hep-ph/9908476

Large or Small Angle MSW from Single Right-Handed Neutrino Dominance [1]

S. F. King

Department of Physics and Astronomy, University of Southampton, Southampton, SO17 1BJ, U.K.

Abstract

In this talk we discuss a natural explanation of both neutrino mass hierarchies *and* large neutrino mixing angles, as required by the atmospheric neutrino data, in terms of a single right-handed neutrino giving the dominant contribution to the 23 block of the light effective neutrino matrix, and illustrate this mechanism in the framework of models with $U(1)$ family symmetries. Sub-dominant contributions from other right-handed neutrinos are required to give small mass splittings appropriate to the MSW solution to the solar neutrino problem. We present three explicit examples for achieving the small angle MSW solution in the framework of $U(1)$ family symmetry models containing three right-handed neutrinos, which can naturally describe all quark and lepton masses and mixing angles. In this talk we also extend the analysis to the large angle MSW solution.

[1] To appear in the proceedings of the 6th San Miniato Topical Seminar on "Neutrino and Astroparticle Physics", Nuclear Physics B (Proc. Suppl.)

There is now strong evidence for atmospheric neutrino oscillations [1]. The most recent analyses of Super-Kamiokande [1] involve the hypothesis of $\nu_\mu \to \nu_\tau$ oscillations with maximal mixing $\sin^2 2\theta_{23} = 1$ and a mass splitting of $\Delta m^2_{23} = 2.2 \times 10^{-3}\ eV^2$. Using all their data sets analysed in different ways they quote $\sin^2 2\theta_{23} > 0.82$ and a mass splitting of $1.5 \times 10^{-3}\ eV^2 < \Delta m^2_{23} < 6 \times 10^{-3}\ eV^2$ at 90% confidence level.

The evidence for solar neutrino oscillations is almost as strong. There are a panoply of experiments looking at different energy ranges, and the best fit to all of them has been narrowed down to two basic scenarios corresponding to either resonant oscillations $\nu_e \to \nu_0$ (where for example ν_0 may be a linear combination of ν_μ, ν_τ) inside the Sun (MSW [2]) or "just-so" oscillations in the vacuum between the Sun and the Earth [3], [4]. There are three MSW fits and one vacuum oscillation fit:

(i) the small angle MSW solution is $\sin^2 2\theta_{12} \approx 5 \times 10^{-3}$ and $\Delta m^2_{12} \approx 5 \times 10^{-6}\ eV^2$;

(ii) the large angle MSW solution is $\sin^2 2\theta_{12} \gtrsim 0.2$ and $\Delta m^2_{12} \approx 1.8 \times 10^{-5}\ eV^2$;

(iii) an additional MSW large angle solution exists with a lower probability [5];

(iv) The vacuum oscillation solution is $\sin^2 2\theta_{12} \approx 0.75$ and $\Delta m^2_{12} \approx 6.5 \times 10^{-11}\ eV^2$ [5].

The standard model has zero neutrino masses, so any indication of neutrino mass is very exciting since it represents new physics beyond the standard model. In this paper we shall assume the see-saw mechanism and no light sterile neutrinos. The see-saw mechanism [6] implies that the three light neutrino masses arise from some heavy "right-handed neutrinos" N_R^p (in general there can be Z gauge singlets with $p = 1, \ldots Z$) with a $Z \times Z$ Majorana mass matrix M_{RR}^{pq} whose entries take values at or below the unification scale $M_U \sim 10^{16}$ GeV. The presence of electroweak scale Dirac mass terms m_{LR}^{ip} (a $3 \times Z$ matrix) connecting the left-handed neutrinos ν_L^i ($i = 1, \ldots 3$) to the right-handed neutrinos N_R^p then results in a very light see-saw suppressed effective 3×3 Majorana mass matrix

$$m_{LL} = m_{LR} M_{RR}^{-1} m_{LR}^T \tag{1}$$

for the left-handed neutrinos ν_L^i, which are the light physical degrees of freedom observed by experiment.

