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# Baryon Mass and Phase Transitions in Large $N$ Gauge Theory

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**Abstract**

We calculate baryon mass in $\mathcal{N} = 4$ large $N$ gauge theory by means of AdS/CFT correspondence, and find that phase transitions occur when temperature or theta angle takes a critical value. Furthermore, we find there are bound states of W-bosons in Higgs phase.

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# 1 Introduction

The string theory is a convenient tool for analyzing non-perturbative properties of Yang-Mills theories. In the last year, Maldacena presented a new approach to an investigation of large $N$ conformal field theory[1]. His idea relies on a fact that a D3-brane with large R-R charge $N$ is well approximated by a classical black brane solution of supergravity. Its near horizon geometry is described by $AdS_5 \times M_5$ where $AdS_5$ is five dimensional anti-de Sitter manifold and $M_5$ is a certain compact space, which is assumed to be $S^5$ in this paper. The Yang-Mills gauge field and its superpartners are regarded to live on the boundary at infinity of the $AdS_5$. These fields couple to supergravity fields in bulk and correlation functions of operators in the Yang-Mills theory are obtained as a classical correlation function of bulk fields on $AdS_5 \times M_5$[2, 3]. Quarks and monopoles are represented by fundamental and D-strings, respectively. By calculating the area of the string world-sheet, we obtain quark-quark potential, monopole-monopole potential and quark-monopole potential etc[4, 5, 6]. Furthermore, it was shown in [7, 8] that baryons correspond to D5-branes wrapped around the manifold $M_5$. The purpose of this paper is to study the baryon configurations. We calculate baryon mass at zero and finite temperature, and we find that, at low temperature, baryons are created from $N$ quarks. At critical temperature $T = T_c$, a phase transition occurs and in high temperature phase, baryons decay into free quarks. Next, we discuss on the Higgs phase where gauge symmetry broken to $SU(N_1) \times SU(N_2)$ and it is shown that bound states of W-bosons exist. Finally, we discuss about a relation of stability of dyons and the theta angle.

# 2 Zero temperature

First, we consider $\mathcal{N} = 4$ $SU(N)$ gauge theory at zero temperature. We assume that the theta angle vanishes in this paper except the section 5. It is realized as a field theory on overlapping $N$ extremal D3-branes. If $N$ is large, the three-brane are well approximated by classical black three-brane solution of supergravity. The metric of the solution of three-branes spreading along $x_{0,\ldots,3}$ is

$$ds^2 = H^{-1/2}(\mathbf{x}) \sum_{\mu=0}^{3} dx_\mu^2 + H^{1/2}(\mathbf{x}) d\mathbf{x}^2, \quad H(\mathbf{x}) = 1 + \frac{r_0^4}{r^4}. \tag{1}$$

where $\mathbf{x} = (x_4, x_5, \ldots, x_9)$ and $r = |\mathbf{x}|$. A radius of the horizon $r_0$ is connected with a R-R charge $N \in \mathbf{Z}$ of the D3-brane by a relation

$$r_0^4 = \frac{2\kappa^2}{4c_5} N T_{\mathrm{D3}} = 4\pi l_s^4 g_{\mathrm{str}} N = 2\eta l_s^4, \tag{2}$$

where $T_{D3} = 1/(2\pi)^3 l_s^4 g_{\rm str}$ is a tension of single extremal D3-brane, $c_5 = \pi^3$ is volume of a five dimensional unit sphere, $2\kappa^2 = (2\pi)^7 l_s^8 g_{\rm str}^2$ is Newton's constant and $\eta = 2\pi g_{\rm str} N$ is an effective coupling constant of the large $N$ theory. We adopt a convention in which the string tension is $T_F = 1/(2\pi l_s^2)$ and the string coupling constant $g_{\rm str}$ is transformed into $1/g_{\rm str}$ by S-duality. In the near horizon region $0 \le r \ll r_0$, eq.(1) is reduced to $AdS_5 \times S^5$ metric:

$$ds^2 = \frac{r^2}{r_0^2} \sum_{\mu=0}^{3} dx_\mu^2 + \frac{r_0^2}{r^2} dr^2 + r_0^2 d\Omega_5^2. \tag{3}$$

