Abstract  In this paper, we present new layouts for the multi-stage interconnection networks such as shuffle, banyan and baseline networks, that are suitable for photonic switching. In these new layouts, we decrease the number of crossovers of the stage links and crossovers between inlet-outlet of stages, which are known as the main bottleneck for the increase in switch capacity when it is realized for integrated photonic switching fabric.

Keywords: Interconnection Networks, Layouts, Photonic Switching

1 Introduction

Advances in the photonic switching systems have been reported in the literature and several new switch architectures are introduced to cope with the need of terabits/s volume of the near future switches for B-ISDN [1,2,3,4]. For example, Nishio et.al. [5] has considered a photonic ATM switch using vertical to surface transmission electro-photonic devices (VSTEPs) to handle optical cell rates up to 1.6 Gbps in the optical buffer memory and self routing with priority control switch. Sawano et.al.[6] has considered polarization independent LiNbO3 matrix switches in their design with a maximum capacity of 128 lines photonic (circuit) switching systems. In both designs, the main bottleneck is the increase in the capacity which is prevented by weakened optical signals from any inlet to any outlet in the switch fabric. Optical amplifiers have been used between the stages to compensate optical signal losses. But even this cannot solve the capacity problem of the photonic switch completely.

This paper presents new layouts for multi-stage interconnection networks by investigating the following two characteristics of the multi-stage shuffle, banyan and baseline interconnection networks that are shown in Figure1.

i) Minimization of the total number of crossovers in a switching network, which is related to the overall complexity of the fabrication process.

ii) Minimization of the maximum number of crossovers between an inlet-outlet pair, which is related to the worst case attenuation that determines the required number of optical amplifiers.

The outline of the paper is as follows. In section 2, we introduced crossover minimization via topological embedding and cyclical drawing of shuffle, baseline and banyan graphs. Finally, in section 3, some conclusions will be drawn.

2 Crossover Minimization via Topological Embedding

In [7], a modular construction scheme was given to design directional-coupler-based switching networks with minimum number of crossovers, which is based on the permutation of stage node numberings without changing the conventional structure of the interconnection networks, such as banyan, baseline shuffle. By the conventional drawing
of an interconnection network, we comprehend that all input nodes in the plane are placed vertically on the left-side while all output nodes are placed also vertically on the right-side and links connecting internal stage nodes are drawn serially. In this paper, we relax locations of the nodes in the interconnection network, so that all nodes can be placed in the plane provided that the adjacency relation of the links will remain the same as, in the shuffle, baseline or banyan interconnection networks. We call topological embedding to such a free embedding of the interconnection network and the result, depending on the structure of the original network, is called banyan graph, shuffle graph or baseline graph, if the underlying network is a multi-stage interconnection. Our aim is to find suitable topologies in terms of crossover minimization without imposing restrictions on the location of input and output nodes. In order to find new layouts, we place the input nodes, denoted by the set \{I_i\} vertically starting from top to bottom while placing the output nodes, denoted by the set \{O_i\} starting from left to right horizontally, where \(i = 1, 2, \ldots, 2^k\). We call cyclical representation of the network for such an embedded multi-stage interconnection network.

We note that, the exact minimum crossing number not only for the class of multipartite graphs but even for the complete bipartite graph \(K_{m,n}\) includes several open problems [8],[9]. For example, R. Guy [10] showed the following theorem:

**Theorem 1:** The crossing number of \(K_{m,n}\) satisfies the inequality:

\[
\text{cr}(K_{m,n}) \leq \left\lfloor \frac{m}{2} \right\rfloor \left\lfloor \frac{m-1}{2} \right\rfloor \left\lfloor \frac{n}{2} \right\rfloor \left\lfloor \frac{n-1}{2} \right\rfloor
\]

where \(m+n\) is the number of nodes of \(K_{m,n}\).

In the above inequality, upper bound has been only proved when if \(m \leq 6\) and \(n\) is arbitrary and then it is conjectured that inequality holds for all \(m\) and \(n\).

We use the notation \(N=2^k\) to denote the size of the network, where \(k\) is the number of node stages. Each input node has two inlets and each output node has two outlets. Any node in the network consists of a 2-by-2 switching element.

### 2.1 Cyclical Drawing of Shuffle Graphs

Shuffle network are widely used in sorting and in the interconnection of multi-processor computer systems. The topology of each link stage is the same but it has more link crossovers than the other interconnection topologies. It can be verified, that the shuffle graph shown in Figure 2 corresponds exactly to the conventional multi-stage shuffle interconnection network.

