The Problem Of Weather Prediction,

as Seen from the Standpoints of Mechanics and Physics

by V. Bjerknes (from Meteorologische Zeitschrift, January 1904)

If, as every scientifically inclined individual believes, atmospheric conditions develop according to natural laws from their precursors, it follows that the necessary and sufficient conditions for a rational solution of the problems of meteorological prediction are the following:

1: The condition of the atmosphere must be known at a specific time with sufficient accuracy

2: The laws must be known, with sufficient accuracy, which determine the development of one weather condition from another.

I.

The job of observational meteorology is to produce knowledge of the condition of the atmosphere at a desired future time. This problem has not been solved for the scientific weather forecaster in even the most felicitous circumstances. Two gaps are especially noticeable. Firstly, the only weather stations reporting daily are on the land. On the sea, which constitutes four fifths of the earth’s surface and therefore exercises an overwhelming influence, no daily weather observations are made. In addition, the regular weather service observations are terrestrial in origin, and lack any information about the condition of the upper atmosphere.

We already possess the technical tools which will make it possible to fill in these two gaps. Steamships which travel fixed routes in the neighborhood of stations and send daily weather reports can contribute with the help of wireless telegraphy. In addition, given the great progress that has been made in aeronautical meteorology during the last few years, it will no longer be impossible to receive daily observations of the upper atmosphere at fixed terrestrial stations or airborne platforms.
Hopefully, the time will also soon come, when a complete statement of atmospheric conditions can be made either daily or on specified dates. At that point, the first condition for scientific weather forecasting will be met.

II.

The second question then presents itself, which is to what extent we have sufficiently detailed knowledge of the laws according to which one atmospheric condition develops into another.

The atmospheric processes are mixtures of a physical and mechanical nature. We can describe each of these processes with one or more mathematical equations. We will have sufficient knowledge of the laws governing the development of atmospheric processes when we can write enough independent equations to calculate all the unknown quantities. The state of the atmosphere at a particular time will be determined, in a meteorological sense, when we can determine the air speed, density, pressure, temperature, and humidity at every point. The wind velocity will be represented by three scalars, the three wind speed components, so that, as a result, we need to be concerned with the calculation of seven unknown quantities.

To calculate these quantities, we set up the following equations:

1. The three hydrodynamic equations of motion. These are differential equations representing relationships between the three wind speed components, the density, and the pressure.

2. The equation of continuity, which states the principle conservation of mass during motion. This relationship is also expressed by a differential equation, more precisely a relationship between the wind speed components and air density.

3. The equation of state for the atmosphere, which is an infinite series involving the density, pressure, temperature, and the humidity of a given air mass.

4. The two major principles of the mechanical theory of heat, which state, in two differential equations, how, as a result of ongoing condition changes, the energy and entropy of a chosen airmass are altered. In addition, these equations introduce no new unknown quantities into the problem, because the energy and entropy express themselves through the same transformations that are found in the equations of state, and tie the changes in these variables with the changes of other known quantities. The other inputs are: firstly, the work done by the air mass, which is determined by the same transformations which are found in the dynamic equations; secondly, the externally determined heat quantities, which will be obtained from physical data concerning radiant energy transfer and the heating of the air caused by the motion of the earth.

It is given that a significant simplification of the problem arises when there are no humidity changes, so that the amount of water present in the air mass remains constant. We then have one fewer variable, and one equation, namely that of the second principle, which can be eliminated. On the other hand, if we have several changes in the condition of the atmosphere, the application of the second principle for each new condition produces a new equation.

In order to calculate the usual seven variables, we need seven independent equations. Insofar as it is possible now to get an overlook of the problem, we must also conclude that we possess a sufficient knowledge of the atmospheric processes upon which a scientific weather forecasting is based. It must also be acknowledged that we may have overlooked important considerations because of the incompleteness of our understanding. The intervention of global processes of an unknown type is conceivable. Further, overall atmospheric changes are accompanied by a long series of accompanying phenomena of an optical and electrical nature and the question remains, how significantly these influence the atmospheric processes. The interconnections are self evident. Rainbows are, for example, a modified refraction of solar energy, and electrical charges have known influences on the condensation processes. Up to this point, however, there are no indications that these ancillary events significantly affect overall atmospheric processes. At any rate, it’s the scientific method to start with the simplest problem which can be stated, which is precisely that with seven equations involving seven unknowns.

