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Application of State Space Averaging Method to Sliding Mode Control of PWM DC/DC Converters

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Abstract - **A novel approach for the analysis and design of sliding mode controllers for PWM DC/DC converters is presented. The main advantage of this non-linear controller is that there is no restriction on the size of the signal variations around the operating point. Small as well as large signal variations around the operating point are considered. Sliding mode controllers for buck, boost, buck-boost, and Cuk converters have been designed and discussed. These controllers have been simulated on a digital computer and their dynamic performances have been shown to be satisfactory. Finally, Lyapunov's second theorem has been used to verify the stability of the designed sliding mode controller for the Cuk converter.**

Keywords: DC/DC converters, buck converters, boost converters, buck-boost converters, Cuk converters, state space averaging, sliding-mode controller.

I. INTRODUCTION

In general, power electronic DC/DC converters are periodic time-variant systems due to their inherent switching operation. Static and dynamic characteristics of these converters have been widely discussed in the literature [1-7]. Classical linear control methods are often used to design regulators for DC/DC converters, and to determine their stability limits around their operating points. However, in order to ensure their large signal stability, and also to improve their large signal dynamic response, it was proposed to use sliding mode control [8-12]. In this paper, instead of using full order state feedback for sliding mode controller, state space averaging models of the converters have been used. It is shown that the use of proposed method will result in a simplified controller. Unlike variable frequency sliding mode controller used in the literature [13], fixed frequency switching PWM technique is used. This simplifies the converter filter design and minimizes converter filters.

In this paper, after representing the state space averaging model [1] for the Buck, Boost, Buck-Boost, and Cuk converters, sliding mode controllers have been designed and the entire closed loop system for every converter has been simulated. Dynamic performances of these converters have been studied. Finally, a simplified controller for the Cuk converter has been introduced and the stability of the system has been verified using the second theorem of Lyapunov.

II. DC/DC CONVERTERS MODELING USING STATE SPACE AVERAGING METHOD

In this section state space averaging method will be used to model the DC/DC converters.

A. *Buck Converter*

The Buck converter of Fig. 1 which is operating with the switching period of T and duty cycle d is considered.

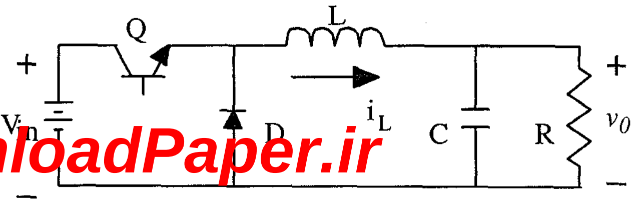

Fig.1 DC/DC Buck converter.

During continuous conduction mode of operation, the state space equations when the switch is ON are given by,

$$\begin{cases} \dfrac{di_L}{dt} = \dfrac{1}{L}(V_{in} - v_0) \\ \dfrac{dv_0}{dt} = \dfrac{1}{C}\left(i_L - \dfrac{v_0}{R}\right) \end{cases} , \quad 0 < t < dT \quad , \quad Q : ON \qquad (1\text{-a})$$

and when the switch is OFF are presented by,

$$\begin{cases} \dfrac{di_L}{dt} = \dfrac{1}{L}(-v_0) \\ \dfrac{dv_0}{dt} = \dfrac{1}{C}\left(i_L - \dfrac{v_0}{R}\right) \end{cases} , \quad dT < t < T \quad , \quad Q : OFF \qquad (1\text{-b})$$

Using the state space averaging method these sets of equations can be shown by,

$$\begin{cases} \dot{x}_1 = -\dfrac{1}{L}x_2 + \dfrac{d}{L}V_{in} \\ \dot{x}_2 = \dfrac{1}{C}x_1 - \dfrac{1}{RC}x_2 \end{cases} \qquad (2)$$

where x_1 and x_2 are the moving averages of i_L and v_0 respectively.

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B. Boost Converter

The Boost converter of Fig. 2 with a switching period of T and a duty cycle of d is given.

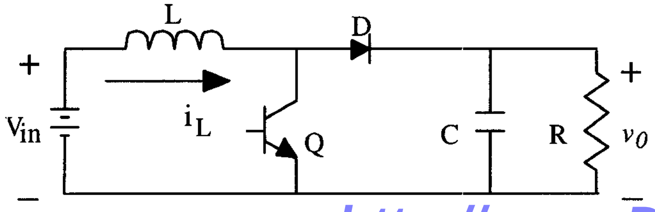

Fig.2 DC/DC boost converter.

