Adaptive Quantization in Min-Sum based Irregular LDPC Decoder

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Abstract—In this paper, we present adaptive quantization schemes in the normalized min-sum decoding algorithm considering scaling effects to improve the performance of irregular low-density parity-check (LDPC) decoder for WirelessMAN (IEEE 802.16e) applications. We discuss the finite precision effects on the performance of irregular LDPC codes and develop optimal finite word lengths of variables over an SNR. For floating point simulation, it is known that in the normalized min-sum or offset min-sum algorithms the performance of a min-sum based decoder is not sensitive to scaling in the log-likelihood ratio (LLR) values. However, when considering the finite precision for hardware implementation, the scaling affects the dynamic range of the LLR values. The proposed adaptive quantization approach provides the optimal performance in selecting suitable input LLR values to the decoder as far as the tradeoffs between error performance and hardware complexity are concerned.

I. INTRODUCTION

Low density parity check (LDPC) codes, first introduced by Gallager decades ago and rediscovered by Spielman, Mackay and many others, can achieve very good performance when decoded with the belief-propagation or the sum-product algorithm [1][2]. The decoding of an LDPC code allows a high degree of parallelism, which makes it very suitable for high data rate applications such as wide-band wireless multimedia communications and magnetic storage systems [3].

There are a variety of decoding algorithms, such as the iterative belief propagation (BP) algorithm, the Log-BP algorithm, the min-sum algorithm, and the normalized or offset min-sum algorithm. The BP algorithm has a good decoding performance but requires a large hardware complexity. The min-sum algorithm can significantly reduce the hardware complexity at the cost of performance degradation, where complex computations at the check nodes are approximated by using comparators and multiplexers, thereby reducing the area and the power consumption of the decoder. Recently, the normalized or offset min-sum algorithm with scaling factors has been preferred for many practical applications since it offers comparable decoding performance compared to that of Log-BP for regular LDPC codes [4].

In [5], [6] and [7], novel versions of the min-sum algorithm and adaptive quantization effects of the Log-BP algorithm are respectively proposed. The two papers [5], [6] apply normalization factors depending on the bit node degree in the extrinsic message or down-scaling factors to the intrinsic message, respectively. The min-sum algorithm with a few additional computations in [5] reduces the magnitude of the extrinsic information in order to avoid early saturation states at the bit nodes. In [6], the variable nodes use the down-scaled intrinsic information iteratively to compensate the quantization errors at the bit nodes caused by finite precision. In other words, using down-scaling factors decreases the prior LLR iteratively as the number of decoding iterations increases since the absolute magnitude of the prior LLR usually grows larger in the high SNR region. In this paper, we propose adaptive quantization schemes in the normalized min-sum decoding algorithm with scaling effects to improve the performance of irregular low-density parity-check (LDPC) decoders.

The rest of the paper is organized as follows. In Section II, the characteristics of IEEE 802.16e LDPC codes are introduced. We provide the background of the normalized min-sum decoding algorithm and the conventional Log-BP algorithm. In Section III, we show the finite precision effects through the normalized min-sum decoding algorithm with a variable number of quantization bits. We then investigate the quantization effects in the min-sum based decoder without estimated channel SNR for the IEEE 802.16e application. In Section IV, we propose an adaptive quantization scheme for the min-sum decoding algorithm to improve the decoder performance. Finally, our conclusions are presented in Section V.

II. REVIEW OF LDPC CODES AND NORMALIZED MIN-SUM DECODING

A. Block Irregular LDPC Codes for WirelessMAN

The IEEE 802.16e, also referred to WirelessMAN [8], is a standard for mobile access where orthogonal frequency division multiplexing (OFDM) is adopted. The LDPC codes standardized in IEEE 802.16e consist of the same style of blocks with different cyclic shifts. The block irregular LDPC

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codes in IEEE 802.16e have competitive performance and provide flexibility and low encoding/decoding complexity.

Each base matrix in the block LDPC codes has 24 block columns and (1 - code rate) × 24 block rows. The expansion factor Z is equal to N/24 for code length N, and Z ranges from 24 to 96 in increments of 4. For example, the code with length N = 1920 has the expansion factor Z=80. There are four code rates (1/2, 2/3, 3/4, and 5/6) and six different code classes spanning four different code rates.

