

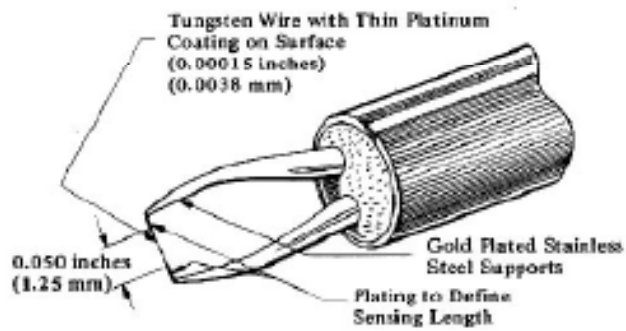
## Hot Wire Measurements



**Purpose:** to measure mean and fluctuating velocities in fluid flows

- **Thermal methods:**

- Compute flow velocity from its relationship between local flow velocity and the convective heat transfer from heated elements.



Hot wire anemometers

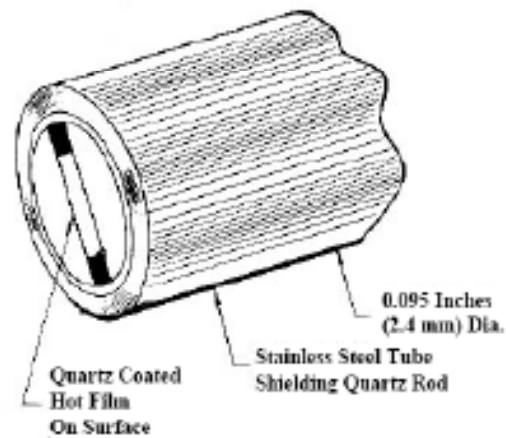


Figure 6: Hot Film Flush Mounted Probe

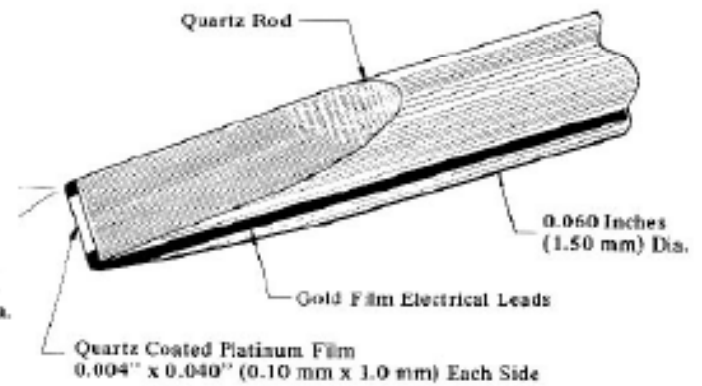
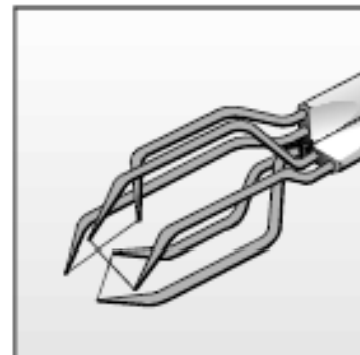
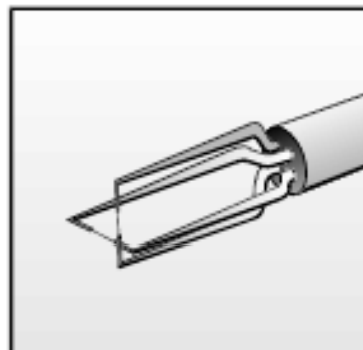
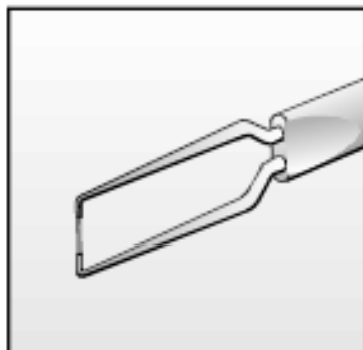


Figure 7: Hot Film Wedge Probe



# Hot Wire Measurements

The hot wire anemometer has been used for many years as a research tool in fluid mechanics.

In spite of the availability of non-intrusive velocity measurement systems (i.e. multi-component laser Doppler velocimetry), it is still widely applied, due to improvements of electronic technology and to increased interest in detailed description of turbulent flow fields.

The hot wire anemometer is still the only instrument delivering at the output a truly analogue representation of the velocity up to high frequencies fluctuations.

# Hot Wire Measurements

The hot wire anemometer consists of a sensor, a small electrically heated wire exposed to the fluid flow and of an electronic equipment, which performs the transformation of the sensor output into a useful electric signal.

The electronic circuitry forms an integral part of the anemometric system and has a direct influence on the probe characteristics.

The sensor itself is very small: typical dimensions of the heated wire are  $5\mu\text{m}$  in diameter and 1 to 3 mm in length, thus giving an almost punctual measurement.

# Hot Wire Measurements

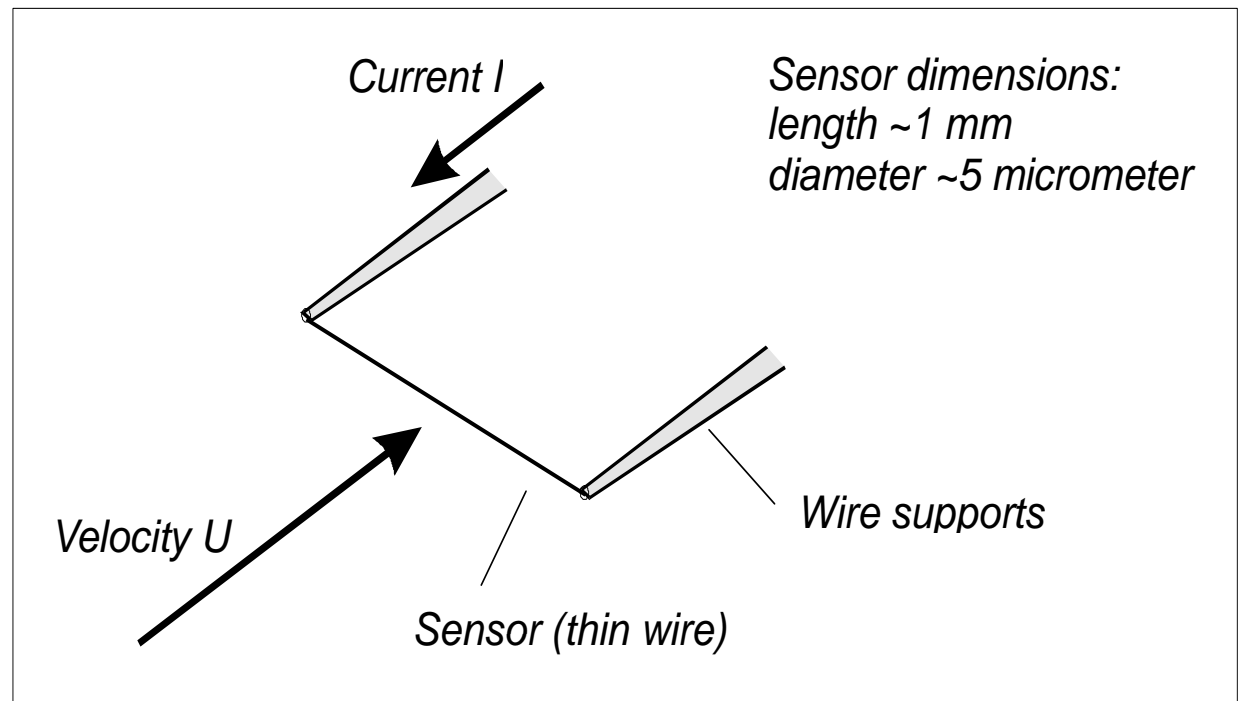
The basic principle of operation of the system is the heat transfer from the heated wire to the cold surrounding fluid, heat transfer which is function of the fluid velocity. Thus a relationship between the fluid velocity and the electrical output can be established.

The purpose of the electronic circuit is to provide to the wire a controlled amount of electrical current, and in the constant temperature method, to vary such a supply so as to maintain the wire temperature constant, when the amount of heat transfer varies.

# Principles of operation

- Consider a thin wire mounted to supports and exposed to a velocity  $U$ . When a current is passed through wire, heat is generated ( $I^2 R_w$ ). In equilibrium, this must be balanced by heat loss (primarily convective) to the surroundings.

- **If velocity changes, convective heat transfer coefficient will change, wire temperature will change and eventually reach a new equilibrium.**



# Probes

## 1. Hot wire sensors:

In practical applications, a material suitable for making a sensor should have some properties:

- a high value of the temperature coefficient of resistance, to increase its sensitivity to velocity variations
- an electrical resistance such that it can be easily heated with an electrical current at practical voltage and current levels.
- possibility of being available as wire of very small diameters
- a high enough strength to withstand the aerodynamic stresses at high flow velocities.

# Governing equation I

- Governing Equation:  $\frac{dE}{dt} = W - H$

$E$  = thermal energy stored in wire

$$E = C_w T$$

$C_w$  = heat capacity of wire

$W$  = power generated by Joule heating

$$W = I^2 R_w$$

recall  $R_w = R_w(T_w)$

$H$  = heat transferred to surroundings



# Governing equation II

- Heat transferred to surroundings

$$H = \sum \begin{array}{l} (\text{convection to fluid} \\ + \text{conduction to supports} \\ + \text{radiation to surroundings}) \end{array}$$

_____ Convection	$\Rightarrow$	$Q_c = Nu \cdot A \cdot (T_w - T_a)$ $Nu = h \cdot d / k_f = f(Re, Pr, M, Gr, \alpha)$ $Re = \rho U / \mu$
_____ Conduction	$\Rightarrow$	$f(T_w, l_w, k_w, T_{supports})$
_____ Radiation	$\Rightarrow$	$f(T_w^4 - T_f^4)$

# Simplified static analysis I

- For equilibrium conditions the heat storage is zero:

$$\frac{dE}{dt} = 0 \quad \therefore W = H$$

and the Joule heating  $W$  equals the convective heat transfer  $H$

- Assumptions
  - Radiation losses small
  - Conduction to wire supports small
  - $T_w$  uniform over length of sensor
  - Velocity impinges normally on wire, and is uniform over its entire length, and also small compared to sonic speed.
  - Fluid temperature and density constant

# Simplified static analysis II

Static heat transfer:

$$W = H \Rightarrow I^2 R_w = hA(T_w - T_a) \Rightarrow I^2 R_w = Nu k_f / d A (T_w - T_a)$$

$h$  = film coefficient of heat transfer

$A$  = heat transfer area

$d$  = wire diameter

$k_f$  = heat conductivity of fluid

$Nu$  = dimensionless heat transfer coefficient

Forced convection regime, i.e.  $Re > Gr^{1/3}$  (0.02 in air) and  $Re < 140 \Rightarrow$

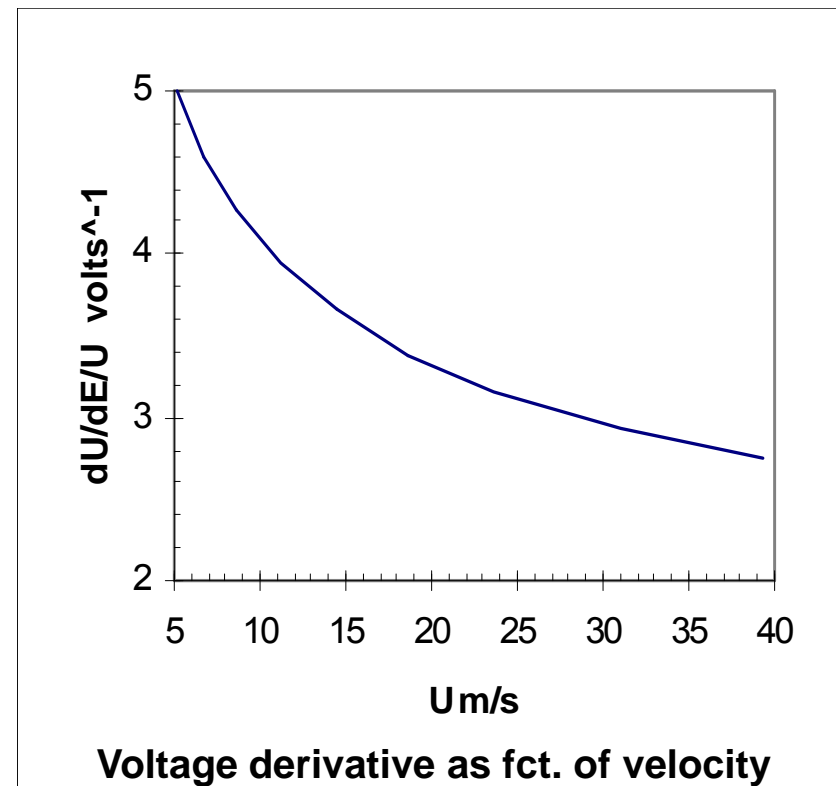
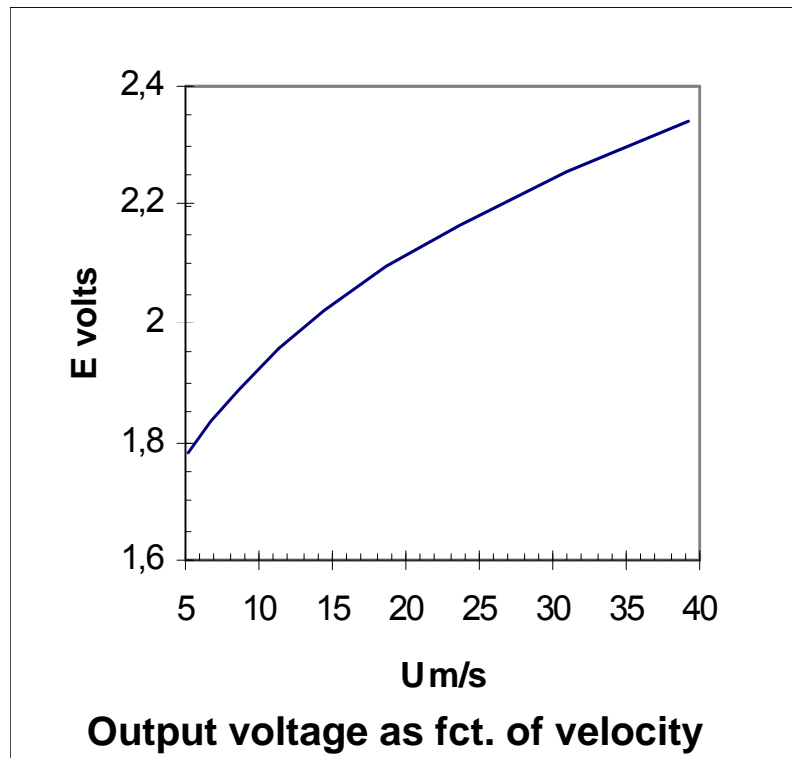
$$Nu = A_1 + B_1 \cdot Re^n = A_2 + B_2 \cdot U^n$$

$$I^2 R_w = E^2 = (T_w - T_a)(A + B \cdot U^n) \quad \text{“King’s law”}$$

The voltage drop is used as a measure of velocity.

# Hot-wire static transfer function

- Velocity sensitivity (King's law coeff.  $A = 1.51$ ,  $B = 0.811$ ,  $n = 0.43$ )



# Probes

## 1. Hot wire sensors:

The materials which are most commonly used are:

tungsten, platinum and platinum-iridium alloys

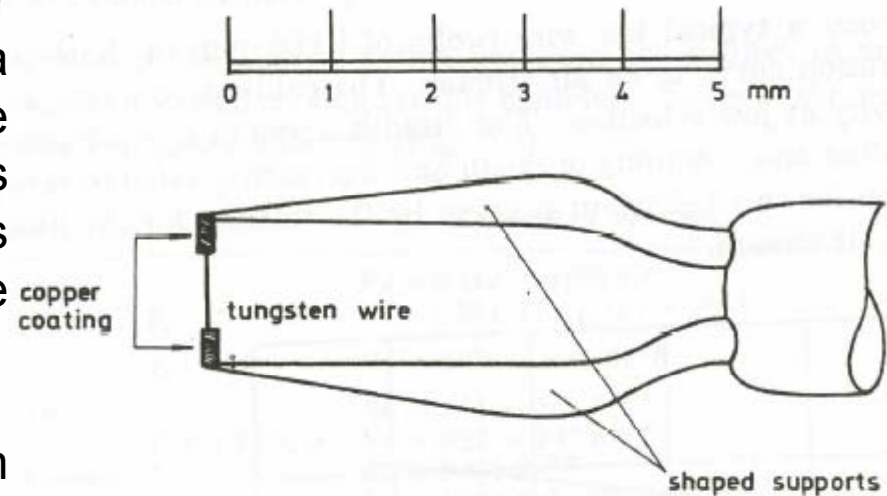
- Tungsten wires are mechanically strong and have a high temperature coefficient of resistance ( $0.004/^\circ\text{C}$ ). However, they can not be used at high temperatures in many gases, because of their poor resistance to oxidation.
- Platinum has good oxidation resistance, has a good temperature coefficient of resistance ( $0.003/^\circ\text{C}$ ), but is mechanically weak, particularly at high temperatures.
- The platinum-iridium alloy is a compromise between tungsten and platinum with good oxidation resistance, and higher tensile strength than platinum, but it has a low temperature coefficient of resistance ( $0.00085/^\circ\text{C}$ )

# Probes

## 1. Hot wire sensors:

Tungsten is presently the most popular hot wire material. When coated with a thin platinum layer, it becomes more resistant to oxidation. In this case, its temperature coefficient ( $0.0032/^\circ\text{C}$ ) does not differ too much from that of pure platinum.