Not surprisingly, following the recent data, there has been a torrent of theoretical papers concerned with understanding how to extend the standard model in order to accomodate the atmospheric and solar neutrino data. Perhaps the minimal extension of the standard model capable of accounting for the atmospheric neutrino data involves the addition of a *single* right-handed neutrino N_R [7], [8]. This is a special case of the general see-saw model with $Z = 1$, so that M_{RR} is a trivial 1×1 matrix and m_{LR} is a 3×1 column matrix where $m_{LR}^T = (\lambda_{\nu_e}, \lambda_{\nu_\mu}, \lambda_{\nu_\tau}) v_2$ with v_2 the vacuum expectation value of the Higgs field H_2 which is responsible for the neutrino Dirac masses, and the notation for the Yukawa couplings λ_i indicates that we are in the charged lepton mass eigenstate basis e_L, μ_L, τ_L with corresponding neutrinos $\nu_{e_L}, \nu_{\mu_L}, \nu_{\tau_L}$. Since M_{RR} is trivially invertible the light effective mass matrix in Eq.1 in the $\nu_{e_L}, \nu_{\mu_L}, \nu_{\tau_L}$ basis is simply given by

$$m_{LL} = \begin{pmatrix} \lambda_{\nu_e}^2 & \lambda_{\nu_e}\lambda_{\nu_\mu} & \lambda_{\nu_e}\lambda_{\nu_\tau} \\ \lambda_{\nu_e}\lambda_{\nu_\mu} & \lambda_{\nu_\mu}^2 & \lambda_{\nu_\mu}\lambda_{\nu_\tau} \\ \lambda_{\nu_e}\lambda_{\nu_\tau} & \lambda_{\nu_\mu}\lambda_{\nu_\tau} & \lambda_{\nu_\tau}^2 \end{pmatrix} \frac{v_2^2}{M_{RR}}. \tag{2}$$

The matrix in Eq.2 has vanishing determinant which implies a zero eigenvalue. Furthermore the submatrix in the 23 sector has zero determinant which implies a second zero eigenvalue associated with this sector. In order to account for the Super-Kamiokande data we assumed [7]:

$$\lambda_{\nu_e} \ll \lambda_{\nu_\mu} \approx \lambda_{\nu_\tau}. \tag{3}$$

In the $\lambda_{\nu_e} = 0$ limit the matrix in Eq.2 has zeros along the first row and column, and so clearly ν_e is massless, and the other two eigenvectors are simply

$$\begin{pmatrix} \nu_0 \\ \nu_3 \end{pmatrix} = \begin{pmatrix} c_{23} & -s_{23} \\ s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} \nu_\mu \\ \nu_\tau \end{pmatrix} \tag{4}$$

where $t_{23} = \lambda_{\nu_\mu}/\lambda_{\nu_\tau}$, with ν_0 being massless, due to the vanishing of the determinant of the 23 submatrix and ν_3 having a mass $m_{\nu_3} = (\lambda_{\nu_\mu}^2 + \lambda_{\nu_\tau}^2)v_2^2/M_{RR}$. The Super-Kamiokande data is accounted for by choosing the parameters such that $t_{23} \sim 1$ and $m_{\nu_3} \sim 5 \times 10^{-2}$ eV. In this approximation the atmospheric neutrino data is then consistent with $\nu_\mu \to \nu_\tau$ oscillations via two state mixing, between ν_3 and ν_0. Note how the single right-handed neutrino coupling to the 23 sector implies vanishing determinant of the 23 submatrix. This provides a natural explanation of both large 23 mixing angles and a hierarchy of neutrino masses in the 23 sector at the same time [7].

