Consistency among Parts and Aggregates: A Computational Model

nectaria tryfona
National Center for Geographic Information and Analysis
Boardman Hall, University of Maine, Orono, ME 04469-5711, U.S.A.
nectaria@spatial.maine.edu

and

max j. egenhofer
National Center for Geographic Information and Analysis
Department of Spatial Information Science and Engineering
and Department of Computer Science
Boardman Hall, University of Maine, Orono, ME 04469-5711, U.S.A.
max@spatial.maine.edu

Abstract
Heterogeneous geographic databases contain multiple views of the same geographic objects at different levels of spatial resolution. When users perceive geographic objects as one spatial unit, although they are physically separated into multiple parts, appropriate methods are needed to assess the consistency among the aggregate and the parts. The critical aspect is that the overall spatial relationships with respect to other geographic objects must be preserved throughout the aggregation process. We develop a systematic model for the constraints that must hold with respect to other spatial objects when two parts of an object are aggregated. We found three sets of configurations that require increasingly more information in order to make a precise statement about their consistency: (1) configurations that are satisfied by the topological relations between the two parts and the object of interest; (2) configurations that need further information about the topological relation between the object of concern and the connector in order to be resolved unambiguously; and (3) configurations that require additional information about the topological relation between the aggregate’s boundary and the boundary or interior of the object of interest to be uniquely described. The formalism extends immediately to relations between two regions with disconnected parts as well as to relations between a region and an arbitrary number of separations.

1. Introduction
Heterogeneous geographic databases contain multiple views of the same geographic objects at different levels of spatial detail. State-of-the-art geographic databases have been designed to contain only a single level of representation. This representation is exactly the one that users can retrieve, query, and display, and no analysis at coarser or more detailed levels is immediately available (Figure 1a).

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To accommodate better analyses at different levels of detail, multiple copies at varying levels of details, covering the same geographic area, could be stored (Bruegger and Frank 1988; Buttenfield 1989; Guptill 1987), which would then be available for retrieval, querying, and display (Figure 1b). In such an environment, the same geographic objects are represented in several different ways, tailored towards the needs of different users and analyses; however, the representations available are
given by what is stored. Storage and update implementations in terms of materialized views of a geographic database and triggers in a database system have been considered (Frank 1991), but not implemented to date. A simpler and more powerful model would derive automatically one level of detail from another such that only the most detailed level would be stored (Beard 1987) and any other less detailed representation could be derived on the fly (Figure 1c). The advantage of the latter model is the opportunity to generate any representation of whatever level of lesser detail users desire. Successful implementations of multi-resolution geographic databases, however, are currently hampered by conceptual and computational limitations (Jones et al 1996).

Multiple geographic representations are often discussed in the context of cartographic generalization (Buttenfield and McMaster 1991; McMaster and Shea 1992). Cartographic generalization is a complex process, which has been seen as involving two types of operations that depend upon each other (Müller et al 1995). Model-based generalization addresses reduction in detail at the representational level, relying on semantic abstraction mechanisms, while graphic-based generalization is motivated by the attempt to remove visual conflicts to better communicate spatial information. Similar roles have been suggested for geographic information systems to manage the object representation, and cartographic databases to administer the presentation part (Frank 1991). We follow this separation into generalizations on the model and graphics level, focusing exclusively on the model level; therefore, this paper does not address the traditional cartographic topics of amalgamation vs. cartographic aggregation, and is not concerned with simplifications, displacements, or exaggerations (McMaster and Shea 1992).

Within the realm of model-based generalization, we focus exclusively on the semantic abstraction of aggregation (Smith and Smith 1977), as it has been established in conceptual modeling and object-oriented database systems. Aggregation refers to the part-whole relationship, which is a fundamental spatial concept rooted in basic human experience (Lakoff and Johnson 1980) and addressed by the mathematical subfield of mereology (Simons 1987).

Among multiple representations, two or more semantically and spatially related objects may be aggregated into a single composite object. A concern in any multiple-representation model is the dependency among the different representations (Egenhofer et al 1994; Müller et al 1995). Users perform such operations as zooming into and out of a map-like representation, and expect that the predominant spatial relations are preserved throughout such changes of resolution. Likewise, a query evaluation algorithm executed at a coarser level should give the same, or at least a very similar, result as a query processed against a more detailed level (Barrera et al 1992).

A key to the successful adoption of multi-resolution geographic databases is consistency across multiple representations. Consistency describes the lack of any logical contradictions within a model. In multiple representations of spatial information, consistency deals with two cases: (1) consistent modeling of spatial objects across different representations and (2) consistent modeling of spatial relations across different representations (Egenhofer et al 1994). Although there has been considerable research in recent years on various aspects of multiple representations (Bruegger and Kuhn 1991; Dettori and Puppo 1996; Kilpeläinen 1992; Puppo and Dettori 1995; Rigaux and Scholl 1995; Timpf et al 1992) and cartographic generalization (Buttenfield and McMaster 1991; Müller et al 1995), we currently lack methods to consistently maintain multiple representations of geographic objects.