Again, assuming continuous conduction mode of operation, the state space equations when the main switch is ON is shown by,

$$\begin{cases} \dfrac{di_L}{dt} = \dfrac{1}{L}(V_{in}) \\ \dfrac{dv_0}{dt} = \dfrac{1}{C}\left(-\dfrac{v_0}{R}\right) \end{cases} \quad , \; 0 < t < dT \quad , \; Q : ON \qquad (3\text{-a})$$

In case the switch is OFF, the state space equations are given by,

$$\begin{cases} \dfrac{di_L}{dt} = \dfrac{1}{L}(V_{in} - v_0) \\ \dfrac{dv_0}{dt} = \dfrac{1}{C}\left(i_L - \dfrac{v_0}{R}\right) \end{cases} \quad , \; dT < t < T \quad , \; Q : OFF \qquad (3\text{-b})$$

Similar to the previous case, the state space averaging model will result in the following equations:

$$\begin{cases} \dot{x}_1 = -\dfrac{1-d}{L}x_2 + \dfrac{1}{L}V_{in} \\ \dot{x}_2 = \dfrac{1-d}{C}x_1 - \dfrac{1}{RC}x_2 \end{cases} \qquad (4)$$

where x_1 and x_2 are the moving averages of i_L and v_0 respectively.

C. Buck-Boost Converter

In Fig. 3 a DC/DC Buck-Boost converter is shown. The switching period is T and the duty cycle is d.

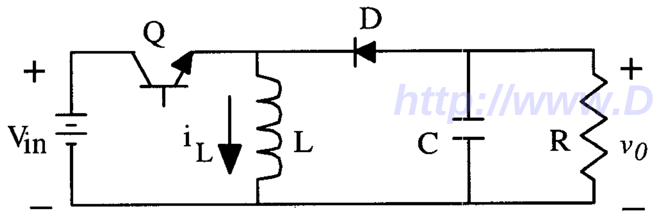

Fig.3 DC/DC Buck-Boost converter.

Assuming continuous conduction mode of operation, when the switch is ON, the state space equations are given by,

$$\begin{cases} \dfrac{di_L}{dt} = \dfrac{1}{L}(V_{in}) \\ \dfrac{dv_0}{dt} = \dfrac{1}{C}\left(-\dfrac{v_0}{R}\right) \end{cases} \quad , \; 0 < t < dT \quad , \; Q : ON \qquad (5\text{-a})$$

while the switch is OFF the state space equations will be presented by,

$$\begin{cases} \dfrac{di_L}{dt} = \dfrac{1}{L}(v_0) \\ \dfrac{dv_0}{dt} = \dfrac{1}{C}\left(-i_L - \dfrac{v_0}{R}\right) \end{cases} \quad , \; dT < t < T \; , \; Q : OFF \qquad (5\text{-b})$$

Application of the state space averaging method to the above equations yields

$$\begin{cases} \dot{x}_1 = \dfrac{1-d}{L}x_2 + \dfrac{d}{L}V_{in} \\ \dot{x}_2 = -\dfrac{1-d}{C}x_1 - \dfrac{1}{RC}x_2 \end{cases} \qquad (6)$$

where similar to the previous cases, x_1 and x_2 are the moving averages of i_L and v_0 respectively.

D. Cuk Converter

The Cuk converter of Fig. 4 with switching period of T and duty cycle of d is considered.

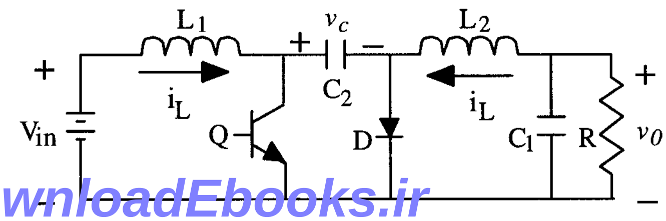

Fig. 4 DC/DC Cuk converter.

During the continuous conduction mode of operation, the state space equations are as follows:

$$\begin{cases} \dfrac{di_{L_1}}{dt} = \dfrac{1}{L_1}(V_{in}) \\ \dfrac{dv_c}{dt} = \dfrac{1}{C_2}(-i_{L_2}) \\ \dfrac{di_{L_2}}{dt} = \dfrac{1}{L_2}(-v_0 + v_c) \\ \dfrac{dv_0}{dt} = \dfrac{1}{C_1}\left(i_{L_2} - \dfrac{v_0}{R}\right) \end{cases} \quad , \; 0 < t < dT \; , \; Q : ON \qquad (7\text{-a})$$

when the switch is OFF the state space equations are represented by

$$\begin{cases} \dfrac{di_{L_1}}{dt} = \dfrac{1}{L_1}\left(V_{in} - v_0\right) \\ \dfrac{dv_c}{dt} = \dfrac{1}{C_2}\left(i_{L_1}\right) \\ \dfrac{di_{L_2}}{dt} = \dfrac{1}{L_2}\left(-v_0\right) \\ \dfrac{dv_0}{dt} = \dfrac{1}{C_1}\left(i_{L_2} - \dfrac{v_0}{R}\right) \end{cases} , dT < t < T \ , \ Q \ : OFF \quad (7\text{-b})$$