In order to account for the solar neutrino data a small mass perturbation is required to lift the massless degeneracy of the two neutrinos ν_0, ν_e. In our original approach [7] [2] we introduced additional right-handed neutrinos in order to provide a subdominant contribution to the effective mass matrix in Eq.2. To be precise we assumed a single dominant right-handed neutrino below the unification scale, with additional right-handed neutrinos at the unification scale which lead to subdominant contributions to the effective neutrino mass matrix. By appealing to quark and lepton mass hierarchy we assumed that the additional subdominant right-handed neutrinos generate a contribution $m_{\nu_\tau} \approx m_t^2/M_U \approx 2\times10^{-3}$ eV, where m_t is the top quark mass. The effect of this is to give a mass perturbation to the 33 component of the mass matrix in Eq.2, which results in ν_0 picking up a small mass, through its ν_τ component, while ν_e remains massless. Solar neutrino oscillations then arise from $\nu_e \to \nu_0$ with the mass splitting in the right range for the small angle MSW solution, controlled by a small mixing angle $\theta_{12} \approx \lambda_{\nu_e}/\sqrt{\lambda_{\nu_\mu}^2 + \lambda_{\nu_\tau}^2}$. The main prediction of this scheme is of the neutrino oscillation $\nu_e \to \nu_3$ with a mass difference $\Delta m_{13}^2 \approx \Delta m_{23}^2$ determined by the Super-Kamiokande data and a mixing angle $\theta_{13} \approx \theta_{12}$ determined by the small an-

[2] Another approach [8] which does not rely on additional right-handed neutrinos is to use SUSY radiative corrections so that the one-loop corrected neutrino masses are not zero but of order 10^{-5} eV suitable for the vacuum oscillation solution.

gle MSW solution. Such oscillations may be observable at the proposed long baseline experiments via $\nu_3 \rightarrow \nu_e$ which implies $\nu_\mu \rightarrow \nu_e$ oscillations with $\sin^2 2\theta \approx 5 \times 10^{-3}$ (the small MSW angle) and $\Delta m^2 \approx 2.2 \times 10^{-3}\ eV^2$ (the Super-Kamiokande square mass difference).

It should be clear from the foregoing discussion that the motivation for single right-handed neutrino dominance (SRHND) is that the determinant of the 23 submatrix of Eq.2 approximately vanishes, leading to a natural explanation of *both* large neutrino mixing angles *and* hierarchical neutrino masses in the 23 sector *at the same time* [7]. Although the explicit example of SRHND above was based on one of the right-handed neutrinos being lighter than the others, it is clear that the idea of SRHND is more general than this.

In [9] we defined SRHND more generally as the requirement that a single right-handed neutrino gives the dominant contribution to the 23 submatrix of the light effective neutrino mass matrix (which can be achieved in other ways than one of the right-handed neutrinos being lighter than the others.) We addressed the following two questions:

1. What are the general conditions under which SRHND in the 23 block can arise and how can we quantify the contribution of the sub-dominant right-handed neutrinos which are responsible for breaking the massless degeneracy, and allowing the small angle MSW solution?

2. How can we understand the pattern of neutrino Yukawa couplings in Eq.3 where the assumed equality $\lambda_{\nu_\mu} \approx \lambda_{\nu_\tau}$ is apparently at odds with the hierarchical Yukawa couplings in the quark and charged lepton sector?

In order to address the two questions above we discuss SRHND in the context of a $U(1)$ family symmetry. In ref.[9] we gave general conditions that theories with $U(1)$

family symmetry must satisfy in order to have SRHND and showed that the models in [10] satisfy these conditions. In this talk we briefly review this approach, giving three examples based on the general analysis in ref.[9].

The Wolfenstein parametrisation of the CKM matrix is roughly

$$V_{CKM} \sim \begin{pmatrix} 1 & \lambda & \lambda^3 \\ \lambda & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix} \tag{5}$$

With a single dominant right-handed neutrino we expect equal neutrino mixing angles in 12 and 13 sectors

$$\theta_{12} \sim \theta_{13} \tag{6}$$

CHOOZ [11] tells us that over most of the interesting mass range $\sin^2 2\theta_{13} < 0.18$, corresponding to $\theta_{13} \leq \lambda$. Thus there are two interesting possibilities for the choice of angle, corresponding to large or small angle MSW with $\theta_{13}^{large} \sim \theta_{12}^{large} \sim \lambda$, or $\theta_{13}^{small} \sim \theta_{12}^{small} \sim \lambda^2$, with the Maki-Nakagawa-Sakata matrix, the leptonic analogue of the CKM matrix, determined in each case:

$$V_{MNS}^{large} \sim \begin{pmatrix} 1 & \lambda & \lambda \\ \lambda & 1 & 1 \\ \lambda & 1 & 1 \end{pmatrix}, V_{MNS}^{small} \sim \begin{pmatrix} 1 & \lambda^2 & \lambda^2 \\ \lambda^2 & 1 & 1 \\ \lambda^2 & 1 & 1 \end{pmatrix} \tag{7}$$