On this background, quarks belonging to fundamental representation of $SU(N)$ are open strings whose one end points are attached on the horizon $r = 0$ of the black brane solution. We adopt a definition of a string orientation that quarks are strings stretched outward from the horizon. Antiquarks in $\overline{N}$ representation are represented by strings with the opposite orientation to that of quarks. Unless we regard the quarks as external particles, we should introduce another D3-brane, which we will call a probe in this paper, to attach another end point of the string. The quark mass is given by

$$M_q = T_F \int_0^{r_{\rm probe}} \sqrt{g_{tt} g_{rr}} dr = \frac{r_{\rm probe}}{2\pi l_s^2}, \tag{4}$$

where $r_{\rm probe}$ represents the position of the probe.

As is shown in [8, 7], junctions of $N$ strings with the same orientation can be constructed in $AdS_5 \times S^5$ and are identified with D5-branes wrapped around $S^5$. The charge of the string end points, which couples to $U(1)$ field on the compact D5-brane, is canceled by the Chern-Simons coupling to the R-R five form field strength wrapped around $S^5$. In terms of the field theory, These configurations are regarded as baryons, bound states of $N$ quarks, or antibarions, bound states of $N$ antiquarks. We identify baryons to wrapped D5-branes from which $N$ strings stretched outward. Antibaryons are D5-brane wrapped around $S^5$ with the opposite direction to that of baryons. A baryon mass is a product of D5-brane tension $T_{D5} = 1/[(2\pi)^5 l_s^6 g_{\rm str}]$, area of $S^5$ and 'gravitational potential' $\sqrt{g_{tt}}$.

$$M_{\rm B}(r) = \sqrt{g_{tt}} \times T_{\rm D5} \times c_5 r_0^5 \ = \frac{r}{2\pi l_s^2} \frac{N}{4} \tag{5}$$

In this expression, the variable $r$ represents the position of wrapped D5-brane. The $r$ dependence of the mass (5) means that wrapped D5-brane feels force $-dM_B/dr$. This is gravitational force due to the black three brane. Furthermore, it is pulled by $N$ strings attached on it. Let us assume $k$ strings are stretched between the wrapped D5-brane and the probe D3-brane and

$N-k$ strings between the D5-brane and the horizon. Then, the total force which D5-brane feels is

$$F = -\frac{dM_B(r)}{dr} + kT_F - (N-k)T_F = T_F\left(2k - \frac{5}{4}N\right). \tag{6}$$

(We assume the D5-brane is between the horizon and the probe.) Therefore, if a condition

$$\frac{5}{8}N \leq k, \tag{7}$$

holds, the D5-brane is pulled to the probe and gets stable on it. On the other hand, if $k$ is smaller than $(5/8)N$, D5-brane move to the horizon and absorbed into it. Consequently, we should identify baryons with wrapped D5-branes at the position of the probe, and their mass is given by $M_B \equiv M_B(r_{\text{probe}})$.

According to the above arguments, it is concluded that if $N$ quarks meet together, they are bound into a baryon. Even if the number of quarks is smaller than $N$, a cost of $N-k$ quark-antiquark pair creations is smaller than baryon binding energy (This condition is the same with (7)), a baryon is generated as well. In terms of the string configurations, the baryon creation process may advance as follows (Fig.1): (a) If $k$ strings meet together, (b) baryon-antibaryon pair, or two wrapped D5-brane with the opposite orientation each other, is created at a point on strings. (c) If the number $k$ of the strings is smaller than $N$, $N-k$ strings with the opposite orientation are generated between baryon and antibaryon to cancel the charge of string end point on the created wrapped D5-brane. (d) Then, the baryon reaches to the position of probe $r = r_{\text{probe}}$ and antibaryon is absorbed into the horizon. The final state consists of a wrapped D5-brane at $r = r_{\text{probe}}$ and $N-k$ antiquarks, and the total energy is

$$E_{\text{Final}} = (N-k)M_q + M_B = \left(\frac{5}{4}NM_q - k\right)M_q. \tag{8}$$

If the condition (7) is satisfied, $E_{\text{Final}}$ is smaller than initial total energy $kM_q$.

