A Constraint Programming Model with Time Uncertainty for Cooperative Flight Departures

Juan J.Ramos,
Nina Schefers, Marko Radanovic,
Miquel A. Piera
Dep. of Logistics and Aeronautics
Universitat Autònoma de Barcelona
Barcelona, Spain
JuanJose.Ramos@uab.es

Pau Folch
Dep. of Research and Innovation
Aslogic
Rubí, Spain
pfolch@aslogic.es

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Content

- PARTAKE context
- Tight trajectory detection
- Tight trajectory resolution: a CP approach
- Results
- Conclusions

PARTAKE context

Improve Air Traffic dynamic Demand Capacity balancing

By identifying The "hot spots" at network level

Rearranging Departure sequence within CTOT margins

Re-adjusting Speed within acceptable tolerance

Short term ATFCM measures, applied at local level and reducing traffic peaks for the whole airspace

PARTAKE context

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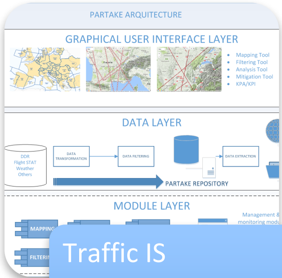
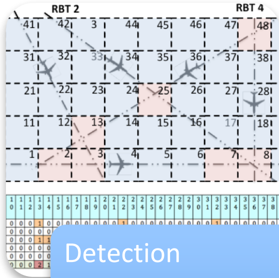
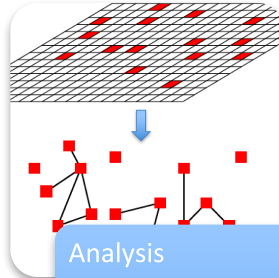


Traffic IS
- Swim-like
- DDR2



Detection
- Mapping
- Filtering



Analysis
- Interdependencies
- Uncertainty

Mitigation
- Operational constraints
- Airport/AU's preferences

CP Model

Obtain and process traffic data from SWIM like networks

Implement the data model and support data exchange among PARTAKE modules

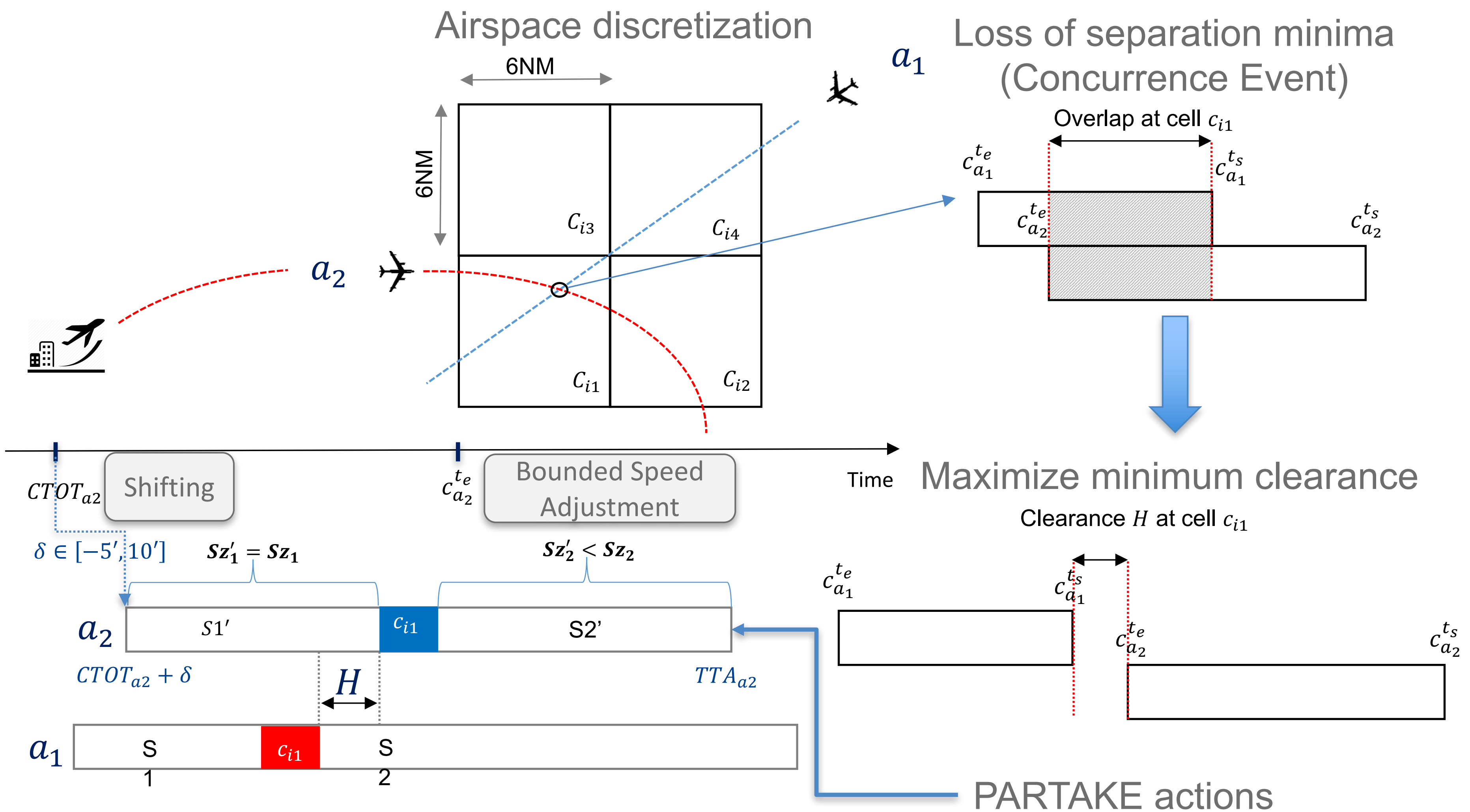
Provide a competitive methods for detecting the potential and proximate concurrence events considering 2 hours of traffic in the ECAC airspace