Note that in the large angle MSW case, we are relying on factors of order unity implicitly present in each element to give us a large enough MSW angle without violating the CHOOZ constraint. Our working assumption is that V_{MNS}^{large} or V_{MNS}^{small} originates from both the neutrino sector and the charged lepton sector in roughly equal measure which, together with Eq.2, gives

$$m_{LL}^{large} \sim \begin{pmatrix} \lambda^2 & \lambda & \lambda \\ \lambda & 1 & 1 \\ \lambda & 1 & 1 \end{pmatrix} m_{\nu_3}, m_{LL}^{small} \sim \begin{pmatrix} \lambda^4 & \lambda^2 & \lambda^2 \\ \lambda^2 & 1 & 1 \\ \lambda^2 & 1 & 1 \end{pmatrix} m_{\nu_3} \tag{8}$$

corresponding to V_{MNS}^{large}, V_{MNS}^{small}, respectively. In general would expect $m_{\nu_2} \sim m_{\nu_3}$ due to large 23 mixing, but with a single right-handed neutrino the vanishing determinant of 23 block solves this problem by setting $m_{\nu_2} = 0$. In order obtain the desired hierarchy between the MSW neutrino mass and the atmospheric neutrino mass, $m_{\nu_2}/m_{\nu_3} \sim \lambda^2$, we must add extra subdominant right-handed neutrinos which contribute to the 23 block at order $O(\lambda^2)$. In the small angle case this would lead to $m_{\nu_1}/m_{\nu_2} \sim \lambda^2$ and a hierarchy of neutrino masses, while in the large angle case we would have $m_{\nu_1} \lesssim m_{\nu_2}$, leading to a semi-hierarchical neutrino mass pattern.

To proceed we introduce a $U(1)$ family symmetry of the kind suggested by Ibanez and Ross [12]. For example a suitable choice of quark, lepton and Higgs charges leads to the quark and charged lepton Yukawa matrices [13]:

$$Y^u \sim \begin{pmatrix} \lambda^8 & \lambda^5 & \lambda^3 \\ \lambda^7 & \lambda^4 & \lambda^2 \\ \lambda^5 & \lambda^2 & 1 \end{pmatrix}, Y^d \sim \begin{pmatrix} \lambda^4 & \lambda^3 & \lambda^3 \\ \lambda^3 & \lambda^2 & \lambda^2 \\ \lambda & 1 & 1 \end{pmatrix} \lambda^n \tag{9}$$

$$Y^e_{large} \sim \begin{pmatrix} \lambda^4 & \lambda^3 & \lambda \\ \lambda^3 & \lambda^2 & 1 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix} \lambda^n, Y^e_{small} \sim \begin{pmatrix} \lambda^4 & \lambda^4 & \lambda^2 \\ \lambda^2 & \lambda^2 & 1 \\ \lambda^2 & \lambda^2 & 1 \end{pmatrix} \lambda^n \tag{10}$$

which lead to the following successful quark and lepton mass relations:

$$\frac{m_u}{m_t} \sim \lambda^8, \quad \frac{m_c}{m_t} \sim \lambda^4, \quad \frac{m_d}{m_b} \sim \lambda^4, \quad \frac{m_s}{m_b} \sim \lambda^2, \tag{11}$$

$$\frac{m_e}{m_\tau} \sim \lambda^4, \quad \frac{m_\mu}{m_\tau} \sim \lambda^2. \tag{12}$$

$$\frac{m_b}{m_t} \sim \lambda^3, \quad \frac{m_b}{m_\tau} \sim 1, \tag{13}$$

where the last relations are valid in the MSSM at the unification scale, where there are two Higgs doublets with vacuum expectation values v_1, v_2 coupling to down-type quarks, up-type quarks, respectively, and $\tan\beta = v_2/v_1 \sim \lambda^{n-3}$. The correct CKM matrix given earlier is also reproduced, and V^{large}_{MNS}, V^{small}_{MNS} are consistent with Y^e_{large}, Y^e_{small}, respectively.

