Abstract. Fuzzy Lyapunov Synthesis is extended for the design of type-1 fuzzy logic controllers for an output regulation problem for a servomechanism with backlash. The problem in question is to design a feedback controller so as to obtain the closed-loop system in which all trajectories are bounded and the load of the driver is regulated to a desired position while also attenuating the influence of external disturbances. Provided the servomotor position is the only measurement available for feedback, the proposed extension is far from trivial because of nonminimum phase properties of the system. Performance issues of the fuzzy regulator constructed are illustrated in an experimental study.

Keywords: Fuzzy Control, Fuzzy Lyapunov Synthesis, Stability, Nonsmooth systems.

1 Introduction

A major problem in control engineering is a robust feedback design that asymptotically stabilizes a plant while also attenuating the influence of parameter variations and external disturbances. In the last decade, this problem was heavily studied and considerable research efforts have resulted in the development of systematic design methodologies for nonlinear feedback systems. A survey of these methods, fundamental in this respect is given in [12].

The design of Fuzzy Logic Systems (FLSs) is a heavy task that FLSs practitioners face every time that they try to use Fuzzy Logic (FL) as a solution to some problem, the design of FLSs implies at least two stages: design of the rule-base and design of the Membership Functions (MFs).

There have been some publications in the design of Type-1 Fuzzy Logic Systems (T1FLS), for example [10] presents Genetic Algorithms (GA) as an optimization method for control parameters, they optimize parameters of the closed-loop system but not of the T1FLS. In [15] GAs are used to optimize all the parameters of a T1FLS. In [19] a hybridization of Neural Networks and GAs are presented to optimize a T1FLS. A Hierarchical Genetic Algorithms is proposed in [2] to optimize rules and MFs parameters of a T1FLS.
In the present paper, the output regulation problem is studied for an electrical actuator consisting of a motor part driven by a DC motor and a reducer part (load) operating under uncertainty conditions in the presence of nonlinear backlash effects and external disturbances. The objective is to drive the load to a desired position while providing the roundedness of the system motion and attenuating external disturbances. Because of practical requirements (see e.g., [14]), the motor’s angular position is assumed to be the only information available for feedback.

This problem was first reported in [1], where the problem of controlling nonminimum phase systems was solved by using nonlinear $H_{\infty}$ control, but the reported results do not provide robustness evidence. In [5], authors report a solution to the regulation problem using a T1FLS. In [6] and [8], authors report solutions using Type-2 Fuzzy Logic Systems Controllers, and making a genetic optimization of the membership function’s parameters, but do not specify the criteria used in the optimization process and GA design. In [7], a comparison of the use of GAs to optimize Type-1 and Type-2 FLS Controllers is reported, but a method to achieve this optimization is not provided. In [9], is presented the designing of the Type-1 FLS following the Fuzzy Lyapunov Synthesis [18] to obtain a FLS for the output regulation of a servomechanism with backlash, but the reported results do not provide robustness evidence in the presence of external disturbances.

In this paper, we extend the results from [9] by designing a Type-1 FLS (Fuzzy Logic Controller - FLC-) extending the Fuzzy Lyapunov Synthesis, a concept that is based on the Computing with Words [20][23] approach of the Lyapunov Synthesis [13], in order to provide robustness evidence in the presence of external disturbances. Additionally, this paper presents a new dynamical model of the case of study, adding a new term that represents the viscous friction present in the mechanism, and results showing the robustness of the designed controllers including noise in the experiments, we also present some measures that allow to make a comparison between the performance of the fuzzy controllers in the experimental results.

The contributions of this paper are as follows:

- We propose a systematic methodology to design Type-1 FLCs via the Fuzzy Lyapunov Synthesis.
- With the proposed methodology we obtain Type-1 FLCs, which due to the nature of designing method, are stable.
- We solve the output regulation problem for an electrical actuator operating under uncertainty conditions in the presence of nonlinear backlash effects and external disturbances.
- We show that the resulting FLSs are so robust that they can deal with the proposed problem.