Using state space averaging method to the above state space equations results in

$$\begin{cases} \dot{x}_1 = -\dfrac{1-d}{L_1} x_2 + \dfrac{1}{L_1} V_{in} \\ \dot{x}_2 = \dfrac{1-d}{C_2} x_1 - \dfrac{d}{C_2} x_3 \\ \dot{x}_3 = \dfrac{d}{L_2} x_2 - \dfrac{1}{L_2} x_4 \\ \dot{x}_4 = \dfrac{1}{C_1} x_1 - \dfrac{1}{RC_1} x_4 \end{cases} \quad (8)$$

where x_1, x_2, x_3, and x_4 are the moving averages of i_{L1}, v_c, i_{L2}, and v_0 respectively.

III. SLIDING MODE CONTROL

Generally, in the DC/DC converters the output voltages need to be regulated. In this paper, sliding mode controller is used to control the converters. To design this type of controllers, the moving averages of their output voltages are used. This will significantly simplify the design. In the Buck, Boost, Buck-Boost DC/DC converters discussed earlier, x_2 is the moving average of the output voltage, and K is the desired output voltage. The sliding surface in the state space is described by the $x_2=K$, and according to the sliding-mode control [8-14] is shown by,

$$\begin{aligned} \dot{x}_2 < 0 \quad if \quad x_2 > K \\ \dot{x}_2 > 0 \quad if \quad x_2 < K \end{aligned} \quad (9)$$

A first order path can be selected based on the following equation and the convergence speed is controlled accordingly,

$$\dot{x}_2 = -\lambda\left(x_2 - K\right) \quad (10)$$

where λ is a positive real number and is called the convergence factor. Figure 5 shows the convergence relation for control of DC/DC converters.

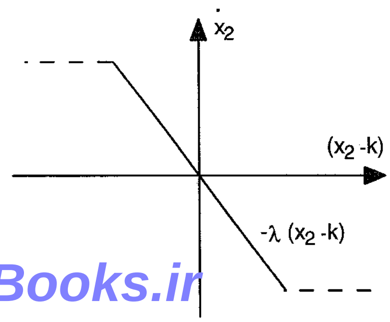

Fig. 5 Convergence relation for control of Buck, Boost, and Buck-Boost converters.

Based on (10), the larger the convergence factor the faster the system will reach its steady state. However, due to limits on the system parameters such as duty cycle, it is not possible to increase the convergence factor beyond a certain value.

In order to design the controller, it is necessary to combine (10) with (2), (4), and (6). This will result in an equation for duty cycle d in terms of the state variables and the system parameters. This equation is important, since it would control the output variable. A main contribution of this paper is the fact that the relationship which is obtained for the duty cycle d, has fewer state variables in it than previous work [13]. The less state variables appear in the duty cycle relation, the fewer feedbacks to the state variables will be needed.

A. *Buck Converter*

The duty cycle as a function of time for sliding mode controller can be obtained by inserting the convergence relation (10), in the state space representation of the Buck converter (2) which is shown by,

$$d(t) = \frac{K + a(x_2 - K)}{V_{in}} \quad (11)$$

where

$$a = LC\lambda^2 - \frac{L}{R}\lambda + 1 \quad (12)$$

At the steady state where the output voltage state variable, x_2, is following the commanded reference, K, the duty cycle is given by

$$d^* = \frac{K}{V_{in}} = \frac{V_0}{V_{in}} \quad (13)$$

where V_0 is constant and it is the steady state value of v_0. Fig. 6 illustrates the sliding-mode controller designed for the Buck converter.

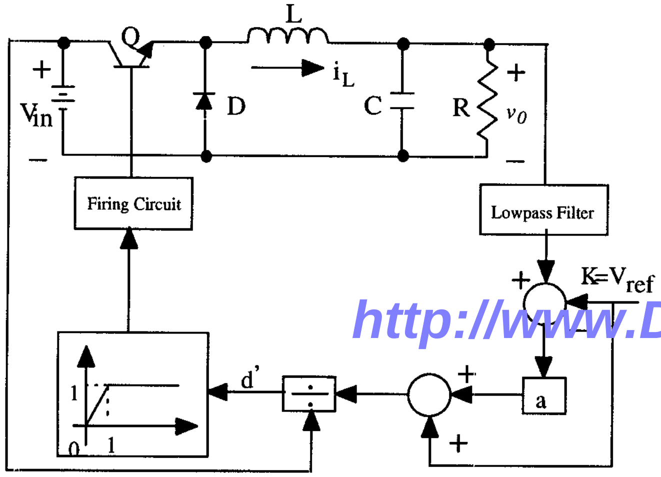

Fig. 6 The designed sliding-mode controller for Buck converter. (d' = 1-d)

A DC/DC Buck converter with the parameters given in Table 1 was designed. This converter was simulated on a digital computer using Matlab package.