This can be realized by using the following input and output node numberings:

\[
I_{2i-1} = \begin{cases} 
2i - 1 & i = 1, 2, \ldots, 2^{k-2} \\
2i - 2^{k-1} & i = 1 + 2^{k-2}, 2 + 2^{k-2}, \ldots, 2^{k-1}
\end{cases}
\]

\[
I_{2i} = \begin{cases} 
2^{k-1} + 2i - 1 & i = 1, 2, \ldots, 2^{k-2} \\
2i & i = 2^{k-2} + 1, 2^{k-2} + 2, \ldots, 2^{k-1}
\end{cases}
\]

and \(O_i = i, i = 1, 2, \ldots, 2^k\).
Property 1: Consider the bipartite graph $G_{(2^k-1)}$ shown in the Figure 3 which consists of node disjoint union of twisted $2^{k-1}$ cycles of length 4. Then the number of crossovers of $G_{(2^k-1)}$ is given by

$$X(G_{(2^k-1)}) = 2^{k-1}(2^{k+1} - 3)$$

Property 2: Let $G(N)$ is the cyclical embedding of the $N$-by-$N$ multi-stage shuffle network. Then the total number of crossovers is given by

$$X(N) = 4\left(\sum_{i=0}^{k-4} 2^{k-4-i} X(G_{[2^i]})\right)$$

where $X(G_{[2^i]}) = 2^{i-1}(2^{i+1} - 3)$

Property 3: The maximum number of crossovers between an inlet $s$ and an outlet $d$ in a $k$-stage cyclical shuffle graph $G(N)$ is given by

$$X^{(k)}(s,d) = 2^k - 3k + 2$$

where $1 \leq s \leq 2^k$ and $1 \leq d \leq 2^k$.

2.2 Cyclical Drawing of Baseline Graphs

Baseline networks have also applications in sorting and in many switching architectures. Cyclical embedding of baseline network is illustrated in Figure 4 for 5-stage, 32-by-32 baseline interconnection network. As it can be seen from the graph, it is decomposed into four identical sub graphs where each sub graph is the baseline network of size 8-by-8. Node numberings for general $N$, for input and output nodes are given by

$$I_i = i, i=1,2,...,2^k,$$

$$O_i = \begin{cases} 
\frac{N}{2} - i + 1 & i = 1,2,...,2^k \\
\frac{3N}{2} - i + 1 & i = \frac{N}{2} + 1, \frac{N}{2} + 2,...,N 
\end{cases}$$

Property 4: The total number of the crossovers in a $k$-stage cyclical baseline graph is given by

$$X(N) = 2^{2k-4} - (k-1)2^{k-2}.$$
Property 5: The maximum number of crossovers between an inlet $s$ and an outlet $d$ in a $k$-stage cyclical baseline graph is given by

$$X^k(s,d)=2^{k-2} - k+1$$

where $1 \leq s \leq 2^k$ and $1 \leq d \leq 2^k$.

2.3 Cyclical Drawing of Banyan Graphs

Banyan networks are widely used in fast Fourier transform in digital signal processing. Cyclical drawing of banyan network, illustrated in Figure 5, considerably reduces the number of crossovers. Input and output node numberings for these graphs are exactly the same as the mappings of the cyclical shuffle graphs (see Section 2.1).

![Figure 5: A 5 Stage 32-by-32 Cyclical Banyan Graph.](image)

Property 6: The total number of crossovers in a $k$-stage cyclical banyan graph is given by

$$X(N)=(3/8)2^{2k-2} - 2^{k-2}(2k-3).$$

Property 7: The maximum number of crossovers between an inlet $s$ and an outlet $d$ in a $k$-stage cyclical banyan graph is given by

$$X^k(s,d)=2^{k-2} - k+1$$

where $1 \leq s \leq 2^k$ and $1 \leq d \leq 2^k$.

3 Conclusions

The number of crossovers between the stage-links in the interconnection networks has an impact on the integrated optical realization, particularly when they are realized with the directional-coupler-based devices. In this paper, we embedded the conventional multi-stage interconnection network in the plane in such a way that the crossovers are minimized. We have summarized in Table 1, the total number of crossovers and the maximum number of crossovers between the inlet-outlet pairs for the conventional multi-stage shuffle, baseline and banyan networks and for the new layouts of the corresponding cyclical interconnection graphs. From Table 1, it can be seen that the cyclical baseline graphs have lower crossover numbers than the others. Moreover, the reduction of the number of crossovers with respect to the conventional drawings is in the order of four. Although we have not attempted to show whether the proposed interconnection layouts result in the minimum number of crossovers, for small values of $k$ (the number of stages) the layouts suggest that the crossover numbers are the minimum possible. Since many networks are based on shuffle, baseline and banyan networks’ topologies, the results of this paper can be applied extensively to the study of crossover minimization for many other switching networks. Arranging alternating fixed-size optical planes and electronic planes in a sandwich fashion can accomplish a package of the new interconnection network layouts to increase the capacity. Similar physical structures have already been implemented by using the three-dimensional optical interconnection concept[11].

References

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Table 1: Number of Crossovers in Conventional Drawing and Cyclical Drawing of Multi-Stage Interconnection Networks

<table>
<thead>
<tr>
<th>Number of stages</th>
<th>Conventional Drawing</th>
<th>Cyclical Drawing</th>
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<td></td>
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<td>X(N), X(s,d)</td>
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<tr>
<td></td>
<td>Baseline Banyan</td>
<td>Baseline Banyan</td>
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