This paper focuses on the computational assessment of topological consistency across multi-resolution geographic databases. It develops a formalism to assess consistency when merging several parts into a whole such that the spatial relations with respect to other objects in the parts’ neighborhood are preserved. In multi-resolution databases, topology often is considered first-class information and its preservation is a high priority in generalization (Lagrange and Ruas 1994, Weibel 1996). Figure 2 shows an example with an area of low pressure over the State of Maine.
When moving towards a less-detailed representation—some of the islands may be dropped and some others may be merged with the mainland—the overall relationship between the set of land masses and the low-pressure zone must be preserved. The goal of this paper is to provide a comprehensive computational formalism to compare such situations and to assess whether the transition from a region with disconnected parts to an aggregate was consistent or not. Unlike other models that consider topological relations with an aggregate at the lowest level of detail only (Clementini et al 1995), we derive from the relations with the parts the possible set of relations with the aggregate. The computational methods for the consistency assessment are complemented by depictions of geometric configurations, which should not be considered cartographic representations. The model developed considers spatial objects and their relations at a level independent of coordinates and, therefore, also independent of the common GIS data structures. This approach is orthogonal to modeling the transformations among different representations, which are usually metric operations (Douglas and Peucker 1973; Müller 1990), while the present methods are topological in nature. This implies that the model introduced is a good candidate for quality control in multi-resolution geographic databases. Such an approach is complementary to empirical and cognitive studies of multi-scale maps (Zhang and Mark 1993), and it may serve as the benchmark for calibrating different series of multi-resolution data sets. This process builds on the existence of operations that perform the actual generalization, for which algorithms have proven to be extremely difficult to design.

![Figure 2](image_url)

**Figure 2:** Two representations of the State of Maine and a low-pressure zone at two different levels of detail.

The remainder of this paper is organized as follows: Section 2 summarizes the 4-intersection model for topological relations, defines objects with disconnected parts, and provides a means to describe a scene of objects with disconnected parts. Section 3 derives the consistency constraints among the relations of an aggregate of two parts that meet. Section 4 builds on these results and shows how the topological relations among regions with disconnected parts can be reduced to the smallest amount of information necessary to describe completely the topology of a scene between one object with two parts and another object with one part. It distinguishes aggregate relations that are uniquely implied from underdetermined aggregate relations, and derives the additional information required to resolve the ambiguities. Section 5 shows how more complex configurations of objects with multiple disconnected parts are developed from the previously introduced formalism. Section 6 concludes with a summary of the results and a discussion of future work.
2. **Topological Relations**

The foundation for the assessment of topological consistency for regions with disconnected parts are the 4-intersection model for binary topological relations and a formal model for regions with disconnected parts. This section describes this background and provides a means to describe a scene of regions with disconnected parts.

2.1 **4-Intersection**

The basic components of a spatial region are its interior and its boundary. The interior of a set \( A \), denoted by \( A^\circ \), is the union of all open sets in \( A \). The closure of \( A \), denoted by \( \overline{A} \), is the intersection of all closed sets containing \( A \). The exterior of \( A \) with respect to the embedded space \( \mathbb{R}^2 \), denoted by \( A^\complement \), is the set of all points of \( \mathbb{R}^2 \) not contained in \( A \). The boundary of \( A \), denoted by \( \partial A \), is the intersection of the closure of \( A \) and the closure of the exterior of \( A \).

A *separation* of a set \( A \) is a pair \( X, Y \) such that \( X \neq \emptyset \) and \( Y \neq \emptyset \); \( X \cap Y = A \); and \( \overline{X} \cap Y = \emptyset \) and \( X \cap \overline{Y} = \emptyset \). If there exists a separation of \( A \), then \( A \) is said to be *disconnected*, otherwise \( A \) is said to be *connected*.

**Definition 1:** Let \( X \) be a connected topological space. A *spatial region* in \( X \) is a non-empty proper subset \( A \) of \( X \) such that \( A^\circ \) is connected, \( A = \overline{A^\circ} \) and \( \partial A \) is connected.

It follows that the interior of each spatial region is non-empty. Furthermore, a spatial region is closed and connected since it is the closure of a connected set.

The topological relation between two spatial regions, \( A \) and \( B \), is characterized by the set intersections of \( A \)'s boundary (\( \partial A \)) and interior (\( A^\circ \)) with the boundary and interior of \( B \) (Equation 1). This model has been called the 4-intersection (Egenhofer and Franzosa 1991).

\[
I(A, B) = \begin{bmatrix} A^\circ \cap B^\circ & A^\circ \cap \partial B \cap \partial A \cap B^\circ & A^\circ \cap \partial B \cap \partial A \cap \overline{B} \cap \overline{A} \end{bmatrix}
\]  

(1)

With each of these four intersections being empty (\( \emptyset \)) or non-empty (\( \neq \emptyset \)), the model has 16 possible topological relations between two point sets. For two regions in \( \mathbb{R}^2 \) (without holes or separations), the categorization shows eight distinct topological relations. They have been called disjoint, contains, inside, equal, meet, covers, coveredBy, and overlap (Figure 3).
More details about topological relations may be expressed by considering additional invariants such as the dimension of an intersection (Egenhofer and Franzosa 1995) For example, for two pairs of regions that are related by a $0$-dimensional meet and $1$-dimensional meet — also called a $0$-meet and $1$-meet, the common boundary has exactly one component that is of dimension $0$ and $1$, respectively.

### 2.2 Regions with Disconnected Parts

A region with disconnected parts is a spatial object that has a connected exterior, $n$ disconnected boundaries, and $n$ disconnected interiors (Figure 4a).

**Definition 2:** A region with disconnected parts, denoted by $A_*$, is a set of regions $A_i$, called parts, such that the union of all parts constitutes $A_*$ (Equation 2) and all parts are mutually exclusive (Equation 3).

$$A_* = \bigcup_{i=0}^{n} A_i \quad \text{with } n \geq 1 \quad (2)$$

$$\forall i, j | i \neq j: \quad A_i \cap A_j = \emptyset \quad (3)$$

During an aggregation, the parts of such a region may be merged, integrating some of the regions’ exteriors with the parts. The portion of the exteriors that will be integrated with the parts is called the connector.