In the above estimation, we assume that the configuration of D5-brane is $S^5$ symmetric under $SO(6)$ rotation. To be precise, this is not true. Because the rotational symmetry of $S^5$ is broken due to the position of probe. In terms of field theory, this corresponds to the fact that $SU(4)_R$ symmetry is broken by Higgs vev. However, even if we use exact solution for the D5-brane configuration, baryon mass is smaller than $(N/4)M_q$ undoubtedly and baryon creation process is essentially not changed. Therefore, we assume that the configuration of D5-brane is simply $S^5$ from now on as well as above.

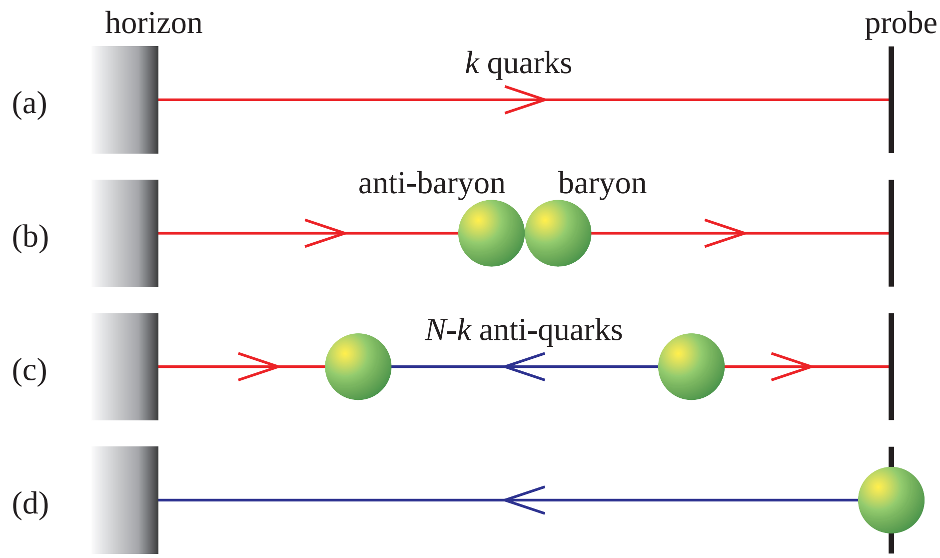


Figure 1: Baryon creation process. (a) If $k$ quarks happen to meet, (b) a baryon-antibaryon pare is created and, (c) if $k$ is smaller than $N$, $N-k$ antiquarks are generated between the baryon and the antibaryon. (d) The baryon pulled by strings moves to the position of the probe and the antibaryon is absorbed into the horizon.

# 3 Higgs phase

Higgs phase where a gauge group $SU(N)$ is broken to direct product of its subgroup is represented by multi-center D3-brane solution[1, 9]. In this section we consider the case that the unbroken gauge group is $SU(N_1)\times SU(N_2)$. Generalization to an arbitrary number of factor groups is straightforward. The classical solution is obtained by replacing the harmonic function $H(\mathbf{x})$ in (1) by

$$H(\mathbf{x})=1+\frac{r_1^4}{|\mathbf{x}-\mathbf{x}_1|^4}+\frac{r_2^4}{|\mathbf{x}-\mathbf{x}_2|^4}, \tag{9}$$

where $\mathbf{x}_1$ and $\mathbf{x}_2$ are positions of two D3-brane and the radii $r_i$ are given by

$$r_i^4=4\pi l_s^4 g_{\rm str}N_i,\quad i=1,2. \tag{10}$$

The near horizon geometry of this classical solution consists of three parts:

- region 1 ($|\mathbf{x}-\mathbf{x}_1|\ll\frac{r_1}{r_1+r_2}|\mathbf{x}_1-\mathbf{x}_2|$),
- region 2 ($|\mathbf{x}-\mathbf{x}_2|\ll\frac{r_2}{r_1+r_2}|\mathbf{x}_1-\mathbf{x}_2|$),
- region 3 ($|\mathbf{x}-\mathbf{x}_{1,2}|\gg|\mathbf{x}_1-\mathbf{x}_2|$).