When dealing with the lepton charges, it is convenient to absorb the physical Higgs charge h_u into the physical lepton charges l_i, whereupon we find the redefined lepton charges [9]

$$\begin{pmatrix} \nu_{1L} \\ e_{1L} \end{pmatrix}, \begin{pmatrix} \nu_{2L} \\ e_{2L} \end{pmatrix}, \begin{pmatrix} \nu_{3L} \\ e_{3L} \end{pmatrix} = (m + l_3, l_3, l_3) \tag{14}$$

where $m = 1, 2$ for Y^e_{large}, Y^e_{small} cases, respectively, and the numerical value of l_3 remains a free choice, which is specified precisely in the examples below.

We now give three examples of $U(1)$ charge assignments for the three lepton doublets and three right-handed neutrinos which satisfies SRHND for the small angle MSW case $m = 2$ [9]. We classify the cases according to the upper block structure of the resulting heavy Majorana matrix:

(i) "Diagonal dominated" upper block of heavy Majorana matrix.

$$\begin{pmatrix} \nu_{1L} \\ e_{1L} \end{pmatrix}, \begin{pmatrix} \nu_{2L} \\ e_{2L} \end{pmatrix}, \begin{pmatrix} \nu_{3L} \\ e_{3L} \end{pmatrix} = (1/2, -3/2, -3/2) \tag{15}$$

$$\bar{\nu}_{1R}, \bar{\nu}_{2R}, \bar{\nu}_{3R} = (0, 1, 2) \tag{16}$$

The resulting mass matrices are:

$$M_{RR} \sim \begin{pmatrix} 1 & \lambda & \lambda^2 \\ \lambda & \lambda^2 & \lambda^3 \\ \lambda^2 & \lambda^3 & \lambda^4 \end{pmatrix} M \tag{17}$$

$$m_{LR} \sim \begin{pmatrix} \lambda^{1/2} & \lambda^{3/2} & \lambda^{5/2} \\ \lambda^{3/2} & \lambda^{1/2} & \lambda^{1/2} \\ \lambda^{3/2} & \lambda^{1/2} & \lambda^{1/2} \end{pmatrix} v_2 \tag{18}$$

$$m_{LL} \sim \begin{pmatrix} \lambda^4 & \lambda^2 & \lambda^2 \\ \lambda^2 & 1 & 1 \\ \lambda^2 & 1 & 1 \end{pmatrix} m_{\nu_3} \tag{19}$$

with the determinant of the lower 23 sub-block of m_{LL} vanishing to order $O(\lambda^2)$ (as required) due to $\bar{\nu}_{3R}$ dominating the contribution to the 23 block. The reason for $\bar{\nu}_{3R}$ dominance in this case is that it is lighter than the next lightest right-handed neutrino $\bar{\nu}_{2R}$ by a factor of λ^2, while the Dirac couplings to the second and third lepton doublets are the same order of magnitude for $\bar{\nu}_{3R}$ and $\bar{\nu}_{2R}$.

(ii) "Off-Diagonal dominated" upper block of heavy Majorana matrix. This is the kind of model discussed in ref.[10].

$$\begin{pmatrix} \nu_{1L} \\ e_{1L} \end{pmatrix}, \begin{pmatrix} \nu_{2L} \\ e_{2L} \end{pmatrix}, \begin{pmatrix} \nu_{3L} \\ e_{3L} \end{pmatrix} = (2, 0, 0) \qquad (20)$$

$$\bar{\nu}_{1R}, \bar{\nu}_{2R}, \bar{\nu}_{3R} = (1, -1, 0) \qquad (21)$$

The resulting mass matrices are:

$$M_{RR} \sim \begin{pmatrix} \lambda^2 & 1 & \lambda \\ 1 & \lambda^2 & \lambda \\ \lambda & \lambda & 1 \end{pmatrix} M \qquad (22)$$

$$m_{LR} \sim \begin{pmatrix} \lambda^3 & \lambda & \lambda^2 \\ \lambda & \lambda & 1 \\ \lambda & \lambda & 1 \end{pmatrix} v_2 \qquad (23)$$