**Definition 3:** The connector between two parts, $A_i$ and $A_j$, denoted by $\Box A_{ij}$, is the region that links $A_i$ with $A_j$, filling the exterior between the two parts such that $A_i$, $\Box A_{ij}$, and $A_j$ are connected (Equations 4a and b).

$$A_i \ 1\text{-meets } \Box A_{ij} \quad (4a)$$

$$A_j \ 1\text{-meets } \Box A_{ij} \quad (4b)$$

This definition is independent of $\Box A_{ij}$’s shape, which may be a slim corridor connecting the two parts, or the convex hull of the two parts (Figure 4b).
**Figure 4:** The constituencies of spatial aggregates: (a) regions with disconnected parts and (b) the shape-independent connector between parts.

**Definition 4:** The generalized region with disconnected parts, denoted by $A^*$, is the union of all parts, $A_i$, all relevant connections $\square A_{jk}$, such that all $A_i$’s are connected through $\square A_{ij}$’s and $A^*$ is homeomorphic to a spatial region $A$ (Equation 5).

$$A^* = \bigcup_{i=0}^{n} A_i \square \bigcup_{i,j=0, i\neq j}^{m} A_{ij}$$  \hspace{1cm} (5)$$

A series of consistency constraints follow from these properties of a region with disconnected parts:

- All parts must be included in the generalized region (Equation 6a),
- All parts are mutually disjoint (Equation 6b), and
- Each connector links a pair of parts (Equation 6c).

\[ \square i: \quad A^* \text{ covers } A_i \]  \hspace{1cm} (6a) \n
\[ \square i, j | i \neq j: \quad A_i \text{ disjoint } A_j \]  \hspace{1cm} (6b) \n
\[ \square i | i \neq j: \quad A_i \text{ meet } \square A_{ij} \quad \square A_{ij} \text{ meet } A_j \]  \hspace{1cm} (6c)
The goal is to determine the relation between \( A \) and another region \( B \) from the relations between \( B \) and \( A \)’s separations, \( A_0 \) and \( A_1 \). This inference can be made recursively, starting with the relations between \( B \) and \( A_0 \), and \( B \) and \( A_0 \cup A_1 \), to derive \( B \)’s relation with respect to \( A_0 \cup A_1 \), and then deriving from the relations between \( B \) and \( A_0 \cup A_1 \), and \( B \) and \( A_1 \) the relation between \( B \) and \( A_0 \cup A_1 \cup A_1 \).

3. **Topological Relations with Aggregates that Meet**

Given a region \( Q \) and two regions \( P_0 \) and \( P_1 \) such that \( P_0 \) and \( P_1 \) meet —like a part \( A_0 \) and its connector \( A_{01} \). We will denote the inference of the topological relations over aggregates as \( r_i \), where \( r_i \) and \( r_j \) are the topological relations between \( P_0 \) and \( Q \), and \( P_1 \) and \( Q \) respectively, and \( \{ r_k \} \) is the set of consistent topological relations implied by \( r_i \) and \( r_j \) for \( P_0 \cup P_1 \). Since \( P_0 \) must meet \( P_1 \), it follows that \( r_i (P_0, Q) \) is an element of the composition of meet \((P_0, P_1)\) with \( r_j (P_1, Q) \). Likewise, \( r_j \) is an element of the composition between \( r_i \) and meet. The result of the composition may be empty, denoted by \( \emptyset \), if \( r_i \) and \( r_j \) cannot be realized under the condition that \( P_0 \) meets \( P_1 \). Such impossible aggregates are determined by checking the consistency of the scene with objects \( P_0 \), \( P_1 \), and \( Q \) (Egenhofer and Sharma 1993).

From among the sixty-four possible combinations for the relations between \( Q \) and \( P_0 \) and \( Q \) and \( P_1 \), thirty-seven were found to be inconsistent. The remaining 27 combinations require a more detailed analysis of the intersections among the parts’ interiors and boundaries.

- If at least one part’s interior intersects with the target’s interior, then the aggregate’s interior will intersect with the target’s interior as well (Equation 7a). Conversely, if they do not intersect, the aggregate’s interior will not intersect with the target’s interior (Equation 7b)

\[
P_i \circ \circ Q = \emptyset \quad P_i \cdot j \circ \circ Q = \emptyset \quad (7a)
\]

\[
P_i \circ \complement Q = \emptyset \quad P_j \circ \complement Q = \emptyset \quad (7b)
\]

- If at least one of the parts’ interiors intersects with the boundary of the target, then the aggregate’s interior must intersect with the target’s boundary (Equation 8a). Otherwise, for non-empty intersections, one has to consider in addition the relations among the interiors: If one part’s interior intersects with the target’s interior, while the other part’s interior does not, the aggregate’s interior must intersect with the target’s boundary (Equation 8b). In these cases, the target’s boundary runs through the aggregate’s interior exactly along the neat line between the two parts. Reversely—if both part’s interiors intersect with the target’s interior, or both do not—the aggregate’s interior cannot intersect with the target’s boundary (Equation 8c).

\[
P_i \circ \complement \partial Q = \emptyset \quad P_i \quad j \circ \complement \partial Q = \emptyset \quad (8a)
\]

\[
P_i \circ \partial Q = \emptyset \quad P_j \circ \partial Q = \emptyset \quad (8b)
\]

\[
\complement P_i \circ \complement Q = \emptyset \quad \complement P_j \circ \complement Q = \emptyset \quad (8c)
\]
\[ P_i \cap \partial Q = \emptyset \quad \square \quad P_j \cap \partial Q = \emptyset \]
\[ \square \left( (P_i \cap Q^o = \emptyset \quad \square \quad P_j \cap Q^o = \emptyset ) \right) \]
\[ (P_i \cap Q^o = \emptyset \quad \square \quad P_j \cap Q^o = \emptyset ) \quad \square \quad P_{ij} \cap \partial Q = \emptyset \]  