Table 1 Buck converter parameters.

V_{in}	L	C	R	f
20 V	1 mH	10 μF	10 Ω	10 kHz

It was previously pointed out that the larger the convergence factor the sooner converter output voltage will follow its commanded value. However, there is an upper limit on the value of convergence factor which has been investigated in the literature. For the buck converter under study a convergence factor of 5000 is selected. Fig. 7 shows the start up of the Buck converter with the designed sliding-mode controller. It is clear that after 2 milliseconds the output is settled down.

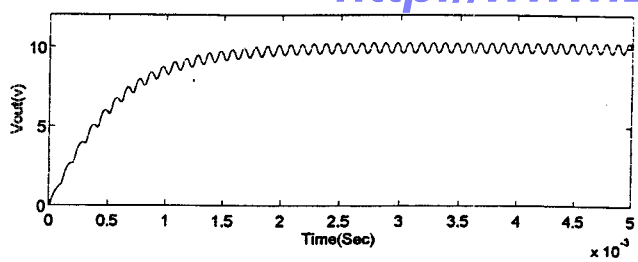

Fig. 7 Start-up performance of the Buck converter with the designed sliding-mode controller.

In order to study the converter dynamic performance under load variation, the load has been increased by 10 times in Fig. 8. Although, the output voltage rises up to 22V, but it is quickly dropped to its set value within 1 millisecond. A step voltage from 10 V to 13 V is illustrated in Fig. 9. Again, after 1 millisecond the output is settled down at its commanded value. Fig. 10 shows the output voltage variation to a step increase of 20% in the input voltage (from 20 V to 24). A satisfactory behavior in the output voltage average is depicted in this figure.

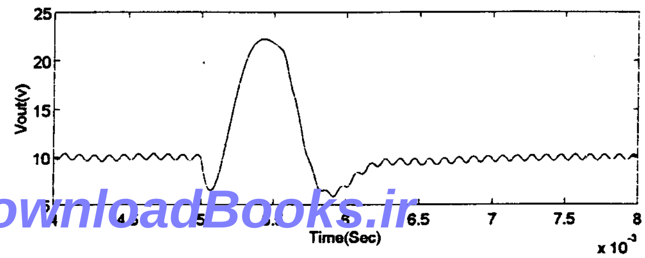

Fig. 8 Output voltage variation of the sliding-mode controlled Buck converter to a 10 times increase in the load.

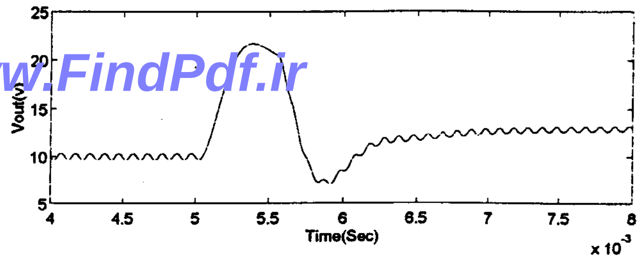

Fig. 9 Output voltage variation of the sliding-mode controlled Buck converter to a 30% increase in the output voltage command .

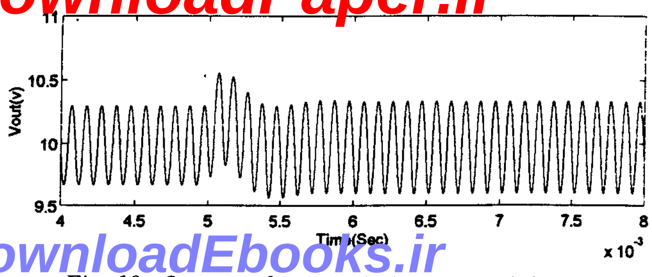

Fig. 10 Output voltage variation of the sliding-mode controlled Buck converter to a 20% increase in the input voltage.

B. *Boost Converter*

In a Boost converter inserting (10) in (4) results in the following equation for the dynamic duty cycle.

$$d(t) = 1 - \frac{V_{in} + \sqrt{V_{in}^2 + a x_2 (x_2 - K)}}{2x_2} \tag{14}$$

where

$$a = -4\lambda\left(LC\lambda - \frac{L}{R}\right) \tag{15}$$

Again, the duty cycle is a function of time. Using (14) to describe the duty cycle, the sliding-mode controlled Boost converter is presented in Fig. 11. This system is simulated using the Matlab package. Fig. 12 shows the output voltage variation to a step voltage change in the commanded output voltage reference. It is clear that after 1.5 milliseconds the output is settled down at its new value. A step increase in the input voltage of about 20% is depicted in Fig. 13. It is shown that the output voltage is reached its set value within 1 millisecond.