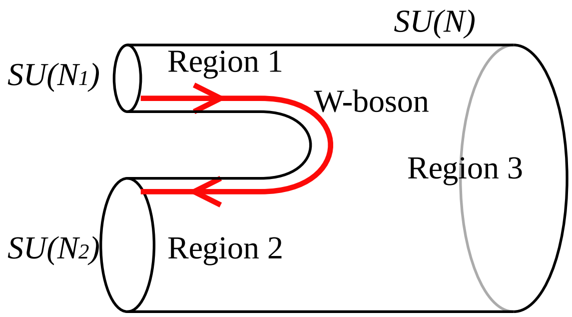


Figure 2: Near horizon geometry of a Higgs phase consist of three regions. W-bosons which belong to $(N_1, \overline{N}_2)$ representation are represented by string stretched from a horizon in region 1 to one in region 2. They are regarded as combinations of $SU(N_1)$ quarks in region 1 and $SU(N_2)$ antiquarks in region 2. W-bosons in $(\overline{N}_1, N_2)$ representation correspond to strings with the opposite orientation.

These three regions are $AdS_5 \times S^5$ with radii $r_1$, $r_2$ and $r_0$, respectively. Though we can discuss about quarks and baryons by introducing another D3-brane probe in the same way as in the last section, we focus on massive W-bosons here. Massive W-bosons belonging to $(N_1, \overline{N}_2)$ representation of unbroken gauge group $SU(N_1) \times SU(N_2)$ are represented by open strings stretched from the horizon in region 1 to that in region 2. This configuration is able to be regarded as a combination of quark configurations in region 1 and antiquark configuration in region 2. W-bosons in $(\overline{N}_1, N_2)$ representation are represented by strings with the opposite orientation. Therefore, we can discuss a dynamics of Higgs phase in the same way with the last section. In each region, W-bosons behave like quarks or antiquarks. If the number of them exceeds the critical value $(5/8)N_1$ or $(5/8)N_2$, they are bound into (anti)baryon. For the concrete argument, let us assume $N_1 < (5/8)N_2$ and $N_2 < (13/8)N_1$. If the number $n_W$ of W-bosons in $(N_1, \overline{N}_2)$ representation satisfies $(5/8)N_1 \leq n_W < (5/8)N_2$, a $SU(N_1)$ baryon is created in the region 1, while antiquarks in region 2 are not bound. (a) in Fig.3 shows a configuration in the case of $n_W = N_1$. It consists of a $SU(N_1)$ baryon and $N_1$ $SU(N_2)$ antiquarks. If the number $n_W$ is larger than $(5/8)N_2$, $SU(N_2)$ antibaryons are created in the region 2. (b) in Fig.3 is a configuration with $n_W = N_2$. It consists of a $SU(N_1)$ baryon, $SU(N_2)$ antibaryon and $N_2 - N_1$ $SU(N_1)$ quarks. These baryons and antibaryons are confined in the region 1 or 2 as is clear due to its topology. Even if $SU(N_1)$ baryons and $SU(N_2)$ antibaryons contact at boundary of region 1 and region 2, they can not merge into one wrapped D5-brane because the D5-branes have opposite wrapping direction each other.

