Abstract—This paper demonstrates a GPS receiver research platform. It can generate digital Immediate Frequency (IF) GPS signal and some signal synchronize functions on high dynamic and weak signal environment. It is based on a mathematical signal model which expresses the digitized IF GPS signal as a function of various errors during propagation. The simulator is able to simulate more than 12 GPS satellite signals due to user. In order to validate the signals and develop more effective algorithm for various GPS signals and different tracking techniques are discussed. Performances of different type tracking loops outputs on particular environment are demonstrated and analyzed. Especially, for weak signal environment, aiming at tracking degraded signal, an errors tracking scheme based on modified Unscented Kalman Filtering (UKF) is designed and implemented. The 30dB/Hz weak signal test result validated the tracking scheme.

Index Terms—Global Positioning System, signal simulation, signal tracking, high dynamic, weak signal, UKF.

I. INTRODUCTION

In 1996, Dr. Akos suggested that some necessary functions which were performed by hardware can be substituted by software for a GPS receiver [1]. From then on, Study on software GPS receiver signal processing techniques has been carried on. However, all algorithms need digital IF signal. The inflexibility of original IF signals, which from a hardware simulator or true GPS receiver, reduce researchers’ efficiency. Especially, Galileo and COMPASS satellite navigation systems are in constructing, it is difficult to develop a new receive equipment for researchers and engineers in laboratory if they have not raw signal. Nowadays, some high performance satellite navigation simulators having been appeared on world markets, such as GSS series GPS/GNSS simulators have been developed by Spirent Communication Corporation in UK, NS series GPS simulators have been developed by OlinkStar Corporation in China. But these products are very expensive, for example, a middle-class GSS simulator will spend more than one hundred million Yuan. This is can not acceptable to a research institute which has not enough budgets.

A receiver research platform should be characterized of GPS signal generator and performing some common baseband signal processing algorithms to develop some advanced methods for some special applications. To develop a high performance GPS receiver, signals with different characteristics are needed first, such as low carrier to noise power spectrum density ratio (CNR) or high dynamic. We are able to gain different outputs convenient from a signal simulator by configuring some key parameters. The simulator has been developed in C and MATLAB languages at Integrated Navigation System Laboratory. It has been applied to develop a high performance receiver on complex environment.

In the first part of the paper, An IF signal simulator is designed. The digital IF signal output from the ADC of a receivers’ front-end is simulated by software only. The multipath, carrier’ high dynamic movement, receiver oscillator are modeled. The IF samples are bit true and the used software architecture provides the possibility to simulate an unlimited number of signals.

The second part of the paper simply describes signal acquisition technique based on FFT.

The third part of the paper implements different baseband signal processing techniques and their performances are discussed on high dynamic environment. When a receiver working on downtown environment, received signal will degraded. A modified UKF carrier error tracking model is estimated to realize perfect parameters estimation for carrier phase error, Doppler error, Doppler rate error and signal amplitude.

II. IF GPS SIGNAL SIMULATOR

A. Immediate frequency signal model

Transmitted GPS analogy signal reaches antenna through atmosphere. After filter and mixing, the immediate frequency signal is modelled as
\[ s_{w}(t) = \sum_{i=1}^{n_i} \left( \sqrt{P_i} d_i (t - T_i - \delta_{\text{tropo}}) c_i(t - T_i - \delta_{\text{tropo}}) \cos(f_{f_i'} - f_{f_{1i}} (T_i - \delta_{\text{tropo}}) + \varphi_i) + MP(t) + n(t) \right) \]

Where, \( T_i = T_i + \delta_{\text{tropo}} - \delta_{\text{tropo}} \), \( P_i \) is signal power, it is expressed by CNR and density of noise power spectrum \( N_s \), that is, \( P = 10^{\frac{C}{10 N_s}} \), \( N \) is the number of visible satellites, \( P_i \) is signal power of satellite \( i \), \( i \) is receive signal time, \( d_i, c_i \) are respectively navigation data and C/A code, \( f_{f_i'}, f_{f_{1i}} \) are respectively immediate frequency and \( L_i \) carrier frequency, \( \varphi_i \) is initial carrier phase, \( MP(t) \) is multipath, \( T_i, \delta_{\text{tropo}}, \delta_{\text{tropo}}, \delta_{\text{tropo}} \) are respectively signal transmit time, troposphere delay, ionosphere delay and satellite clock error, \( n(t) \) is a white Gaussian noise (WGN) with zero. The Doppler shift changes the rate of the PRN code and carrier frequency \( f_{f_{1i}} \). These affections must be taken into consideration [2].