(8c)

- The aggregate’s boundary does not intersect with the target’s interior if neither part’s boundary intersects with the target’s interior (Equation 9a); however, it does intersect if one of the two parts’ boundaries intersects with the target’s interior, while the other does not (Equation 9b). In case both parts’ boundaries intersect with the target’s interior, one has to take into account the relation between all three boundaries to determine the actual relation: If at least one part’s boundary does not intersect with the target’s boundary, the aggregate’s boundary does intersect with the target’s interior (Equation 9c); on the other hand, if all three boundaries intersect, the boundary-interior intersection between the aggregate and the target is undetermined and can be either empty or non-empty (Equation 9d).

\[ \partial P_i \cap \partial Q = \emptyset \quad \square \quad \partial P_j \cap \partial Q = \emptyset \quad \square \quad \partial P_{ij} \cap \partial Q = \emptyset \]
\[ \square \quad \partial P_i \cap \partial Q = \emptyset \quad \square \quad \partial P_j \cap \partial Q = \emptyset \]
\[ \square \quad \partial P_{ij} \cap \partial Q = \emptyset \]

(9a)

\[ \partial P_i \cap \partial Q = \emptyset \quad \square \quad \partial P_j \cap \partial Q = \emptyset \quad \square \quad \partial P_{ij} \cap \partial Q = \emptyset \]
\[ \square \quad \partial P_i \cap \partial Q = \emptyset \quad \square \quad \partial P_{ij} \cap \partial Q = (\emptyset \quad \emptyset ) \]

(9b)

\[ \partial P_i \cap \partial Q = \emptyset \quad \square \quad \partial P_j \cap \partial Q = \emptyset \]
\[ \square \quad \partial P_{ij} \cap \partial Q = \emptyset \quad \square \quad \partial P_{ij} \cap \partial Q = (\emptyset \quad \emptyset ) \]

(9c)

\[ \partial P_i \cap \partial Q = \emptyset \quad \square \quad \partial P_j \cap \partial Q = \emptyset \]
\[ \square \quad \partial P_{ij} \cap \partial Q = \emptyset \quad \square \quad \partial P_{ij} \cap \partial Q = (\emptyset \quad \emptyset ) \]

(9d)

- The boundaries of the aggregate and the target do not intersect if neither parts’ boundaries intersect with the target’s boundary (Equation 10a). If one of the parts’ boundaries intersects with the target’s boundary, while the other part does not, then the boundary-boundary intersection between the aggregate and the target are non-empty (Equation 10b). In case both parts’ boundaries intersect with the target’s boundary, the boundary-boundary intersection is undetermined (i.e., either empty or non-empty) if the target is a true subset of the aggregate (Equation 10c and d), and otherwise the boundary-boundary intersection is non-empty (Equation 10e-g).

\[ \partial P_i \cap \partial Q = \emptyset \quad \square \quad \partial P_j \cap \partial Q = \emptyset \quad \square \quad \partial P_{ij} \cap \partial Q = \emptyset \]
\[ \square \quad \partial P_i \cap \partial Q = \emptyset \quad \square \quad \partial P_j \cap \partial Q = \emptyset \quad \square \quad \partial P_{ij} \cap \partial Q = \emptyset \]
\[ \square \quad \partial P_i \cap \partial Q = \emptyset \quad \square \quad \partial P_{ij} \cap \partial Q = (\emptyset \quad \emptyset ) \]

\[ \partial P_{ij} \cap \partial Q = \emptyset \quad \square \quad \partial P_{ij} \cap \partial Q = (\emptyset \quad \emptyset ) \]

(10a)

\[ \partial P_{ij} \cap \partial Q = \emptyset \quad \square \quad \partial P_{ij} \cap \partial Q = \emptyset \]
\[ \square \quad \partial P_{ij} \cap \partial Q = \emptyset \quad \square \quad \partial P_{ij} \cap \partial Q = (\emptyset \quad \emptyset ) \]

(10b)

\[ \partial P_{ij} \cap \partial Q = \emptyset \quad \square \quad \partial P_{ij} \cap \partial Q = \emptyset \]
\[ \square \quad \partial P_{ij} \cap \partial Q = \emptyset \quad \square \quad \partial P_{ij} \cap \partial Q = (\emptyset \quad \emptyset ) \]

(10c)
\[ \partial P_i \triangleleft \partial Q = \emptyset \quad \partial P_j \triangleleft \partial Q = \emptyset \]
\[ \quad P_i \cap Q^c = \emptyset \quad P_j \cap Q^c = \emptyset \]
\[ \partial P_i \cap \partial Q = \emptyset \quad \partial P_j \cap \partial Q = \emptyset \quad \partial P_{i\downarrow j} \cap \partial Q = (\emptyset \cap \emptyset) \quad (10d) \]
\[ \partial P_i \triangleleft \partial Q = \emptyset \quad \partial P_j \triangleleft \partial Q = \emptyset \]
\[ \quad P_i \cap Q^c = \emptyset \quad P_j \cap Q^c = \emptyset \quad \partial P_{i\downarrow j} \cap \partial Q = \emptyset \quad (10e) \]
\[ \partial P_i \cap \partial Q = \emptyset \quad \partial P_j \cap \partial Q = \emptyset \]
\[ \quad P_j \cap Q^c = \emptyset \quad P_j \cap Q^c = \emptyset \quad \partial P_{i\downarrow j} \cap \partial Q = \emptyset \quad (10f) \]
\[ \partial P_i \triangleleft \partial Q = \emptyset \quad \partial P_j \triangleleft \partial Q = \emptyset \]
\[ \quad P_i \cap Q^c = \emptyset \quad P_j \cap Q^c = \emptyset \quad \partial P_{i\downarrow j} \cap \partial Q = \emptyset \quad (10g) \]

The successive application of the constraints among boundaries and interiors leads to patterns of valid 4-intersection matrices. By matching them with the definitions of the eight region-region relations (Figure 3), one obtains the set of possible aggregate inferences (Table 1). All aggregate inferences are symmetric, such that the sequence of the parts does not matter. Twenty-one combinations are unique, while six are underdetermined leaving five with a choice of two possible relations and one with three possible relations. For all 27 inferred combinations, geometric interpretations can be found.