The actual tendency is to use a platinum coated tungsten wire coated at the extremities by a thick copper or gold deposit so that the resulting probe has good mechanical and aerodynamic characteristics. Typical dimensions are 1 to  $10\ \mu\text{m}$  in diameter ( $5\ \mu\text{m}$  being the most used choice) and a length of 1 to 3 mm for the heated wire.



Typical hot wire probe geometry

# Probes

## 2. Hot film sensors:

The hot film sensor is essentially a conducting film laid on a ceramic or quartz substrate. The sensor can be, typically, a quartz rod coated with a platinum film.

When compared with hot wires, the cylindrical hot film sensor has many advantages, the first being that it is less susceptible to accumulation of materials on its surface and easier to clean. However, the double structure (film plus substrate) makes its frequency response more complex.

The metal film thickness on a typical film sensor is less than  $0.1 \mu\text{m}$ . Thus, the mechanical strength and the effective thermal conductivity of the sensor are determined almost entirely by the substrate material.

Most films are made of platinum due to its good oxidation resistance and the resulting long term stability.

Because of their stability, film sensors have been used for many measurements which were otherwise very difficult with the more fragile and less stable hot wires.

# Probes

## 2. Hot film sensors:

Some manufacturers utilize special film coating techniques:

For measurement in air, hot films are coated with a thin layer of high purity alumina. This is highly resistant and has high thermal conductivity to minimize loss of frequency response.

Hot film sensors made for water or conducting fluids have a heavier coating of quartz which provides complete electrical insulation.



# Probes

## 2. Hot film sensors:

Even though film sensors have basic advantages, hot wire sensors give superior performance in many applications.

The diameter of cylindrical film sensor is typically 0.025 mm or larger. When compared with the dimensions of a hot wire, this is quite large.

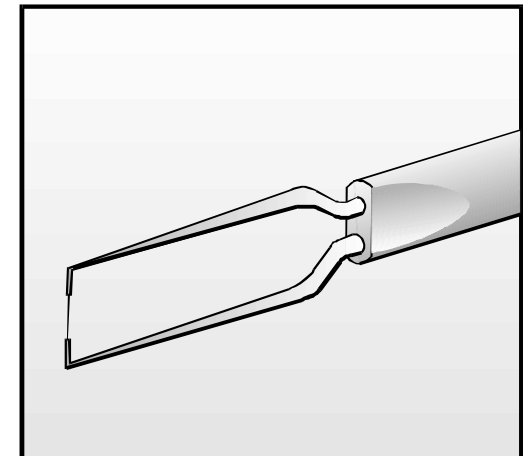
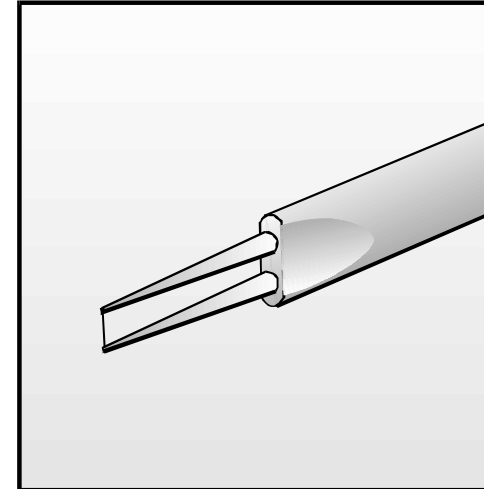
In addition, the film sensor generally has a lower sensitivity. Therefore, in application requiring maximum frequency response, minimum noise level and very close proximity to a surface, the platinum-coated tungsten hot wire sensor is superior.

# Probe types

- Miniature Wire Probes  
Platinum-plated tungsten,  
5  $\mu\text{m}$  diameter, 1.2 mm length
- Gold-Plated Probes  
3 mm total wire length,  
1.25 mm active sensor  
copper ends, gold-plated

## Advantages:

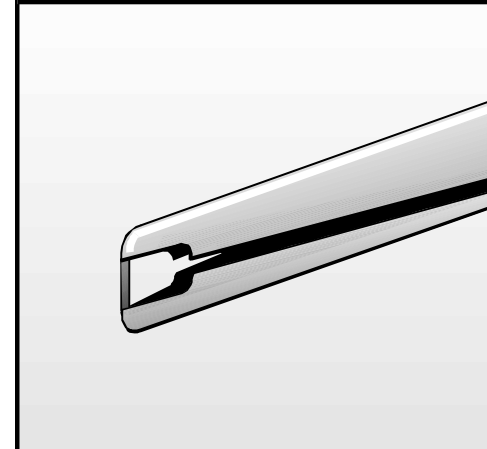
- accurately defined sensing length
- reduced heat dissipation by the prongs
- more uniform temperature distribution along wire
- less probe interference to the flow field



# Probe types

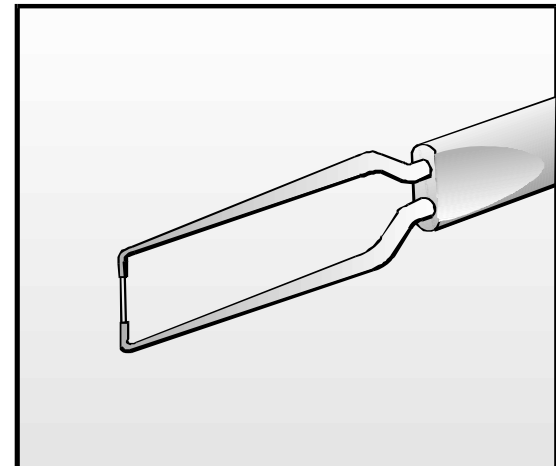
- Film Probes

Thin metal film (nickel) deposited on quartz body. Thin quartz layer protects metal film against corrosion, wear, physical damage, electrical action



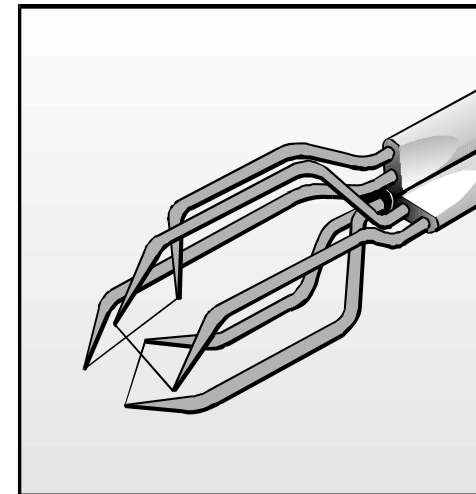
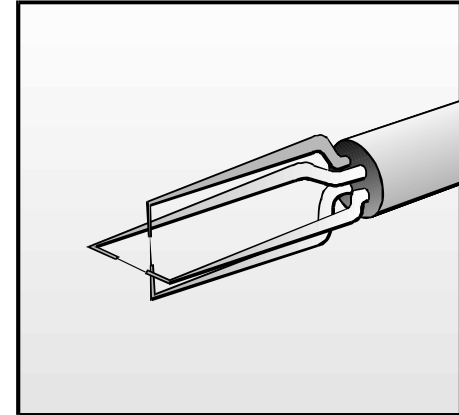
- Fiber-Film Probes

“Hybrid” - film deposited on a thin wire-like quartz rod (fiber) “split fiber-film probes.”



# Probe types

- X-probes for 2D flows  
2 sensors perpendicular to each other. Measures within  $\pm 45^\circ$ .
- Split-fiber probes for 2D flows  
2 film sensors opposite each other on a quartz cylinder. Measures within  $\pm 90^\circ$ .
- Tri-axial probes for 3D flows  
3 sensors in an orthogonal system. Measures within  $70^\circ$  cone.



# Hints to select the right probe

- Use wire probes whenever possible
  - ✓ relatively inexpensive
  - ✓ better frequency response
  - ✓ can be repaired
- Use film probes for rough environments
  - ✓ more strong
  - ✓ worse frequency response
  - ✓ cannot be repaired
  - ✓ electrically insulated
  - ✓ protected against mechanical and chemical action

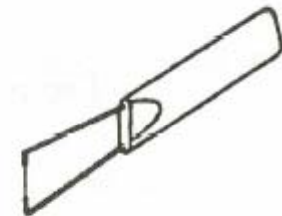
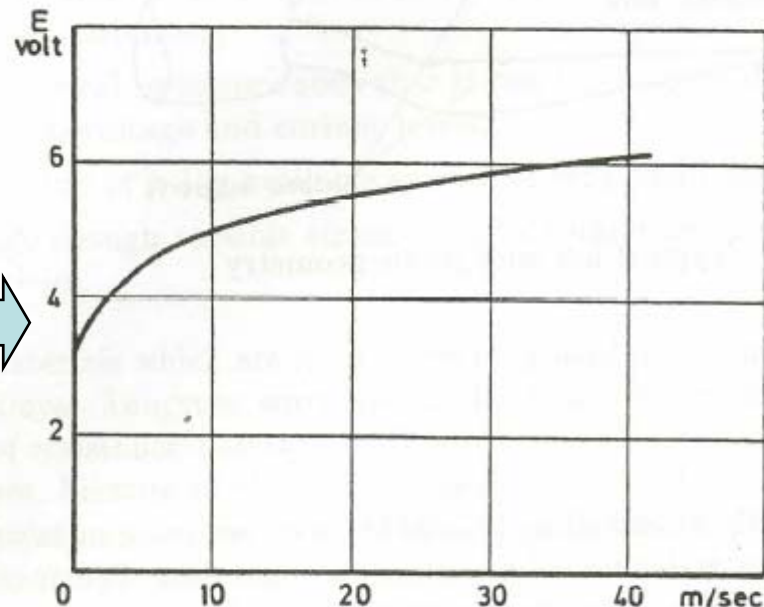
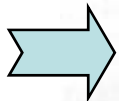
# Calibration Characteristics

Figure shows the velocity calibration curve in air stream of a typical hot wire probe of 0.005 mm in diameter.

The calibration curve is non-linear, with maximum sensitivity at low velocities.

The disadvantages of a non-linear output in terms of convenient reading and recording of mean and fluctuating velocity components are well known. The reason for this behavior is given by the relation for the heat transfer from a body in a flowing air stream.

*Hot wire calibration curve*



## Calibration Characteristics

Writing such an equation for the hot wire, if  $I$  is the heating current flowing in the wire,  $R_w$  is the resistance at operating temperature  $T_w$ ;  $D$  and  $l$  its diameter and length, the steady state energy balance takes the form:

$$\text{Supply} = I^2 R_w = \pi D l h (T_w - T_a) = \text{Dissipation}$$

where  $h$  is the heat transfer coefficient related to the other thermodynamic properties to the fluid in the Nusselt number:

$$Nu = \frac{hD}{k}$$

where  $k$  is the thermal conductivity coefficient for the fluid.

By substitution, we obtain:

$$I^2 R_w = \pi l k (T_w - T_a) Nu$$

## Calibration Characteristics

The problem is now to obtain a relation between the Nusselt number and the other thermodynamic properties of the fluid and the characteristics of the flow around the thin wire.

$$Nu = f(Re, Pr)$$

where

$$Re = \text{Reynolds number } \frac{UD}{\nu}$$

$$Pr = \text{Prandtl number } \frac{c_p \mu}{k}$$

The determination of the above relation has been the subject of many investigations trying to determine a universal cooling law for cylinders.



# Calibration Characteristics

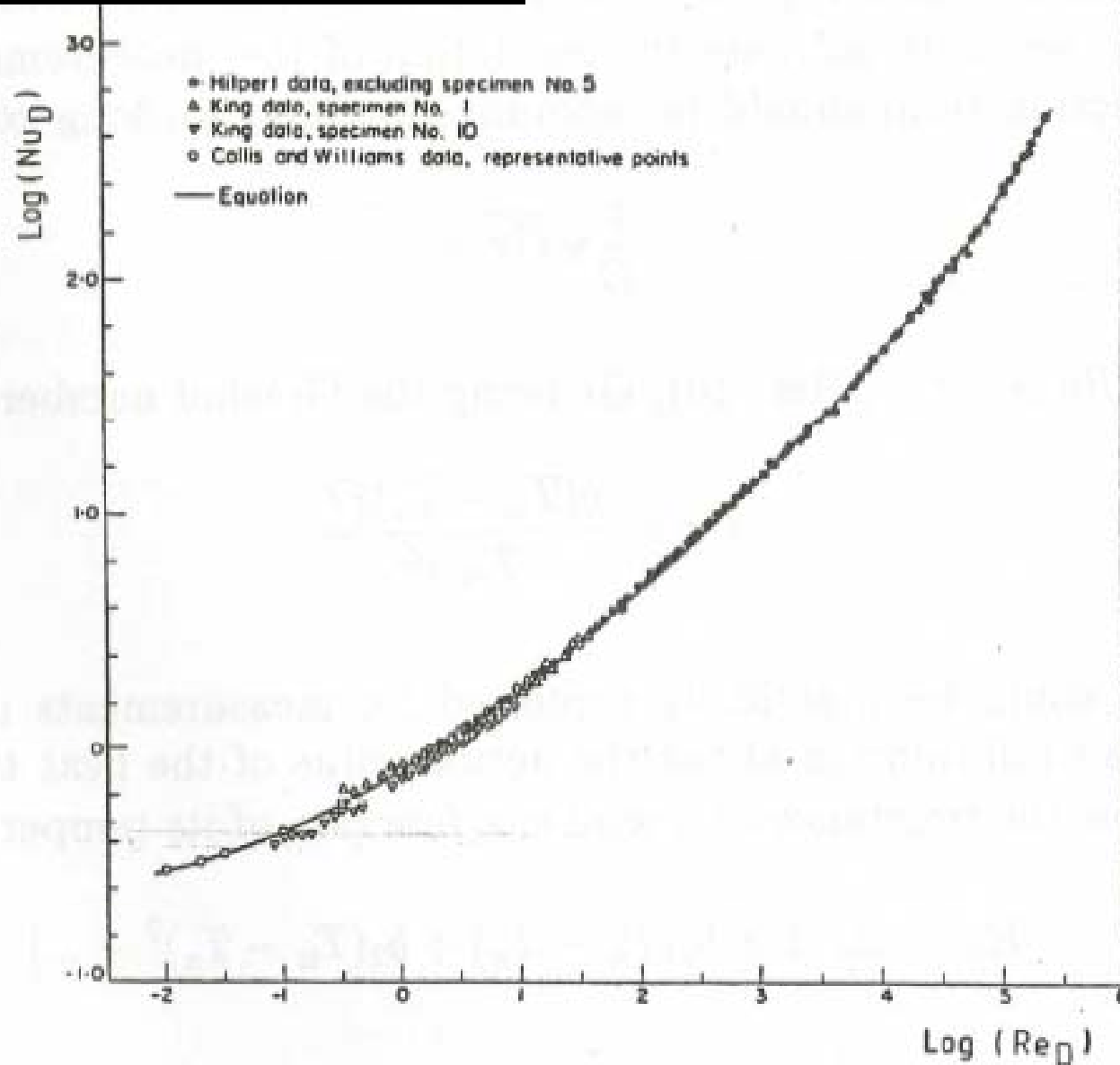
## Hot wire correlations for heat transfer

Author	$k, \rho, \eta$ at $T$	Correlation equation	$Re$
King		$Nu = 0.318 + 0.690 Re^{0.5}$	$0.055 < Re < 55$
Hilpert	$T_m$	$Nu = 0.891 [Re(T_w/T_g)^{0.25}]^{0.330}$	$1 < Re < 4$
Kramers	$T_m$	$Nu = 0.039 + 0.51 Re^{0.5}$	$0.01 < Re < 10^4$
Ulsamer			
Eckert and Soehngen		$Nu = 0.43 + 0.48 Re^{0.5}$	$1 < Re < 4 \times 10^3$
McAdams	$T_m$ but $T_g$ for $\rho$	$Nu = 0.32 + 0.43 Re^{0.52}$	$0.1 < Re < 10^3$
Scadron and Warshawsky	$T_g$	$Nu = 0.431 Re^{0.5}$	$250 < Re < 3 \times 10^4$
Churchill and Brier	$T_g$	$Nu = 0.535 Re^{0.5} (T_g/T_w)^{0.12}$	$300 < Re < 2300$
Douglas and Churchill	$T_m$	$Nu = 0.46 Re^{0.5} + 0.00128 Re$	$500 < Re < 3 \times 10^5$
Van Der Hegge Zijnen	$T_m$	$Nu = 0.35 + 0.5 Re^{0.5} + 0.001 Re$	$0.01 < Re < 10^4$
Glawe and Johnson	$T_g$	$Nu = 0.428 Re^{0.5}$	$400 < Re < 3 \times 10^3$
Collis and Williams	$T_m$	$Nu (T_m/T_g)^{-0.17} = 0.24 + 0.56 Re^{0.45}$	$0.02 < Re < 44$
Baldwin <i>et al.</i>	$T_g$	Graphical correlation	$0.02 < Re < 3 \times 10^5$
Laurence and Sandborn	$T_g$		$(0.001 < Kn < 37)$
Davies and Fisher	$T_g$	$Nu = (2.6/\gamma\pi) Re^{0.33}$	$0 < Re < 50$
Richardson		$Nu = 0.3737 + 0.37 Re^{0.5} + 0.056 Re^{0.66}$	$1 < Re < 10^5$
Parnas	$T_g$	$Nu = 0.823 Re^{0.5} (T_w/T_g)^{0.085}$	$10 < Re < 60$
Ahmad	$T_m$	$Nu(v_g/v_m)^{0.15} = 0.2068 + 0.4966 Re^{0.45}$	$4 < Re < 40$
Present	$T_m$	$N_c = 0.34 + 0.65 Re^{0.45}$	$0.015 < Re < 20$ $l/d > 400$

It is noted that the Prandtl number does not appear in these relations because they are established for air at ambient temperature. The most widely accepted low is that of Collis and Williams.