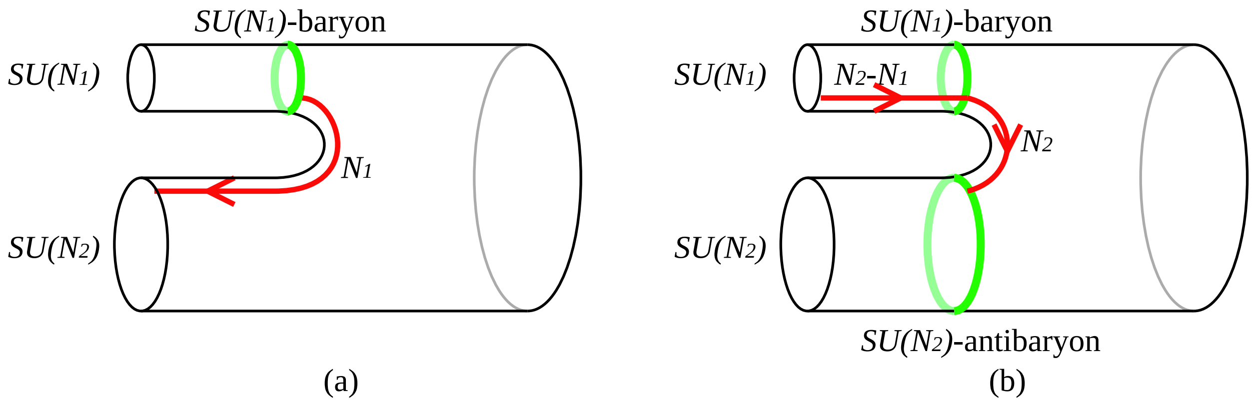


Figure 3: If $k$ W-bosons belonging to $(N_1, \overline{N}_2)$ representation meet together, a bound state is created if $k$ is larger than $(5/8)N_1$ or $(5/8)N_2$. (a) When $(5/8)N_1 \leq k \leq (5/8)N_2$, it is regarded as combination of a $SU(N_1)$ baryon and $SU(N_2)$ antiquarks. (b) When $k$ is larger than both of $(5/8)N_1$ and $(5/8)N_2$, the bound state is regarded as combination of $SU(N_1)$ baryon and $SU(N_2)$ antibaryon.

## 4 Finite temperature

Next, we consider baryon configuration at finite temperature. We restrict our argument to unbroken $SU(N)$ theory with a probe at $r = r_{\rm probe}$ again. A large $N$ gauge theory at finite temperature is realized as a field theory on non-extremal D3-branes. The classical non-extremal black three-brane solution of supergravity is given by Horowitz and Strominger[10]. In our convention, Euclidean version of the metric can be written as

$$ds^2 = f_+ f_-^{-1/2} dt^2 + f_+^{-1} f_-^{-1} dr^2 + r^2 d\Omega_5^2 + f_-^{1/2} \sum_{i=1}^{3} dx_i^2, \quad f_\pm = 1 - \frac{r_\pm^4}{r^4}, \tag{11}$$

where $r_\pm$ is represented by energy density $E$ and pressure $P$ on the three-brane* as follows.

$$r_+^4 = \frac{2\kappa^2}{4c_5} \frac{3E + P}{4}, \quad r_-^4 = \frac{2\kappa^2}{4c_5} \frac{-E + 5P}{4}. \tag{12}$$

Their geometric mean $\sqrt{r_+ r_-}$ is equal to $r_0$ given by eq.(2). If $r_-$ and $r_+$ is different, this manifold is everywhere smooth and its topology is $R^3 \times D$, where $D$ is two dimensional disc. The center of disc $D$ corresponds to the horizon $r = r_+$.

Because unique dimensionless parameter of this theory is $M_q/T$, massless quark limit is equivalent to high temperature limit. In this limit, the probe approaches to the center of the disc $r_{\rm probe} \to r_+$ and world sheets of strings which represent quarks become small disc. The quark mass is proportional to the area of the disc. On the other hand, baryon mass

*$E$ and $P$ is defined as diagonal components of energy-momentum tensor. Namely, $T_{\mu\nu} = {\rm diag}(E, P, P, P)$.

is proportional to the circumference of the boundary of the small disc. Therefore, if quark mass is smaller than some critical value, baryon mass get larger than $N$ times of quark mass. If so, quarks do not make bound states. In other words, baryons, which are stable at zero temperature, decay into free quarks at certain critical temperature $T_c$.