These kinds of inferences can be combined for more than two parts, such that \( r_i \quad r_j \quad r_k \) derives the relation between two pairs of adjacent regions, \( P_0 \) and \( P_1 \), and \( P_1 \) and \( P_2 \). Such combined inferences must be associative, i.e., they must hold independently of the sequence in which the aggregates are formed; therefore, the consistent aggregate-relation inference for three parts is the common relation that can be realized in both directions (Equation 11).

\[ r_{ijk} = (r_i \quad r_j) \quad r_k \triangleright r_i \quad (r_j \quad r_k) \quad (11) \]

This method will be used to infer which combinations are possible, and which are impossible.
Table 1: Aggregate-relation inferences for $P_0 \ r_i \ Q$ and $P_1 \ r_j \ Q$ implies $(P_0 \sqcup P_1) \ r_k \ Q$. 
4. Consistent Topological Relations between Regions with Disconnected Parts

Ideally, no knowledge about the relation between $\Box A_0$ and $B$ would be required to derive the topological relation between $A_*$ and $B$. Since the topological relation between the connector and $B$ is unknown, inferences are calculated as the aggregate-relation inference over the universal relation $U$, i.e., \{disjoint, meet, overlap, equal, covers, contains, coveredBy, inside\} (Equation 12).

$$r_{A_\ast B} = (r_{A_0 B} U \Box A_0 B) \ R_{A_0 B} \ L r_{A_1 B} (U \Box A_1 B) r_{A_1 B}$$

(12)

For this inference, a total of 64 combinations are possible; however, due to the constraints among the parts in the scene, only a subset of them can be realized.

- 33 combinations are impossible, because they would construct an inconsistent scene description. For example, if $A_0$ equal $B$ then $A_1$ cannot be equal to $B$ as well. These configurations are not of further interest.

- 31 combinations are consistent. For example, if $A_0$ is disjoint from $B$, and $A_1$ contains $B$, then $A_\ast$ overlaps with $B$. These 31 combinations will be the subject of further examination.

4.1 Relations Uniquely Implied

Among the 31 consistent combinations of part-relations, there are 10 combinations that provide a unique result, i.e., the relation between $A_\ast$ and $B$ is uniquely determined by $A_0$ $r_i$ $B$ and $A_1$ $r_j$ $B$. The 10 unique combinations form five pairs of converse combinations, such that $r_i U r_j = r_j U r_i$ (Table 2).
Table 2: The five consistent combinations of part-relations that provide a unique result (without converse combinations).

<table>
<thead>
<tr>
<th>( r_i )</th>
<th>( r_j )</th>
<th>( r = r_i \ U \ r_j )</th>
</tr>
</thead>
<tbody>
<tr>
<td>disjoint</td>
<td>inside</td>
<td>Overlap</td>
</tr>
<tr>
<td>disjoint</td>
<td>equal</td>
<td>Covers</td>
</tr>
<tr>
<td>disjoint</td>
<td>contains</td>
<td>Contains</td>
</tr>
<tr>
<td>meet</td>
<td>inside</td>
<td>Overlap</td>
</tr>
<tr>
<td>inside</td>
<td>overlap</td>
<td>Overlap</td>
</tr>
</tbody>
</table>

4.2 Underdetermined Relations

The remaining 21 consistent combinations provide underdetermined results of various degrees, i.e., the composition is not crisp, but results in a set of possible relations. While these inferences serve as a mechanism to determine which generalizations are inconsistent, they are insufficient to make a precise statement about the generalizations’ consistencies. For example, if \( A_0 \) and \( A_1 \) both overlap with \( B \), it would be inconsistent if \( A_* \) was inside \( B \). On the other hand, \( A_* \) overlaps with \( B \) would be one of three possible configurations — \( A_* \) could also cover or contain \( B \) — and a more precise assessment is only possible if additional information is available. Among these 21 combinations, there are eight pairs of converse pairs of relations, for which \( r_i \ U \ r_j \) is the same as \( r_j \ U \ r_i \).

4.3 Relations Implied by Parts-Relations and Connection-Relation

More detail may be obtained about the 21 consistent but underdetermined configurations by also considering the relation between the connector \( \sqcap A_{01} \) and \( B \). If all eight topological relations were possible between \( \sqcap A_{01} \) and \( B \) for all 21 configurations, a total of \( 21 \times 8 = 168 \) cases could be found. Due to the constraints among the parts and the connectors, only 73 relation triples are consistent and therefore worthwhile to be considered. Among them there are 48 combinations with a unique inference, including 15 pairs of converse combinations. Table 3 displays the remaining 33 cases. The consideration of the relation with respect to the connector resolves four underdetermined part-relations completely, and leaves the remaining 17 part-relations partially underdetermined.
<table>
<thead>
<tr>
<th>$r_i$</th>
<th>$r_j$</th>
<th>$r_k$</th>
<th>$r = r_i \lor r_k \lor r_j$</th>
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<td>disjoint</td>
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<td>covers</td>
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<td>disjoint</td>
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**Table 3:** The 33 combinations implied uniquely by the part-relations *and* the connection-relation (without converse cases).
4.4 Relations that Require Additional Information

The 25 underdetermined configurations of parts-connection relations include 10 triples of relations that are converse, reducing the number of remaining ambiguous cases to 15. These 15 combinations have only four different results for $A_r^s B$: three pairs (contains or covers; covers or overlap; and coveredBy or equal) and one triple (contains, covers, or overlap). To resolve the remaining ambiguities, it is sufficient to identify the criteria that allow us to distinguish between the relations in each set. This distinction follows immediately from the differences in the 4-intersection matrices.