$$m_{LL} \sim \begin{pmatrix} \lambda^4 & \lambda^2 & \lambda^2 \\ \lambda^2 & 1 & 1 \\ \lambda^2 & 1 & 1 \end{pmatrix} m_{\nu_3} \qquad (24)$$

with determinant of the lower 23 sub-block again vanishing to order $O(\lambda^2)$ (as desired) due to $\bar{\nu}_{3R}$ dominating the contribution to the 23 block. The reason for $\bar{\nu}_{3R}$ dominance in this case is that its Dirac couplings to the second and third lepton doublets is larger by a factor of $1/\lambda$ compared to those of $\bar{\nu}_{2R}$, $\bar{\nu}_{1R}$, while all right-handed neutrinos have roughly equal masses.

(iii) "Democratic" upper block of heavy Majorana matrix.

$$\begin{pmatrix} \nu_{1L} \\ e_{1L} \end{pmatrix}, \begin{pmatrix} \nu_{2L} \\ e_{2L} \end{pmatrix}, \begin{pmatrix} \nu_{3L} \\ e_{3L} \end{pmatrix} = (3/2, -1/2, -1/2) \qquad (25)$$

$$\bar{\nu}_{1R}, \bar{\nu}_{2R}, \bar{\nu}_{3R} = (0, 0, 1) \qquad (26)$$

The resulting mass matrices are:

$$M_{RR} \sim \begin{pmatrix} 1 & 1 & \lambda \\ 1 & 1 & \lambda \\ \lambda & \lambda & \lambda^2 \end{pmatrix} M \tag{27}$$

$$m_{LR} \sim \begin{pmatrix} \lambda^{3/2} & \lambda^{3/2} & \lambda^{5/2} \\ \lambda^{1/2} & \lambda^{1/2} & \lambda^{1/2} \\ \lambda^{1/2} & \lambda^{1/2} & \lambda^{1/2} \end{pmatrix} v_2 \tag{28}$$

$$m_{LL} \sim \begin{pmatrix} \lambda^4 & \lambda^2 & \lambda^2 \\ \lambda^2 & 1 & 1 \\ \lambda^2 & 1 & 1 \end{pmatrix} m_{\nu_3} \tag{29}$$

with determinant of the lower 23 sub-block once again vanishing to order $O(\lambda^2)$ due to $\bar{\nu}_{3R}$ dominating the contribution to the 23 block. The reason for $\bar{\nu}_{3R}$ dominance in this case is similar to case (i), namely that it is lighter than the two other (in this case) degenerate right-handed neutrinos by a factor of λ^2, while the Dirac couplings to the second and third lepton doublets are the same for all right-handed neutrinos.

Although we have focussed on the small angle MSW case for definiteness, similar examples may readily be constructed for the large angle MSW case. For example case (ii) above may trivially be extended to the large angle MSW case by taking

$$\begin{pmatrix} \nu_{1L} \\ e_{1L} \end{pmatrix}, \begin{pmatrix} \nu_{2L} \\ e_{2L} \end{pmatrix}, \begin{pmatrix} \nu_{3L} \\ e_{3L} \end{pmatrix} = (1, 0, 0) \tag{30}$$

$$\bar{\nu}_{1R}, \bar{\nu}_{2R}, \bar{\nu}_{3R} = (1, -1, 0). \tag{31}$$

The resulting mass matrices in the large angle case become:

$$M_{RR} \sim \begin{pmatrix} \lambda^2 & 1 & \lambda \\ 1 & \lambda^2 & \lambda \\ \lambda & \lambda & 1 \end{pmatrix} M \tag{32}$$

$$m_{LR} \sim \begin{pmatrix} \lambda^2 & 1 & \lambda \\ \lambda & \lambda & 1 \\ \lambda & \lambda & 1 \end{pmatrix} v_2 \tag{33}$$

$$m_{LL} \sim \begin{pmatrix} \lambda^2 & \lambda & \lambda \\ \lambda & 1 & 1 \\ \lambda & 1 & 1 \end{pmatrix} m_{\nu_3} \tag{34}$$

with determinant of the lower 23 sub-block again vanishing to order $O(\lambda^2)$ (as desired) due to $\bar{\nu}_{3R}$ dominating the contribution to the 23 block. The reason for $\bar{\nu}_{3R}$ dominance in this case is that its Dirac couplings to the second and third lepton doublets is again larger by a factor of $1/\lambda$ compared to those of $\bar{\nu}_{2R}$, $\bar{\nu}_{1R}$, while all right-handed neutrinos have roughly equal masses, as before.

