On the Effectiveness of Dynamic Spectrum Management Algorithms in xDSL Networks

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Abstract— **This work proposes a method for the evaluation of the effectiveness of adopting dynamic spectral management (DSM) algorithms in different DSL scenarios. In the last years several DSM algorithms emerged in the literature but their comparison has been typically conducted within few scenarios and considering specific operating points. This work proposes the adoption of the DSM effectiveness factor (DEF) as a figure of merit capable of comparing the volumes of the whole rate regions, which expresses the set of operating points in the Pareto front. A random scenario generator was used to obtain four hundred DSL scenarios and compared flat power back-off (PBO) and DSM algorithms of levels 1 and 2. Besides confirming well-known facts, such that the effectiveness of DSM is significant in near-far scenarios, the results based on the proposed DEF allow to quantify the gains in bit rate that DSM can bring.**

Keywords— Dynamic spectral management, digital subscriber line, discrete multitone modulation.

I. INTRODUCTION

Dynamic Spectrum Management (DSM) algorithms are established techniques known to mitigate crosstalk and improve performance of digital subscriber line (DSL) systems.

The DSM algorithms are usually divided into three different levels of coordination: DSM level 1, 2 and 3. For each level, there is a different set of related algorithms, each one with specific requirements. For the DSM level 3 (the highest level of coordination), there is the *crosstalk cancellation* set, also called vectoring DSM [1], which aims to provide a near crosstalk-free transmission for DSL systems. These techniques have a relatively high computational cost and coordination requirements, and are out of the scope of this work. For DSM level 1 (autonomous mode of operation) and level 2 (multiuser spectrum coordination), there is the *spectrum balancing* (SB) set, which consists of algorithms based on multiuser cooperation that dynamically adapt the transmit signals according to variations in channel conditions [2]–[9].

There is a difficulty in comparing and measuring the improvements gained with the use of DSM algorithms. Sometimes, DSM algorithms are told to always boost the DSL system's performance. What they actually do is to reduce the crosstalk effects. When those reductions are significant, high improvements can be achieved. But, in scenarios with high direct gains and/or low crosstalk, the use of DSM may not be beneficial enough to justify its practical implementation. There is also the question about how much one would gain by adopting DSM level 2 instead of DSM level 1 algorithms. In summary, it is not clear throughout the literature which kind of DSL topologies would benefit the most from DSM algorithms.

The DSM performance is usually presented using the near-far scenarios (*e.g.* VDSL upstream or ADSL/2/+ downstream with a remote terminal). In such scenarios, the DSM performance is known to be very high, but they do not cover the entire DSL topologies faced by DSL carriers. The objective of this work is to develop a method to evaluate the effectiveness of adopting dynamic spectral management (DSM) algorithms in different DSL scenarios. This is achieved through the adoption of the proposed *DSM effectiveness factor* (DEF) as a figure of merit.

Section II brings a brief description of the DSL system model. The proposed DSM effectiveness factor is explained in Section III. The description of the random scenario generator and numerical results can be found in Section IV. Finally, the conclusions are presented in Section V.

II. THE DSL SYSTEM MODEL

Most DSL standards adopt discrete multi-tone (DMT) modulation, which is a technique that divides the available spectrum in K parallel sub-channels (or tones). Assuming independent transmission on different tones (no intersymbol or intercarrier interferences) and considering a N-lines interference channel, where each line treats interference from the others as noise, the achievable transmit bitloading of modem n on tone k can be written as [10]

$$b_n^k = \log_2\left(1 + \frac{1}{\Gamma\gamma_n}\frac{\left|h_{n,n}^k\right|^2 p_n^k}{\sum_{m\neq n}\left|h_{m,n}^k\right|^2 p_m^k + \sigma_n^k}\right), \qquad (1)$$

where

- $|h_{n,n}^k|^2$ is the the square-magnitude of the direct channel gain for user n at tone k;
- $|h_{m,n}^k|^2$ denotes the square-magnitude of the far-end crosstalk channel from transmitter m to receiver n at tone k;

- p_m^k denotes the power transmitted by user m at tone k;
- σ_n^k represents the background noise power on tone k at receiver n;
- Γ is the signal-to-noise (SNR) ratio gap, which is a function of the desired bit error rate;
- γ_n is the noise margin of user n.

The total used power P_n^{tot} and the total data rate R_n can be calculated as:

$$P_n^{\text{tot}} = \sum_{k=1}^{K} p_n^k \qquad R_n = f_s \sum_{k=1}^{K} b_n^k, \tag{2}$$

where f_s is the DMT symbol rate [10].