The paper is organized as follows. The dynamic model of the nonminimum phase servomechanism with nonlinear backlash and the problem statement are presented in Sections 2 and 3, respectively. Section 4 addresses fuzzy sets and systems theory. The design of Fuzzy Logic Controllers using the Fuzzy Lyapunov Synthesis is presented in Section 5. The numerical simulations for the designed FLSs are presented in Section 6. Conclusions are presented in Section 7.

2 Dynamic model

The dynamic model of the angular position $q_i(t)$ of the DC motor and the $q_o(t)$ of the load are given according to

$$J_0^{-1}\ddot{q}_0 + f_0^{-1}\ddot{q}_0 = T + w_0,$$
$$J_i\ddot{q}_i + f_i\dot{q}_i + T = \tau_m + \Gamma_c(q_i) + w_i,$$

hereafter, $J_0$, $f_0$, $\ddot{q}_0$, and $\dot{q}_0$ are, respectively, the inertia of the load and the reducer, the viscous output friction, the output acceleration, and the output velocity. The inertia of the motor, the viscous motor friction, the motor acceleration, and the motor velocity are denoted by $J_i$, $f_i$, $\dot{q}_i$, and $\ddot{q}_i$, respectively. The input torque $\tau_m$ serves as a control action, and $T$ stands for the transmitted torque. $\Gamma_c(q_i)$ is the Coulomb friction torque which affects the motor dynamics. The external disturbances $w_i(t)$, $w_0(t)$ have been introduced into the driver equation (1) to account for destabilizing model discrepancies due to hard-to-model nonlinear phenomena, such as friction and backlash.

The transmitted torque $T$ through a backlash with amplitude $j$ is typically modeled by a dead-zone characteristic [p. 7, 22]:

$$T(\Delta q) = \begin{cases} 0, & \text{if } |\Delta q| \leq j \\ K\Delta q - K j(\Delta q), & \text{otherwise} \end{cases}$$
with
\[ \Delta q = q_i - Nq_0, \quad (3) \]
where \( K \) is the stiffness, and \( N \) is the reducer ratio. Such a model is depicted in Fig. 1. Provided the servomotor position \( q_i(t) \) is the only available measurement on the system, the above model (1)-(3) appears to be non-minimum phase because along with the origin the unforced system possesses a multivalued set of equilibria \( (q_i, q_0) \) with \( q_i = 0 \) and \( q_0 \in [-j, j] \).

![Fig. 1. Dead-zone model of backlash and its monotonic approximation](image)

To avoid dealing with a non-minimum phase system, we replace the backlash model (2) with its monotonic approximation:
\[ T = K \Delta q - K \eta(\Delta q), \quad (4) \]
where
\[ \eta = -2j \frac{\exp\left(-\frac{\Delta q}{j}\right)}{1+\exp\left(-\frac{\Delta q}{j}\right)}. \quad (5) \]

The present backlash approximation is inspired from [21]. Coupled to the drive system (1) subject to motor position measurements, it is subsequently shown to continue a minimum phase approximation of the underlying servomotor, operating under uncertainties \( w_i(t) \) \( w_0(t) \) to be attenuated. As a matter of fact, these uncertainties involve discrepancies between the physical backlash model (2) and its approximation (4) and (5).

### 3 Problem Statement

To formally state the problem, let us introduce the state deviation vector \( x = [x_1, x_2, x_3, x_4]^T \) with
\[ x_1 = q_0 - q_d, \]
\[ x_2 = \dot{q}_0, \]
\[ x_3 = q_i - Nq_d, \]
\[ x_4 = \dot{q}_i, \]
where \( x_1 \) is the load position error, \( x_2 \) is the load velocity, \( x_3 \) is the motor position deviation from its nominal value, and \( x_4 \) is the motor velocity. The nominal motor position \( Nq_d \) has been pre-specified in such a way to guarantee that \( \Delta q = \Delta x \), where \( \Delta x = x_3 - Nx_1 \).