# Calibration Characteristics

## Hot wire correlations for heat transfer



# Calibration Characteristics

It is noted that most of the relations are of the form  $A+BU^n$  so the energy balance may be rewritten as:

$$I^2 R_w = \{A + BU^n\} (T_w - T_a) = H(U) (T_w - T_a)$$

**A** represents the natural convection term and **BU<sup>n</sup>** represents the forced convection term

Taking into account the uncertainty of the values A, B and n, and the fact that the problems are made even worse by the finite length of the wire, and the effect of the support, one may say that **a direct calibration of the actual probe has to be made every time before a test.**

Furthermore, because of the dependence on fluid properties, **the calibration has to be made in the same fluid at the same temperature as for the actual test.**

## Calibration Characteristics

The output contains information both for the natural convection and for the forced convection. If one wants to increase the resolution of the measurements, the influence of the natural convection term should be minimized.

Good working conditions are reached when:

$$\frac{l}{D} \sqrt{Gr} < 1$$

$Gr$  being the Grashof number:

$$Gr = \frac{g(T_w - T_a)D^3}{T_m \nu^2}$$

$$T_m = \frac{T_w + T_a}{2}$$

$\nu$  is the kinematic viscosity of air

# Calibration Characteristics

The resistance of a wire is a function of its temperature.

For a metallic conductor:

$$R_w = R_a [1 + b_1(T_w - T_a) + b_2(T_w - T_a)^2 + \dots]$$

where the coefficients  $b_1$  and  $b_2$  have the following typical values:

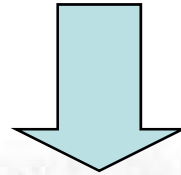
$$\begin{array}{ll} \text{-platinum: } b_1 = 3.5 \times 10^{-3}/^{\circ}\text{C} & ; \quad b_2 = -5.5 \times 10^{-7}/^{\circ}\text{C}^2 \\ \text{-tungsten: } b_1 = 5.2 \times 10^{-3}/^{\circ}\text{C} & ; \quad b_2 = 7 \times 10^{-7}/^{\circ}\text{C}^2 \end{array}$$

It could be seen that for usual operating temperatures (up to 200°C), the expression could be linearized to:

$$R_w = R_a [1 + b_1(T_w - T_a)]$$

## Calibration Characteristics

$$I^2 R_w = \{A + BU^n\} (T_w - T_a) \equiv H(U) (T_w - T_a)$$



$$b_1 \frac{I^2 R_w R_a}{R_w - R_a} = A + BU^n$$

Thus, the information about the actual value of the heat transfer could be obtained either

- as the value of  $R_w$  if  $I$  is kept constant (constant current method)
- or as the value of  $I$  if  $R_w$  is kept constant (constant temperature method)

# Modes of anemometer operation

**Constant Current (CCA)**

**Constant Temperature (CTA)**

# Constant current anemometer CCA

- **Principle:**

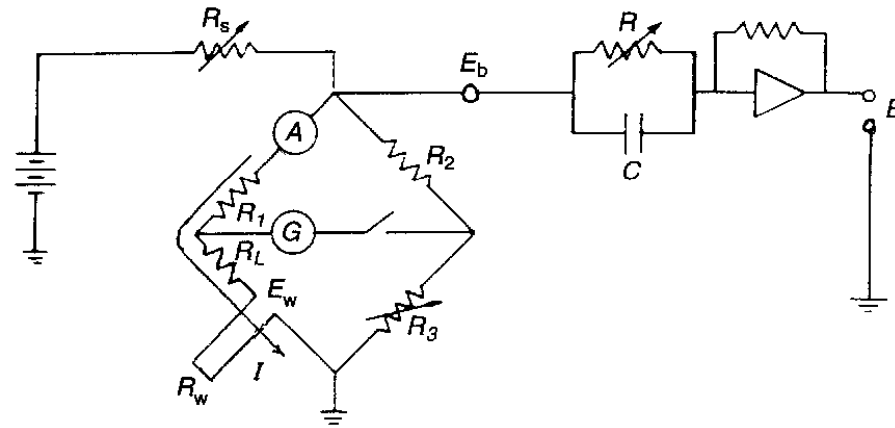
Current through sensor is kept constant

- **Advantages:**

- High frequency response

- **Disadvantages:**

- Difficult to use
- Output decreases with velocity
- Risk of probe burnout





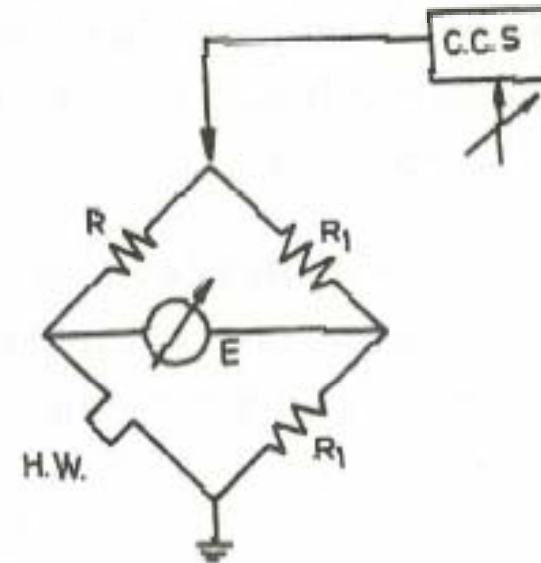
# Control Circuits

## Constant current:

For a useful application of hot wire sensors, the selection of the probe is of primary importance; however, the sensor must be controlled by an electronic circuit to obtain the best possible performance.

In its basic form, the control circuit may be reduced to a source of constant current feeding a calibration and measurement bridge.

The two resistors  $R_1$  are chosen to be equal, the value of  $R$  is chosen to be equal to the hot resistance of the wire (usually 1.8 times the cold wire resistance) and the supply current is increased until, for zero wind velocity, balance is obtained at the bridge output.



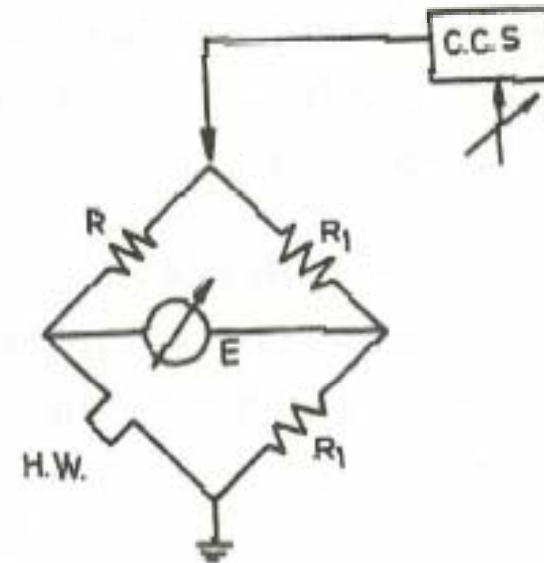
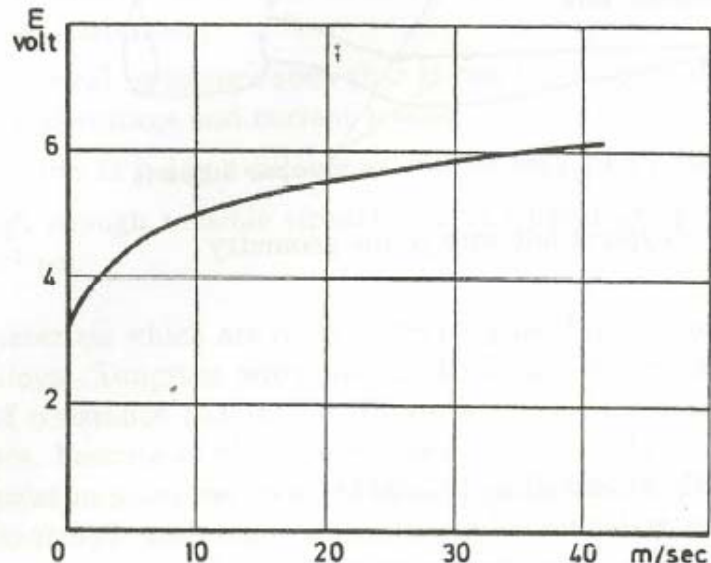
Hot wire control circuit

# Control Circuits

## Constant current:

Any change in wind velocity will change the heat transfer, and thus the wire temperature and resistance and cause an unbalanced voltage to appear at the bridge output.

This can be calibrated against flow velocity to obtain the wire calibration curve.



Hot wire control circuit

# Control Circuits

## Constant current:

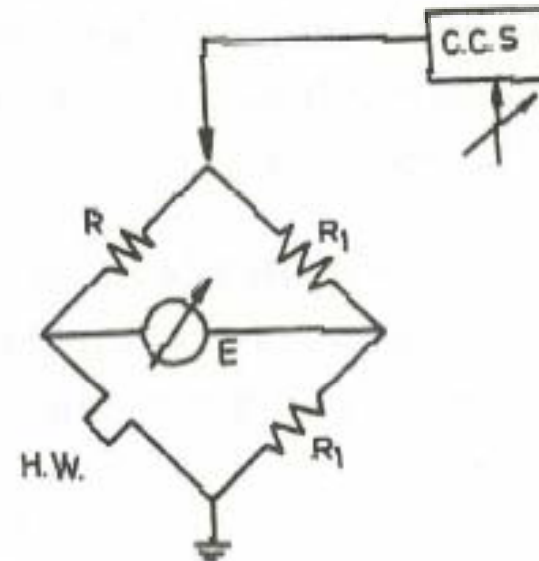
One of the major reasons for using a hot wire anemometry is its ability to detect and follow **fast fluctuations of velocity**. In the case of this circuit, if the velocity changes takes place very rapidly, the response of the sensor will lag behind the actual change in velocity due to its own thermal inertia.

Therefore, the equation for the instantaneous heat transfer can be written as:

$$I^2 R_w = H(U)(T_w - T_c) + C_w \frac{dT_w}{dt}$$

where  $C_w$  represents the thermal inertia of the wire itself.

$$C_w \approx \rho C \frac{\pi D^2 \ell}{4}$$



Hot wire control circuit

# Control Circuits

## Constant current:

Equation can be linearized and solved for small velocity perturbations, assuming:

$$H(U, t) = \bar{H}(U) + \tilde{H}(U, t)$$

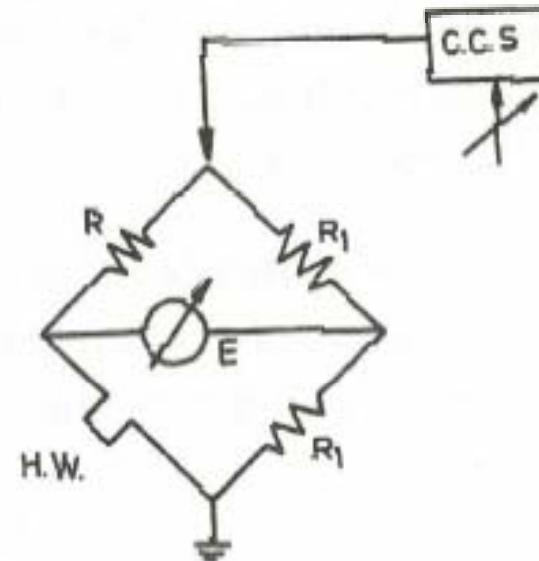
and expressing the output as:

$$E(t) = \bar{E}(t) + e(t)$$

to obtain for the fluctuating output,

$$M \frac{de}{dt} + e = -\tilde{H}$$

where M is the wire time constant:  $M = \frac{CR_w}{R_o H}$



Hot wire control circuit

$R_w$ : hot wire resistance  
 $R_o$ : cold wire resistance

# Control Circuits

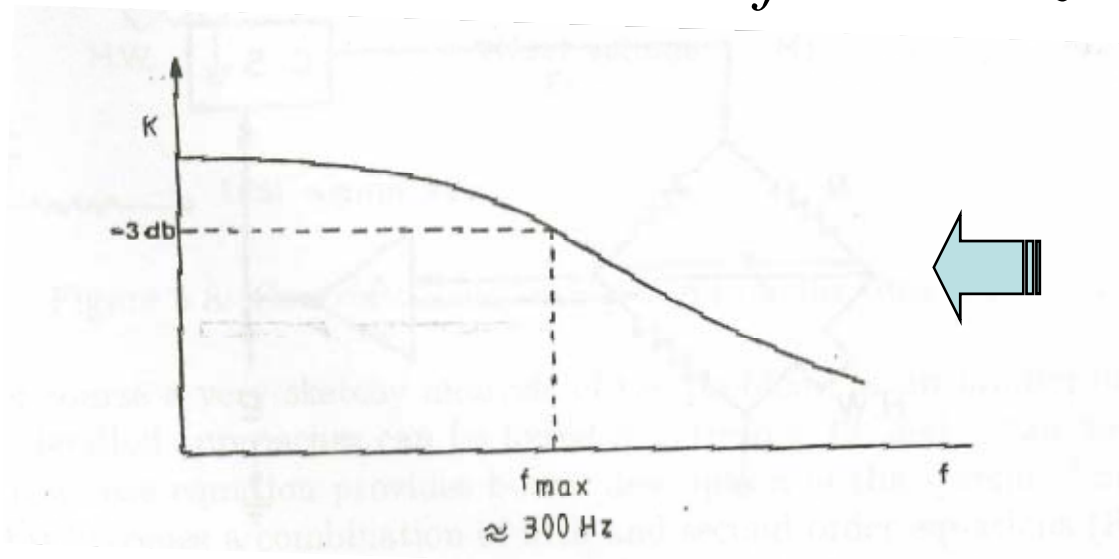
## Constant current:

Frequency limit for the hot wire response to sinusoidal velocity fluctuations:

$$f_{\max} = \frac{1}{2\pi M}$$

The frequency response is function of the fluid velocity through H, so that a universal value cannot be associated to these parameters. Typical values are:

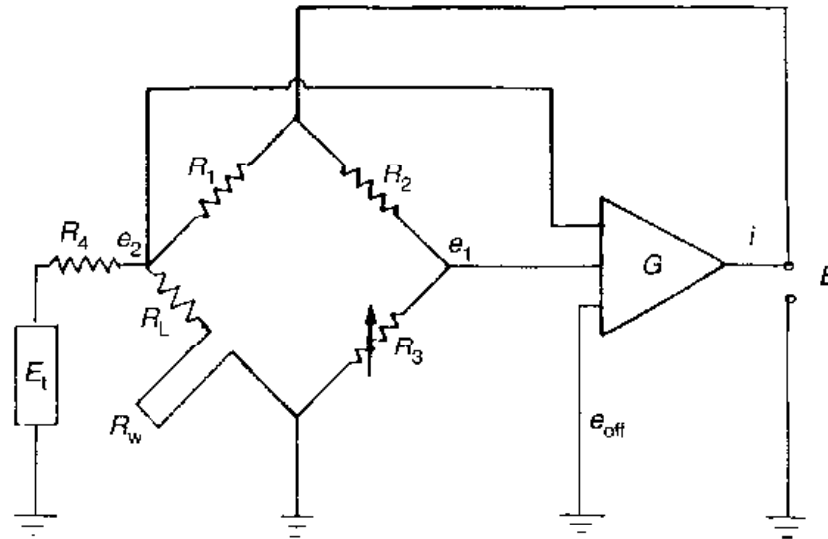
$$f \approx 300\text{Hz}$$



**Hot wire frequency response**

# Constant Temperature Anemometer CTA

- Principle:  
Sensor resistance is kept constant by servo amplifier
- Advantages:
  - Easy to use
  - High frequency response
  - Low noise
  - Accepted standard
- Disadvantages:
  - More complex circuit



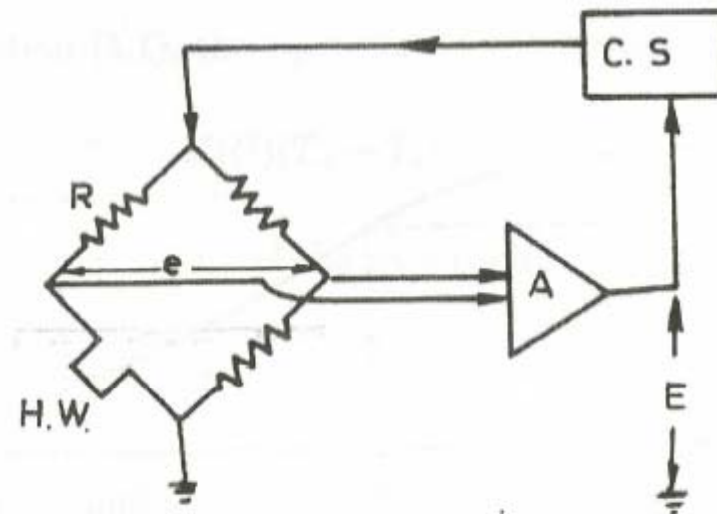
# Control Circuits

## Constant temperature:

It is a system in which the output from the bridge is amplified and used to control the supply voltage such as to **maintain the wire temperature constant**.

The amplifier output  $E$ , required to maintain the wire at a constant temperature is a function of the flow velocity.