To discuss a near horizon geometry of the manifold (11), it is convenient to introduce new coordinate $\rho$ by

$$\rho^4 = \frac{r^4 - r_-^4}{r_+^4 - r_-^4}. \tag{13}$$

After proper rescaling of the coordinates $x_{1,2,3}$ and $t$, we get a near horizon metric:

$$ds^2 = r_+^2 \left[ \left(\rho^2 - \frac{1}{\rho^2}\right) dt^2 + \left(\rho^2 - \frac{1}{\rho^2}\right)^{-1} d\rho^2 + \rho^2 \sum_{i=1}^{3} dx_i^2 + d\Omega_5^2 \right]. \tag{14}$$

Requiring smoothness at the horizon $\rho = 1$, the period of time $t$ is fixed as follows.

$$0 \le t < \pi. \tag{15}$$

On this manifold, The quark mass $M_q$ and baryon mass $M_B$ are

$$\beta M_q = \frac{1}{2\pi l_s^2} \int_0^\pi dt \int_1^{\rho_{\rm probe}} d\rho \sqrt{g_{tt} g_{\rho\rho}} = \frac{1}{2\pi l_s^2} r_+^2 \pi (\rho_{\rm probe} - 1) = \sqrt{\frac{\eta}{2}} (\rho_{\rm probe} - 1), \tag{16}$$

$$\beta M_B = \frac{1}{(2\pi)^5 l_s^6 g_{\rm str}} \times \pi^3 r_0^5 \times \pi r_0 \sqrt{\rho^2 - \frac{1}{\rho^2}} = \frac{N}{4} \sqrt{\frac{\eta}{2}} \sqrt{\rho^2 - \frac{1}{\rho^2}}. \tag{17}$$

(Inverse temperature $\beta$ and masses $M_q$ and $M_B$ should be defined by using metric in the asymptotic flat region at infinity. However, their products $\beta M_q$ and $\beta M_B$ are dimensionless and they does not depend upon the metric.) And a ratio of baryon mass and quark mass is

$$\frac{M_B}{M_q} = \frac{\sqrt{\rho^2 - \frac{1}{\rho^2}}}{4(\rho - 1)} N. \tag{18}$$

In the low temperature limit $\rho \to \infty$ this expression reduced to $M_B/M_q = N/4$, which coincides to the result of the previous section. The ratio (18) is monotonously decreasing function of $\rho$ and cross the line $M_B/M_q = N$ at $\rho \sim 1.23$ (Fig.4). This means a phase transition occurs at a critical temperature $T_c$ determined by

$$\sqrt{\frac{2}{\eta}} \frac{M_q}{T_c} \sim 0.23. \tag{19}$$

In the high temperature side $T > T_c$, baryons are unstable and decay into $N$ quarks.

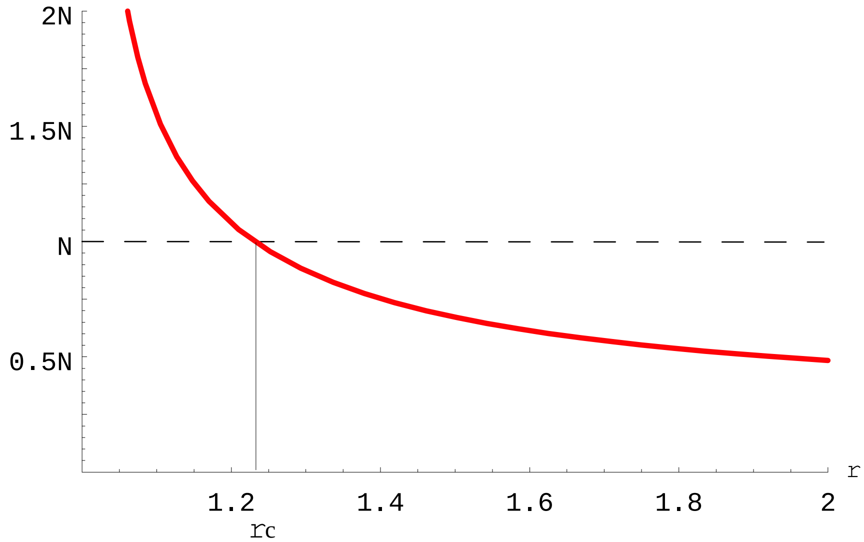


Figure 4: $\rho$-dependence of the ratio $M_B/M_q$.