Determine the required properties to ensure the mitigation problem solvability

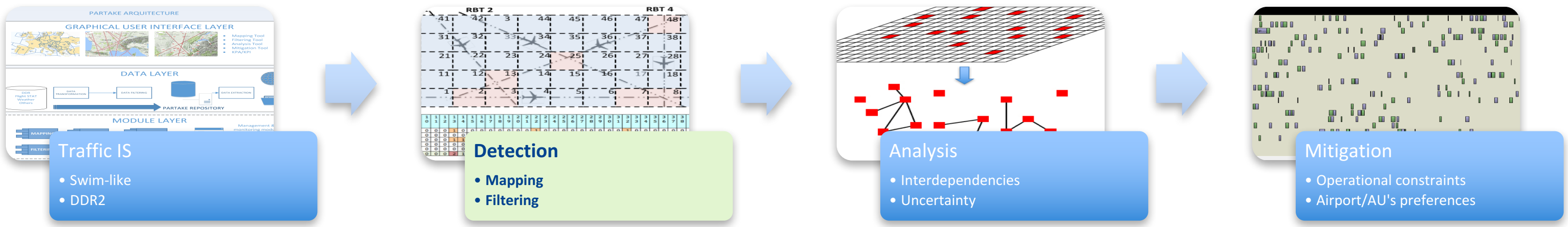
Consider the effects of both spatial and temporal uncertainty

Mitigate (reduce to a maximum) the concurrence events by determining the most suitable departure coordination

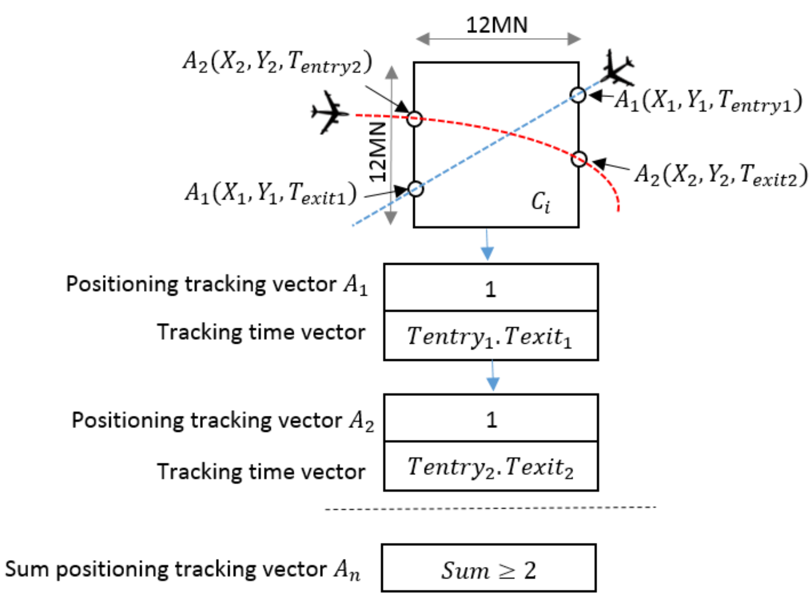
PARTAKE main concepts



Detection Module: tight trajectories

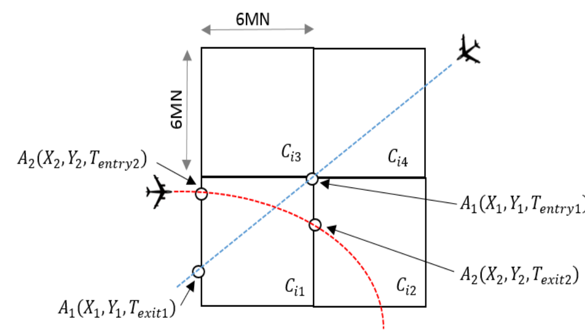


Macro-mapping process



Macro-cell (square bin of 12 NM) with potential concurrence events is divided into four microcells

Micro-mapping process



The temporal looseness (size of overlap or clearance) is recorded in $[\mathbf{t}_e^a, \mathbf{t}_s^a]$, where $\mathbf{t}_e^a$represents the entry time and $\mathbf{t}_s^a$ the exit time of aircraft a.