The wire temperature is again fixed by the choice of the resistance  $R$  of the bridge (usually 1.8 times the cold wire resistance)



Constant temperature control circuit

# Control Circuits

## Constant temperature:

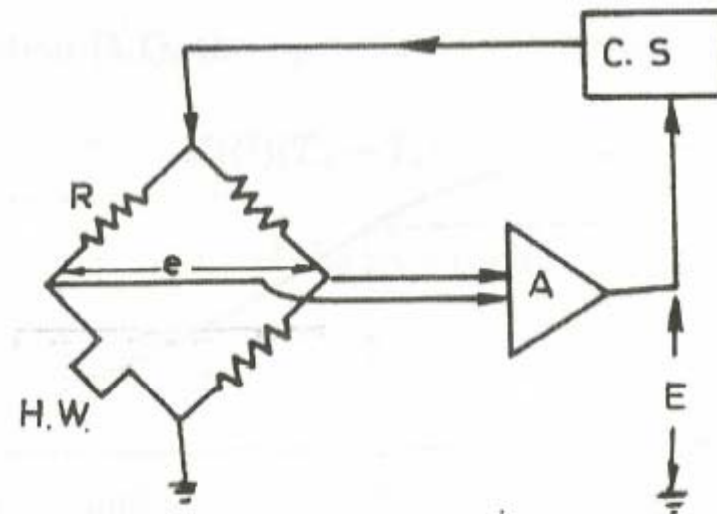
If the amplifier gain  $K$  is large enough, the bridge unbalance is given by:

$$e \approx \frac{E}{K}$$

and the wire is constantly kept at a fixed temperature (or at least its temperature

fluctuations are  $\approx \frac{1}{K+1}$  times smaller

than in the constant current mode.)



Constant temperature control circuit



# Control Circuits

## Constant temperature:

If the system is reduced to below circuit, a second order equation can be written and the resonant frequency becomes:

$$\omega = f \left( \frac{R_w}{R_o}, H, E, M_1, M, K, L \right)$$

$M_1$ : time constant of electronic circuit

$M$ : time constant of hot wire and cables

$L$ : compensation inductance used to minimize the effects of  $M$  and  $M_1$

$E$ : the amplifier offset voltage

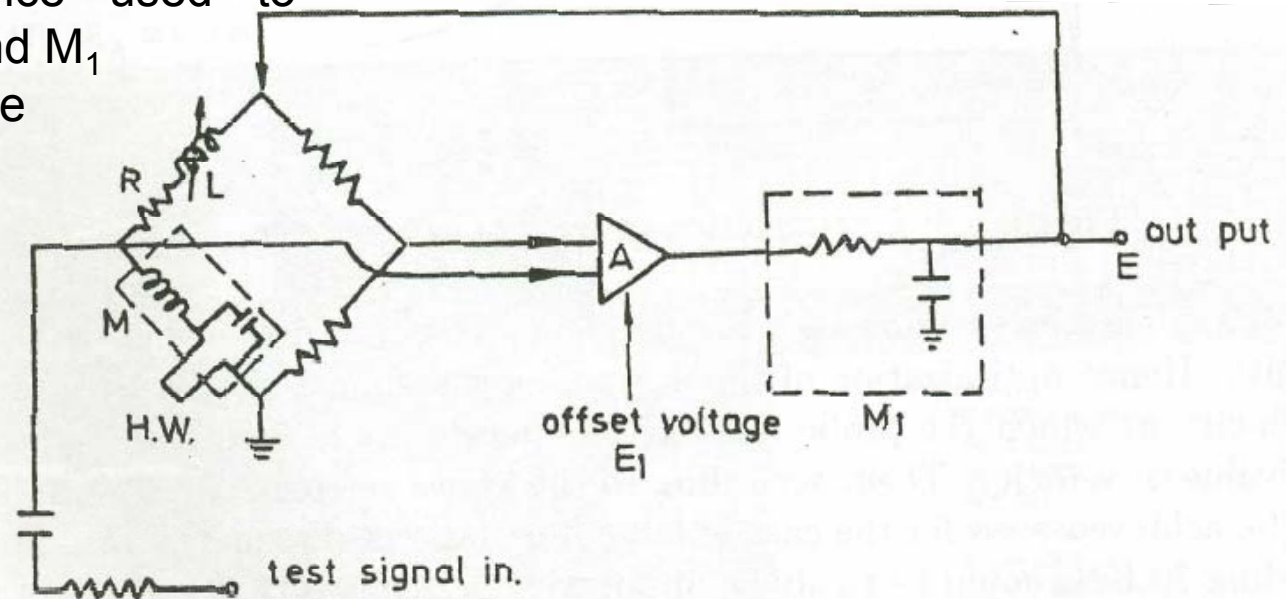


Figure 5.8: Control circuit taking into account time constants

# Control Circuits

## Constant temperature:

Frequency response depends on  $H(U)$ , that is on fluid velocity.

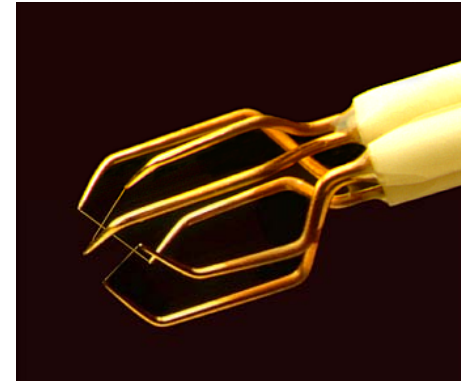
Hence optimization of the anemometer response should be done for the mean flow velocity at which the probe is likely to operate, or in case of impulsive flows, for the mean value of velocity.

Then, according to the above reference a good anemometer response will be achieved even for the case of large fluctuations.

Because the temperature of the wire remains almost constant, all non linearities introduced by thermal lag effect are substantially smaller and in most cases negligible.

# Constant temperature anemometer CTA

- 3-channel  
StreamLine with  
Tri-axial wire probe  
55P91

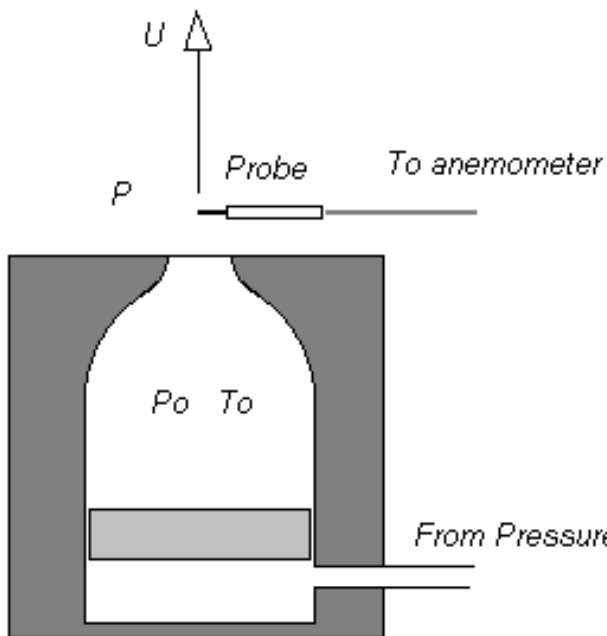


# Velocity calibration (Static cal.)

- Despite extensive work, no universal expression to describe heat transfer from hot wires and films exist.
- For all actual measurements, direct calibration of the anemometer is necessary.

# Velocity calibration (Static cal.)

- Calibration in gases (example low turbulent free jet):



Velocity is determined

isentropic expansion:

$$P_0/P = (1 + (\gamma - 1)/2M^2)^{\gamma / (\gamma - 1)}$$

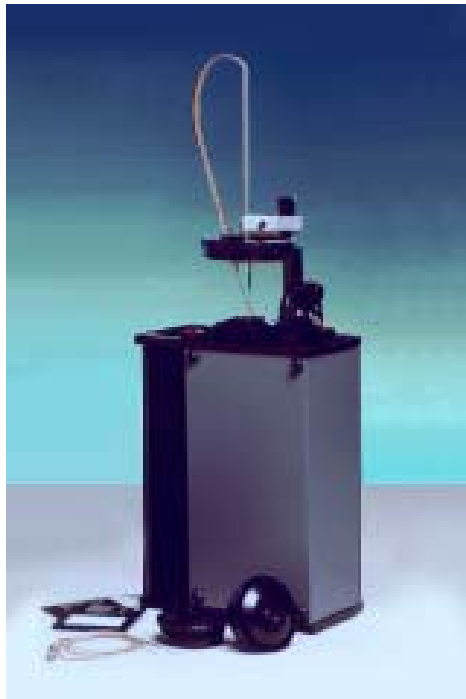
$$a_0 = (\gamma P T_0)^{0.5}$$

$$a = a_0 / (1 + (\gamma - 1)/2M^2)^{0.5}$$

$$U = Ma$$

## Hot-wire Calibrator

The Dantec Dynamics Hot-Wire Calibrator is a simple, but accurate, device for 2-point calibration of most hot-wire probes used with Constant Temperature Anemometers. The calibrator produces a free jet, where the probe is placed during calibration. It requires a normal pressurized air supply and is able to set velocities from 0.5 m/s to 60 m/s.

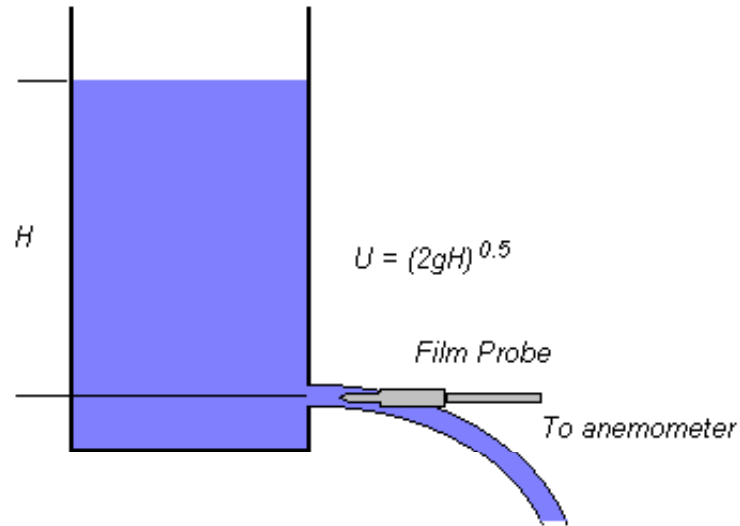


## StreamLine calibrator

The StreamLine calibration system is intended for calibration of probes in air from a few cm/sec up to Mach 1

The flow unit creates a free jet and requires air from a pressurised air supply. The probe to be calibrated is placed at the jet exit. The full velocity range is covered by 4 different nozzles. The flow unit can be equipped with a pitch-yaw manipulator that allows 2-D and 3-D probes to be rotated for calibration of directional sensitivity.

# Velocity calibration (Static)



- **Film probes in water**
  - Using a free jet of liquid issuing from the bottom of a container
  - Towing the probe at a known velocity in still liquid
  - Using a submerged jet

## Water calibrator for CTA film probes

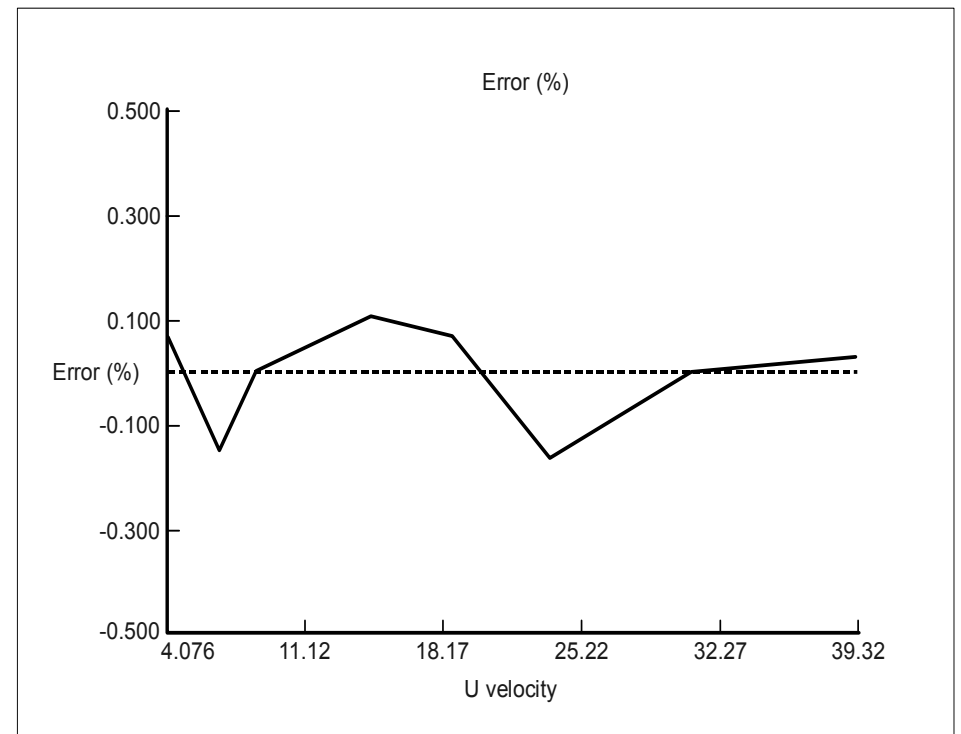
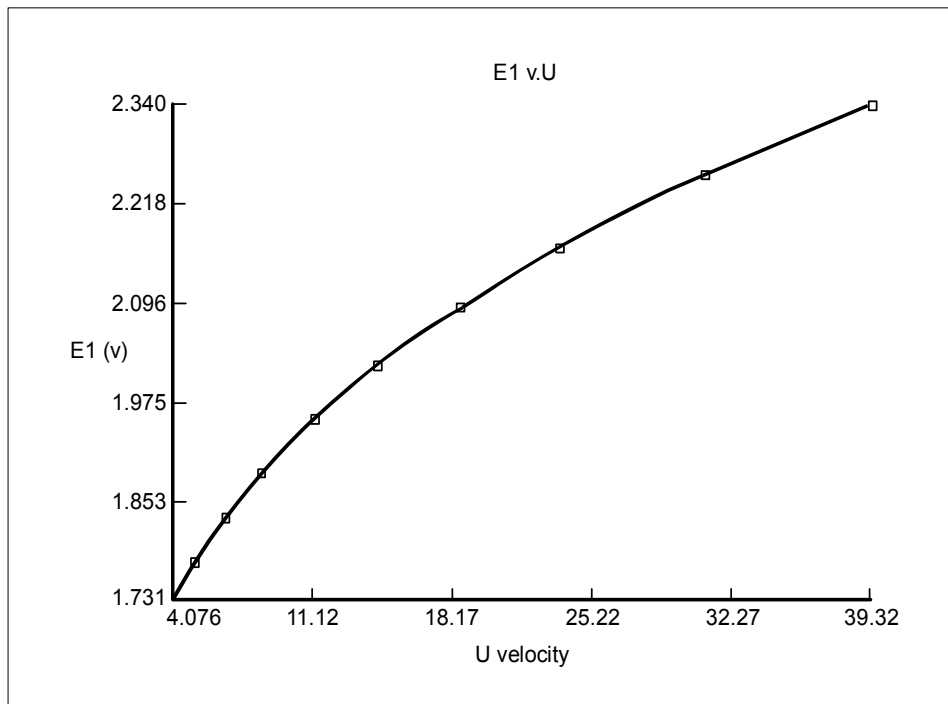
In many water applications, the use of film probes may still be an attractive option in special cases. The calibrator is a recirculating water tunnel with a submerged jet in a water reservoir in front of the jet. It covers velocities from 0.005 to 2 m/s with combinations of three different nozzles (600, 120 and 60 mm<sup>2</sup>). Velocity is calculated on the basis of a precision-variable area flowmeter reading.



# Typical calibration curve

- Wire probe calibration with curve fit errors

(Obtained with Dantec 90H01/02)Calibrator)



Curve fit (velocity  $U$  as function of output voltage  $E$ ):

$$U = C_0 + C_1E + C_2E^2 + C_3E^3 + C_4E^4$$



# Dynamic calibration/tuning

- Direct method

Need a flow in which sinusoidal velocity variations of known amplitude are superimposed on a constant mean velocity

- Microwave simulation of turbulence (<500 Hz)
- Sound field simulation of turbulence (>500 Hz)
- Vibrating the probe in a laminar flow (<1000Hz)

All methods are difficult and are restricted to low frequencies.

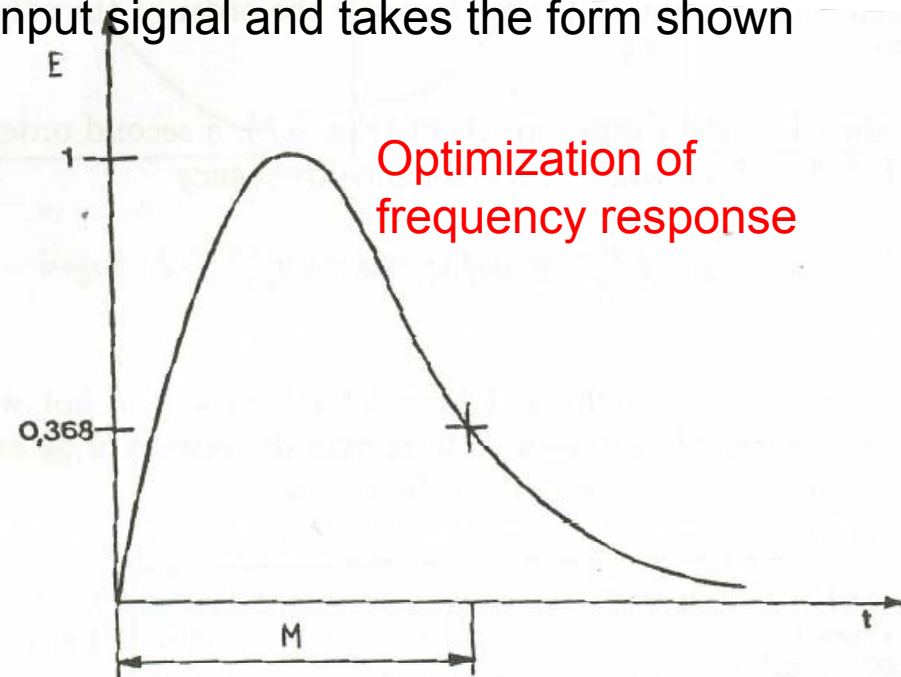
# Control Circuits

The usual approach to evaluate the result obtained is by means of the so called square wave test:

A square wave test current is injected at the wire terminal and the response of the system monitored on an oscilloscope.

