11)]

$$\Phi_S^{(n)}(\rho;z_e,z_h)=\phi_{nn}(\rho)[a_1\zeta_s(z_e)\xi_s(z_h)+a_2\zeta_a(z_e)\xi_a(z_h)], \tag{19}$$

$$\Phi_A^{(n)}(\rho;z_e,z_h)=\phi_{nn}(\rho)[b_1\zeta_s(z_e)\xi_a(z_h)+b_2\zeta_a(z_e)\xi_s(z_h)]. \tag{20}$$

Then, for a DQW, we deal with the two independent two-level problems, i.e., the coefficients $\{a_1,a_2\}$ and $\{b_1,b_2\}$ are to be determined from 2×2 secular equations, each giving the two sets of solutions: the symmetric direct (indirect) excitons $E_{\rm SD}^{(n)}$, $\Phi_{\rm SD}^{(n)}$ [$E_{\rm SI}^{(n)}$, $\Phi_{\rm SI}^{(n)}$] and the antisymmetric direct (indirect) excitons $E_{\rm AD}^{(n)}$, $\Phi_{\rm AD}^{(n)}$ [$E_{\rm AI}^{(n)}$, $\Phi_{\rm AI}^{(n)}$]. We are interested in the excitonic symmetric-antisymmetric splittings $\Delta_{\rm XD}^{(n)}=E_{\rm AD}^{(n)}-E_{\rm SD}^{(n)}$ and $\Delta_{\rm XI}^{(n)}=E_{\rm SI}^{(n)}-E_{\rm AI}^{(n)}$ in the two-level approximation $\Delta_{\rm XD}^{(n)}=\Delta_{\rm XI}^{(n)}\equiv\Delta_X^{(n)}$. To reveal what physical factors are responsible for $\Delta_X^{(n)}$, let us go over to the combinations

$$\zeta_{1(2)}(z_e)=\frac{\zeta_s(z_e)\pm\zeta_a(z_e)}{\sqrt{2}}, \tag{21}$$

$$\xi_{1(2)}(z_h)=\frac{\xi_s(z_h)\pm\xi_a(z_h)}{\sqrt{2}}. \tag{22}$$

Under the appropriate choice of the phase factors of $\zeta_s(z_e)$, $\zeta_a(z_e)$ and $\xi_s(z_h)$, $\xi_a(z_h)$, the (real) wave functions $\zeta_1(z_e)$ [$\xi_1(z_h)$] describe the electron (hole), which is predominantly in the left (1) well, while $\zeta_2(z_e)$ [$\xi_2(z_h)$] — in the right (2) well. Solving the corresponding secular equation, in the representation (21), (22), we obtain a number of the Coulomb matrix elements,

$$W_{kl}^{k'l'}=\int_{-\infty}^{\infty}dz_e\int_{-\infty}^{\infty}dz_h\int d^2\rho\,\frac{e^2}{\epsilon\sqrt{\rho^2+(z_e-z_h)^2}}\times\phi_{nn}^2(\rho)\zeta_k(z_e)\zeta_{k'}(z_e)\xi_l(z_h)\xi_{l'}(z_h), \tag{23}$$

where the indices k,k' $(l,l')=1,2$ describe the electron (hole) either in the left (1) or the right (2) well; here and below, we omit for brevity the Landau-level index n.

Let us clarify the physical meaning of different matrix elements: (i) $W_{11}^{11}=W_{22}^{22}\equiv W_D$ is the binding energy of the direct (intrawell) exciton, (ii) $W_{12}^{12}=W_{21}^{21}\equiv W_I$ is the binding energy of the indirect (interwell) exciton; this is correct up to exponentially small terms which can be neglected (as it will be clear below). (iii) $W_{11}^{21}=W_{21}^{11}=W_{22}^{12}=W_{12}^{22}\equiv W_e$ correspond to a process in which, due to the Coulomb e-h interaction, the initial direct state (in either the left or the right well) transforms into the indirect exciton, due to the *electron* transfer to the adjacent well; (iv) $W_{11}^{12}=W_{12}^{11}=W_{22}^{21}=W_{12}^{11}\equiv W_h$ is similar to (iii), but correspond to the *hole* transfer. (v) $W_{11}^{22}=W_{22}^{11}\equiv W_{\rm eh}$ describe the process in which the initial direct exciton in the right (left) well transforms to the direct exciton in the left (right) well, due to a *simultaneous* transfer of e and h; (vi) $W_{12}^{21}=W_{21}^{12}$ describes the transformation between the two indirect excitons, also with a simultaneous transfer of e and h; clearly, $W_{12}^{21}=W_{11}^{22}=W_{\rm eh}$.