B. Normalized Min-Sum decoding Algorithm

In the normalized min-sum algorithm, which can be considered as an approximation of the BP algorithm, there are two kinds of computation units, check node units (CNUs) and variable node units (VNUs). Messages are denoted by $R_{cv}$ for extrinsic messages from the check node $c$ to the variable node $v$, and by $L_{cv}$ for extrinsic messages from the variable node $v$ to the check node $c$. The update operation at the check nodes in the normalized min-sum algorithm can be expressed as follows:

$$R_{cv} = \alpha \cdot S_{cv} \cdot \min_{n \in N(c) \cap v} |L_{nc}|$$  \hspace{1cm} (1)

$$S_{cv} = \prod_{n \in N(c) \cap v} \text{sign}(L_{nc})$$  \hspace{1cm} (2)

where $\alpha$ is a correction factor, $S_{cv}$ stands for the sign part of $R_{cv}$, and $N(c)$ denotes the set of variable nodes connected to the check node $c$. The update extrinsic message ($R_{cv}$) from a check node to a variable node is equal to the minimum reliability of the incoming $L_{nc}$ extrinsic messages from other nodes. In the case of an implementation using the normalized min-sum algorithm, the memory storage element corresponding to CNU stores the smallest value, second smallest value, and the index of the edge providing the incoming message of least value. Compared to the BP algorithm, the CNU in the normalized min-sum algorithm has the advantage of reducing the size of a Look Up Table (LUT), which is required to implement Eq. (3) in Log-BP algorithm.

$$|R_{cv}| = 2 \tanh^{-1}\left(\prod_{n \in N(c) \cap v} \tanh\left(\frac{|L_{nc}|}{2}\right)\right)$$  \hspace{1cm} (3)

The update operation at the variable nodes is the same as in BP and can be expressed as follows:

$$L_{nv} = \sum_{c \in M(v)} R_{nc} - y_v$$  \hspace{1cm} (4)

$$L_{vn} = \sum_{c \in N(v)} R_{cv} - y_v$$  \hspace{1cm} (5)

where $M(v)$ denotes the set of check nodes connected to the variable node $v$ and $y_v$ is the prior LLR given by $2r_v/\sigma^2$, where $r_v$ is the AWGN channel output and $\sigma^2$ is the noise variance. For the AWGN channel, it is known that the min-sum based algorithms such as the normalized min-sum or offset min-sum are insensitive to scaling the VNU computations in (4), (5) because the scaling factor $\sigma$ does not affect the output of the CNU in (1). Therefore, the prior LLR, $y_v$, values can be computed as the received value $r_v$. The decoding algorithm stops if either the estimated codewords satisfy all the parity check equations or the maximum number of iterations is reached.

III. Finite Precision Effects on Normalized Min-Sum Decoder for Irregular LDPC Codes

In the implementation of normalized min-sum LDPC decoding, effects due to finite precision should be considered because they degrade the error performance of systems. The quantization effects are related to the fixed-point number format that is used in the processing of intrinsic and extrinsic messages in the decoder. Moreover, the hardware complexity and decoding performance depend on the fixed number format. In this work, we assume that irregular LDPC codes are modulated by BPSK and transmitted over an AWGN channel. For simplicity, we use one correction factor for all check nodes in the normalized min-sum algorithm.

We use the notation $(q, f)$ to represent a quantization scheme in which q bits are used for total bit size and f bits are used for fractional values. In a uniform quantization scheme, a signed fixed-point number format has a quantization precision of $2^f$ with a maximum value of $2^{q-f-1} - 2^f$ and a minimum value of $-2^{q-f-1}$. To analyze the quantization effects of the normalized min-sum algorithm, we consider the received values to be clipped symmetrically at a given maximum and minimum value in the uniform $(q, f)$ quantization scheme. Values above the maximum or below the minimum are clipped in both the CNU and VNU.

The performances of the (1920, 1280) irregular LDPC code with floating point and several quantization schemes are shown in Fig. 1. In this paper, we limit the word length as 6 to investigate the saturation and quantization effects. It is shown that for $q = 6$, the (6, 1) quantization has the best performance. We can see that the difference between (6, 1) and (6, 2)
quantization is quite significant in high SNR region. In other words, the performance gain using (6, 1) quantization compared with (6, 2) is more than 0.4dB at BER=7×10^{-3}. It is known that the normalized or offset min-sum decoding does not need any channel information and works with just the received values as inputs. In order to analyze the effect of channel information on the normalized min-sum decoding, we select the input to the decoder to be 2r/\sigma^2. As the SNR increases, (6, 2) quantization scheme is not sufficient to cover the distribution of 2r/\sigma^2 because the dynamic range of 2r/\sigma^2 is larger than that of (6, 2) quantization scheme.