**B. Implemented structure of the simulator**

The implementation structure of the simulator is shown in figure 1. The navigation data file provided information to calculate satellite position and velocity on receive signal moment. If a carrier movement on the earth is simulated, the altitude and azimuth of visible satellites are gained. Signal propagation time and introduced errors along the real GPS signal propagation path should be simulated [3]. These errors include Doppler, satellite clock error, atmosphere delay, receiver oscillator error and so on. The simulator is the simulation of various errors behaves in essentially the same manner. They all produce errors on the propagation and carrier phase measurements, namely, a code delay, or a carrier phase delay or an advanced error. The effect on the carrier phase is also indicated by the Doppler shift. Finally, the output signal is filtered and quantified to four value, \( \pm 1, \pm 3 \), and saved. In this paper, the multipath affection, Doppler shift caused by carrier movement and local oscillator error will be discussed in detail.

![Figure 1. Implementation structure of the simulator](image1)

**C. Multipath**

Multipath occurs when reflected signals, in addition to the direct signal, reach the antenna. It depends greatly on the properties of the reflector, the antenna gain pattern, and the type of correlator used in the GPS receiver [4]. Multipath interferes with the correlator’s ability to precisely determine the time instant of signal reception. Usually, an antenna will at least receive two types multipath [5]. One is scattered multipath, it is defined that parts of the reflected signal reach antenna by coarse reflecting surface. It doesn’t affect signal tracking and positioning. The other is mirror multipath, it is reflected by wall, water or ground [6]. Power of mirror multipath signal is quite larger than scatter multipath signal. The figure 2 is showing a mirror multipath.

In figure 2, \( d \) is vertical distance between wall and antenna. If the reflecting surface is water or ground, \( d \) is the height of antenna. Satellite signal \( L_i \) reach antenna directly and \( L_i \) is reflected by wall. \( \sigma \) is reflection angle. Therefore, \( S_i = d / \sin \sigma \), \( S_2 = -S_1 \cos(2\sigma) \)

Assuming \( S = S_1 + S_2 \), then the phase delay between direct and reflected signal is

\[ Q = \frac{S}{\lambda} = \frac{(S_1 + S_2)2\pi}{\lambda} = 2d \sin \sigma \frac{2\pi}{\lambda} = \frac{4\pi d}{\lambda} \sin \sigma \]  

Here, \( \lambda \) is the wave length of \( L_i \) (0.19m).

The vectors, which are formed by direct signal path and reflected signal path, accord with parallelogram. Assuming \( C \) is amplitude of combined signal by direct signal, amplitude is \( A \), and reflected signal, amplitude is \( B \). The reflected signal phase is delayed \( \Psi \) relative direct signal:

\[ \Psi = \arctan \frac{A \sin Q}{B + A \cos Q} \]  

Define \( \alpha = A / B \) ( \( \alpha < 1 \) ) is amplitude attenuation factor. After \( Q \) is substituted, we can get

\[ \Psi = \arctan \frac{\alpha \sin(4\pi d \sin \sigma / \lambda)}{1 + \alpha \cos(4\pi d \sin \sigma / \lambda)} \]
Equation (4) shows that the phase delay changes with reflecting angle and vertical distance. The phase delay with $d$ and $\alpha$ is shown in figure 3. On intensity reflecting environment, multipath will become very complex. For the simulator, one can get approximate phase delay through parameters configure in equation (4).