The temporal looseness is given by:

$$H = \min\left(t_s^{a_1}, t_s^{a_2}\right) - \max\left(t_e^{a_1}, t_e^{a_2}\right)$$

Detection Module: outcomes:

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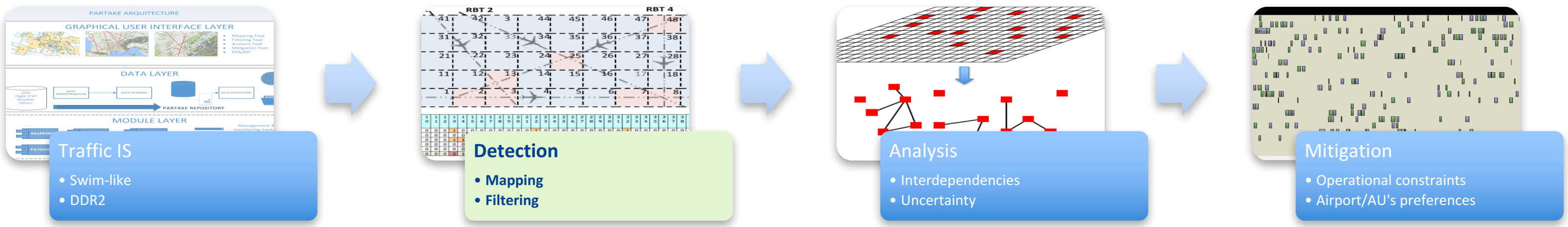
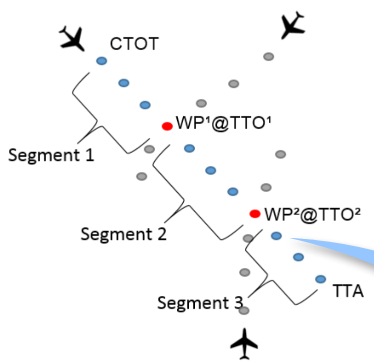
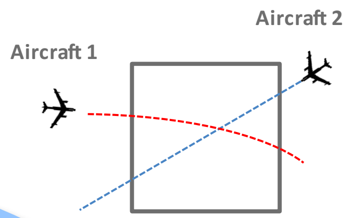
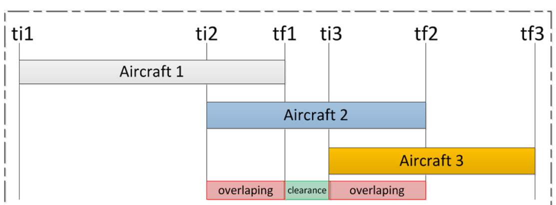




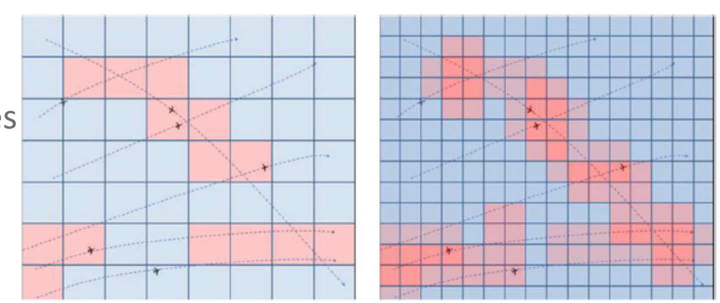


Discretization

Occupancy time window analysis



Interdependencies



The mapping process is performed over each 4D trajectory described as a set of point defined every second.

The mapping process of one day of traffic takes less than 5' in a standard desktop computer.

Detection within 2h of traffic takes less than 2''

Detection algorithm has $O(n \times m)$ complexity.

Detection Module: outcomes:

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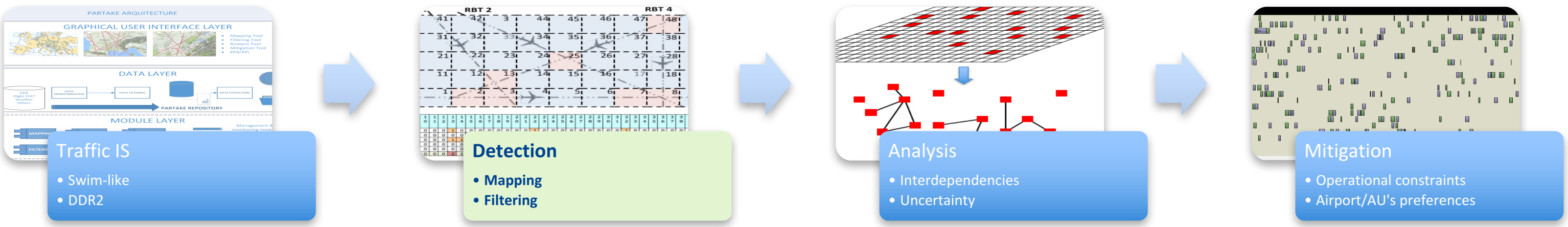
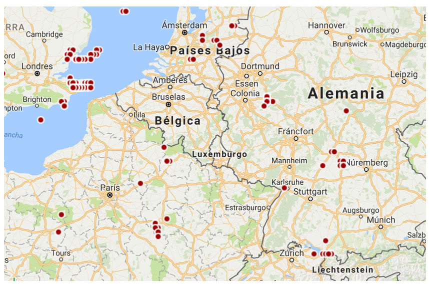
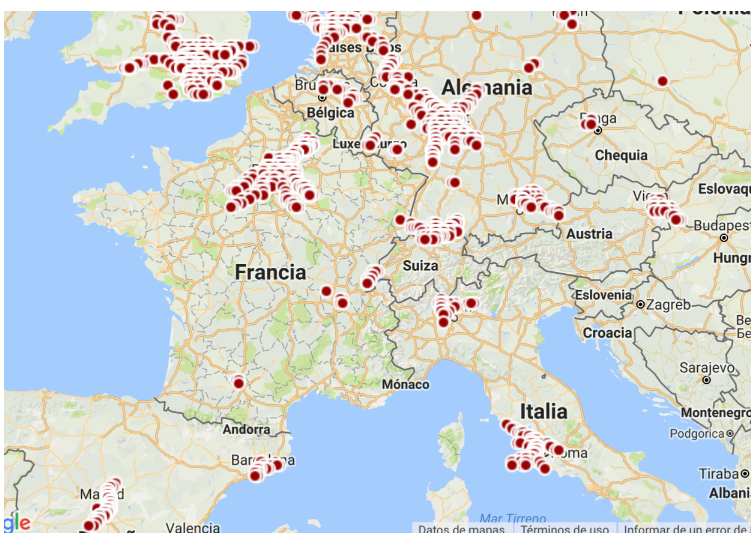






The mapping process is performed over each 4D trajectory described as a set of point defined every second.