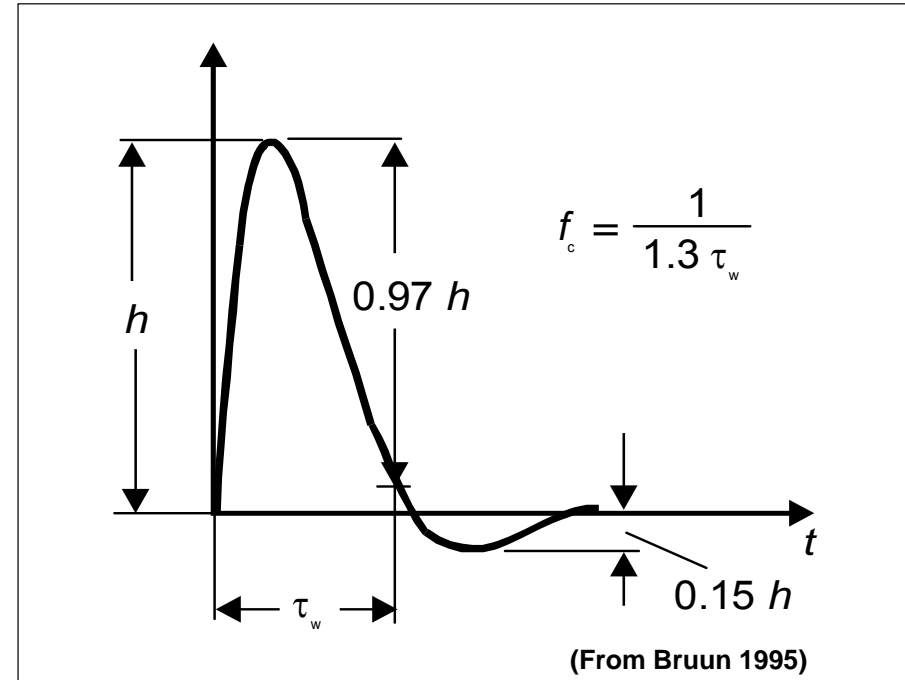
Because of the feedback system involved, the resulting output is (theoretically should be) equal to the derivative of the input signal and takes the form shown in the Figure.

If the order of the system is higher than the second there is some difficulty in translating the information from the square wave test into frequency response of the hot wire.



# Dynamic calibration/tuning

- Indirect method “SQUARE WAVE TEST”



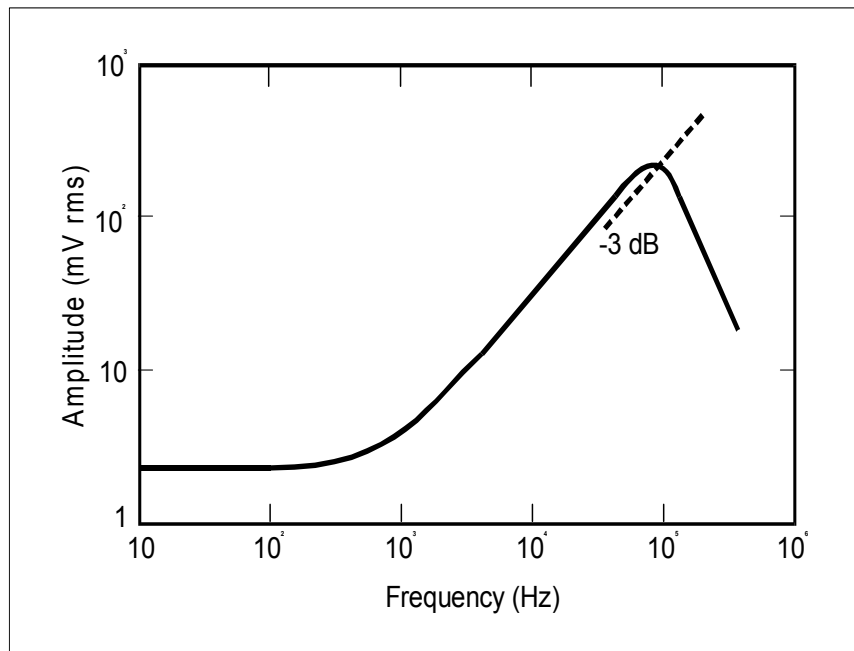
For a wire probe (1-order probe response):

Frequency limit (- 3dB damping):  $f_c = 1/1.3 \tau$

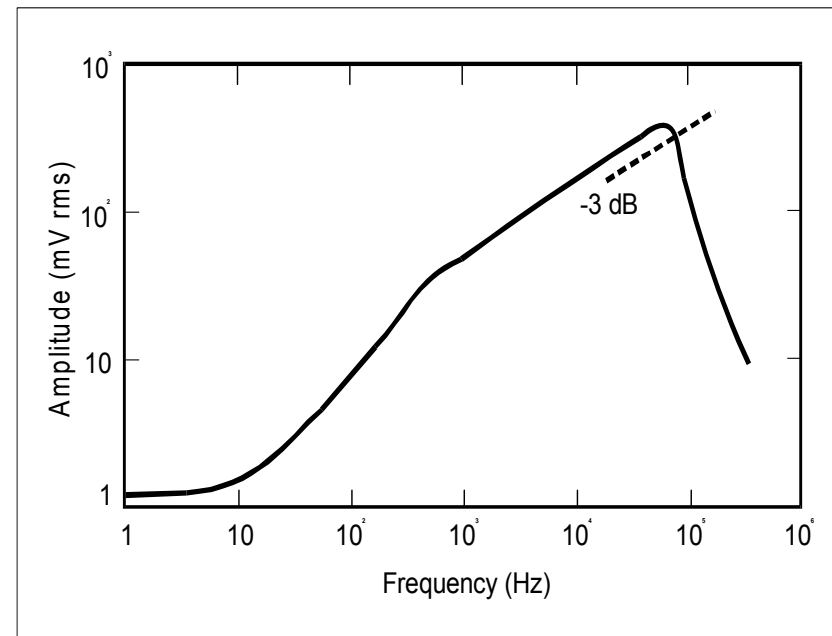
# Dynamic calibration/tuning

- Indirect method, “SINUS TEST”

Subject the sensor to an electric sine wave which simulates an instantaneous change in velocity and analyse the amplitude response.



**Typical Wire probe response**



**Typical Fiber probe response**

# Control Circuits

Constant temperature:

Sine and square wave tests for Hot wire

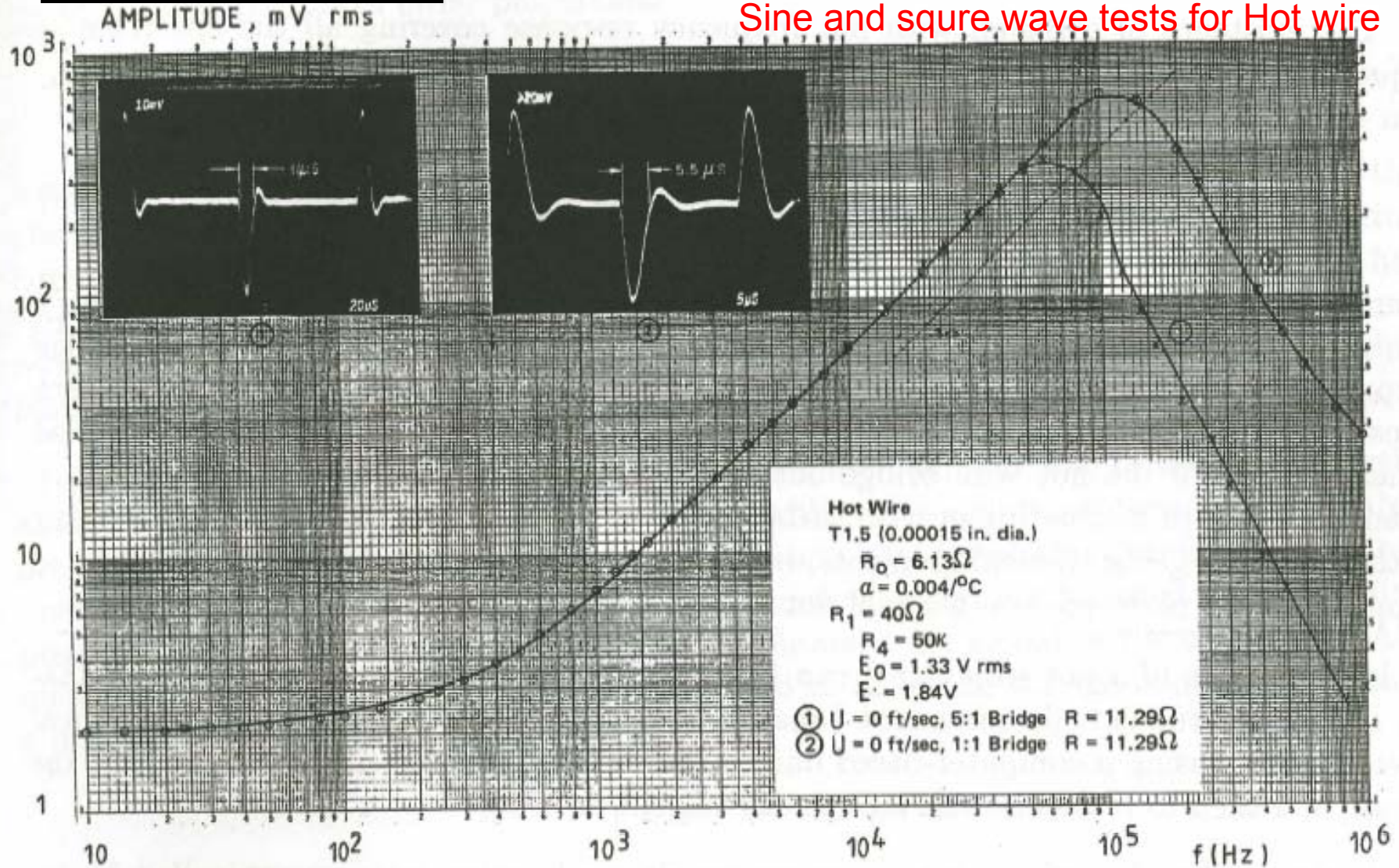


Figure 5.10: Sine and square wave tests on TSI T 1.5 hot wire at zero flow angle [12]



# Control Circuits

Constant temperature:

Sine and square wave tests for Hot film

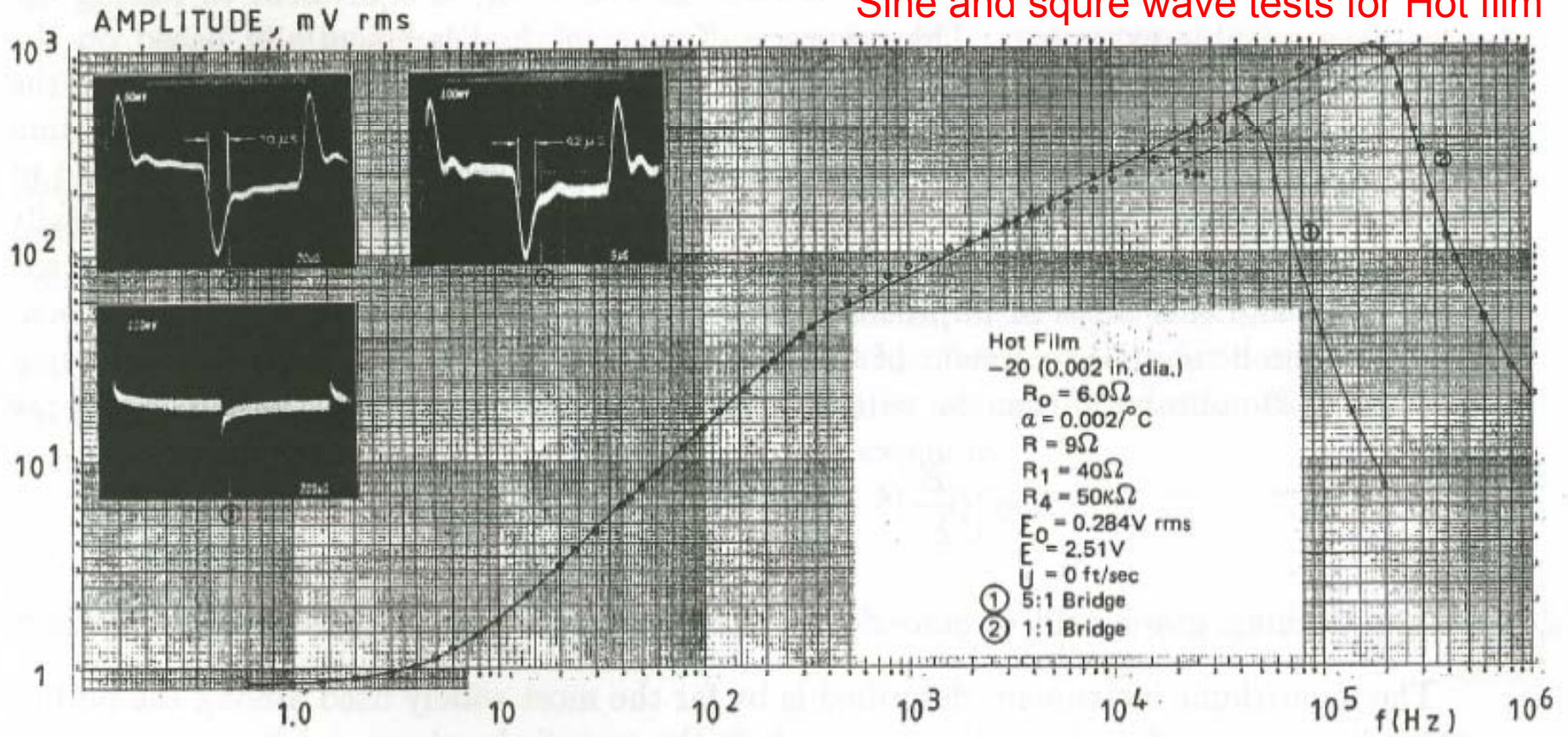


Figure 5.11: Sine and square wave tests on TSI-20 hot film sensor with no flow [12]

Optimum adjustment could be obtained in a matter of minutes

# Dynamic calibration

## Conclusion:

- Indirect methods are the only ones applicable in practice.
- Sinus test necessary for determination of frequency limit for fiber and film probes.
- Square wave test determines frequency limits for wire probes. Time taken by the anemometer to rebalance itself is used as a measure of its frequency response.
- Square wave test is primarily used for checking dynamic stability of CTA at high velocities.
- Indirect methods cannot simulate effect of thermal boundary layers around sensor (which reduces the frequency response).

# Linearization

The existence of a region with flat frequency response covering all the spectrum of frequencies of interest, allows the instantaneous response of the hot wire to be written even for unsteady flows, in an algebraic form as:

$$E^2 = A + B(U)^n$$

where A and B are constants obtained by a calibration.

This equation, known as **King's law**, is non-linear, and must be taken into account when interpreting the hot wire bridge output as a velocity signal.

However, it would be more practical to have an instrument whose output is directly proportional to velocity. This is possible by performing a so called 'linearization' of the hot wire bridge output.

A linearization is necessary when dealing with fluctuating or with turbulent flows, because in such cases the processing of the signal to get statistical moments, or just average quantities, will lead to wrong results if performed on a signal not related linearly to velocity.



# Linearization

Linearization of a hot wire signal can be performed in two ways, either using an analog on-line instrument (analog linearizer) or by performing a numerical linearization using a computer-based data acquisition system to sample and digitize the signal, and then to recalculate instantaneous velocities.

Analog on-line linearization is more commonly made using instruments called “logarithmic linearizers”, that are non-linear electronic instruments having an output to input dependence exactly inverse to the voltage-velocity calibration curve of a hot wire anemometer.

The value of the exponent  $n$  can easily be obtained from the voltage-velocity calibration curve, by plotting it in the form:

$$\text{Log} \left[ \left( \frac{E}{E_0} \right)^2 - 1 \right] = \text{Log} \frac{B}{E_0^2} + n \text{Log} U$$

and determining, graphically or numerically the slope of the almost straight line obtained.

# Linearization

The logarithmic instrument described is by far the most widely used among the on-line analog linearizers.

An alternative approach is the so-called polynomial linearizer, which approximates the inverse of the King's law.

$$U = f(E) = \left(\frac{E^2 - A}{B}\right)^{1/n}$$

by a (usually) fourth order polynomial

$$U = f(E) = k_1 + k_2E + k_3E^2 + k_4E^3 + k_5E^4$$

whose coefficients are determined by a best fit calculation.

A polynomial relation of the above kind can be easily obtained by electronic means, using multipliers and squaring circuits.

It is easier to obtain a higher frequency response with polynomial linearizers.

# Linearization

Numerical linearization has become nowadays quite common, due to the general availability of low cost compatible personal computers and associated plug-in data acquisition boards.

The data acquisition board performs sampling of the non-linear hot wire signal at a sufficiently high frequency, according to usual data sampling practices and the samples obtained are stored in the computer RAM memory.

A generalized regression method is used to fit to the voltage-velocity calibration a mathematical expression of the kind:

$$E^2 = A + B(U)^n \quad \text{King's law}$$

OR

$$E^2 = A + B(U)^n + CU \quad \text{Gaulier's modified law}$$

OR

$$U = k_1 + k_2 E + k_3 E^2 + k_4 E^3 + \dots + k_{i+1} E^i \quad \text{polynomial fit}$$

## Directional Response of a Hot Wire Probe

The output of a hot wire anemometer, besides being a function of the velocity magnitude  $U$ , is also a function of the incoming flow direction.

However, if the flow direction is unknown, the hot wire output can always be interpreted as the velocity of an hypothetical flow directed perpendicularly to the wire.