D. High dynamic

Under high dynamic condition, the carrier’s movement trajectory need to simulation. The trajectory used in this paper is similar with [7], which consists of positive and negative going jerk pulse of 1 second duration and magnitude of 50g/sec, separated by 1s of constant acceleration, as shown in figure 2. The corresponding acceleration and velocity trajectories are shown in the figure. The initial conditions for acceleration were chosen for symmetric 20g excursions. The velocity trajectory is converted to an equivalent Doppler frequency trajectory as

$$f_d(t) = \frac{f_c}{c} v_d(t)$$

(5)

Here, $v_d(t)$ is the Doppler velocity, $f_c$ denotes the carrier frequency and $c$ is the speed of light.

E. Local oscillator error

A GPS receivers use local oscillator to convert an input frequency to an intermediate frequency before the signal is demodulated. In the ideal receiver, these frequency conversions would not distort the input signal, and all information on the signal could be recovered. In a real-world receiver, both the mixer used for converting the signal’s frequency and the local oscillator will distort the signal and limit the receiver’s ability to recover the modulation on a signal [8].

The oscillator frequency error is easily modeled using Allan variance [9]:

$$\sigma_i^2(\tau) = \frac{2\pi^2}{3} h_2 + 2\ln 2 h_1 + \frac{h_0}{2\tau}$$

(6)

Here, $\sigma_i^2$ is Allan variance, $h_2$ is Allan parameter specifying the random walk, $h_1$ is Allan parameter specifying the flicker frequency, $h_0$ is Allan parameter specifying the white frequency, and $\tau$ is sampling interval of the Allan variance.

Those terms specify the long term accuracy of an oscillator, which is not of interest in equation (6). Typical oscillator TCXO and OCXO using by GPS receiver Allan parameters are listed in table 1. This paper’s oscillator simulating model and parameters configuration can be found in [9]. The Outputs are shown in figure 5.

Doppler frequency, multipath, oscillator and other errors have been implemented in the simulator. It can generate GPS 12 satellites signals. Central frequency is 4.092MHz and the sampling frequency is 12.276MHz. Signal of PRN 3 includes Doppler shift caused by
vehicle movement with high velocity. The vehicle’s state accords with figure 4.

III. SIGNAL ACQUISITION AND TRACKING

A. Signal acquisition

![Figure 6. Signal acquisition](image)

The acquisition algorithm is used to determine whether a certain satellite is in the input signal, and if so determine its code phase and carrier frequency. The Fast Fourier Transform (FFT) approach will be used to acquire GPS C/A code. The FFT approach performs circular convolution in the frequency domain. A non-coherent correlator in frequency domain can be adapted to the acquisition of GPS signals. The Discrete Fourier Transform (DFT) and its inverse are used to calculate the correlation value. More details about DFT signal acquisition are in [10], [11] and [12].

The advantage of the FFT version is that it calculates the correlation for an entire range dimension (selected Doppler) in a single step. The disadvantage is that when Doppler is non-zero the convolved reference signal produces some errors. As carrier movement with high velocity, the code and carrier Doppler change quickly, this change can not be captured for long data. 1 ms data is used for PRN 3. The acquisition result is showed in figure 6.

Sometimes, one has to acquisition weak signal. For example, a receiver is working in indoor or downtown etc. In order to get more signal processing gain, the longer integrating time are needed. Such as non-coherent, differential coherent acquisition techniques, etc.

B. Carrier tracking for high dynamic signal

The acquisition approach gives the initial estimates of the Doppler and code offset, and then control will be handed over to tracking loops to track the variations of carrier phase and code offset due to the line of sight movement between satellites and the receiver. Conventional Delay Locked Loop (DLL), PLL and FLL were implemented to test their performance for high dynamic signal in the software GPS receiver.