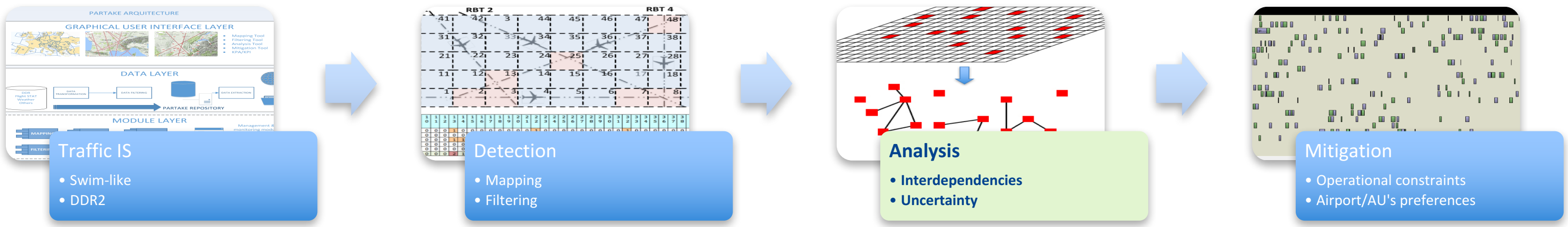
The mapping process of one day of traffic takes less than 5’ in a standard desktop computer.

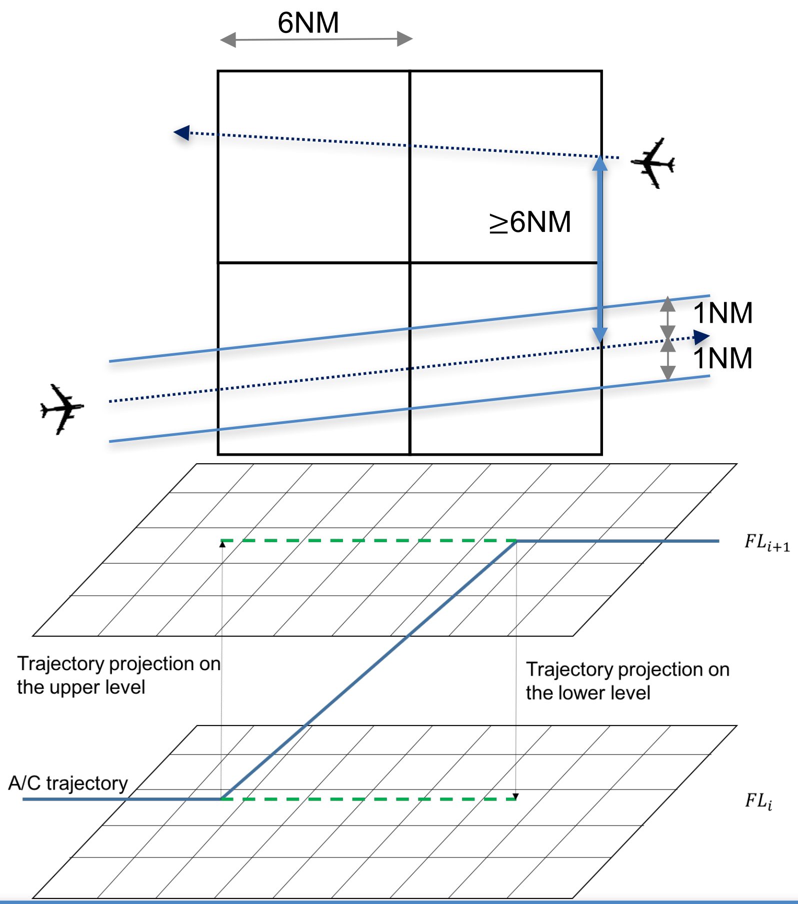
Detection in 2h of traffic takes less than 2’’

Detection algorithm has $O(n \times m)$complexity.

The application is implemented in Java according to a server-client architecture.