III.

Of the seven equations, only one is expressed as an infinite series. The other six are partial differential equations. Of the seven unknowns, one can be eliminated with the help of the equation of state, and the solution then becomes one of the integration of a system of six partial differential equations with six unknowns and initial values obtained from observations of the beginning atmospheric conditions.

It's not possible to obtain a rigorous mathematical integration of this system of equations. Even the calculation of the motion of three objects, which are mutually influenced according to simple Newtonian law, goes substantially beyond today's mathematical tools. It's self evident that calculation of the substantially more complicated interactions of atmospheric molecules is hopeless. The exact analytic solution would not be what we need, even if we were able to obtain it. In order for such a solution to be useful, it would of necessity include all conceivable conditions, something which would introduce an infinite number of singularities into the analytic solution. The predictions need only concern themselves with average values over large areas and long periods of time, for example hourly reports by degrees of longitude as opposed to every second by millimeters.

Accordingly, we abandon all thoughts of an analytic solution, and restate the problem of the weather forecaster in the following practical form:

Because of independent observations, the initial state of the atmosphere is represented by a number of tables, which specify the division of the seven variables from layer to layer in the atmosphere. With these tables as the initial values, one can specify similar new tables which represent the new values from hour to hour.

Graphical or mixed graphical and numerical methods are needed in order to solve the problem in this form, whether from the partial differential equations, or from the physical/ dynamic principles which underlie the equations. The effectiveness of such methods can't be questioned on an a priori basis. Everything depends on successfully separating a single overwhelmingly difficult problem into a series of sub problems, none of which present impossible difficulties.

IV.

In order to implement this partition into sub problems, we must call on the general principle, which underlies the Calculus of Several Variables. For computational purposes, we can replace the simultaneous changes of several variables with sequential changes of individual variables or groups of variables. If we proceed to very small intervals, we approach the exact methods of differential calculus. If we retain a finite interval, then we'll make use of the numerical analysis approximation techniques of finite differences and numerical integration.

This principle must not be blindly applied, because the practical utility of the methods will, above all, depend on the natural grouping of the variables which are contained in the mathematically and physically well defined sub problems. Most importantly, the primary division is fundamental. It must follow the major problem in a natural order.

Such a natural order must also be specified. It lies on the border between specific dynamic and physical processes, out of which the atmospheric processes are synthesized. A segmentation along these boundaries provides a decomposition of the main problem into purely hydrodynamic and thermodynamic sub problems.

The link between the hydrodynamic and thermodynamic problems is very easy to separate, so simple, in fact, that theoretical Hydrodynamicists have used it to avoid all serious contact with Meteorology, because the connection is made by the equations of state. If one assumes that the temperature and humidity are not involved in these equations, then we arrive at the conventionally applied “supplementary equations” of hydrodynamics, which are only relations between density and pressure. In that manner, we are led to the study of fluid flow under the circumstances that every explicit accounting of thermodynamic processes falls away by itself.

Instead of allowing the temperature and humidity of the equations of state to disappear entirely, we can regard them as fixed values for small periods of time, with values given either by observations or from previous computations. When the dynamic problem is solved for this time interval, further calculations can be made according to purely thermodynamic methods to obtain temperature and humidity. These values can be used as known quantities in the solution of the hydrodynamic problem in the next time interval, etc.

V.

The general principle of the first decomposition of the main problem has been stated. In the practical follow through, we have the choice of several different paths, each according to technical considerations, which introduce hypotheses about temperature and humidity. To go into these considerations in greater detail would be meaningless in such a general discussion.

The next major question will be to what extent the hydrodynamic and thermodynamic partial problems can be solved in a sufficiently simple manner.

We first consider the hydrodynamic problem, which is the real major problem, as the dynamic equations provide the primary predictions. Only in this manner can time be introduced into the problem as an independent variable, as the thermodynamic equations don’t involve time.