DLL is used to track the C/A code and de-spread the incoming signal. PLL consists of Numerically Controlled Oscillator (NCO), carrier loop filter and a discriminator [13]. PLL receives signal that is only modulated by navigation message. The NCO generates a carrier frequency based on the Doppler frequency computed during the acquisition process. The signal generated is divided into in-phase ($I$) and quadrature ($Q$) components. The input signal is correlated with $I$ and $Q$ channel signals. The outputs of the correlators are filtered and the phase is analyzed suing a discriminator. The discriminator used in this algorithm is arc tangent discriminator that is insensitive to the phase transition. A PLL that uses arc tangent discriminator is similar to Costas Loop. The output of the discriminator is used to generate a control signal to tune the frequency of the oscillator (NCO) so that the loop can continual de-modulate the incoming signal.

Generally, the carrier loop is a weaker loop because the carrier wavelength is much shorter than the chip length and the carrier loop needs to track all dynamics while the code loop needs only to track the dynamic difference between carrier loop and code loop. Because carrier aiding technique has been applied to code loop in this simulator, following analysis emphasizes on carrier tracking for high dynamic environment.

Assuming code loop has been synchronized and $c_k(t_k)$ is local code. An approximate Doppler shift $\hat{f}_d$. Doppler rate $\hat{f}_d$, phase $\hat{\theta}_k$ are obtained. They are used to construct the following model for the in-phase and quadrature local signals

\[
\begin{align*}
I_k(t) &= c_k(t_k) \cdot \cos(2\pi f_s + \hat{f}_d t_k + \pi \hat{f}_d t_k^2 + \hat{\theta}_k) \\
Q_k(t) &= c_k(t_k) \cdot \sin(2\pi f_s + \hat{f}_d t_k + \pi \hat{f}_d t_k^2 + \hat{\theta}_k)
\end{align*}
\]

The $I$ and $Q$ components from correlators are

\[
\begin{align*}
I_k &= A_k \cdot d_k \cdot \cos(\theta_k) + n_{I,k} \\
Q_k &= A_k \cdot d_k \cdot \sin(\theta_k) + n_{Q,k}
\end{align*}
\]

Where, $d_k$ is the data bit associated with sample $k$. $\theta_k$ is the phase error. $n_{I,k}$ and $n_{Q,k}$ are the noise for the $I$ and $Q$ samples, which is assumed to be additive white Gaussian noise (AWGN). $A_k$ is the signal level.

Assuming that the acceleration along the line-of-sight is constant, the Doppler frequency, in the incoming signal can be expressed as

\[
f_d(t) = f_v + f_a t
\]

Where, $d_f$ is the overall Doppler frequency in the incoming signal, $v_f$ is the frequency shift caused by the relative velocity and $a_f$ is the change rate of the frequency shift caused by the acceleration along the line-of-sight between the satellite and the receiver. The Doppler frequency over a period of $\Delta t$ is obtained as

\[
f_d(t + \Delta t) = f_d(t) + f_a \Delta t
\]
The carrier phase variation caused by the Doppler frequency in the incoming signal over a period of $\Delta t$ is

$$\Delta \theta = \int_0^\infty (f(t + \tau) - f(t)) d\tau = f_0 \Delta t + \frac{1}{2} f_0 \Delta t^2$$ \hspace{1cm} (11)$$

Under high dynamic condition, $\Delta \theta$ will be affected by noise, it is caused by carrier movement. Tracking ability of PLL mostly depends on the loop’s orders [14]. Tracking results for carrier’s acceleration and jerk using second, third order PLL and FLL are shown in figure 7. Bandwidth was set to a proper value for tracking high dynamic signal. Note that tracking time corresponds with figure 2.

From these tracking results, we can find out: The first order loop (no loop filter) cannot track a frequency ramp excitation signal (acceleration); the second order loop can track but with a constant phase difference (from 0 to 500 ms in figure 7a) and the third order loop can track a frequency ramp excitation signal (acceleration) with no phase difference (Figure 7b). A higher order loop can track carrier’s jerk with no phase difference, but it will frequently lose lock under worse environment. Conventional FLL is also able to track these varies (Figure 7c). So, under high dynamic environment, one can using FLL cooperated with PLL in order to reduce phase error and the receiver has to switch between a FLL and PLL.