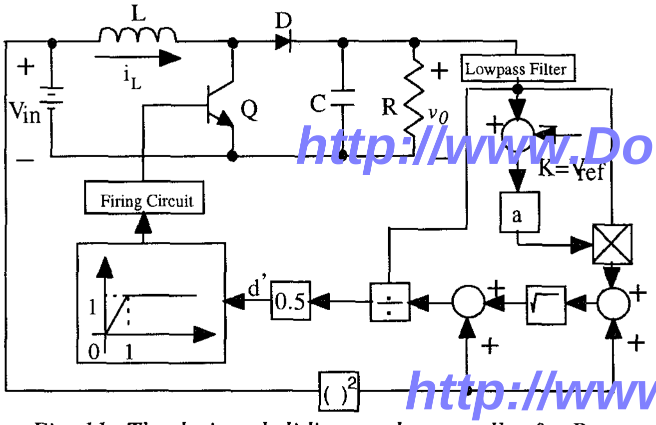

Fig. 11 The designed sliding-mode controller for Boost converter. (d'=1-d)

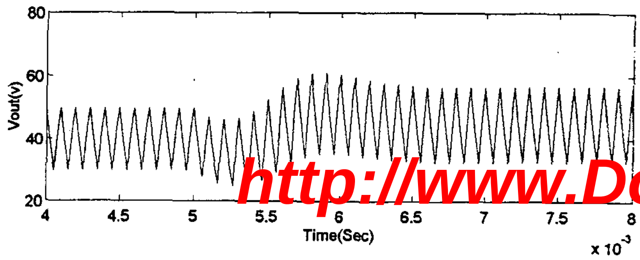

Fig. 12 Output voltage variation of the sliding-mode controlled Boost converter to a 5V increase in the output voltage command.

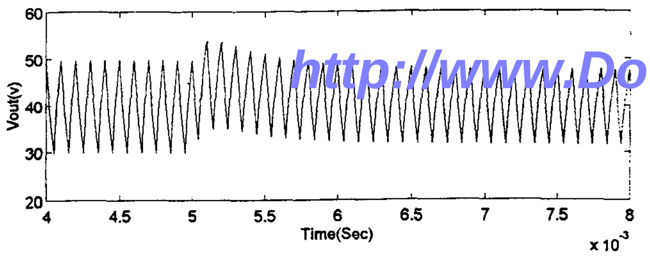

Fig. 13 Output voltage variation of the sliding-mode controlled Buck converter to a 20% increase in the input voltage.

C. *Buck-Boost Converter*

Substituting (10) in (6) results in the following relation for the dynamic duty cycle:

$$d(t)=1+\frac{V_{in}+\sqrt{V_{in}^2+a(x_2-V_{in})(x_2-K)}}{2(x_2-V_{in})} \qquad (16)$$

where

$$a=-4\lambda\left(LC\lambda-\frac{L}{R}\right) \qquad (17)$$

Fig. 14 illustrates the designed sliding-mode controlled Buck-Boost converter. Notice that there is no need for a feedback from the inductor current in this design.

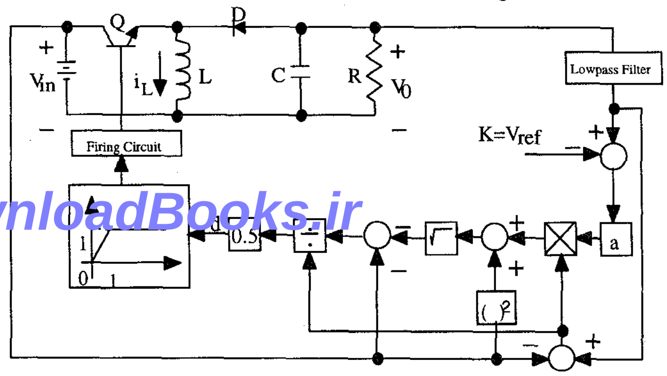

Fig. 14 The designed sliding-mode controller for Buck-Boost converter. (d'=1-d)

The designed sliding-mode Buck-Boost converter with the parameters presented in Table 1 was simulated on a digital computer. A convergence factor equal to 5000 was used. The reference output voltage was set at K=20 V. Fig. 15 shows the output variation to a large signal change in the load. The load has been increased by 10 times in this case. It is shown that, although the load variation is drastically large, the output voltage has been settled down to its prescribed value within 1 millisecond. A step decrease in the input voltage equal to 4 V is depicted in Fig. 16. Again, a satisfactorily performance was obtained.