Such velocity is denominated the 'effective cooling velocity'. In other words, the hot wire output is, by definition, just a function of the effective cooling velocity.

Using two angles  $\alpha$  and  $\beta$ , to define the velocity direction, one may write:

$$U_{eff} = F(U, \alpha, \beta)$$

## Directional Response of a Hot Wire Probe

$$U_{eff} = F(U, \alpha, \beta)$$

Such relation has to be obtained experimentally, by an angular calibration, made by varying  $\alpha$  and/or  $\beta$  at constant velocity  $U$  of the calibration apparatus.

The linearized hot wire output  $E$  will directly yield  $U_{eff}$ .

***U<sub>eff</sub>*** is the effective cooling velocity sensed by the wire and deducted from the calibration expression, while ***U*** is the velocity component normal to the wire

# Directional Response of a Hot Wire Probe

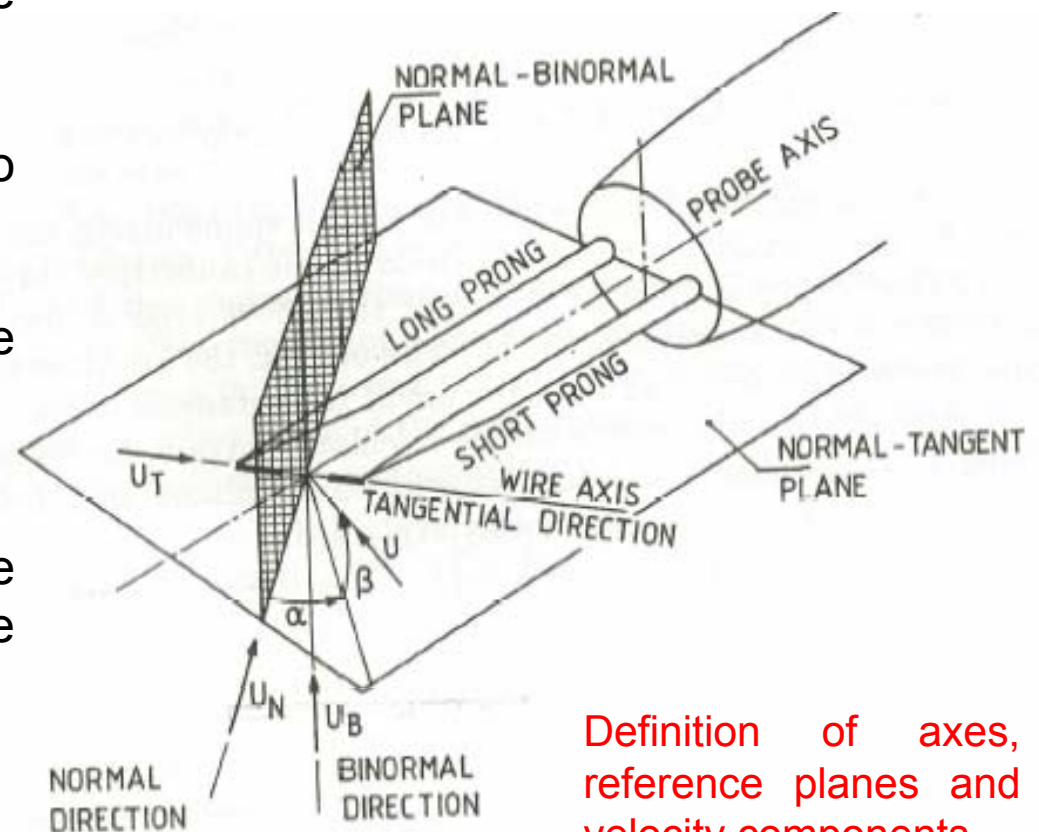
One has to define the angles  $\alpha$  and  $\beta$ . This is generally done by defining two reference planes, the plane of the wire (containing the axis of the wire and the axis of the prongs) and the plane normal to the wire axis.

Also three reference directions can be defined as shown in the Figure.

-The tangential direction, (parallel to the wire axis)

-The normal direction (normal to the wire, and parallel to the wire plane)  
[Normal-Tangential]

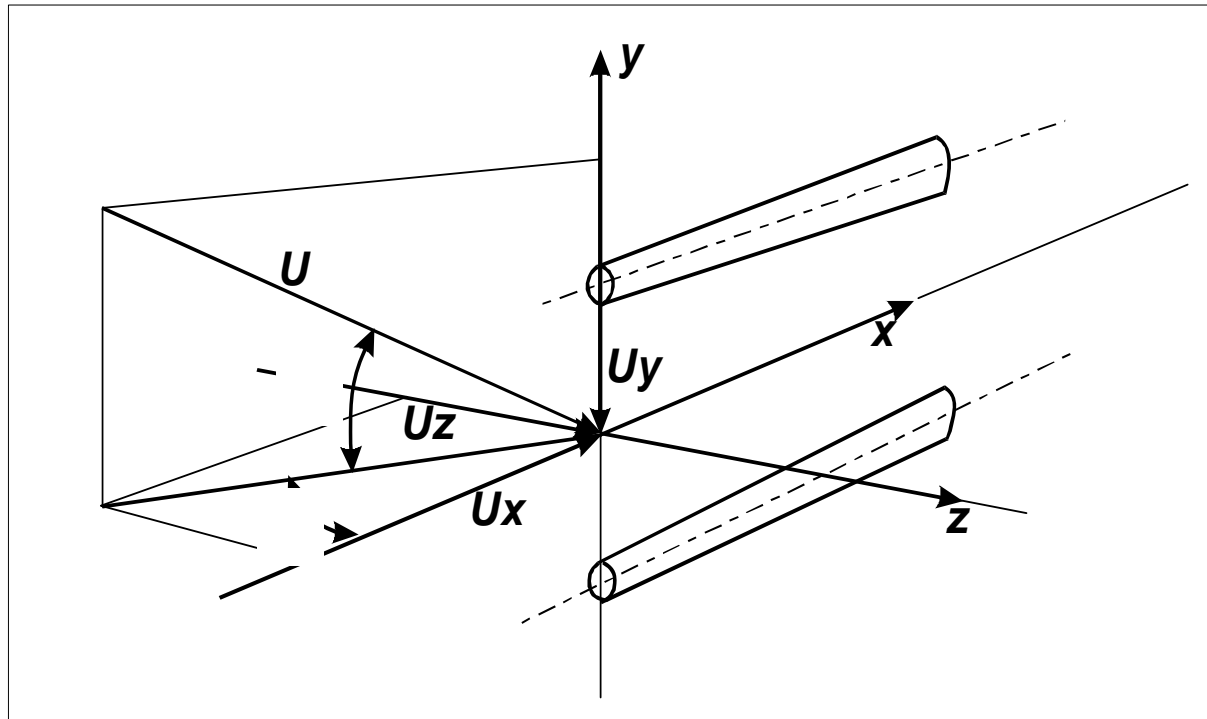
-The binormal direction (normal to the previous two, i.e. normal to the plane of the wire.)  
[Normal-Binormal]



Definition of axes, reference planes and velocity components.

# Directional response

## Probe coordinate system



Velocity vector  $U$  is decomposed into normal  $U_x$ , tangential  $U_y$  and binormal  $U_z$  components.

# Directional Response of a Hot Wire Probe

Several definitions of  $\alpha$  and  $\beta$  exist in the literature. Here we define:

$\beta$  : the angle between the velocity and the N-T plane

$\alpha$  : the angle (in the N-T plane) between the velocity projection on the N-T plane and the normal direction.

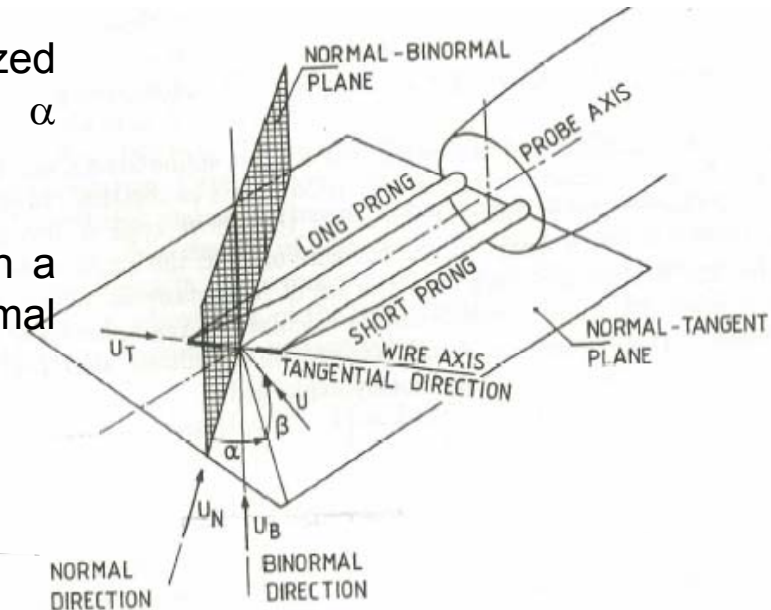
**In the N-T plane**, the angular response is characterized by a strong decrease of effective velocity when  $\alpha$  increases.

This is due to the fact that the heat transfer rate on a heated cylinder is much larger when the flow is normal to the cylinder than when it is parallel to it.

The phenomenon is represented by Champagne law:

$$U_{eff}^2 = U^2(\cos^2 \alpha + k_T^2 \sin^2 \alpha) = U_N^2 + k_T^2 U_T^2$$

with  $k_T$  depending of the specific wire, typically of the order of 0.1





# Directional Response of a Hot Wire Probe

In the N-B plane, the angular response is can be represented by a relation proposed by Gilmore:

$$U_{eff}^2 = U^2 (\cos^2 \beta + k_B^2 \sin^2 \beta) = U_N^2 + k_B^2 U_B^2$$

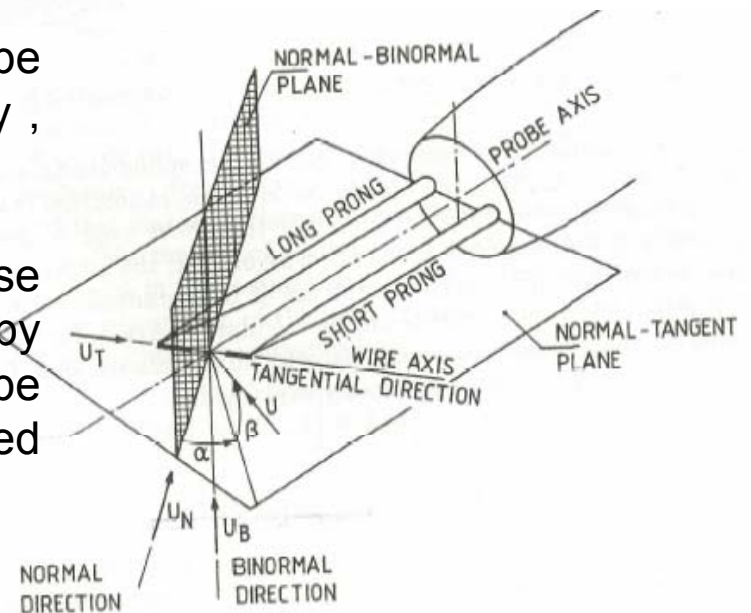
The coefficient  $k_B$  is around 1 to 1.1, and this value, slightly larger than 1, is physically due to the effect of the wire prongs that, when subjected to a flow from the binormal direction, cause an acceleration of the flow over the wire located between them.

Finally the two relations are often combined in a single expression, as proposed by Jorgensen in 3D flows:

$$U_{eff}^2 = U_N^2 + k_T^2 U_T^2 + k_B^2 U_B^2$$

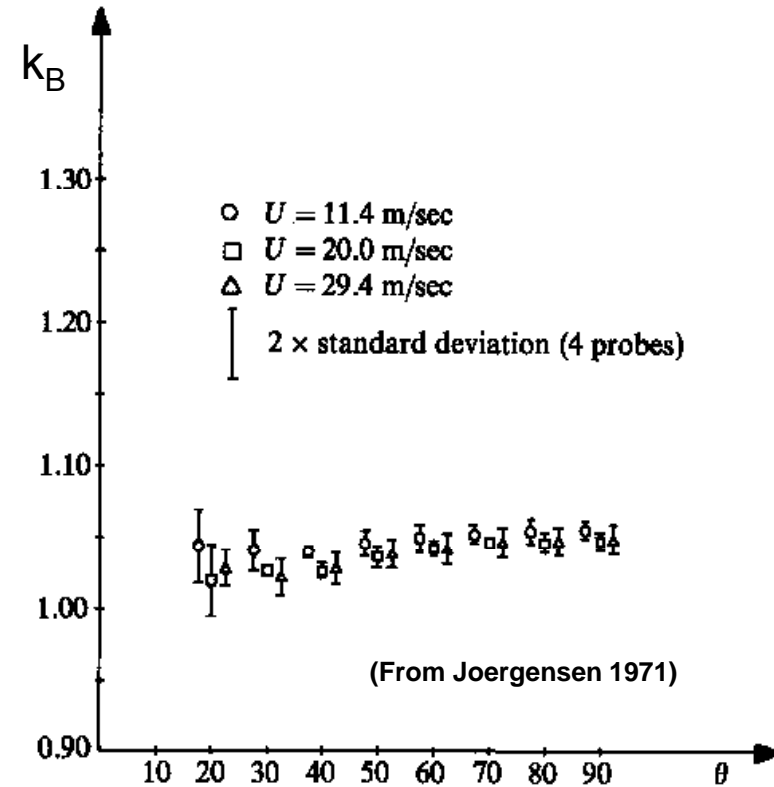
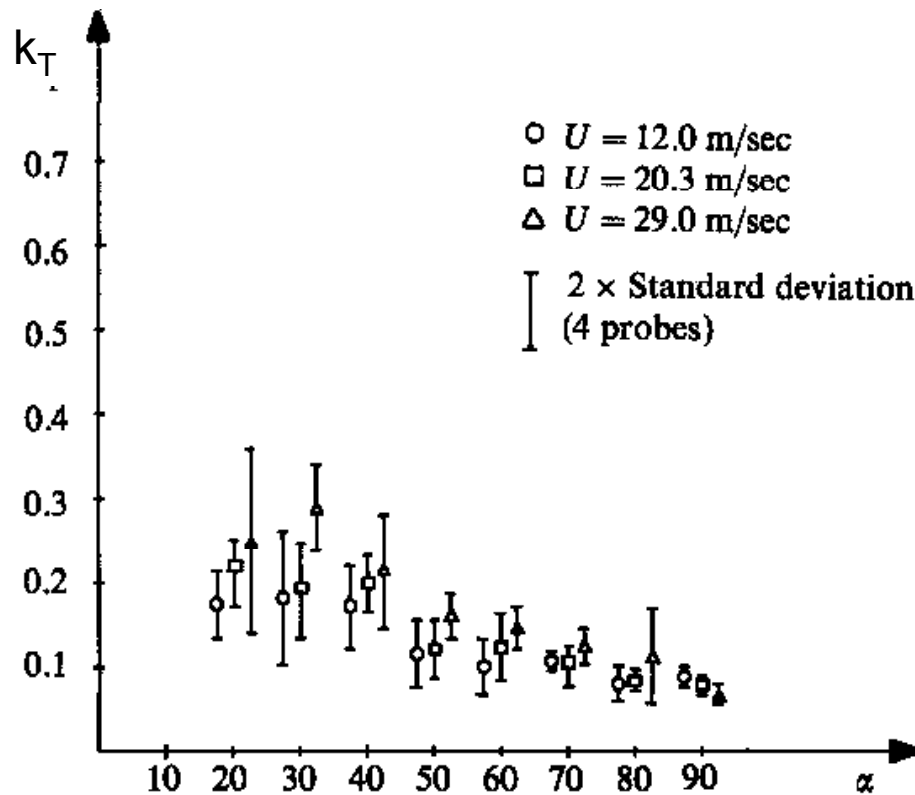
A suggestion is of individually calibrating each wire to be used, and to employ the resulting values, either directly , or after having fitted a specifically adapter curve.

This recommendation is even more important in the case of multiple wire probes, where the presence of near-by prongs can cause the directional response of a wire to be extremely different from the response of an isolated similar wire.