C. Carrier tracking for Weak GPS signal

Common GPS signal is between 35dB/Hz and 55dB/Hz. If received signal is lower than 35dB/Hz, one can say this is a weak signal.

The Kalman filter is essentially a recursive algorithm that implements a predictor corrector type estimator. The predictor is based on a system model and the corrector is based on the measurement model. The Kalman filter is optimal in the sense that it minimizes the estimated error covariance. Ping have established adaptive Kalman Filtering model to track high dynamic GPS signal [15]. It has a linear measurement model based on oscillator output. In this paper, and Psiaki realized weak GPS signal tracking using EKF [16]. But the EKF algorithm has some potential drawbacks such as Jacobi matrix calculation, linear error to solve nonlinear equation. So, in this paper, we established a modified UKF method tracking weak signal due to its precision is equal to EKF’s 2nd Taylor series expansion. An error tracking scheme is designed, it is figure 8. The phase discriminator is substituted by UKF in order to get better parameters estimation.

The estimated parameters are phase difference, Doppler frequency error and Doppler rate error. That is

$$x = [\theta_e, f_e, \alpha_e, A_c]$$

We can establish the error tracking model as follows:

$$\begin{bmatrix}
\theta_e \\
f_e \\
\alpha_e \\
A_c
\end{bmatrix} =
\begin{bmatrix}
1 & \Delta t & \frac{1}{2} \Delta t^2 & 0 \\
0 & 1 & \Delta t & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
\theta_e \\
f_e \\
\alpha_e \\
A_c
\end{bmatrix}
+ \begin{bmatrix}
w_{\theta_e} & 0 & 0 & 0 \\
0 & w_{f_e} & 0 & 0 \\
0 & 0 & w_{\alpha_e} & 0 \\
0 & 0 & 0 & w_{A_c}
\end{bmatrix}
$$

\hspace{1cm} (12)
Here, $\Delta t$ is UKF update time. Disturbances term are assumed to be independent, variances can be defined as:

$$E[w_{i}w_{j}^{T}] = Q_{i} , E[w_{i}w_{j}^{T}] = Q_{j} , E[w_{i}w_{j}^{T}] = Q_{a}$$

(13)

This leads to the process noise:

$$Q = \begin{bmatrix} Q_{0} & 0 & 0 & 0 \\ 0 & Q_{f} & 0 & 0 \\ 0 & 0 & Q_{a} & 0 \\ 0 & 0 & 0 & Q_{1} \end{bmatrix}$$

(14)

Since the algorithm processes discrete observations, the discrete process noise can be calculated using the transition matrix $Q = \int_{0}^{\Delta t} \Phi_{x,t}Q_{x,t} \Phi_{x,t}^{T} dt$.

The measurement equation (15) is established based on (8). Measurement noise is defined as $R_{k}$:

$$z_{k} = \begin{bmatrix} I_{p,k} \\ P_{q,k} \\ n_{I_{p,k}} \\ n_{P_{q,k}} \end{bmatrix} = \begin{bmatrix} \bar{A}d_{i} \hat{d}_{i} \cos(\hat{\theta}_{t,k}) \\ \bar{A}d_{i} \hat{d}_{i} \sin(\hat{\theta}_{t,k}) \end{bmatrix} + R_{k}$$

(15)

Where, $\bar{A}$ is signal mean level during loop update, $\hat{d}_{i}$ is message bit estimation, if $I_{p,k} > 0$, $\hat{d}_{i} = 1$, else $\hat{d}_{i} = -1$.