Detection Module: spatial uncertainty



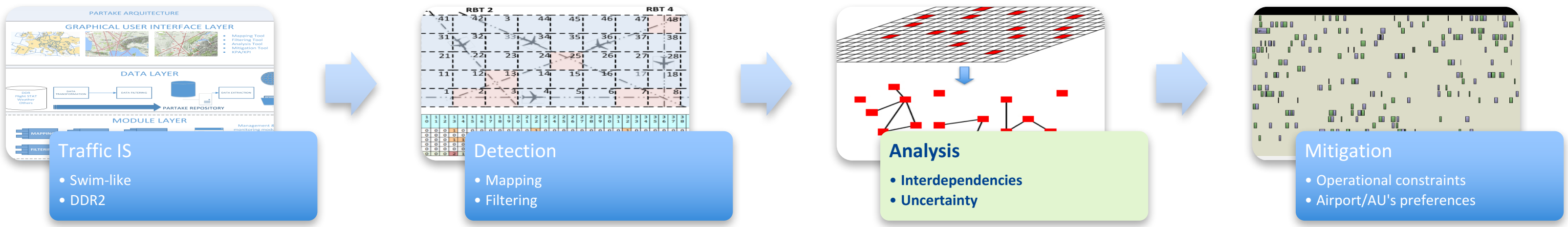


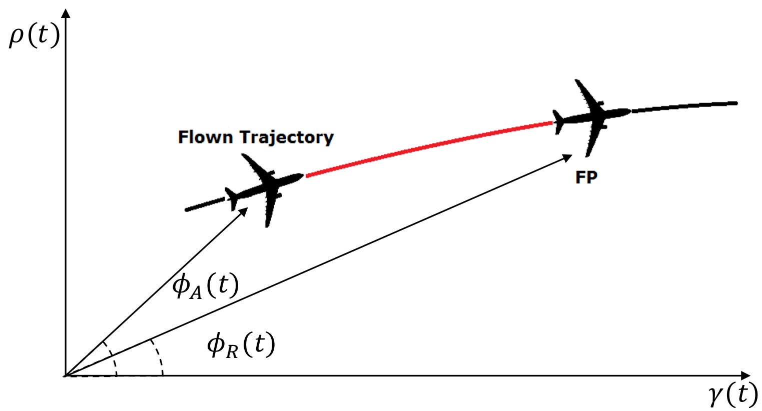
Uncertainty and Disturbances

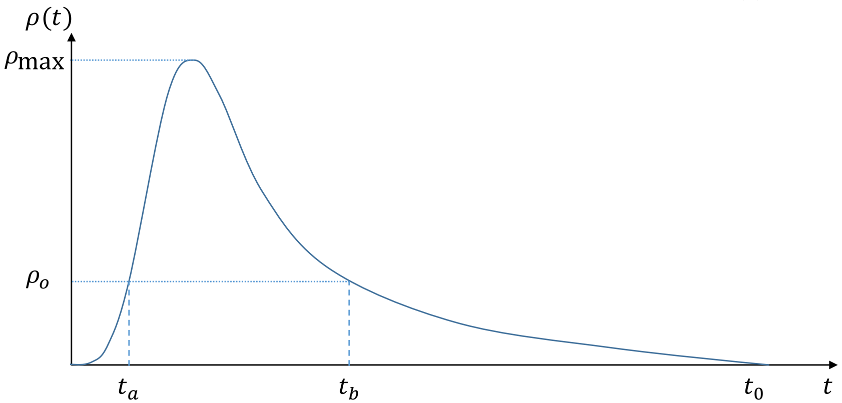
Lateral deviations: The size of 6NM has been defined by considering that, under TBO concept, the aircraft will be within a RBT envelope of 1 NM (e.g. slight diversions because of adverse wind condition)

Vertical deviations: a conservative approach is adopted during climb or descend by considering the aircraft to be in both flight levels during this manoeuvre.