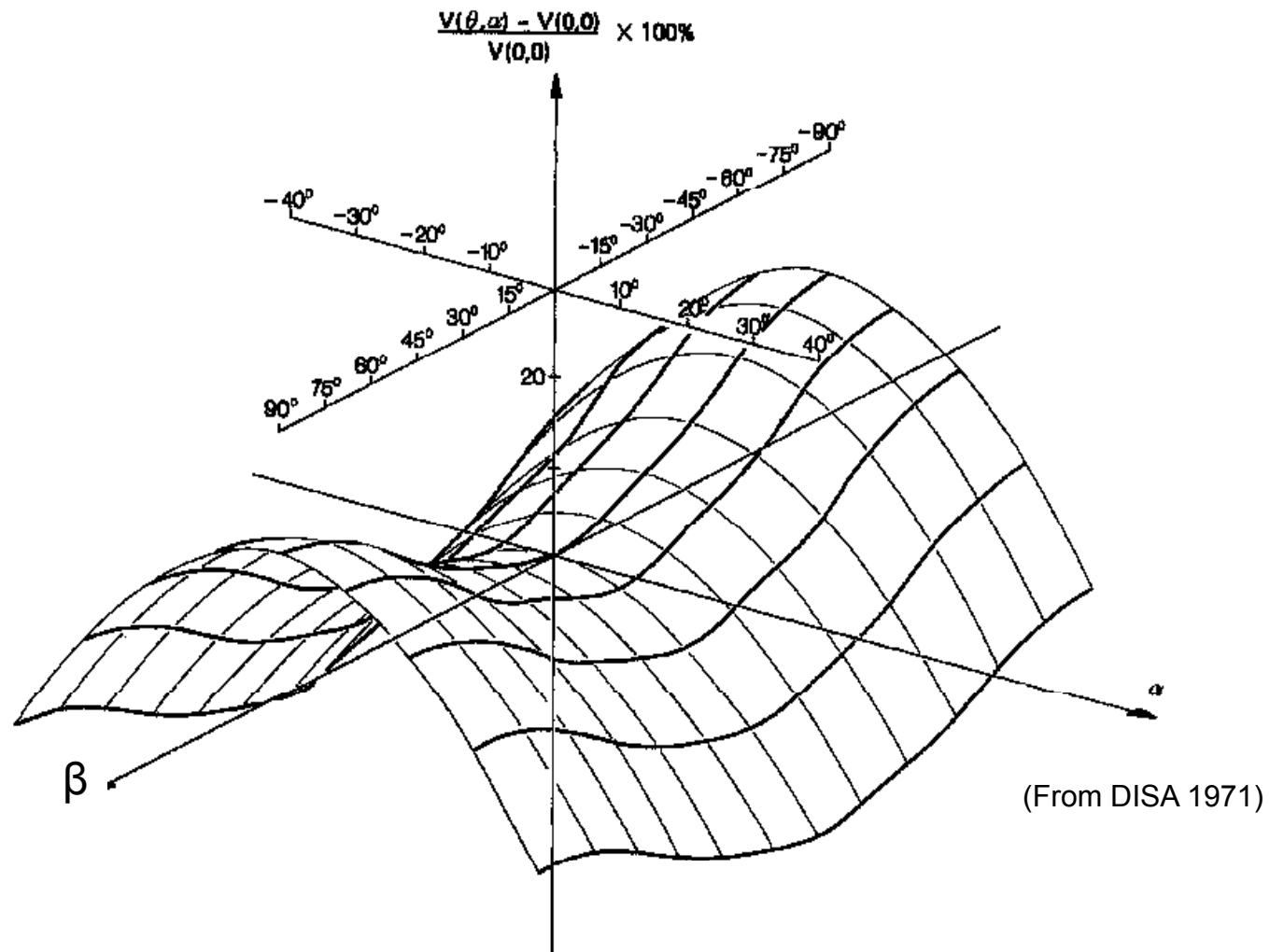


# Directional response

- Yaw and pitch factors  $k_T$  and  $k_B$  depend on velocity and flow angle



- Typical directional response for hot-wire probe



## Directional Response of a Hot Wire Probe

The directional sensitivity of the hot wire shown in the Figure may be used to determine the direction and magnitude of the velocity in an unknown two-dimensional or three-dimensional flow field.

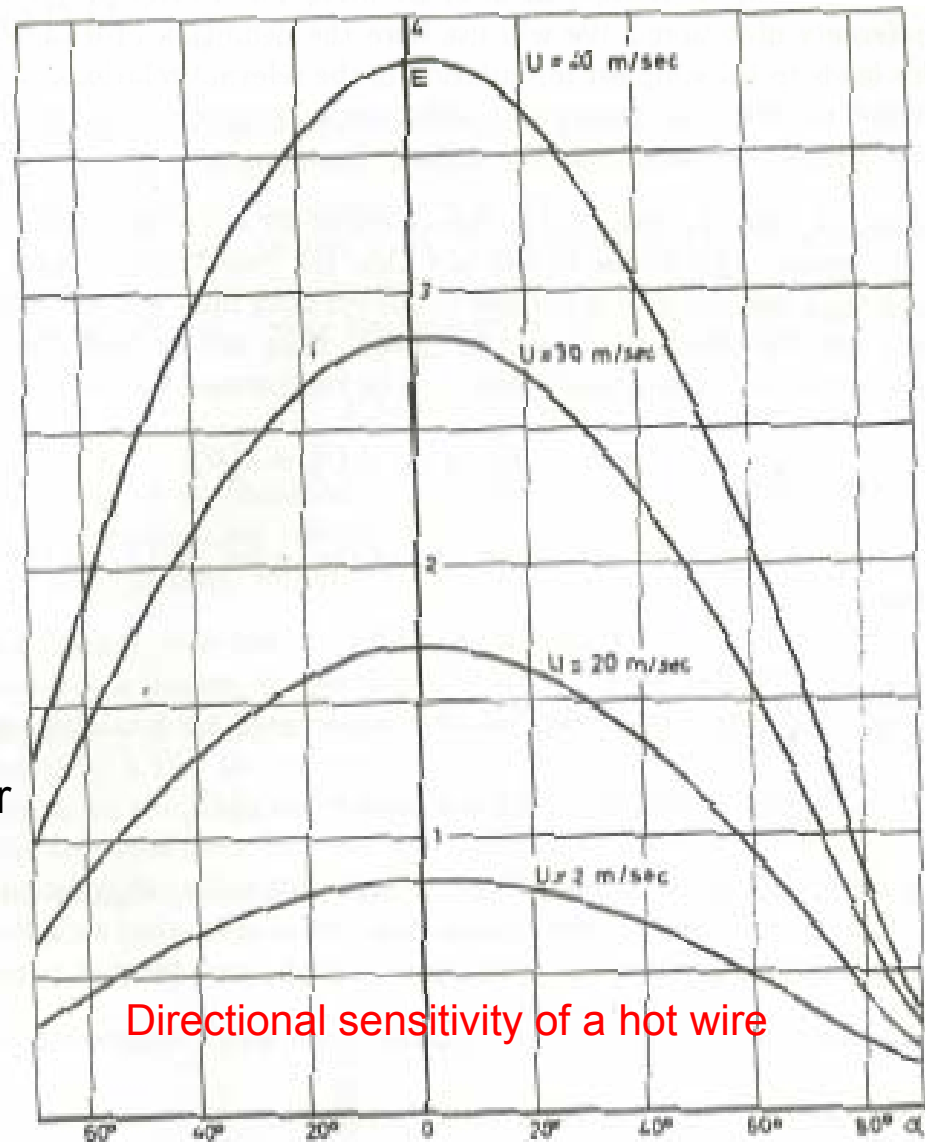
For instance, in a two-dimensional case, making two measurements at  $\alpha$  and  $\alpha+\theta$ , it is possible to write:

$$U_{eff_1} = E_{l1} = U f(\alpha)$$

$$U_{eff_2} = E_{l2} = U f(\alpha + \theta)$$

where  $E_{l1}$  and  $E_{l2}$  are the two anemometer outputs. Thus,

$$\frac{E_{l1}}{E_{l2}} = \frac{f(\alpha)}{f(\alpha + \theta)} = F(\alpha)$$



# Directional Response of a Hot Wire Probe

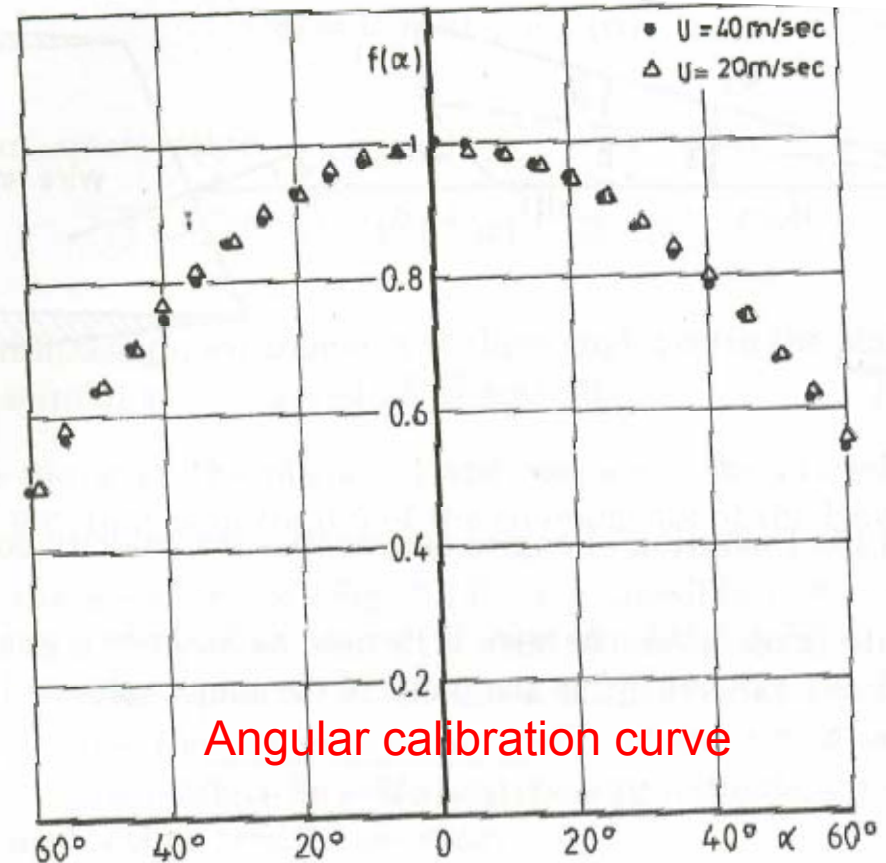
$F(\alpha)$  can be obtained from the wire angular calibration curve  $f(\alpha)$

Equation 
$$\frac{E_{\ell 1}}{E_{\ell 2}} = \frac{f(\alpha)}{f(\alpha + \theta)} = F(\alpha)$$

allows the determination of the unknown angle  $\alpha$ .

Equation 
$$U_{eff_1} = E_{\ell 1} = U f(\alpha)$$
$$U_{eff_2} = E_{\ell 2} = U f(\alpha + \theta)$$

allows the determination of the absolute velocity  $U$



Angular calibration curve

## Directional Response of a Hot Wire Probe

To increase the accuracy, a measurement can also be made at  $(\alpha-\theta)$  to obtain:

$$E_{e3} = U f(\alpha - \theta)$$

and

$$\frac{E_{e3}}{E_{e1}} = \frac{f(\alpha - \theta)}{f(\alpha)} = F(\alpha)$$

The only condition which must be satisfied is that the angles  $\alpha$ ,  $\alpha-\theta$ ,  $\alpha+\theta < 60^\circ$ , for the calibration curve to be valid. If these values are exceeded in the measurements, the results give an estimate of the true flow angle, which can be used to relocate the hot wire.

## Measurement of Turbulence

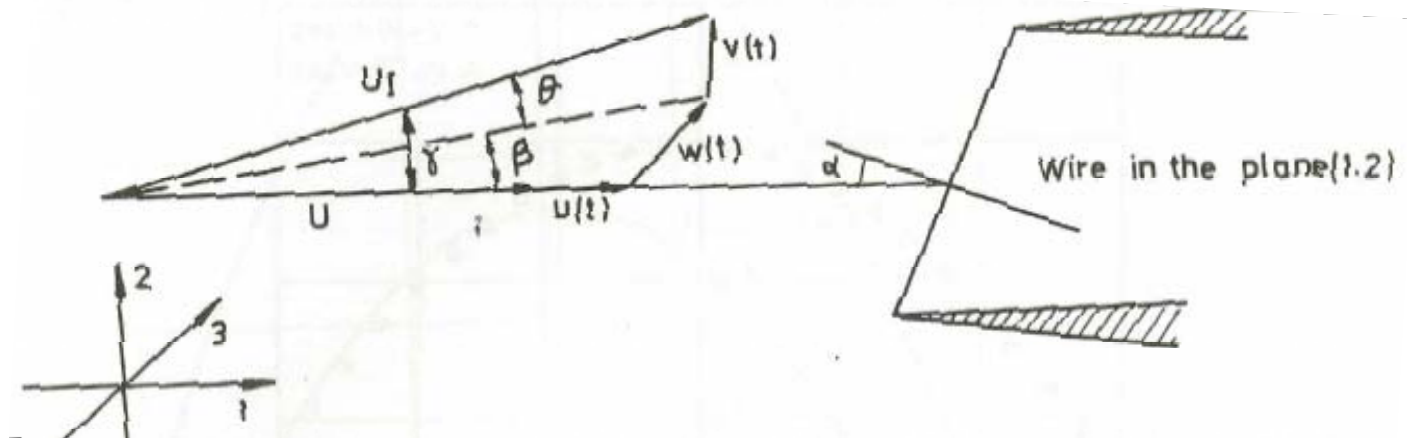
The directional sensitivity of the hot wire is also used to resolve the fluctuating components of the velocity field.

The wire is located at a known angle  $\alpha$  (which can be determined as shown previously) in the plane of the mean velocity  $U$ .

The instantaneous velocity  $U_I$  has the value:

$$U_I = \sqrt{(U + u)^2 + v^2 + w^2}$$

and forms a solid angle  $\gamma$  with the mean velocity.



## Measurement of Turbulence

If  $\theta$  and  $\beta$  are the two projections of  $\gamma$  in the plane of the wire and in the plane normal to it, then

$$U_{eff} = U_I f(\alpha + \theta) g(\beta)$$

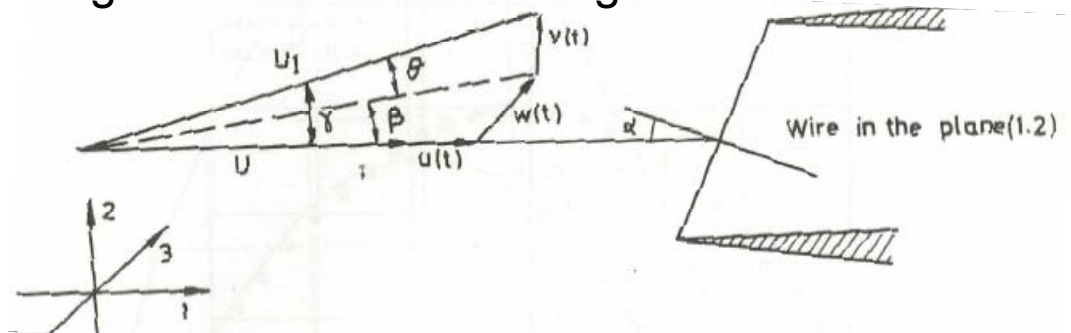
If the fluctuation components are small,  $\beta$  constitutes a small rotation around the wire and can be neglected, thus

$$E_{eff} \rightarrow U_{eff} = U_I f(\alpha + \theta)$$

with Taylor series expansion gives

$$E_{eff} \rightarrow U_I f(\alpha) + U_I \left. \frac{\partial f}{\partial \alpha} \right|_{\alpha} \theta = U f(\alpha) + u f(\alpha) + v f'(\alpha)$$

if the substitution  $u/U = \theta$  is made and higher order terms are neglected





## Measurement of Turbulence

$$E_{\theta} \rightarrow U_I f(\alpha) + U_I \left| \frac{\partial f}{\partial \alpha} \right|_{\alpha} \theta = U f(\alpha) + u f(\alpha) + v f'(\alpha)$$

This equation may be used to determine the fluctuating velocity components considering only its fluctuating part

$$e_{\theta} = u f(\alpha) + v f'(\alpha)$$

and taking its mean square value

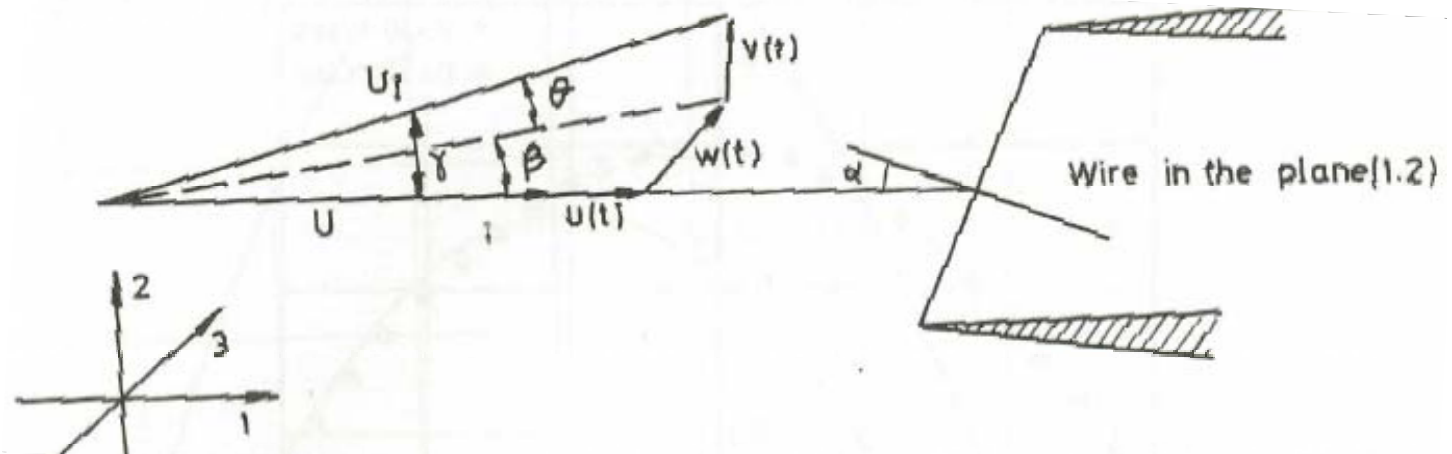
$$\overline{e_{\theta}^2} = \overline{u^2} f^2(\alpha) + \overline{v^2} f'^2(\alpha) + \overline{uv} f(\alpha) \cdot f'(\alpha)$$

## Measurement of Turbulence

Making measurements at three angles  $\alpha$  (in the plane 1,2) it is possible to determine the values of  $\overline{u^2}$ ,  $\overline{v^2}$  and  $\overline{uv}$

By having the probe in the plane 1,3 and repeating the procedure, it is possible to determine  $\overline{w^2}$  and  $\overline{vw}$  ; that is in total 5 of the components of the Reynolds stress tensor.

A more common procedure is to use x-probes, that is double wire probes with the wires already fixed at predetermined angles on a single support. Then measurements can be obtained without rotating probes and simultaneously so that more accurate data reduction techniques could be applied.