Outputs of the UKF are used to calculated $\hat{\theta}_{t,k}$, and this value will be used to control local NCO, make the phase constituted. The UKF carrier tracking processing of flow:

Initialize:

$$\hat{x}_{0} = E[x_{0}]$$

$$P_{0} = E[(x_{0} - \hat{x}_{0})(x_{0} - \hat{x}_{0})^{T}]$$

Calculate sigma points:

$$X_{i,k}^{e} = [\hat{x}_{i,k}^{0} + (\eta_{i} \sqrt{P_{i,k}})]$$

Calculate weight coefficients:

$$W_{0}^{(i)} = \frac{\lambda}{\lambda + \beta}$$

$$W_{i}^{(i)} = \frac{1}{\lambda + \beta} (1 - \alpha^{2} + \beta)$$

$$W_{i}^{(i)} = \frac{1}{2} \beta$$

Time update:

$$x_{i+1}^{e} = f(X_{i,k}^{e})$$

$$\hat{x}_{i} = \sum_{i=0}^{2L} W_{0}^{(i)} X_{i+1}^{e}$$

$$P_{i} = \sum_{i=0}^{2L} W_{i}^{(i)} [X_{i+1}^{e} - \hat{x}_{i}] [X_{i+1}^{e} - \hat{x}_{i}]^{T} + \bar{Q}_{i}$$

$$X_{i+1}^{e} = [\hat{x}_{i+1}^{0} + (\eta_{i} \sqrt{P_{i,k}})]^{T}$$

$$z_{i} = \sum_{i=0}^{2L} W_{0}^{(i)} z_{i}^{e}$$

Measurement update:

$$P_{z} = \sum_{i=0}^{2L} W_{i}^{(i)} [z_{i+1}^{e} - \hat{z}_{i}] [z_{i+1}^{e} - \hat{z}_{i}]^{T} + \bar{R}_{i}$$

$$P_{z} = \sum_{i=0}^{2L} W_{i}^{(i)} [X_{i+1}^{e} - \hat{x}_{i}] [z_{i+1}^{e} - \hat{z}_{i}]^{T}$$

$$K_{i} = P_{z} / \bar{R}_{i}$$

$$\hat{x}_{i} = \hat{x}_{i+1}^{e} + K_{i} (z_{i} - \hat{z}_{i})$$

$$P_{i} = P_{i+1} - K_{i} P_{z} K_{i}^{T}$$

Compare with conventional UKF algorithm, one can find that a resample processing has been considered for the modified UKF version:

$$\bar{X}_{i+1} = [\hat{x}_{i+1}^{0} + (\eta_{i} \sqrt{L + \lambda})] \sqrt{L + \lambda}$$

Thus, affections of additive noise are induced in state vector, reduce the vector’s dimensions. The calculation load will be decreased.

Usually, on weak signal environment carrier is in low dynamic movement, such as downtown. So, a third carrier tracking loop is employed. The errors estimation history using UKF loop for 30dB/Hz weak GPS signal are demonstrated in figure 9. The initial value is as follows: loop update time 1ms, initial frequency error 12Hz, noise bandwidth 15Hz, receiver oscillator type TCXO, carrier dynamic 0.1g.

As can be seen from figure 9, the outputs of UKF maintaining a stable error during weak GPS signal tracking. The mean phase error is equal to -0.1432 degree. The error standard deviation is equal to 33.3784 degree. This value is smaller than carrier tracking threshold 45 degree (3 $\sigma$). So, the 30dB/Hz weak signal can be tracking. We can also see that, the output of the phase error is somewhat noisy. This undesired result due to the signal power. If one want to track the weaker GPS signal. The tracking loop update time should be set to more than 1 ms. Such as 20ms, to suppress more noise.

Figure 9. Weak signal errors tracking history using third order UKF loop
IV. CONCLUSION

An IF GPS signal simulator has been developed on MATLAB and C platform. It is able to simulate some conventional errors during signal propagation and processing. Carrier’s high dynamic movement and weak GPS signal environments which lead to signal variation can be simulated. Besides, the paper confirmed the conventional PLL-assisted-FLL can tracking high dynamic signal using simulated signal. An error tracking model is established by using modified Unscented Kalman Filtering for tracking weak signal. These are validated in the test. Certainly, this platform is a simplified scheme; some necessary modified tasks need to be done, such as complex multipath environment, RF interference. We will further develop some other useful algorithms for special environment application on it.

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