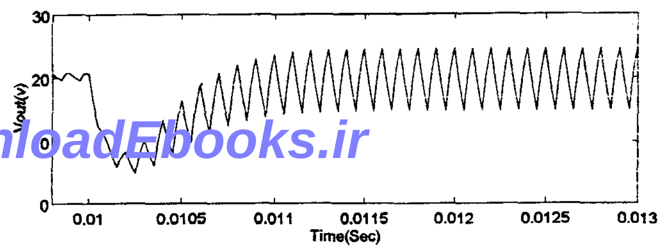

Fig. 15 Output voltage variation of the sliding-mode controlled Buck-Boost converter to a 10 times increase in the load.

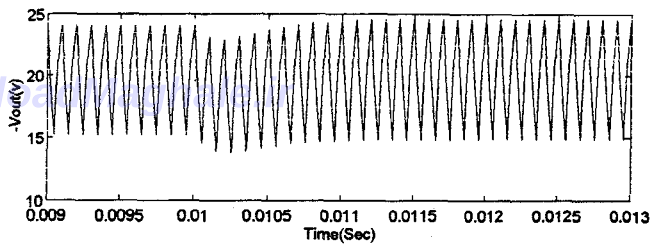

Fig. 16 Output voltage variation of the sliding-mode controlled Buck-Boost converter to a 4V decrease in the input voltage.

D. Cuk Converter

In a Cuk converter, the state variable x_4 is the moving average of the output voltage and K is the output voltage reference. The sliding surface in the state space is defined by $x_4=K$, and according to the sliding-mode controlled [8-14]

$$\dot{x}_4 < 0 \quad \text{if} \quad x_4 > K$$
$$\dot{x}_4 > 0 \quad \text{if} \quad x_4 < K \qquad (18)$$

A first order path for the sliding-mode controller which is given by (19) was selected. Again, λ is a positive real number and is called the convergence factor.

$$\dot{x}_4 = -\lambda\left(x_4 - K\right) \qquad (19)$$

Substituting (19) in (8) results in the following dynamic duty cycle:

$$d = \frac{V_{in} + 2K + a\left(x_4 - K\right) - g}{2\left[\left(V_{in} + K\right) + b\left(x_4 - K\right)\right]} \qquad (20)$$

where a, b, and g are presented in the Appendix. At the steady state where $x_4=K$, the duty cycle is presented by

$$d^* = \frac{K}{V_{in} + K} \qquad (21)$$

and the input-output relation for the Cuk converter at the steady state is given by,

$$\frac{V_0}{V_{in}} = \frac{d^*}{1 - d^*} \qquad (22)$$

Fig. 17 shows the sliding-mode controlled Cuk converter which results from (20).

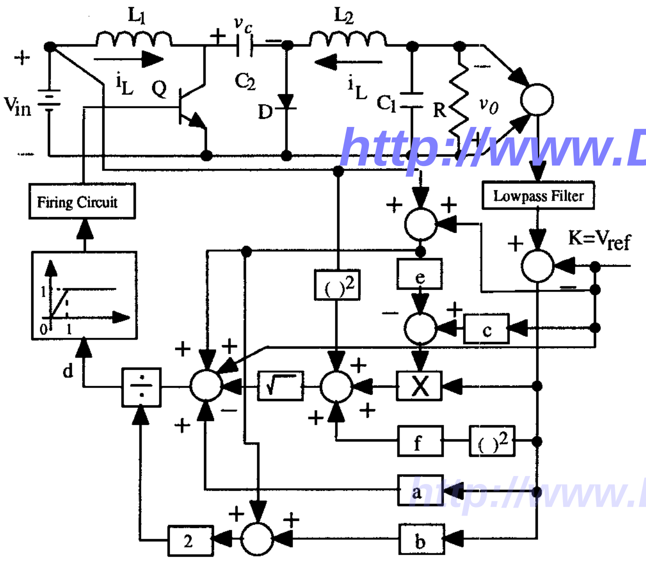

Fig. 17 The designed sliding-mode controller for Cuk converter.

The sliding-mode controlled Cuk converter shown in Fig. 17 with the parameters described in Table 2 was simulated on a digital computer. It was previously pointed out that the convergence factor needs to be selected such that a satisfactory performance is achieved. In this case a conversion factor equal to 270 was used. Fig. 18 shows the start-up performance of the designed Cuk converter. It is clear that the converter arrives at its steady state within 10 milliseconds. The dynamic response of the converter output voltage to an increase in the load by 100% is depicted in the Fig. 19. Again, within 40 milliseconds the output is reached its steady state. In Fig. 20, the commanded output voltage reference was reduced by 2 V. The new operating point is reached after 40 milliseconds. The converter response to a 3V reduction in the input voltage is illustrated in Fig. 21. It is shown that within a reasonable time, 40 milliseconds, the output is reached its steady state value.