In order to simplify the notation, we define the $N \times K$ power matrix

$$\mathbf{P} = \begin{bmatrix} p_1^1 & \cdots & p_1^K \\ \vdots & \ddots & \vdots \\ p_N^1 & \cdots & p_N^K \end{bmatrix} \tag{3}$$

that puts together in a single variable the information about the transmitted power of all N users on all K tones.

III. THE DSM EFFECTIVENESS FACTOR

The spectrum balancing formulations used by DSM algorithms are usually posed as an single-objective problem, *e.g.* maximize a weighted-sum of the bit rates of a set of users. But those problems are multiobjective in nature, as one could enforce more a weight for one user (leading to higher rates for it) than for another (which results in performance reduction for this last one). Then, in practice, there is no unique optimal solution, but a set of non-dominated optimal solutions, often called as *Pareto* solutions [11].

In this section we describe a new figure of merit to assess the performance improvements possible by the DSM algorithms. This metric is called *DSM effectiveness factor* (DEF), which is based on computational geometry [12] and can be considered a rough estimate of the gains obtained by the DSM algorithms. This figure of merit takes in account the multiobjective aspects of the spectrum balancing optimization.

In order to measure how "effective" a DSM solution can be, we analyze the geometry of the *Pareto* solutions. As in this work we focus only on rate maximization mode of DSM algorithms, we use computational geometry [12] to calculate the volume of rate regions, which characterize all Pareto optimal data rate combinations among users.

Assume that a given rate optimization method (DSM level 1, level 2, etc.) provides Z operating points that describes the achievable rates of N users. These Z points are the vertices of a N-dimensional (typically concave) *polytope*, which has (N-1)-dimensional *facets*. These facets are themselves polytopes, whose facets are (N-2)-dimensional *ridges* (also called *subfacets*) of the original polytope. Ridges are once again polytopes whose facets give rise to (N-3)-dimensional boundaries of the original polytope, and so on. For example, when $N = 2$, a facet is an edge, i.e., a line segment while when $N = 3$ a facet is a polygon (see Figure 1). The intuition is that the larger the volume of the rate region the better, giving the more flexibility the DSL network operator has for setting the operating points.

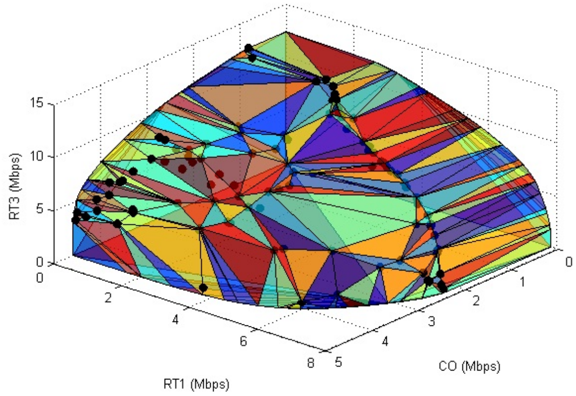

Fig. 1. Example of rate region for 3 users denoted as CO, RT1 and RT3, with rates in Mbps. Each circle represents an operating point and the facets were obtained with computational geometry.

To obtain the rate region volume $V_{\text{algorithm}}$ of a given algorithm, before calculating its convex hull, $N+1$ shaping points are always added to the Z points generated by the algorithm. The first point is located at the origin $(0, 0, \ldots, 0)$ and the others have only the i-th element different from zero, $i = 1, \ldots, L$. For example, when solving, the shaping point $(r_1^{\max}, 0, 0, \ldots, 0)$ is included, where $r_1^{\max}$ is the rate obtained by user 1 when all other users are inactive. The rate $r_2^{\max}$ of the second user imposes another shaping point $(0, r_2^{\max}, 0, \ldots, 0)$ and so on.

Besides $V_{\text{algorithm}}$, it is useful to calculate the volume $V_{\text{no-xtalk}}$ of the rate region corresponding to the ideal operating points obtained with the assumption of no crosstalk among the users[1]. This rate regions consists of a N-dimensional cuboid formed by 2^N points representing the combinations of the maximum rates. For example, when $N = 3$, these points are $(0, 0, 0), (r_1^{\max}, 0, 0), \ldots, (r_1^{\max}, r_2^{\max}, r_3^{\max})$.

The DEF is then calculated as follows

$$\text{DEF} = \left(\frac{V_{\text{algorithm}}}{V_{\text{no-xtalk}}} \right)^{1/N},$$

with the two volumes obtained by an algorithm such as Quickhull [13]. Because no user can operate at a rate larger than the one that can be obtained without crosstalk, the DEF is limited to the range $[0, 1]$.