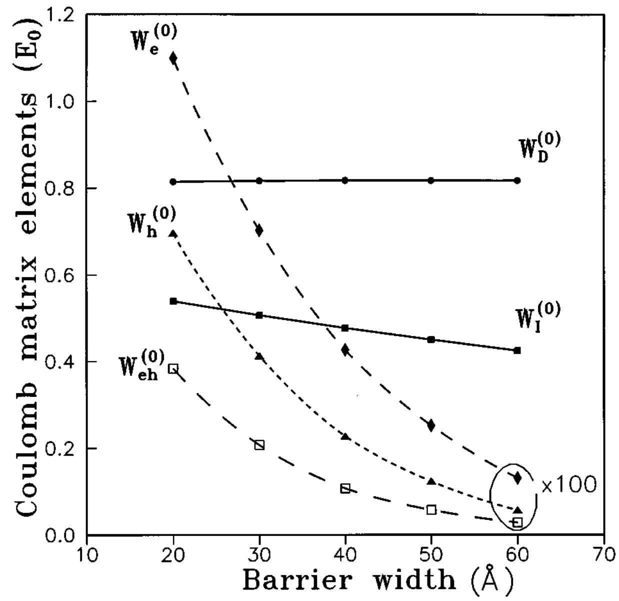

FIG. 9. The dependences of the Coulomb matrix elements Eq. (23) (given in units of $E_0=\sqrt{\pi/2}e^2/\epsilon\ell_B\sim B^{1/2}$, the binding energy of the strictly 2D magnetoexciton in the zero Landau level, Ref. 25) for a symmetric $In_{0.2}Ga_{0.8}As/GaAs$ DQW, with $L_1=L_2=60$ Å versus the barrier width L_b. The deviation of $W_D^{(0)}$ from E_0 is connected with a quasi-two-dimensional character of the wave functions involved into Eq. (23).

When the wells widths $L_1=L_2$ are kept constant, these matrix elements behave rather differently with increasing the barrier width L_b (see Fig. 9 for $n=0$ case). W_D is practically independent on L_b (in fact, there is a very slight initial increase of W_D with L_b, due to a decreasing leakage of the wave function through the barrier). W_I decreases, for the case considered, superlinearly with L_b (following the increasing separation between e and h in the indirect exciton). The matrix elements W_e, W_h and $W_{\rm eh}$ fall off exponentially with L_b. Figure 9 shows that $W_e > W_h$, which is due to the fact that the barrier is more transparent for a lighter electron; the matrix element $W_{\rm eh}$ is the smallest describing the two-particle e-h transfer.

From the solutions of the secular equations, in the wide-barrier regime, when $\delta E_{\rm DI}\equiv W_D-W_I\gg\Delta_e,\Delta_h$, we obtain the excitonic symmetric-antisymmetric splitting in the form

$$\Delta_X=2W_{\rm eh}+\frac{1}{\delta E_{\rm DI}}[\Delta_e\Delta_h+2(\Delta_eW_h+\Delta_hW_e)+4W_eW_h]+O\left(\frac{\Delta_e^3\Delta_h}{\delta E_{\rm DI}^3}\right) \tag{24}$$

[the leading remaining term is given for a situation of interest, when $\delta E_{\rm DI} > \Delta_e \gg \Delta_h$, and Eq. (24) is still applicable]. A physical interpretation of Eq. (24) is that the exciton symmetric-antisymmetric splitting Δ_X is determined, in a wide-barrier regime, by the *two-particle e-h* transfer between the wells.

Indeed, the first term in (24), $2W_{\rm eh}$, describes a *direct* process of a simultaneous *e-h* transfer, due to the Coulomb *e-h* interaction. The remaining set of terms in brackets of Eq. (24) describes all possible different processes of the second order through an intermediate state: The first term, $\Delta_e \Delta_h / \delta E_{\rm DI}$, corresponds to a successive transfer of e and h, due to the single-particle tunnelings through the barrier with a formation of the intermediate state. [Which is the indirect (direct) exciton for the direct (indirect) exciton splitting. Thus, the energy denominator $\delta E_{\rm DI}^{-1}$ appears.] Analogously, the terms $\sim(\Delta_e W_h + \Delta_h W_e)/\delta E_{\rm DI}$ describe the process in which one particle is transferred, due to the single-particle tunneling, while the other one — by the Coulomb effects. Finally, the term $\sim W_e W_h / \delta E_{\rm DI}$ is the successive transfer of e and h entirely due to the Coulomb effects. Considering the typical values for Δ_e, Δ_h and the different Coulomb matrix elements [Fig. (9)], we conclude that $\Delta_X \simeq \Delta_e \Delta_h / \delta E_{\rm DI}$, while other terms are negligibly small.[36] (This formula, at high B, is in good quantitative agreement with the results of full numerical calculations of $\Delta_X^{(n)}$.) It is now clear that it is due to the denominator $\delta E_{\rm DI}^{-1}$ that the excitonic effects *suppress* Δ_X.