- The difference between contains and covers is the boundary-boundary intersection, i.e., $\partial A_r^s \cap \partial B$ will resolve these ambiguities.
- The difference between covers and overlap is the boundary-interior intersection ($\partial A_r^s \cap B^o$).
- The difference between coveredBy and equal is the boundary-interior intersection ($\partial A_r^s \cap B^o$) as well.
- There is no single criterion that would allow us to make the distinction between contains, covers and overlap; therefore, they need two conditions that must both hold true. The conditions of contains or covers, and covers or overlap are one possible solution, i.e., $\partial A_r^s \cap \partial B$ and $\partial A_r^s \cap B^o$.

Table 4 shows how the 15 ambiguous combinations of part-connection relations are resolved by considering the relation of the aggregate’s boundary with the boundary and interior of the other region.
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<th>Diagram 2</th>
<th>Diagram 3</th>
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### Diagrams

- **Diagram 1**: $A_0$ disjoint $B$, $A_1$ meet $B$, $\partial A_0 \cap \partial B \neq \emptyset$.
- **Diagram 2**: $A_0$ disjoint $B$, $A_1$ cover $B$, $\partial A_0 \cap \partial B \neq \emptyset$.
- **Diagram 3**: $A_0$ disjoint $B$, $A_1$ meet $B$, $\partial A_0 \cap \partial B \neq \emptyset$.

**Explanation**

- **Diagram 1**: $A_0$ disjoint $B$ means $A_0$ and $B$ do not overlap. $A_1$ meet $B$ means $A_1$ and $B$ intersect but do not overlap, and $\partial A_0 \cap \partial B \neq \emptyset$ indicates that the boundaries of $A_0$ and $B$ intersect.
- **Diagram 2**: Similar to Diagram 1, but $A_0$ disjoint $B$ means $A_0$ and $B$ are independent, and $A_1$ cover $B$ means $A_1$ completely covers $B$.
- **Diagram 3**: $A_0$ disjoint $B$, $A_1$ meet $B$ means $A_0$ and $B$ do not overlap, and $A_1$ and $B$ intersect, with $\partial A_0 \cap \partial B \neq \emptyset$ indicating intersection of boundaries.

**Note**: The diagrams illustrate spatial relationships between $A_0$, $A_1$, and $B$, highlighting disjoint, meet, and cover conditions.

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**Symbols**

- $A$: Set
- $B$: Set
- $\partial$: Boundary
- $\cap$: Intersection
- $\cup$: Union
- $\subset$: Subset
- $\supset$: Superset
- $\emptyset$: Empty set
- $\neq$: Not equal to
- $\neq \emptyset$: Not equal to empty set
- $\neq \emptyset$: Not equal to empty set
A₀ meet B  
A₁ overlap B  
△A₀₁ overlap B

A₀ meet B  
A₁ overlap B  
△A₀₁ coveredBy B

A₀ meet B  
A₁ coveredBy B  
△A₀₁ coveredBy B

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**Table 4:** The 15 combinations of part-relations that require the connection-relation and the relation to the boundary of the generalized region in order to obtain a unique result (without converse cases).
4.5 A Sketch of the Algorithm to Determine Consistency

The consistency constraints captured in Tables 2-4 provide the basis for a comprehensive, systematic assessment of the consistency of two scenes containing a region with disconnected parts. Figure 5a shows two configurations, $S_X$ and $S_Y$, where $S_Y$ is less detailed than $S_X$, i.e., $S_Y \subseteq S_X$. Object $A^X$ has two parts $A_0^X$ and $A_1^X$, and object $B^X$ one part. In the less detailed scene $S_Y$, $A_0^X$ and $A_1^X$ have been merged into $A^*$. An algorithm to assess the consistency among the relations of objects of two such scenes starts with checking whether the relation between $A^*$ and $B$ can be derived completely from the relations between $A_0$ and $B$, and between $A_1$ and $B$. If necessary, the relation between $\partial A_0$ and $B$ is added to determine the consistency or inconsistency of the relation between $A^*$ and $B$. In case the configuration is still not uniquely describable, the empty/non-empty intersections of $\partial A^* \cap B^*$ and $\partial A^* \cap \partial B$ are added in a final test.

5. Consistency of $n$-Aggregates

While the previous investigations focused on a particular scene configuration—one region with two disconnected parts and another region without disconnected parts—the results generalize to more complex configurations involving objects with more parts. This section demonstrates these extensions. First, the scenario of a 2-aggregate and a simple region is extended to an $n$-aggregate and a region by successively applying the algorithm described in Section 4.5. Then it is shown how the consistency of two 2-aggregates is assessed. The combination of the two extensions allows us to treat scenes of two arbitrarily complex regions with disconnected parts.