The hydrodynamic problem is an excellent candidate for a graphical solution. In order to solve the three dynamic equations, one has to construct simple parallelograms for a suitable number of chosen points, and solve graphically for intervening points. The principal difficulty will be found in accounting for the limits of the freedom of motion, which come from the equation of continuity and the boundary conditions. The test of whether the equation of continuity is satisfied is also left to graphical methodology and, in this manner, no consideration can made of the earth’s topography, because the constructions are carried out on charts, that represent this topography in a conventional manner.
There are also no substantial mathematical difficulties to be found in the solution of the hydrodynamic partial problems. A perceptible gap in the knowledge of the factors , with which we are required to calculate, is also present insofar as we have an incomplete knowledge of the viscous friction in the motion of the air, because the friction depends on the difference in the speed of small molecules, while meteorologists are compelled to compute the average movement of extended masses of air. None the less, one can’t utilize the laboratory obtained coefficients of friction for the frictional elements in the hydrodynamic equations. On the contrary, empirical results must be obtained of the effective resistance to motion of large masses of air. We already have sufficient data of this type to make the first attempts at predictions of the motion of air, which will be supplemented and corrected over time.

The thermodynamic partial problem is significantly simpler to look at than the hydrodynamic one. One only takes out of the solved hydrodynamic problem the work which the air masses have performed during the displacements. With the knowledge of this work, and the

additional knowledge of the thermal changes caused by radiant energy flux during the given time period, one can obtain a new distribution of temperature and humidity from known thermodynamic principles. From a mathematical standpoint, the calculations are no more difficult than similar computations made in laboratory experiments, which are made with air masses at rest in enclosed spaces. Extensive preparatory work was done in the studies of Hertz, V. Bezold, and others.
As in the hydrodynamic problem, the biggest difficulty in carrying out the calculations is the incomplete nature of our understanding of several of the factors. Initially, there will be uncertainty in the estimation of the quantities of heat which the air mass receives from radiant energy flux, and in the mass of water which evaporates from the earth's surface or which condenses from clouds and falls as rain. We have sufficient knowledge for the experimental initiation of the first calculations. With further work, we will obtain increasingly exact values of the constants which are associated with different continents and seas, different atmospheric heights, differing weather conditions, and varying degrees of cloud cover.

VI.

It's certain that, if we proceed in the indicated manner, that no intractable mathematical difficulties will be encountered. When the graphical methods have been worked out and the necessary tools implemented, the individual operations will be easy to carry out. In addition, the number of single operations do not need to become excessive. The number will depend on the length of the time interval which is being used for the solution of the dynamic partial problem. The smaller the time interval chosen, the more complex the work, but the results are also more accurate. Larger time intervals yield faster, less accurate results. Initial attempts will provide experience. Only when substantial accuracy is required might intervals of at most an hour be useful. It's highly unusual that air masses will traverse more than a degree of longitude in an hour's time and that, during this time, their paths will substantially change. Therefore, the conditions are fulfilled under which one can construct a simple parallelogram with straight sides. When enough experience has been obtained, it will also be easier to work with larger time intervals, say of approximately six hours. For a 24 hour weather prediction, it would be necessary to carry out four hydrodynamic constructions and to calculate the thermodynamic corrections of temperature and humidity four times.

It might then be possible that, sometime in the future, a technique of this sort could be put into practical, daily weather service use. No matter how this procedure worked, sooner or later it will be necessary to undertake a deeper scientific study of atmospheric processes that is founded on the laws of Mechanics and Physics. In this manner, one will necessarily arrive at a method that is sketched here.

While this is understood, there is also a general plan for dynamic meteorological research.

The principal task of observational meteorology is to work out the most comprehensive picture of the physical and dynamic conditions of the atmosphere from observations. This picture must certainly have such a form as to be useful as a starting point for weather predictions based on rational dynamic-physical methods.

Clearly, this first introductory exercise is not of limited generality. It is self evident that it is certainly more involved to represent all heights of the atmosphere that what is now done at sea level. In addition, we realize that our opportunity for direct observation of the upper atmosphere will always remain very constrained. It is therefore of greatest importance that data from the upper atmosphere be most thoroughly utilized. From the directly measured quantities we must calculate all associated elements in the widest area. For that purpose, we must utilize the mathematical relationships between the different data elements. Also, if we want to use these sporadic observations to construct a thorough picture of the state of the atmosphere, we must use dynamic-physical methods in large areas.

The second and most important task of theoretical meteorology will ultimately be to take this picture of the condition of the atmosphere as a starting point and construct future states, whether with the methods outlined here or with comparable methodology. The comparison of the constructed states with the observed ones will in part yield verification of the validity of the methodology, and in another part provide indications for better values of constants and improvements of technique.
I will return at future opportunities to the major points of this program.