One may argue that for semiconductor structures, the long-range (dipole) part[23] of the *e-h exchange* interaction [though containing, at high B, a small factor $\sim(a_0/\ell_B)^2 \ll 1$ in comparison with the direct *e-h* interaction; a_0 is the lattice constant] would give the matrix elements coupling the wells which fall off algebraically rather than exponentially. It can be shown, however, that the corresponding matrix elements (for singlet magnetoexcitons, e.g., in the zero Landau level), analogous to (23) but between states with a finite exciton momentum $\mathbf{K}$, are

$$\tilde{W}_{kl}^{k'l'}(\mathbf{K}) = \frac{|\boldsymbol{\mu}\cdot\hat{K}|^2}{\epsilon \ell_B^3} K \ell_B \exp\left(-\tfrac{1}{2}K^2 \ell_B^2\right) \times \int_{-\infty}^{\infty} dz_1 \int_{-\infty}^{\infty} dz_2 \exp(-K|z_1 - z_2|) \times \zeta_k(z_1)\,\xi_{l'}(z_1)\,\xi_l(z_2)\,\zeta_{k'}(z_2), \qquad (25)$$

where $\hat{K} = \mathbf{K}/|\mathbf{K}|$ and $\boldsymbol{\mu} = e\int d^3 r u_c^*(\mathbf{r})\mathbf{r} u_v(\mathbf{r})$ is the interband dipole matrix element (cf. Ref. 37). It is seen from Eq. (25) that for $\mathbf{K}=0$ excitons, the matrix elements vanish (cf. with the case of spherically symmetric quantum dots[38]). Consequently, there is no contribution of the long-range part of the exchange interaction to Δ_X. When a finite value of the momentum of excitons involved in the absorption or emission of light is taken into account, the long-range exchange interactions lead to a broadening rather than to a splitting of lines.[37]

IV. CONCLUSIONS

We have theoretically considered the binding energies and optical properties of direct and indirect excitons in $In_xGa_{1-x}As/GaAs$ coupled DQW's with simple valence band in the perpendicular quantizing magnetic field B and electric field $\mathcal{E}$. The theory uses the expansion of s-excitonic states in terms of orthonormal wave functions of noninteracting quasi-two-dimensional *e-h* pairs in a DQW and in the magnetic and electric fields. Such approach can be also considered as a variational treatment of the first several eigenstates and gives the upper bounds for the ground and excited states. Theoretical results for the energies and oscillator strengths are in overall good agreement with the experimental data of Ref. 10. Also, the theory describes well both the direct-indirect crossover (induced by increasing $\mathcal{E}$ at a fixed B) and the indirect-direct crossover (induced by increasing B at a sufficiently strong fixed $\mathcal{E}$). Theoretical overestimations of the slopes of lines in the Landau fans at high fields B suggest that possibly the used electron and hole effective masses are too light and/or the parameters describing heavy-hole nonparabolicity (see Appendix) are underestimated.

We theoretically considered the nature of the *excitonic* symmetric-antisymmetric splittings Δ_X in symmetric DQW's. It was shown that, in a wide-barrier regime, a *two-particle e-h* transfer between the two wells forming a DQW structure determines Δ_X. An explanation was given to a fact that the excitonic effects suppress Δ_X. This picture is also relevant, with modifications due to the Pauli exclusion principle, for spatially symmetric and antisymmetric electron states, e.g., in coupled quantum dots.

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APPENDIX: NONPARABOLICITY

In the presence of nonparabolicity, which is mainly due to the effect of the split-off light-hole subband, the in-plane energy-dependent density of state heavy-hole mass can be described as $m_{h\parallel}(\varepsilon) = m_{h0}(1 + 2C\varepsilon/\Delta)$, where m_{h0} is the mass at zero energy, Δ is the energy splitting (in meV) between the heavy- and light-hole subbands, and C is constant of order unity (see Ref. 20 and the references therein); strictly speaking, this formula is applicable when $\varepsilon \ll \Delta$. We adopt the following values for these parameters: $m_{h0} = 0.08 m_0$, $\Delta = 33$ meV, $C = 1.7$.[20] To obtain the positions of Landau levels, we first determine the energies ε_n, $n = 0,1,2,\ldots$ separating in ε space the regions, which contain $\mathcal{N}_0 = (2\pi\ell_B^2)^{-1}$ states (i.e., the number of states in one Landau level per unit area)

$$\varepsilon_0 = 0, \qquad (A1)$$

$$\varepsilon_{n+1}=\frac{[1+2A(\frac{1}{2}A\varepsilon_n^2+\varepsilon_n+\hbar\omega_0)]^{1/2}-1}{A}, \qquad (A2)$$

where $A\equiv 2C/\Delta$ and $\hbar\omega_0=\hbar eB/m_{h0}c$ is the bare cyclotron energy. The positions of free heavy-hole Landau levels in the presence of nonparabolicity are then determined as

$$\varepsilon_{ch}(n)=\frac{\varepsilon_{n+1}+\varepsilon_n}{2}, \quad n=0,1,2,\ldots. \qquad (A3)$$

Clearly, such a consideration may be applicable only below the region of energies where crossings with the light-hole terms occur.

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