Detection Module: temporal uncertainty







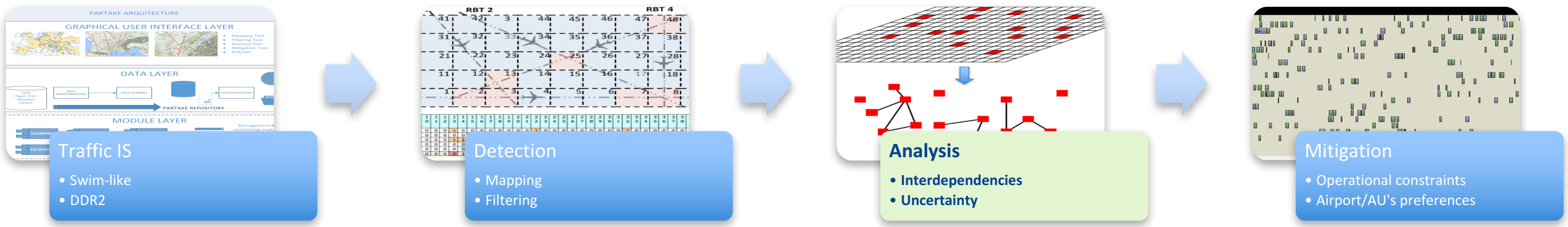
Uncertainty and Disturbances

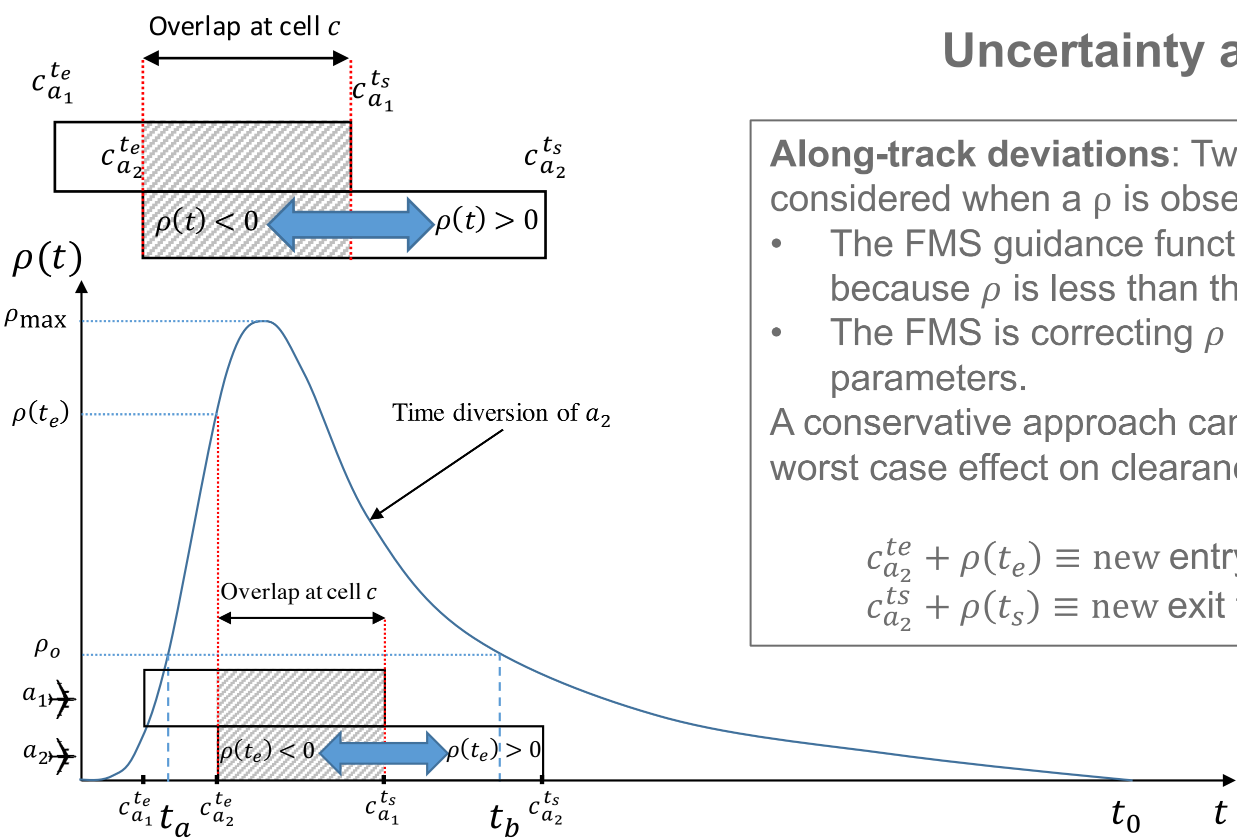
Along-track deviations: let be $\gamma(t) \in \mathbb{R}^3$ the RBT and $\bar{\gamma}(t) \in \mathbb{R}^3$ the actual flown trajectory. Then, under the TBO concept we will expect that $\|\gamma(t) - \bar{\gamma}(t)\|_2 \approx 0$ at least in most cases. if $\|\gamma(t) - \bar{\gamma}(t)\|_2 \neq 0$ is observed, then $\rho \in \mathbb{R}$ will be defined satisfying:

$$\|\gamma(t) - \bar{\gamma}(t+\rho)\|_2 = 0$$

The objective is to identify $\rho(t)$

Detection Module: temporal uncertainty





Uncertainty and Disturbances

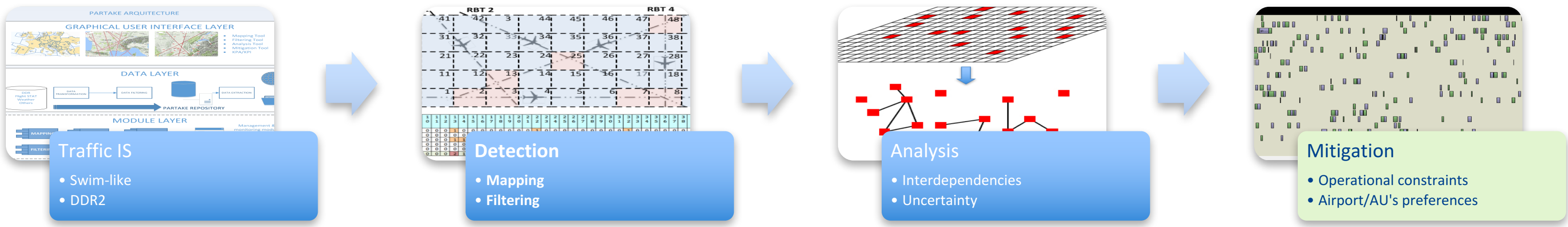
Along-track deviations: Two possibilities can be considered when a ρ is observed:

- The FMS guidance functionality has not yet acted because ρ is less than the alert value set on it.
- The FMS is correcting ρ changing some aircraft flight parameters.