Table 2 DC/DC Cuk converter's parameters.

V_{in}	L_1	L_2	C_1	C_2	R	f
24 V	3 mH	19 mH	47μF	100μF	15 Ω	10 kHz

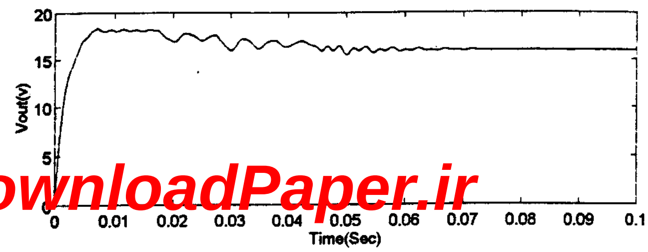

Fig. 18 Start-up performance of the Cuk converter with the designed sliding-mode controller.

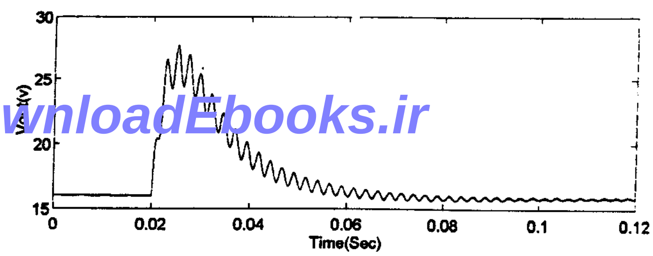

Fig. 19 Output voltage variation of the sliding-mode controlled Cuk converter to a 100% reduction in the load..

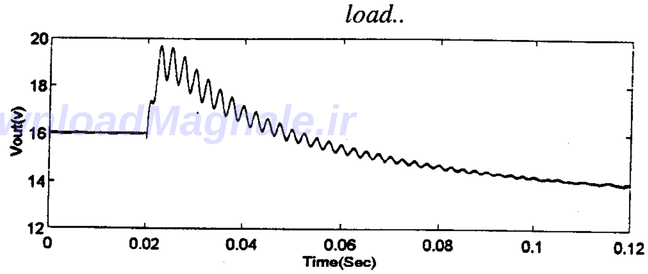

Fig. 20 Output voltage variation of the sliding-mode controlled Cuk converter to a 2 V decrease in the output voltage command .

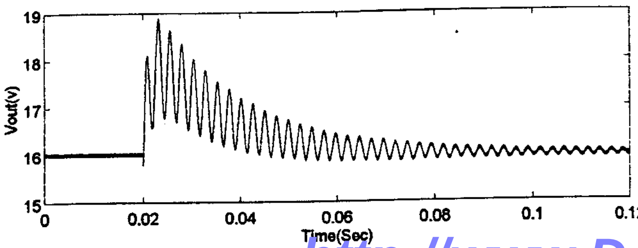

Fig. 21 Output voltage variation of the sliding-mode controlled Cuk converter to a 3 V decrease in the input voltage.

IV. SIMPLIFIED SLIDING-MODE CONTROLLER FOR THE CUK CONVERTER

In order to reduce the complexity of the designed sliding-mode controller for the Cuk converter, the following approximation was considered.

$$V_{in}^2 + \left[cK - e\left(V_{in} + K\right) + f\left(x_4 - K\right)\right]\left(x_4 - K\right) \cong V_{in}^2 \quad (23)$$

This approximation is valid based on the fact that for stable systems at the steady state, x_4 will approach its steady state value, K. Therefore, $(x_4\text{-}K)$ will be very small. Using this approximation, a simplified relation for the switch duty cycle was obtained.

$$d(t) = \frac{K + (a/2)\left(x_4 - K\right)}{\left(V_{in} + K\right) + b\left(x_4 - K\right)} \quad (24)$$

Fig. 22 shows the resultant simplified sliding-mode controlled Cuk converter.

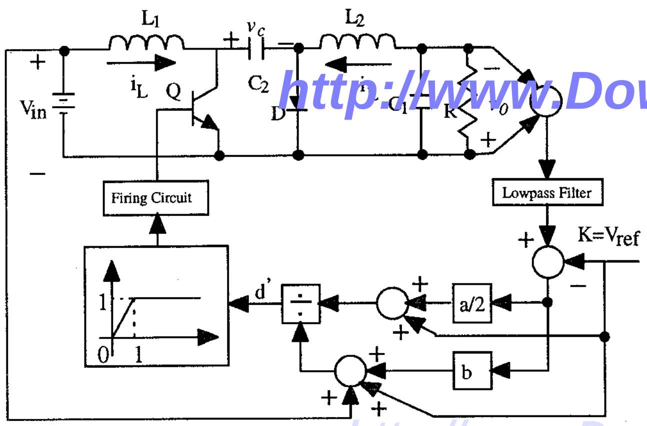

Fig. 22 Simplified sliding-mode controlled Cuk converter.