Then, system (1)-(5), represented in terms of the deviation vector \( x \), takes the form
\[ \dot{x}_1 = x_2, \]
\[ \dot{x}_2 = J_{0}^{-1}\left[-KNx_3 - KN^2x_1 - f_0x_2 + KN\eta(\Delta x) + w_0\right], \]
\[ \dot{x}_3 = J_{0}^{-1}\left[\sigma + KNx_1 - Kx_3 - f_0x_4 + KN\eta(\Delta x) + \Gamma_0(\dot{q}_i) + w_0\right]. \quad (6) \]

The zero dynamics
\[ \dot{x}_1 = x_2, \]
\[ \dot{x}_2 = J_{0}^{-1}\left[-KN^2x_1 - f_0x_2 + KN\eta(-Nx_1)\right], \quad (7) \]
of the undisturbed version of system (6) with respect to the output
\[ y = x_3, \quad (8) \]
is formally obtained ([11] for details) by specifying the control law that maintains the output identically to zero.

The objective of the Fuzzy Control output regulation of the nonlinear driver system (1) with backlash (4) and (5), is thus to design a Fuzzy Controller so as to obtain the closed-loop system in which all these trajectories are bounded and the output \( q_0(t) \) asymptotically decays to a desired position \( q_d \) as \( t \to \infty \) while also attenuating the influence of the external disturbances \( w_i(t) \) and \( w_0(t) \).

### 4 Type-1 Fuzzy Sets and Systems

A Type-1 Fuzzy Set (T1FS), denoted \( A \), is characterized by a Type-1 membership function (T1MF) \( \mu_A(z) \) [3], where \( z \in \mathbb{Z} \), and \( \mathcal{F} \) is the domain of definition of the variable, i.e.,
where \( \mu(z) \) is called a Type-1 membership function of the T1FS \( A \). The T1MF maps each element of \( X \) to a membership grade (or membership value) between 0 and 1.

Type-1 Fuzzy Logic Systems (T1FLS) - also called Type-1 Fuzzy Inference Systems (T1FIS), are both intuitive and numerical systems that map crisp inputs into a crisp output. Every T1FIS is associated with a set of rules with meaningful linguistic interpretations, such as:

\[
R^I: \text{IF } y \text{ is } A^I_y \text{ AND } \dot{y} \text{ is } A^I_{\dot{y}} \text{ THEN } u \text{ is } B^I_u, \tag{10}
\]

which can be obtained either from numerical data, or from experts familiar with the problem at hand. In particular (10) is in the form of Mamdami fuzzy rules [16]—[17]. Based on this kind of statements, actions are combined with rules in an antecedent/consequent format, and then aggregated according to approximate reasoning theory to produce a noninear mapping from input space \( U = U_1 \times U_2 \times \cdots \times U_m \) to output space \( W \), where \( A^I_k \subset U_k, k = 1,2, \ldots, m \) and the output linguistic variable is denoted by \( \tau_m \).

A T1FIS consists of four basic elements (see Fig. 2: the Type-1 fuzzifier, the Type-1 fuzzy rule-base, the Type-1 inference engine, and the Type-1 defuzzifier. The Type-1 fuzzy rule-base is a collection of rules in the form of (10), which are combined in the Type-1 inference engine, to produce a fuzzy output. The Type-1 fuzzifier maps the crisp input into a T1FS, which are subsequently used as inputs to the Type-1 inference engine, whereas the Type-1 defuzzifier maps the T1FSs produced by the Type-1 inference engine into crisp numbers.

\[
A = \{ (z, \mu(z)) | \forall z \in Z \}, \tag{9}
\]

\[
\tau_m = u_{\text{COA}} = \int \frac{\mu_A(u) \, du}{\int \mu_A(u) \, du}, \tag{11}
\]

where \( \mu_A(u) \) is the aggregated output of the T1MF. This is the most widely adopted defuzzification strategy, which is reminiscent of the calculation of expected values in probability distributions.

5 Design of the Fuzzy Logic Controllers

To apply the Fuzzy Lyapunov Synthesis, we assume the following:

- The system may have really two degrees of freedom referred to as \( x_1 \) and \( x_2 \) respectively. Hence by (6), \( \dot{x}_1 = x_2 \).
- The states \( x_1 \) and \( x_2 \) are the only measurable variables.
- The exact equations (1)-(5) are not necessarily known.
- The angular acceleration \( \dot{x}_2 \) is proportional to \( \tau_m \), that is, when \( \tau_m \) increases (decreases), \( \dot{x}_2 \) increases (decreases).
- The initial conditions \( x(0) = (x_1(0), x_2(0))^T \) belong to the set \( \mathbb{N} = \{ x \in \mathbb{R}^2 : \| x - x^* \| \leq \varepsilon \} \) where \( x^* \) the equilibrium point is.