In [5] and [6], two modified versions of the normalized min-sum algorithm are presented and a behavioral analysis on extrinsic message states is studied as the number of iterations increases. The degree distribution polynomials of our code are \lambda(x) = 0.2917x^2 + 0.5x^3 + 0.2083 x^6 with respect to the variable nodes and \rho(x) = x^{10} with respect to the check nodes. Fig. 2 shows the number of bit errors at the variable nodes of degree {2, 3, 6} after some number of iterations at SNR = 2.75 dB. From Fig. 2, the convergence speed of correcting bit errors at variable nodes of degree 6 is faster than other variable nodes of degree 2 and 3, although variable nodes of degree 3 contain more bit errors than that of the others. The percentage reduction in bit errors on variable nodes of degree {2, 3, 6} is shown in Fig. 3. After 10 iterations, the percentage reduction in bit errors in both floating and fixed point implementations decreases. In other words, the number of bit errors remains unchanged after a certain number of iterations and the extrinsic messages have no effect on decoding performance.

From the above observation, distinct down-scaling factors [6], which are determined by the degree of the variable nodes, is used on the intrinsic messages (2r/\sigma^2) in order to reduce the strength effects of the intrinsic magnitude on the extrinsic information L_v. Instead of using the down-scaling factors on the intrinsic messages, our proposed quantization scheme uses the received value (r_v) as the inputs to intrinsic messages in a high SNR region. This quantization method helps to improve the performance of irregular LDPC decoders without additional hardware complexity, which will be discussed in Section IV.

IV. ON IMPLEMENTATION OF ADAPTIVE QUANTIZATION IN NORMALIZED MIN-SUM ALGORITHM

In this section, we analyze quantization effects on the performance of an irregular LDPC decoder. The study presented in the last section led us to consider the dynamic range of the prior LLR in order to take more precisely into account the effect of finite precision on the intrinsic data. Quantization of incoming prior LLR data significantly affects the decoding performance and it should be analyzed in order to design an efficient LDPC decoder in terms of hardware complexity and decoding performance. In the case of floating point simulations, the error performance of a normalized or offset min-sum algorithm does not vary with SNR estimation. However, when considering the finite precision of a hardware implementation, scaling affects the dynamic range of LLR values. At high SNR, quantization effects are reduced by used r_v rather than 2r/\sigma^2. In that case, a (6, 2) quantization scheme is sufficient.

Fig. 4 shows a VNU architecture for simulating various quantization schemes on the normalized min-sum decoding algorithm. The architecture needs additional hardware ((2q + 1)-bits adders and shift operators) so that it holds the precision of intrinsic message computations in order to use down-scaling factors. In our work, we use the same VNUs excluding down-scaling factors with shift operations and add blocks. In Fig. 5, we present the performances of the normalized min-sum algorithm with the (6, 2) quantization scheme without SNR estimation and down scaling factors. Based on our simulation results, the SNR estimation does not help the normalized min-sum decoding performance at high SNR levels while the min-sum based algorithms need to know the channel state information at low SNR region. Considering dynamic range of LLR received inputs, an adaptive quantization scheme can be used in a normalized min-sum based decoder.

With the adaptive quantization in the normalized min-sum algorithm, y_v, in Eq. (4) can be expressed as

\[
y_v = \begin{cases} 
2r_v/\sigma^2, & \text{SNR} \leq C \ dB \\
 r_v, & \text{SNR} > C \ dB 
\end{cases}
\] (6)
finite word-lengths of variables for the normalized min-sum algorithm. In the simulations, up to $2\times10^6$ block codewords are simulated for each high SNR data point. We have proposed an adaptive quantization for the normalized min-sum algorithm. Computer simulation results show that the proposed quantization scheme, which depends on the dynamic range of LLR input values and uses suitable LLR input values to the decoder, achieves much better performance than the conventional (6, 2) quantization scheme.

VI. ACKNOWLEDGMENT

This research was supported by grant No. R01-2006-000-10596-0 from the Basic Research Program of the Korea Science & Engineering Foundation.

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