In conclusion SRHND provides an elegant mechanism for yielding both large 23 neutrino mixing angles and hierarchical 23 neutrino masses simultaneously by virtue of the approximately vanishing 23 subdeterminant of m_{LL} in these models. Such models hence provide a natural explanation of the atmospheric neutrino data. U(1) family symmetry is discussed as an organising principle which leads to a controlled expansion in the Wolfenstein parameter λ, capable of providing a complete explanation of the quark and lepton spectrum in general. We give some explicit examples of U(1) charge assignments in the lepton sector which lead to SRHND in the 23 block of m_{LL}. In these examples, the subdeterminant vanishes to order λ^2, and gives rise to a non-zero mass ratio $m_{\nu_2}/m_{\nu_3} \sim \lambda^2$ capable of accounting for the solar neutrino data via the large or small angle MSW effect. In the large angle MSW case we must rely on numerical factors of order unity to slightly enhance θ_{12} relative to θ_{13} in order to give a large MSW angle without violating the CHOOZ constraint.

References

[1] Y. Fukuda et al., Super-Kamiokande collaboration, Phys. Lett. B433 (1998) 9; Phys. Lett. B436 (1998) 33; Phys. Rev. Lett. 81 (1998) 1562; T Kajita (these proceedings).

[2] L. Wolfenstein, Phys. Rev. **D17** (1978) 2369;

S. Mikheyev and A. Yu. Smirnov, Sov. J. Nucl. Phys. **42** (1985) 913.

[3] V.N. Gribov, B.M. Pontecorvo, *Phys. Lett.* **B 28** (1969) 493.

[4] more recently, see, *e.g.*

V. Barger, K. Whisnant, R.J.N. Phillips, *Phys. Rev.* **D24** (1981) 538; S. L. Glashow, L. M. Krauss, *Phys. Lett.* **190B** (1987) 199; V. Barger, R.J.N. Phillips, K. Whisnant, *Phys. Rev. Lett.* **65** (1990) 3084, *ibid* **69** (1992) 3135; N.Hata, P.G. Langacker, *Phys. Rev.* **D 56** (1997) 6107; P.I. Krastev, S.T. Petcov, *Nucl. Phys.* **B 449** (1995) 605. S. L. Glashow, P. J. Kernan, L. M. Krauss, hep-ph/9808470.

[5] J.N. Bahcall, P.I. Krastev and A.Y. Smirnov, *Phys. Rev.* **D58** (1998) 096016; and references therein.

[6] M. Gell-Mann, P. Ramond and R. Slansky in Sanibel Talk, CALT-68-709, Feb 1979, and in *Supergravity* (North Holland, Amsterdam 1979);

T. Yanagida in *Proc. of the Workshop on Unified Theory and Baryon Number of the Universe*, KEK, Japan, 1979.

[7] S. F. King, Phys. Lett. **B439** (1998) 350, hep-ph/9806440.

[8] S. Davidson and S. F. King, Phys. Lett. **B445** (1998) 191, hep-ph/9808296.

[9] S. F. King, hep-ph/9904210, to appear in Nucl. Phys. B.

[10] G. Altarelli and F. Feruglio, hep-ph/9807353, G. Altarelli and F. Feruglio, hep-ph/9809596, G. Altarelli and F. Feruglio, hep-ph/9812475.

[11] M. Apollonio *et al* hep-ex/9711002.

[12] L. Ibanez and G.G. Ross, Phys. Lett. B332 (1994) 100.

[13] J. Elwood, N. Irges and P. Ramond, Phys. Rev. Lett. 81 (1998) 5064, hep-ph/9807228; N. Irges, S. Lavignac and P. Ramond, Phys. Rev. D58 (1998) 035003, hep-ph/9802334.