The control objective is to design the rule-base as a fuzzy controller \( \tau_m = \tau_m(x_1, x_2) \) to stabilize the system (1)-(5).

Theorem 1 that follows establishes conditions that help in the design of the fuzzy controller to ensure asymptotic stability. The proof can be found in [13].

**Theorem 1 (Asymptotic stability [13])** Consider the nonlinear system (1)-(5) with an equilibrium point at the origin, i.e., \( f(0) = 0 \). Let \( x \in \mathbb{N} \), then the origin is asymptotically stable if there exists a scalar Lyapunov function \( V(x) \) with continuous partial derivatives such that

- \( V(x) \) is positive definite
- \( \dot{V}(x) \) is negative definite

The fuzzy controller design proceeds as follows. Let us introduce the Lyapunov function candidate

\[
V(x_1, x_2) = \frac{1}{2} (x_1^2 + x_2^2), \tag{12}
\]

which is positive-definite and radially unbounded function. The time derivative of \( V(x_1, x_2) \) results in:
Designing Type-1 Fuzzy Logic Controllers via Fuzzy Lyapunov Synthesis for...

To guarantee stability of the equilibrium point $(x_1, x_2) = (x_1, x_2) = (x_1 x_2 + x_2 x_2)$, we require that:

$$x_1 x_2 + x_2 x_2 \leq 0. \quad (14)$$

We can now derive sufficient conditions so that inequality (14) holds: If $x_1$ and $x_2$ have opposite signs, then $x_1 x_2 < 0$ and (14) will hold if $x_1 x_2 = 0$; if $x_1$ and $x_2$ are both positive, then (14) will hold if $x_1 x_2 \leq -x_1$; if $x_1$ and $x_2$ are both negative, then (14) will hold if $x_1 x_2 \geq -x_1$

We can translate these conditions into the following fuzzy rules:

- If $x_1$ is positive and $x_2$ is positive then $\dot{x}_2$ must be negative big.
- If $x_1$ is negative and $x_2$ is negative then $\dot{x}_2$ must be positive big.
- If $x_1$ is positive and $x_2$ is negative then $\dot{x}_2$ must be zero.
- If $x_1$ is negative and $x_2$ is positive then $\dot{x}_2$ must be zero.

However, using our knowledge that $\dot{x}_2$ is proportional to $u$, we can replace each $\dot{x}_2$ with $u$ to obtain the following fuzzy rule-base for the stabilizing controller:

- If $x_1$ is positive and $x_2$ is positive then $u$ must be negative big.
- If $x_1$ is negative and $x_2$ is negative then $u$ must be positive big.
- If $x_1$ is positive and $x_2$ is negative then $u$ must be zero.
- If $x_1$ is negative and $x_2$ is positive then $u$ must be zero.

This fuzzy rule-base can be represented as in Table 1.

<table>
<thead>
<tr>
<th>No.</th>
<th>Error</th>
<th>Change of Error</th>
<th>Control</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>positive</td>
<td>positive</td>
<td>negative big</td>
</tr>
<tr>
<td>2</td>
<td>negative</td>
<td>negative</td>
<td>positive big</td>
</tr>
<tr>
<td>3</td>
<td>positive</td>
<td>negative</td>
<td>zero</td>
</tr>
<tr>
<td>4</td>
<td>negative</td>
<td>positive</td>
<td>zero</td>
</tr>
</tbody>
</table>