Note that if a rate region has a volume V, the rate $R = V^{1/N}$ generates a corresponding cuboid with all edges having the same length or, equivalently, a DSL system will all users operating at the same rate. Hence, if one represents each rate region with cuboids of the same volume, the DEF

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[1] This upper bound is used for simplicity. A tighter upper bound could be obtained using the theoretical capacity of the channel with crosstalk.

can be written as

$$\text{DEF} = \frac{R_{\text{algorithm}}}{R_{\text{no-xtalk}}},$$

where $R_{\text{algorithm}}$ and $R_{\text{no-xtalk}}$ are the corresponding rates.

To illustrate the metric, Fig. 2 shows an simple example of the performance of a hypothetical DSM algorithm in a 2 users scenario in the multiobjective sense. To calculate the DEF, we first calculate the area of the rate region of the algorithm and divide it by the *no crosstalk* area, which denotes the highest performance case, and then take the square root, as follows

$$\text{DEF} = \left(\frac{0.5}{1}\right)^{1/2} = \frac{\sqrt{2}}{2}.$$

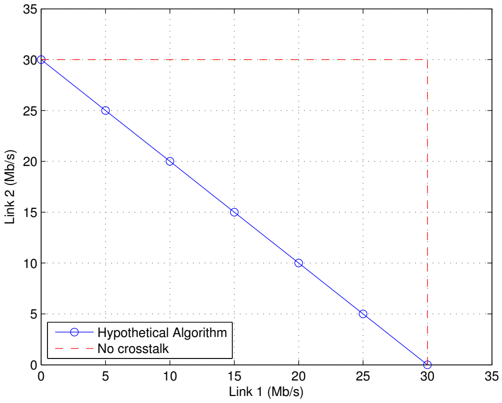


Fig. 2. Example of rate region areas for the 2-users case.

Fig. 3 shows another example of the rate regions of DSM level 1 and 2 algorithms, Flat PBO and the *no crosstalk* case in a 2 users scenario. In this example the DEFs found for each algorithm were: $\text{DEF}_{\text{DSM2}} = 0.9862$, $\text{DEF}_{\text{DSM1}} = 0.9104$, and $\text{DEF}_{\text{flatPBO}} = 0.8117$.

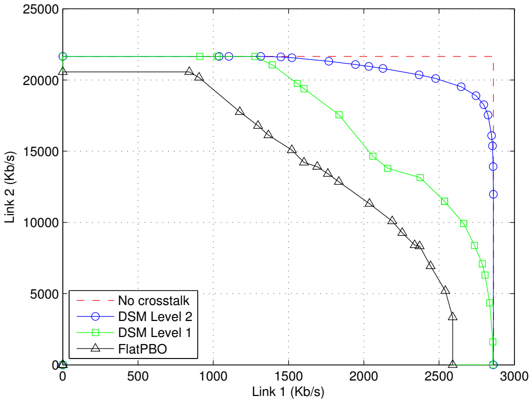


Fig. 3. Rate region area.

IV. SIMULATION RESULTS

This section presents the results of simulations for different scenarios generated based on the topologies of Fig. 4.

It uses as reference the ideal case (with no alien crosstalk), and, as base line, the power backoff methods, which are simple procedures of power control currently used by DSL systems.

A. The Random Scenario Generator

The Random Scenario Generator was developed to allow the generation of different scenarios using the ADSL2+ technology, in order to evaluate the performance of the DSM algorithms. Two types of topologies were considered in this work. The topology 1 consist of a CO (Central Office) with 2 users. The topology 2 consist of a CO, a RT (Remote Terminal) and 2 users. Both topologies are depicted in Fig. 4. The choice of a 2-user scenario is justified by the possibility of better visualization of the rate region, since with 2 users the rate region is a 2-dimensional plot. However, the random generator can handle a larger number of DSL users.

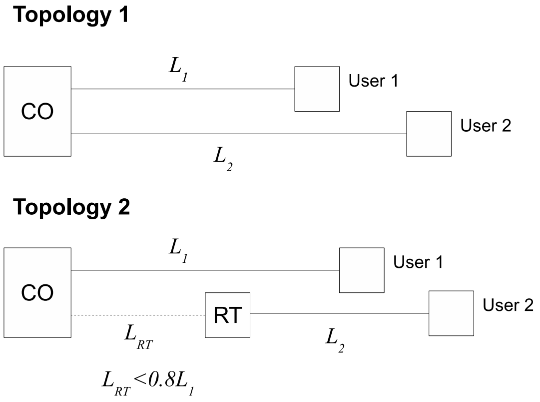


Fig. 4. Topologies 1 and 2.