5.1 Relations Involving a Region and a Region with Three (or More) Disconnected Parts

An $n$-aggregate is formed from a region with $n$ disconnected parts. The consistency of scenes containing configurations with $n > 2$ separations can be derived from the consistency of pairs of objects. For example, a region with three parts (Equation 13) is aggregated by integrating two connectors.

$$A^* = A_0 \cap A_1 \cap A_2$$ (13)

Depending on the geometric configuration, there may be a choice of connection (Figure 5b) establishing a connector to link (1) $A_0$ with $A_1$, forming $A^*_{01}$, and a connector between $A_{01}$ with $A_2$, or (2) $A_0$ with $A_2$, forming $A^*_{02}$, and a connector between $A_1$ with $A_{02}$, or (3) $A_1$ with $A_2$, forming $A^*_{12}$, and a connector between $A_0$ with $A_{12}$ (Equations 14a-c). The actual choice of the connectors depends on the semantics and importance of the parts, as well as their geometric configuration.

$$A^* = A^* \cap A_{01} \cap A_2$$ (14a)

$$A^* = A^* \cap A_{02} \cap A_1$$ (14b)
\[ A_v^* = A_v \bigcap A_{12} \bigcap A_0 \]  \hspace{1cm} (14c)

Since the different configurations can be obtained by permutations of the connectors’ indices, it is sufficient to select one configuration to demonstrate the approach.

We use the approach for the relation between a region and a 2-aggregate (Section 4) by applying the following four steps:

- Connect \( A_0 \) to \( A_1 \) via \( \bigcap A_{01} \), forming the aggregate \( A_{01}^* = A_0 \bigcap A_1 \bigcap A_{01} \).
- Applying the algorithm sketched in Section 4.5, one obtains \( A_{01}^* \bigcap B \).
- Connect \( A_2 \) to \( A_{01}^* \) via \( \bigcap A_{012}^* \), forming the aggregate \( A_* = A_{01}^* \bigcap A_2 \bigcap A_{012}^* \).
- Applying the algorithm of Section 4.5, one obtains \( A_* \bigcap B \).

Figure 5c shows an example of this process, integrating a region with three parts, \( A_* \), into a single object, \( A_* \). This region is related to another object, \( B \), such that one part of \( A_* \) overlaps with \( B \), while the other parts of \( A_* \) are disjoint from \( B \). \( A_* \) is consistent with \( A_* \) if \( A_* \) overlaps with \( B \). This result is obtained from \( A_0 \) disjoint \( B \), \( A_1 \) overlap \( B \), \( \bigcap A_{01} \) overlap \( B \), \( \partial A_1 \bigcap B^o = \bigcap \emptyset \), and \( \partial A_1 \bigcap \partial B = \bigcap \emptyset \), which imply that \( A_{01}^* \) overlap \( B \) (Table 4). This intermediate result is used in the next inference step where \( A_{01}^* \) overlap \( B \), \( A_2 \) disjoint \( B \), \( \bigcap A_{012}^* \) overlap \( B \), \( \partial A_* \bigcap B^o = \bigcap \emptyset \), and \( \partial A_* \bigcap \partial B = \bigcap \emptyset \) imply that \( A_* \) overlap \( B \) (Table 4).

In order to ensure that aggregation is associative, it is required that the sequence in which the aggregates are formed does not matter (Equation 15).

\[ ((A_0 r_i^X B A_1 r_j^X B A_2 r_j^X B) = (A_0 r_i^X B (A_1 r_j^X B A_2 r_j^X B)) \]  \hspace{1cm} (15)

Further extensions to account for consistency of configurations with a greater number of parts are achieved by recursively applying this process over intermediate aggregates.

### 5.2 Relations between two Regions each with two Parts

The formalism for a 2-aggregate and a region also can be applied to assessing the consistency of two 2-aggregates. If \( A_* = A_0 \bigcap A_1 \) and \( B_* = B_0 \bigcap B_1 \), then \( A_* = A_* \bigcap A_{01} \) and \( B_* = B_* \bigcap B_{01} \). The aggregates are topologically consistent if \( A_0 \), \( A_1 \), \( B_0 \), and \( B_1 \) conform with \( A_* \) and \( B_* \). This assessment can be done in two ways: (1) by forming the aggregate \( A_* \), followed by the formation of the aggregate \( B_* \), or (2) by forming \( B_* \) prior to forming \( A_* \). The method for either step follows immediately from the consistency assessment for one 2-aggregate. First, the two corresponding parts are aggregated to form a generalized region with disconnected parts. This aggregate must be consistent with respect to each of the other region’s parts, involving two consistency assessments of the type presented in the algorithm sketched in Section 4.5. Finally, the second region with
disconnected parts gets aggregated and its relation must be consistent with respect to the first aggregate. This step involves another assessment with the algorithm of Section 4.5.

With two regions with disconnected parts, there is a choice of sequence in which the aggregates are formed. For instance, one may first assess the consistency of aggregate $A_*$ prior to forming $B_*$, and then integrate $A_*$ and $B_*$. Conversely, one could first assess $B_*$’s consistency, then form $A_*$, and finally aggregate $A_*$ and $B_*$. Forming aggregates in different sequences does not influence the result (Equation 16).

$$((A_0r_i^X B_0 \quad A_0r_i^X B_0) \quad (A_0r_i^X B_1 \quad A_0r_i^X B_1)) =$$

$$((A_0r_i^X B_0 \quad A_0r_i^X B_1) \quad (A_0r_i^X B_2 \quad A_0r_i^X B_1))$$

(16)