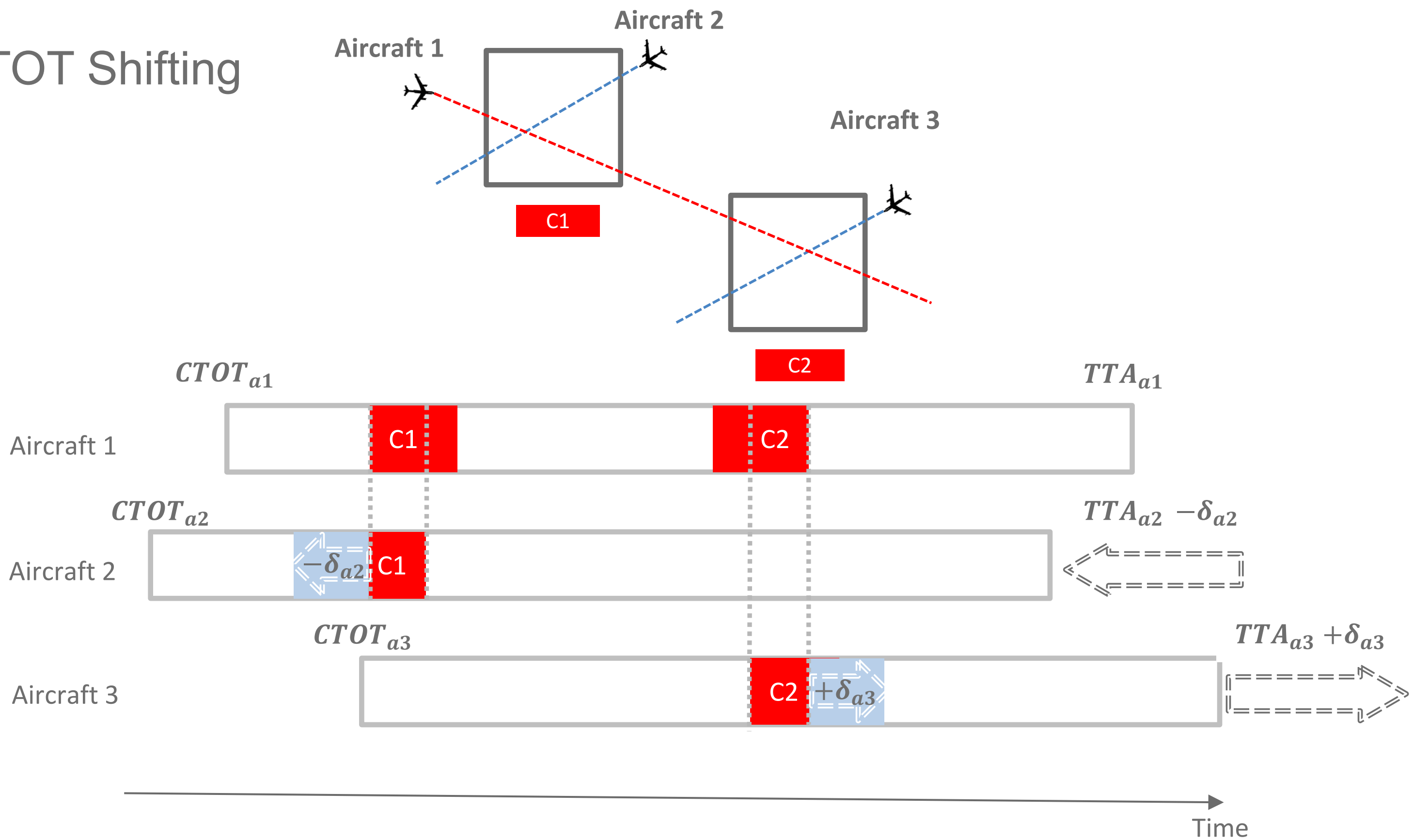
A conservative approach can be adopted to consider the worst case effect on clearance of the observed delay:

$$c_{a_2}^{te} + \rho(t_e) \equiv \text{new entry time}$$
$$c_{a_2}^{ts} + \rho(t_s) \equiv \text{new exit time}$$

Mitigation: tight trajectories resolution



CTOT Shifting



CTOT Shifting model

Parameters

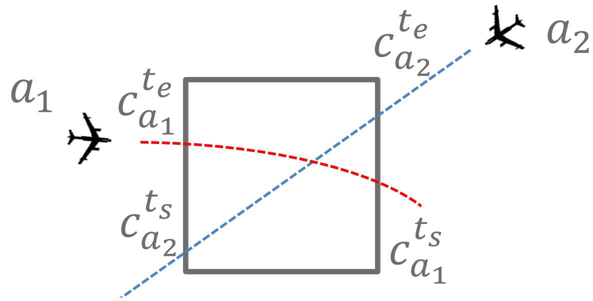
$C_A = \{< c, a > \mid \forall c \in C, \forall a \in A\}$