In the topology 1, the lines lengths L_1 and L_2 are chosen randomly. The length of each line is a random variable uniformly distributed over the interval [0.2 km, 6 km] (we assume 0.2 km as the minimum distance from the CO to the user and 6 km as the maximum distance, since it is the maximum distance that the ADSL2+ standard allows [14]).

In the topology 2, the length of the line without the RT (L_1) is chosen in the same way as the lines in topology 1. However, for the line with the RT, we assume that the length L_{RT} from the CO to the RT must be uniformly distributed between zero and $0,8L_1$ (the term 0.8 is to ensure that RT is not positioned after *User 1*). The length L_2 is also uniformly distributed in the same way L_1 is.

B. Numerical Results

This section shows the calculated DSM effectiveness factors (described in Section III) for randomly generated scenarios for the topologies 1 and 2 indicated in Fig. 4.

Two hundred scenarios were generated for each topology described in Section IV-A. The SCAWF [6] was used to represent the results achieved by DSM level 1 algorithms, while SCALE [6] (proposed in the same work) was used as the DSM level 2 algorithm.

Fig. 5 and Fig. 7 show the DEF for topologies 1 and 2, respectively, calculated for the 200 randomly generated scenarios (for each topology type). The results were ordered for better visualization. In Fig. 6 and Fig. 8, the same results for topologies 1 and 2 are shown in the form of histograms, respectively.

In Fig. 5 and Fig. 6, it can be observed that the algorithms of DSM level 1 and 2 guarantee efficiency of over 80% for all the generated scenarios, while the Flat PBO ensures a DEF of over 80% in only 60% of the scenarios. It also can be observed that for most of the scenarios, DSM levels 1 and 2 obtain similar results.

For topology 2, shown in Fig. 7 and Fig. 8, the difference between the performance of DSM level 1 and 2 is larger, suggesting that level 2 algorithms are more useful in scenarios with remote terminals.

The average DEF values obtained for both topologies are shown in Table I. It can be noticed that topology 2 using DSM level 2 achieves the same DEF as topology 1 when using DSM level 1. This is expected as scenarios with RTs usually have higher crosstalk levels. Also, for both topologies, there is a considerable improvement on the average performance compared to the case where only Flat PBO is used.

TABLE I
DEF AVERAGE FOR TOPOLOGIES 1 AND 2.

Algorithms	Topology 1	Topology 2
DSM level 2 (Scale)	0.9631	0.9343
DSM level 1 (Scawf)	0.9363	0.8823
Flat PBO	0.7878	0.7713

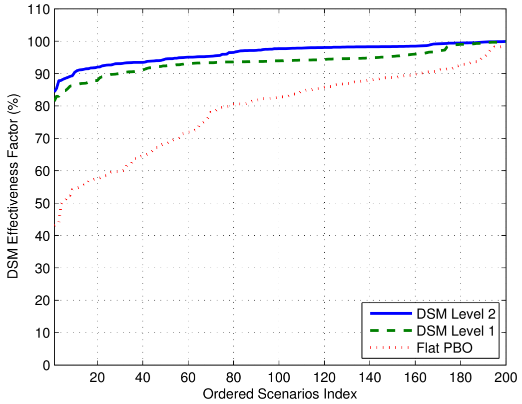


Fig. 5. DEF for topology 1, considering the 2-users case.

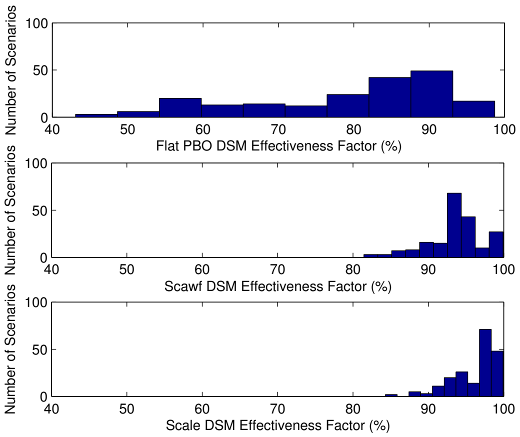


Fig. 6. Histogram of DEF for topology 1, considering the 2-users case.