# Disturbing effects (problem sources)

- Anemometer system makes use of heat transfer from the probe

$$Q_c = Nu \cdot A \cdot (T_w - T_a)$$
$$Nu = h \cdot d/k_f = f(Re, Pr, M, Gr, \alpha),$$

- Anything which changes this heat transfer (other than the flow variable being measured) is a “PROBLEM SOURCE!”
- Unsystematic effects (contamination, air bubbles in water, probe vibrations, etc.)
- Systematic effects (ambient temperature changes, solid wall proximity, eddy shedding from cylindrical sensors etc.)

# Problem sources

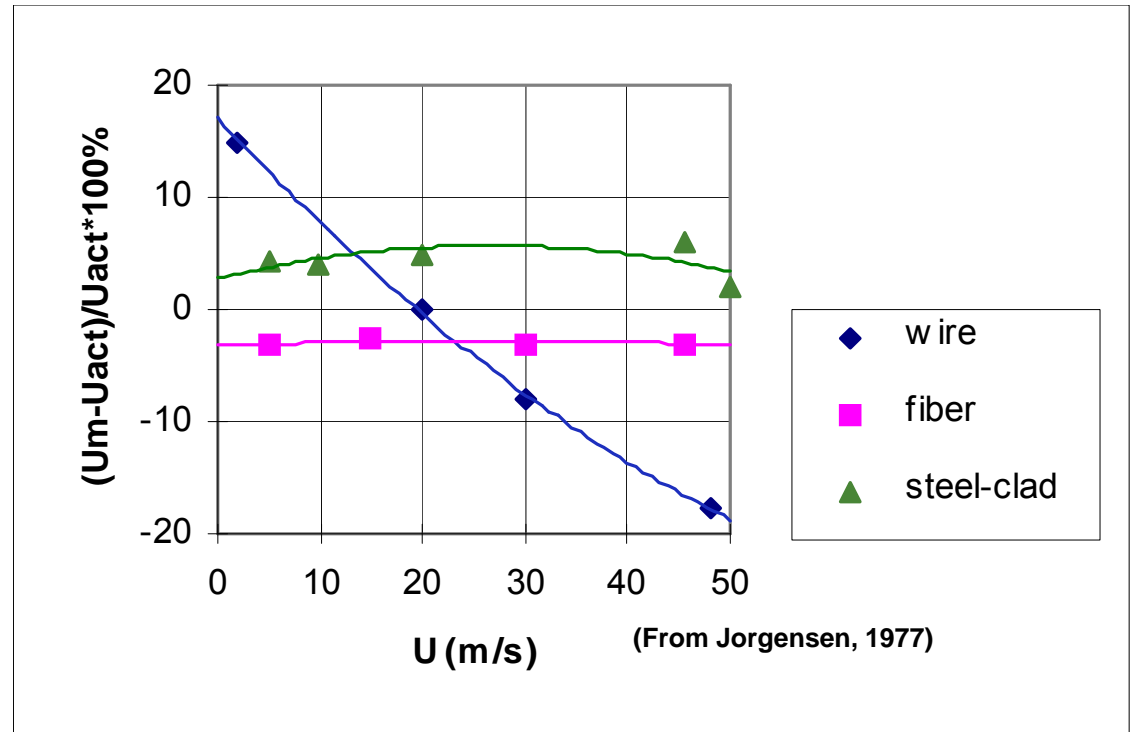
## Probe contamination

- Most common sources:
  - dust particles
  - dirt
  - oil vapours
  - chemicals
- Effects:
  - Change flow sensitivity of sensor (DC drift of calibration curve)
  - Reduce frequency response
- Cure:
  - Clean the sensor
  - Recalibrate

# Problem Sources

## Probe contamination

- Drift due to particle contamination in air  
5  $\mu\text{m}$  Wire, 70  $\mu\text{m}$  Fiber and  
1.2 mm SteelClad Probes



**Wire and fiber exposed to unfiltered air at 40 m/s in 40 hours**

**Steel Clad probe exposed to outdoor conditions 3 months during winter conditions**

# Problem Sources

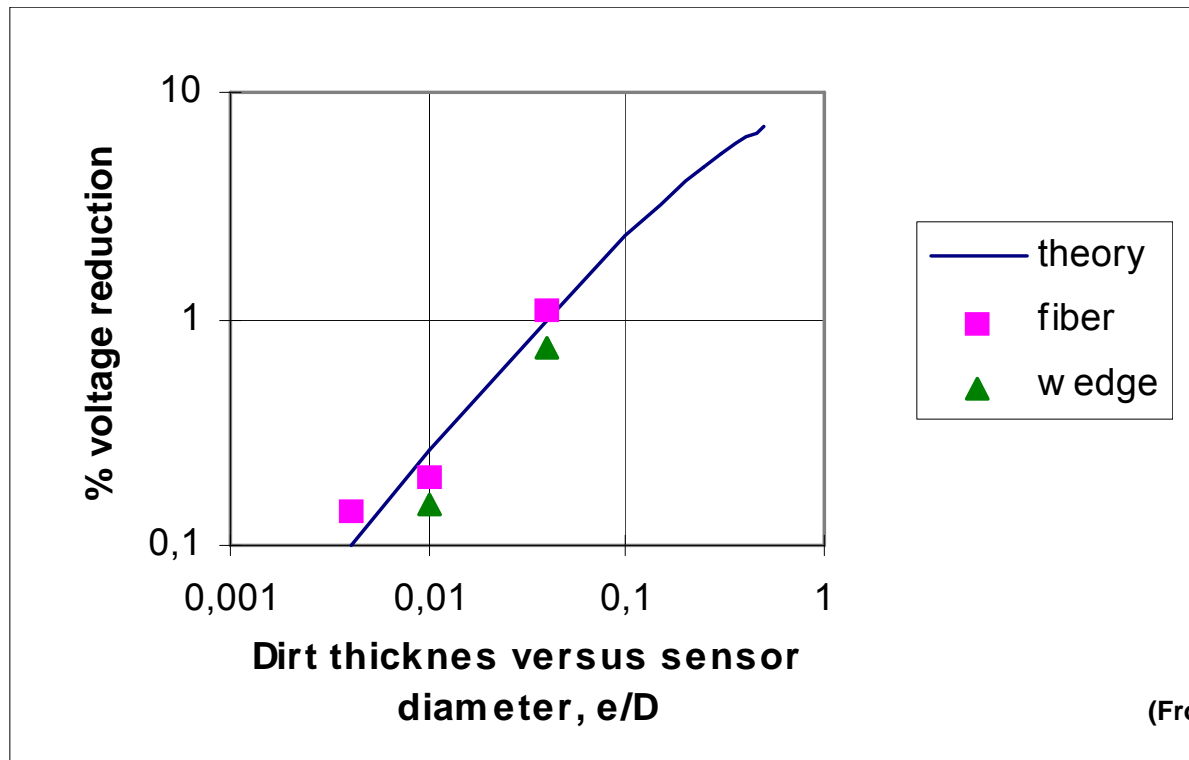
## Probe contamination

- Low Velocity
    - slight effect of dirt on heat transfer
    - heat transfer may even increase!
    - effect of increased surface vs. insulating effect
  - High Velocity
    - more contact with particles
    - bigger problem in laminar flow
    - turbulent flow has “cleaning effect”
  - Influence of dirt INCREASES as wire diameter DECREASES
  - Deposition of chemicals INCREASES as wire temperature INCREASES
- \* FILTER THE FLOW, CLEAN SENSOR AND RECALIBRATE

# Problem Sources

## Probe contamination

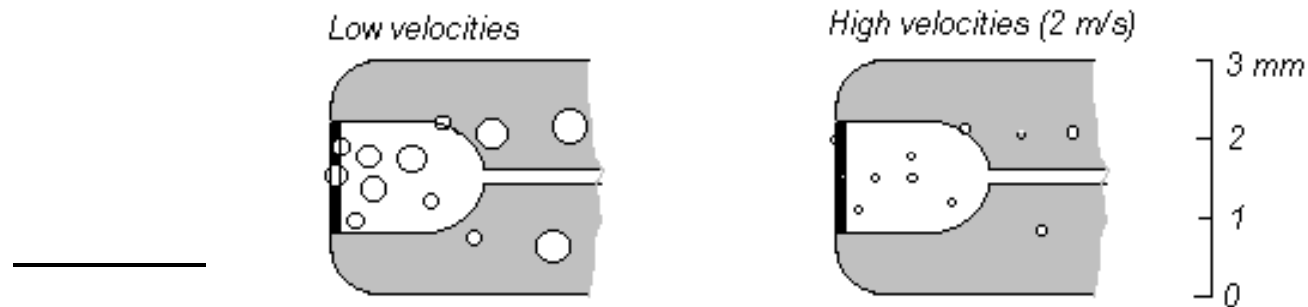
- Drift due to particle contamination in water  
Output voltage decreases with increasing dirt deposit



# Problem Sources

## Bubbles in Liquids

- Drift due to bubbles in water



(From C.G.Rasmussen 1967)

In liquids, dissolved gases form bubbles on sensor, resulting in:

- reduced heat transfer
- downward calibration drift

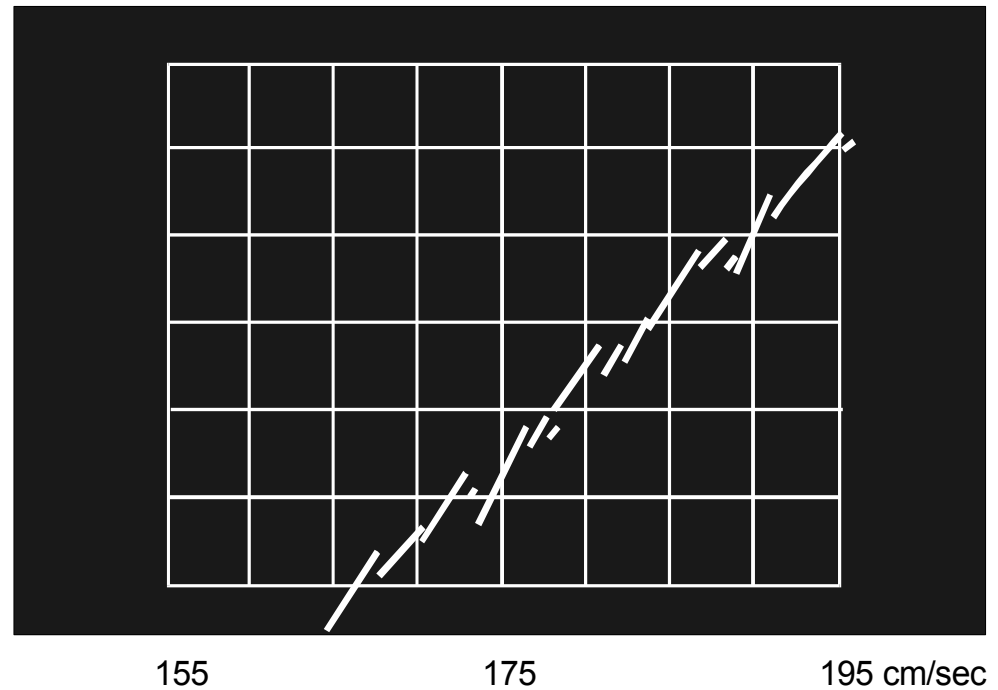


# Problem Sources

## Bubbles in Liquids

- Effect of bubbling on portion of typical calibration curve
- Bubble size depends on
  - surface tension
  - overheat ratio
  - velocity
- Precautions
  - Use low overheat!
  - Let liquid stand before use
  - Don't allow liquid to cascade in air
  - Clean sensor

e



(From C.G.Rasmussen 1967)

## HOT-WIRE CALIBRATION

The hot-wire responds according to King's Law:

$$E^2 = A + Bu^n$$

where  $E$  is the voltage across the wire,  $u$  is the velocity of the flow normal to the wire and  $A$ ,  $B$ , and  $n$  are constants. You may assume  $n = 0.45$  or  $0.5$ , this is common for hot-wire probes (although in a research setting, you should determine  $n$  along with  $A$  and  $B$ ).

$A$  and  $B$  can be found by measuring the voltage,  $E$ , obtained for a number of known flow velocities and **performing a least squares fit for the values of  $A$  and  $B$**  which produce the best fit to the data.

By defining  $u^n = x$  and  $E^2 = y$ , this least squares fit becomes simply a linear regression for  $y$  as a function of  $x$ .

The values of  $A$  and  $B$  depend on the settings of the anemometer circuitry, the resistance of the wire you are using, the air temperature, and, to a lesser extent, the relative humidity of the air.

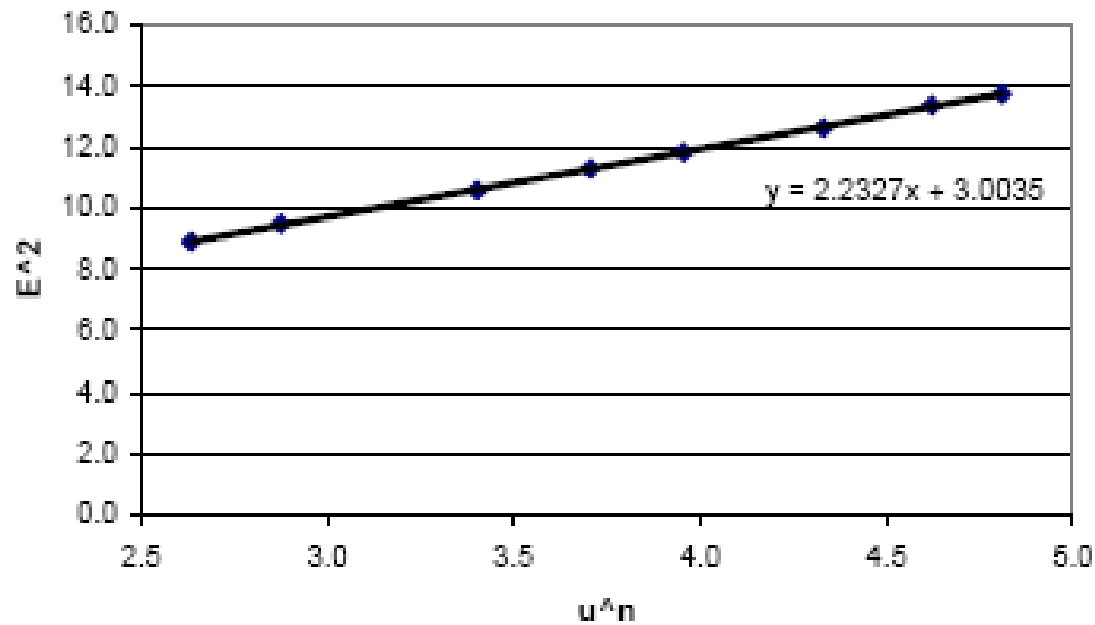
### Calibration data analysis

$\Delta p$ (in. H <sub>2</sub> O)	$\Delta p$ (Pa)	$u$ (m/s)	$E$ (volts)	$E^2$	$u^n$	$u$ pred
0.17	42.346	8.600	2.979	8.874	2.633	8.572
0.251	62.523	10.450	3.076	9.462	2.875	10.595
0.53	132.021	15.185	3.249	10.556	3.401	15.001
0.776	193.298	18.374	3.360	11.290	3.706	18.433
1.036	258.063	21.230	3.440	11.834	3.955	21.231
1.55	386.098	25.968	3.554	12.631	4.330	25.728
2.07	515.628	30.009	3.655	13.359	4.621	30.252
2.47	615.266	32.780	3.707	13.742	4.809	32.794

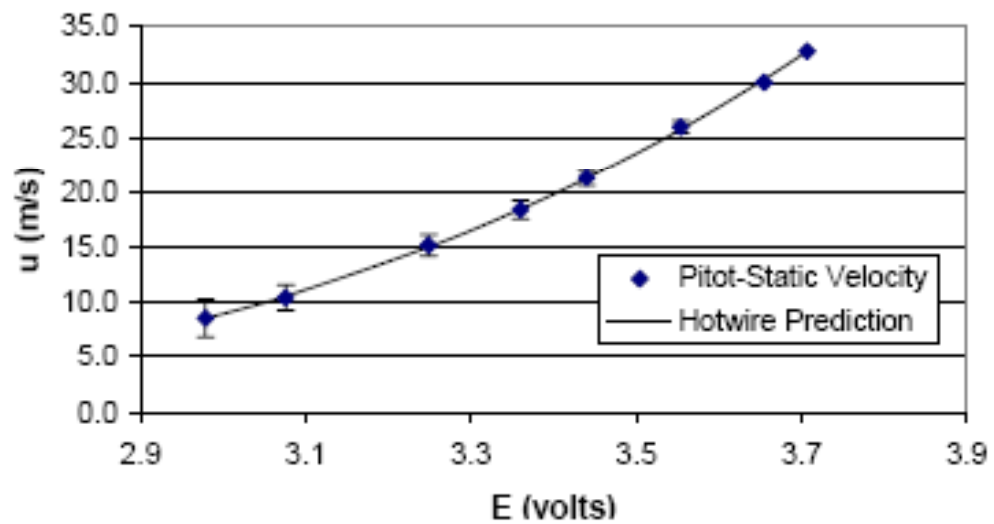
Pitot tube measurement and velocity calculation

voltage measurement obtained from hotwire system

$$E^2 = A + Bu^n$$



Trendline plot to calibration data determining  $A$  and  $B$ .



King's law prediction for velocity using the values of  $A$  and  $B$  determined in above figure

## **Multi-Channel CTA System of Dantec in the Department**

The Multichannel CTA offers a solution for mapping of velocity and turbulence fields in most air flows. Up to 16 points can be monitored simultaneously, reducing or eliminating probe traverse.



# Measurement principles of CTA