## 5 Theta angle

Until now, we have been discussing only on quarks and their bound states, baryons. In $\mathcal{N}=4$ supersymmetric Yang-Mills theory, in addition to quarks, $(p,q)$-dyons exists and we can consider their bound states, $(p,q)$-baryons. ($p$ and $q$ are relatively prime integer.) In terms of brane configurations, they are represented by $(p,q)$ strings stretched between the horizon and the probe and $(p,q)$ five-branes wrapped around $S^5$, respectively. Their tensions are proportional to $|p+q\tau|$ ,where $\tau=\theta/(2\pi)+i/g_{\rm str}$, and masses of $(p,q)$-dyons and $(p,q)$-baryons are given by

$$M_{(p,q)-{\rm dyon}}=|p+q\tau|M_q,\quad M_{(p,q)-{\rm baryon}}=|p+q\tau|M_B. \tag{20}$$

They satisfy a relation

$$M_{(p,q)-{\rm baryon}}=\frac{N}{4}M_{(p,q)-{\rm dyon}}. \tag{21}$$

Because the relation (21) is identical to that between the quark mass and the usual baryon mass, it might seems that the story in the section 2 is the case for $(p,q)$-dyons and $(p,q)$-baryons also. However, it is not true.

For example, let us consider the case of $(N,1)$-dyon. It can be regarded as bound state of one monopole and $N$ quarks, and the $N$ quarks can be bound into a baryon. If baryon are created, total energy of the state is

$$\frac{N}{4}M_q=M_{\rm monopole}. \tag{22}$$

If $N$ is large enough, this energy is smaller than original dyon mass $[(NM_q)^2 + M_{\text{monopole}}]^{1/2}$. Therefore, $(N,1)$-dyons are unstable and decay into one monopole and one baryon.

If $\theta$ angle are turned on, similar phenomena get happen for dyons with small (namely, not order $N$) charges. By means of S-duality transformation

$$\begin{pmatrix} 1 & -N \\ 0 & 1 \end{pmatrix} \in SL(2,\mathbf{Z}), \tag{23}$$

$(N,1)$-dyons in vanishing $\theta$ background transformed into monopoles with $\theta = -2\pi N$ and these monopoles are also unstable. Because monopoles are stable in $\theta = 0$ background, it is expected that a phase transition occurs at cirtain critical theta angle $-\theta_c$ between 0 and $-2\pi N$. In the same way, the existence of critical value $+\theta_c$ between 0 and $2\pi N$ is shown. It is determined as a point on which mass of one monopole and total mass of the state containing one baryon and one $(-N,1)$-dyon.

$$\left|\frac{i}{\eta} + \frac{\theta_c}{2\pi N}\right| = \frac{1}{4} + \left|\frac{i}{\eta} + \frac{\theta_c}{2\pi N} - 1\right|. \tag{24}$$

If the absolute value of theta angle get larger than $\theta_c$, a monopole get unstable and decay into a $(-N,1)$-dyon and a baryon.

Here, we give the critical theta angle on which monopoles get unstable. The critical points for other dyons are also given in the similar way.

# 6 Conclusion

We calculate mass of baryon configurations in $AdS_5 \times S^5$ spacetime and obtain the value smaller than constituent quark mass. This means quarks are bound into baryons. The binding energy per one quark is of order of quark mass. In the Higgs phase, where gauge symmetry is broken to $SU(N_1) \times SU(N_2)$, W-bosons in $(N_1, \overline{N}_2)$ representation are regarded as combination of $SU(N_1)$ quarks and $SU(N_2)$ antiquarks and they are also create bound states. Then we calculate baryon mass at finite temperature and we find that at a critical temperature, which is same order with quark mass, baryon decay into free quarks. Finally, we show that stability of dyons depends upon $\theta$-angle. As a example, we show that monopoles are stable if absolute value of $\theta$ is smaller than $\theta_c$, which is determined by eq.(24). If $|\theta| > \theta_c$, a monopoles decays into one baryon and one $(-N,1)$-dyon.

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**Note Added:** After this work was completed, a paper [11] appeared which discusses the same issue.

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