The example in Figure 5d will be used to demonstrate that the two sequences may require different resources, though they will obtain the same result. Since $A_0 \text{ disjoint } B_0$ and $A_1 \text{ disjoint } B_0$, we need knowledge about the topological relation between $\Box A_0$ and $B_0$ to assess the consistency of $A_*$. From $\Box A_0 \text{ disjoint } B_0$ it follows that $A_* \text{ disjoint } B_0$ (Table 3). The same inference steps find that $A_* \text{ disjoint } B_1$. The last step is the aggregation of $B_*$. $B_0 \text{ disjoint } A_* \text{ and } B_1 \text{ disjoint } A_*$, therefore, we need additional knowledge about $\Box B_0$ and $A_*$. Since $\Box B_0 \text{ overlaps } A_*$, it follows that $B_* \text{ overlaps } A_*$ (Table 3). The converse combination starts with forming the aggregate $B_*$ first. Since $B_0 \text{ disjoint } A_0$ and $B_1 \text{ disjoint } A_0$, additional information about the relation between $\Box B_0$ and $A_0$ is necessary. $\Box B_0 \text{ overlap } A_0$ implies that $B_* \text{ overlap } A_0$ (Table 3). This inference step is repeated for the relation between the $B$ aggregate and $A_1$. From $B_0 \text{ disjoint } A_1$, $B_1 \text{ disjoint } A_1$, and $\Box B_0 \text{ overlap } A_1$ follows that $B_* \text{ overlap } A_1$ (Table 3). The aggregation of $A_0$ and $A_1$ is based on $B_* \text{ overlap } A_0$ and $B_* \text{ overlap } A_1$. Since $\Box A_0$ overlaps with $B_*$, we need information about the relations between the boundaries of $A_*$ and $B_*$, and between $A_*$’s boundary and $B_*$’s interior. Since both intersections are non-empty, it follows that $B_* \text{ overlaps } A_*$ (Table 4).
Figure 5: Various forms of aggregates: (a) two pairs of objects at two different representations levels, (b) two different ways to establish connectors between a region with three disconnected parts, (c) recursively forming a 3-aggregate, and (d) two 2-aggregates.
6. Conclusions

Since aggregates change their conceptual representation from one level of detail to another, it is important to determine whether an aggregate complies spatially with respect to its constituent parts. The major result of this investigation is a comprehensive formalism to assess consistencies and inconsistencies among aggregates of two-dimensional spatial objects.

The target application for this consistency assessment are multi-representation geographic databases in which users perform spatial analysis tasks at different levels of detail. Consistency among aggregates applies to the generalization of a large variety of geographic objects. Our results are a contribution in the area of model-oriented generalization towards releasing cartographers from tedious manual quality control when maintaining multi-resolution geographic databases.

At the coarsest level, we identified consistent and inconsistent configurations when a set of disconnected regions gets merged into a single component. Approximately half of the configurations that can be described through a complete combination of topological relations, lead to a consistent aggregate; the remainder cannot be realized in a 2-dimensional space between 2-dimensional objects. At a more detailed level, we identified under which constraints certain aggregates can be formed. We developed a formal model to assess the consistency of 2-aggregates, which identified three levels of increasing knowledge about the aggregate: (1) the strongest configurations are uniquely determined by the two relations between the two parts and the reference object, (2) by adding information about the connector between the two parts, more configurations can be uniquely assessed, and (3) the weakest consistent configurations require information about details of how the aggregate is formed, referring to the boundary of the aggregate.

The formalism developed applies recursively to more complex configurations, including the formation of two aggregates consisting of n and m disconnected parts. The algebra over such complex regions is associative, because the combinations are independent of the sequence in which the aggregates are formed. Varying costs may incur, however, if different sequences are chosen to assess the aggregates’ compositions. This provides opportunities for optimizing the assessment of aggregates, particularly if they involve a large number of parts.

We have implemented a software prototype that assesses the consistency of the relation between two regions with two disconnected parts. Users construct interactively two objects with separations at the most detailed level, and then draw the corresponding aggregates. The consistency checker evaluates whether the two configurations are topologically compatible. The prototype was written in C++ using the MetroWerks code development environment for Macintosh and PowerMacintosh computers. The core of the algorithm, with the exception of the user interface code, is portable to other platforms and C++ compilers.

Future work includes:

- A limitation on aggregates was imposed by the fact that disconnected parts had to be truly disconnected (i.e., the part relation must be disjoint). Occasionally, it may be necessary to consider parts that are connected through a common boundary part to be aggregated. An example is an urban area in which buildings touch “kitty-corner” and get integrated into a city block. To account for such situations, it will be necessary to extend the concept of a region with disconnected parts to allow for regions with disconnected interiors, but connected boundaries.

- The topological relations among parts, and those between the aggregate and the reference object, were based on the empty/non-empty intersection of boundaries and interiors. This
most generic distinction (Egenhofer and Franzosa 1991) may be insufficient at times when detailed topological details are desired (Egenhofer and Franzosa 1995); therefore, the level of granularity of topological relations will have to be enhanced beyond the basic eight region-region relations to include also knowledge about the component relations.

- Complementary to consistency for aggregates, we developed a formalism to determine the consistency of multi-representation objects that contain holes (Paiva and Egenhofer 1995). Both configurations—regions with disconnected parts and regions with holes—may co-exist, such as an island with a lake within which another island is located (i.e., a region with disconnected parts such that one part is located in the hole of the other part). An integration of the formalisms for regions with disconnected parts and regions with holes would allow for inferences in more complex settings.

- The current method provides a Boolean answer with respect to the consistency of two configurations, i.e., two configurations are found to be consistent or inconsistent. In many situations, however, it is necessary to assess the degree at which configurations differ from each other. Such similarity assessments, based on the methods presented, may be developed by relaxing some of the constraints about equivalence. Formal descriptions of similarity may lead to new methods for change detection—whether aggregates observed at different times are the same or not.

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