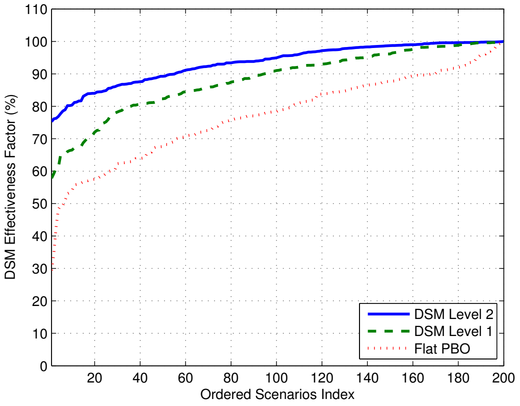


Fig. 7. DEF for topology 2, considering the 2-users case.

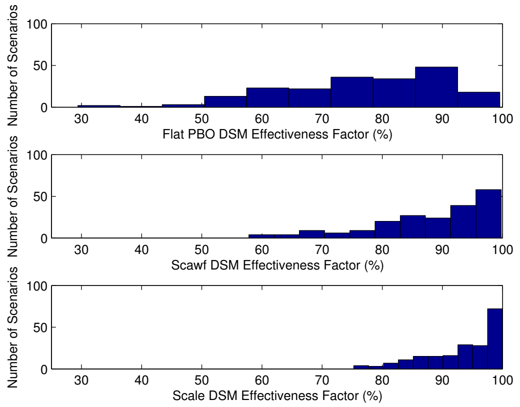
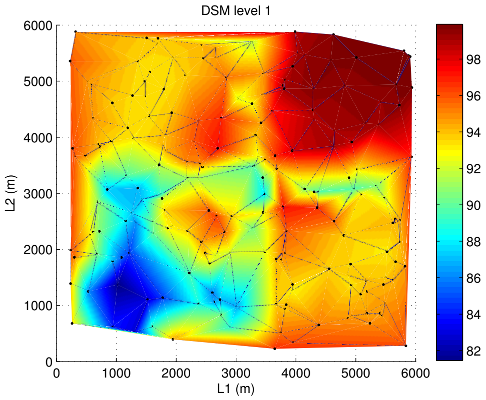
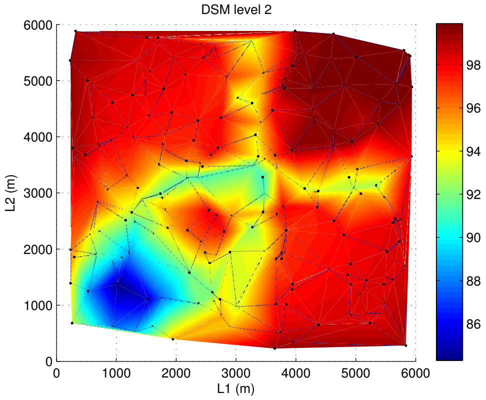


Fig. 8. Histogram of DEF for topology 2, considering the 2-users case.

In Fig. 9, it is illustrated how the DEF is distributed according to the lines' lengths for topology 1, considering DSM level 1 in Fig. 9(a), and level 2 in Fig. 9(b), algorithms. For both algorithms, the higher DEFs (and, therefore, higher gains in using DSM) are achieved when at least one of the lines is longer than about 3500 meters. When both lines are short, the benefits of using DSM algorithms are lower.



(a) DSM level 1.



(b) DSM level 2.

Fig. 9. DEF for topology 1, considering the 2-users scenarios. The DEF values are indicated by the colors in the colorbar. The dots indicate the DEF values obtained during the simulations.

V. CONCLUSIONS

The paper proposed a new figure of merit to evaluate the effectiveness of DSM algorithms. Such metric was used in many different DSL scenarios in order to investigate common patterns of scenarios where DSM is more effective. The results have shown that DSM level 2 algorithms are better suited for near-far scenarios, while DSM level 1 algorithm shows satisfactory performance for ordinary topologies (i.e, without remote terminal).

For some specific scenarios, the difference in the calculated DEF was as high as 50% for both topologies. This shows that the use of DSM algorithms can bring significant benefits, but the improvements can vary a lot depending on the scenario. It was also observed that DSM level 2 always performed better than DSM level 1 algorithms, but not by a large margin in most scenarios.

As the DSM level 2 algorithms always present better performance, one could ask: why do not always use DSM level 2? The answer for that relies on the fact that the prerequisites to perform DSM level 2 are higher, requiring, *e.g.* a spectrum management center (SMC) [8] and complete knowledge of the direct and crosstalk transfer functions. For DSM level 1, which is an autonomous mode of operation, only the direct channel and measurements of the background noise are required, hence they are much easier to implement in practice. Then, if both DSM levels 1 and 2 perform similarly, for both practical and economical reasons, the DSM level 1 is more recommended to be used. The proposed DEF allows to quantify these facts and can be adopted in many distinct applications of DSM and related methods.

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