6 Results

To perform simulations we use the dynamical model (1)-(5) of the experimental testbed installed in the Robotics & Control Laboratory of CITEDI-IPN (see Fig. 4, which involves a DC motor linked to a mechanical load through an imperfect contact gear train [1]. The parameters of the dynamical model (1)-(5) are given in Table 2, while $N = 3, j = 0.2 \text{[rad]}, K = 5 \text{[N-m/rad]}, \text{and } \Gamma_i(q_i) = 0.2$. These parameters are taken from the experimental testbed.
Performing a simulation of the closed-loop system (1)-(4) with the Type-1 FLC designed in Section 5 and \( w_0 = w_1 = 0 \), and we have the following results: the control surface of Fig. 5 and we obtain the system's response of Fig. 6, showing that the load reaches the desired position, although we only have feedback from the motor position, \( x_1 \) and \( x_2 \) trajectories are in Fig. 7, in which can seen that \( q_0 \to q_d \) while \( x_0 \to 0 \). In Fig. 8 the trajectories for (12) and (13) are depicted, satisfying Theorem 1.
6.2 Random Noise

In order to verify the robustness of the proposed FLC, we perform a simulation of the closed-loop system (1)-(4) with the Type-1 FLC designed in Section 5 and $w_0 = w_i = \text{random}()$ (in Fig. 9 is depicted the noise signal), and we have the following results: system's response of Fig. 10 showing that the load reaches the desired position although we only have feedback from the motor position, $x_1$ and $x_2$ trajectories are in Fig. 11, in which can be seen that $q_0 \rightarrow q_d$ while $x_0 \rightarrow 0$. In Fig. 12 the trajectories for (12) and (13) are depicted satisfying Theorem 1.
6.3 Sinusoidal Noise

Again, in order to verify the robustness of the proposed FLC, we perform a simulation of the closed-loop system (1)-(4) with the Type-1 FLC designed in Section 5 and $w_0 = w_i = \sin(\omega t)$ with amplitude of 0.1 [rad] and frequency of 10 (in Fig. 13 is depicted the noise signal), and we have the following results: system’s response of Fig. 14 showing that the load reaches the desired position, although we only have feedback from the motor position, $x_1$ and $\dot{x}_1$ trajectories are in Fig. 15, in which can seen that $q_0 \to q_d$ while $x_1 \to 0$. In Fig. 16 the trajectories for (12) and (13) are depicted, satisfying Theorem 1.
6.4 Comparison

The three experiments (noise free, with random noise and sinusoidal noise) were performed in a satisfactory fashion; the load was regulated in the set point in all the three cases. Now Table 3 concentrates some measures that can help us make a comparison between them, and as can be seen, the performance of the three experiments are very close in most measurements, but the settling time increases with both kind of noise.

<table>
<thead>
<tr>
<th>Measurement</th>
<th>Noise Free</th>
<th>Random Noise</th>
<th>Sinusoidal Noise</th>
</tr>
</thead>
<tbody>
<tr>
<td>Settling time</td>
<td>13.500</td>
<td>18.000</td>
<td>17.600</td>
</tr>
<tr>
<td>Overshoot</td>
<td>2.200</td>
<td>2.200</td>
<td>2.185</td>
</tr>
<tr>
<td>ISE</td>
<td>(4.7395 \times 10^4)</td>
<td>(4.3779 \times 10^4)</td>
<td>(3.9373 \times 10^4)</td>
</tr>
<tr>
<td>IAE</td>
<td>(1.3413 \times 10^4)</td>
<td>(1.3679 \times 10^4)</td>
<td>(1.2425 \times 10^4)</td>
</tr>
<tr>
<td>ITSE</td>
<td>(8.5851 \times 10^5)</td>
<td>(9.5549 \times 10^5)</td>
<td>(9.5377 \times 10^5)</td>
</tr>
<tr>
<td>ITAE</td>
<td>(2.6025 \times 10^5)</td>
<td>(3.0120 \times 10^5)</td>
<td>(3.0164 \times 10^5)</td>
</tr>
</tbody>
</table>

7 Conclusions

The main goal of this paper was to propose a systematic methodology to design T1FLCs to solve the output regulation problem of a servomechanism with nonlinear backlash.

The proposed design strategy results in a controller that guarantees that the load reaches the desired position. The regulation problem was solved as was predicted; this affirmation is supported with simulations where the T1FLC designed by following the Fuzzy Lyapunov Synthesis achieve the solution to the regulation problem.

Moreover, the results show that the designed fuzzy control systems are capable of dealing with uncertainties (disturbances) like noise and viscous friction added to the system.
References


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Designing Type-1 Fuzzy Logic Controllers via Fuzzy Lyapunov Synthesis for...


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