The system presented in Fig. 22 was simulated on a digital computer. A similar results to the system shown in Fig. 17 was obtained. However, when the load was increased by 100% the response was not satisfactory. A better response was obtained by reducing the convergence factor at the expense of increasing the settling time.

VI. STABILITY OF THE CUK CONVERTER

The sliding surface for the Cuk converter is defined as follows:

$$S = \left\{ x \mid x_4 - K = 0 \right\} \quad (25)$$

The controller needs to keep the variables along the sliding surface, therefore,

$$x_4 = K \quad , \quad \dot{x}_4 = 0 \quad (26)$$

The above conditions result in the following state space equations:

$$\begin{cases} \dot{x}_1 = -\frac{1-d}{L_1}x_2 + \frac{1}{L_1}V_{in} \\ \dot{x}_2 = \frac{1-d}{C_2}x_1 - \frac{d}{C_2}x_3 \\ \dot{x}_3 = \frac{d}{L_2}x_2 - \frac{1}{L_2}K \\ 0 = \frac{1}{C_1}x_3 - \frac{1}{RC_1}K \end{cases} \quad (27)$$

In order to prove the stability of the system, a continuously differentiable positive definite function, V(x), needs to be determined. Let V(x) to be presented by the following quadratic function:

$$V(x) = \frac{1}{2}\left(x - x_e\right)^T p\left(x - x_e\right) \quad (28)$$

where x is the vector representation of the state variables given by

$$x = \left[x_1 \quad x_2 \quad x_3\right]^t$$

and x_e is the state variable equilibrium point shown by

$$x_e = \left[x_{1e} \quad x_{2e} \quad x_{3e}\right]^t$$

and P is given by

$$P = diag\left[a_1 \quad a_2 \quad a_3\right] \quad , \quad a_1, a_2, a_3 > 0$$

In order to determine the stability of the system, the derivative of (28) needs to be negative definite. Where,

$$\dot{V}(x) = -(1-d)\left(\frac{a_1}{L_1} - \frac{a_2}{C_2}\right)x_1x_2 + \frac{V_{in}}{L_1}a_1\left(x_1 - x_{1e}\right) - \frac{d}{RC_2}Ka_2\left(x_2 - x_{2e}\right) - (1-d)\left(\frac{a_2}{C_2}x_{2e}x_1 - \frac{a_1}{L_1}x_{1e}x_2\right) + \frac{a_3}{L_2}\left(x_3 - x_{3e}\right)\left(dx_2 - K\right) \quad (29)$$

If a_1=1000, and a_2=a_3=1 are selected, $\dot{V}(x)$ for all $x \neq x_e$ is negative. Therefore, $\dot{V}(x)$ is a negative definite function and consequently *V(x)* is a Lyapunov function. It is clear that the closed-loop system is asymptotically stable and the arbitrary operating point x_e is a stable equilibrium point. In this paper, only stability of Cuk converter was considered. The stability analysis of other types of DC/DC converters follows the same procedure.

VII. CONCLUSION

In this paper, the state space averaging method was used to design sliding mode controller for several DC/DC converters. At first a sliding-mode controller based on this method was designed for Buck converter. This controller was simulated using digital computer and its performance was shown to be satisfactory. The same procedure was later be extended to other types of DC/DC converters such as Boost, Buck-Boost, and Cuk converters. Again, satisfactory results were obtained. To show that the designed controllers were stable, Lyapunov second theorem was used. A positive definite function was obtained to prove the stability of the system.

APPENDIX

The variables a, b, and g used in (20) are defined as follows:

$$a = 2\left[L_2 C_1 \lambda\left(\lambda - \frac{1}{RC_1}\right) + 1\right]$$

$$b = \lambda C_1\left(\lambda - \frac{1}{RC_1}\right)(L_1 + L_2) + 1$$

$$g = \left\{V_{in}^2 + \left[cK - e(V_{in} + K)\right](x_1 - K) + f(x_1 - K)^2\right\}^{1/2}$$

where

$$c = -4 L_1 C_1 \lambda\left(\lambda - \frac{1}{RC_1}\right)$$

$$e = 4\lambda^2 L_1 L_2 C_1 C_2\left[\lambda\left(\lambda - \frac{1}{RC_1}\right) + \frac{1}{L_2 C_1}\right]$$

$$f = -e\left(b + 1 - \frac{1}{RC_1 \lambda}\right)$$

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