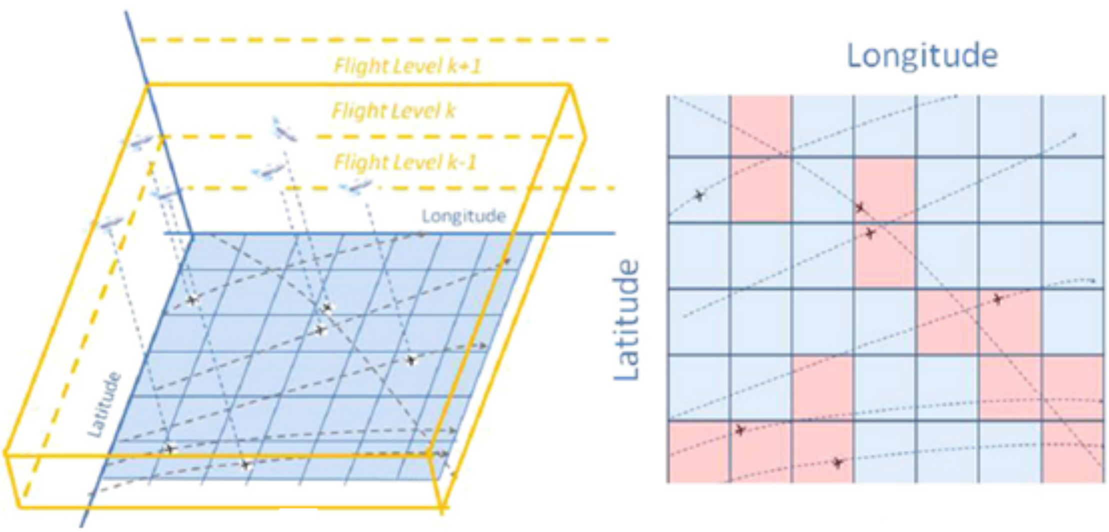
$c_a^{te} \equiv entry\ time$

$c_a^{ts} \equiv exit\ time$

A set of aircrafts

C set of cell with concurrence events





CTOT Shifting model

Parameters

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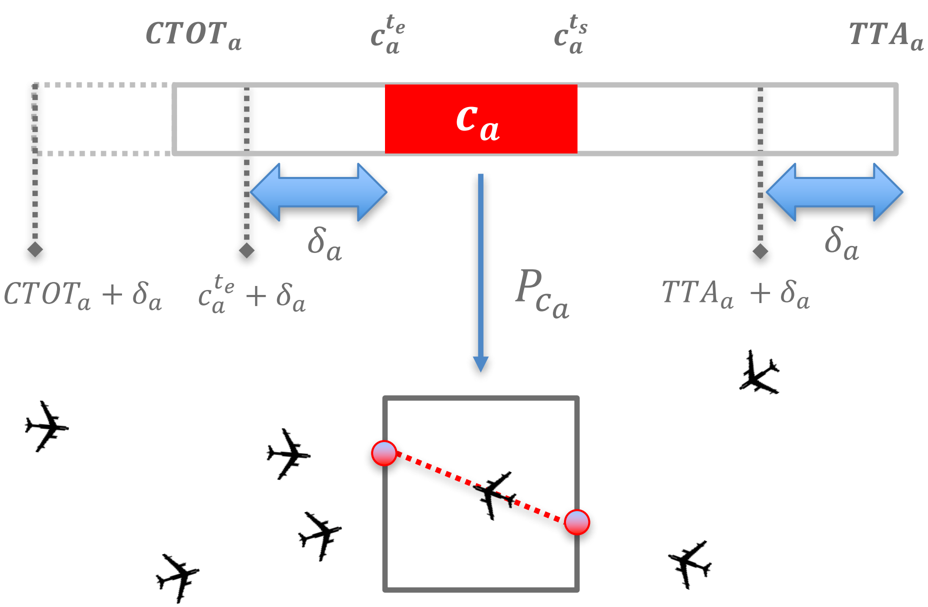

Decision Variables

$$\delta_a \in [-\delta_{min}, \delta_{max}],\ \forall a \in A$$

$$\forall c_a \in C_a$$

$$P_{ca} = [s_{c_a}, e_{c_a}), \qquad \forall\ c_a \in C_A$$

$$sz(P_{c_a}) = e_{c_a} - s_{c_a} (= c_a^{ts} - c_a^{te})$$

$$P_{c_a} \in [c_a^{te} - \delta_{min}, c_a^{ts} + \delta_{max}]$$

$$\forall c \in C$$

$$F_c = \{P_{c_a} | c_a \in C_A\}$$

$$\pi: F_c \rightarrow [1, m],\ m = |F_c|$$

$$\forall\ P_{c_{ai}}, P_{c_{aj}} \in F_c$$

$$P_{c_{ai}} \neq P_{c_{aj}} \Rightarrow \pi(\ P_{c_{ai}}) \neq \pi\left(P_{c_{aj}}\right)$$

CTOT Shifting model

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Parameters

$$C_A = \{< c, a > \mid \forall c \in C, \forall a \in A\}$$
$$c_a^{te} \equiv entry\ time$$
$$c_a^{ts} \equiv exit\ time$$

Constraints

$$s(P_{c_a}) = c_a^{te} + \delta_a, \forall c_a \in C_A$$
$$\forall P_{c_i}, P_{c_j} \in F_c$$
$$NO(F_c) \Leftrightarrow \pi(P_{c_i}) < \pi(P_{c_j})$$
$$\Rightarrow e(P_{c_i}) \leq s(P_{c_j})$$

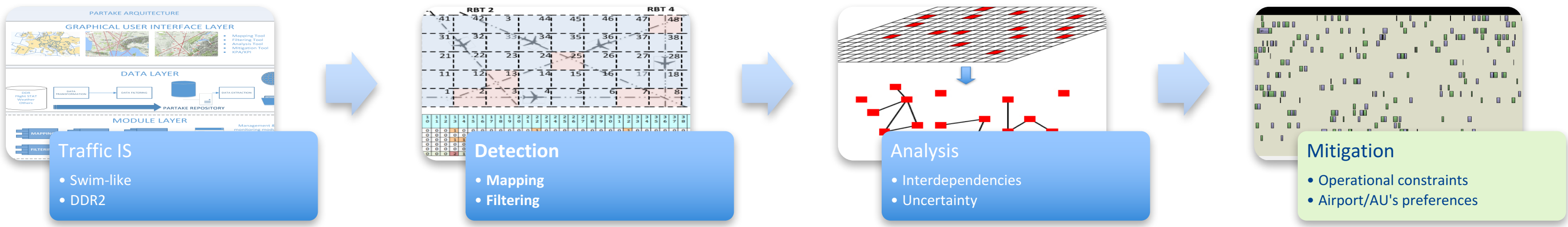
Objective

$$\min_{a \in A} \sum_{a=1}^{n} |\delta_a|$$

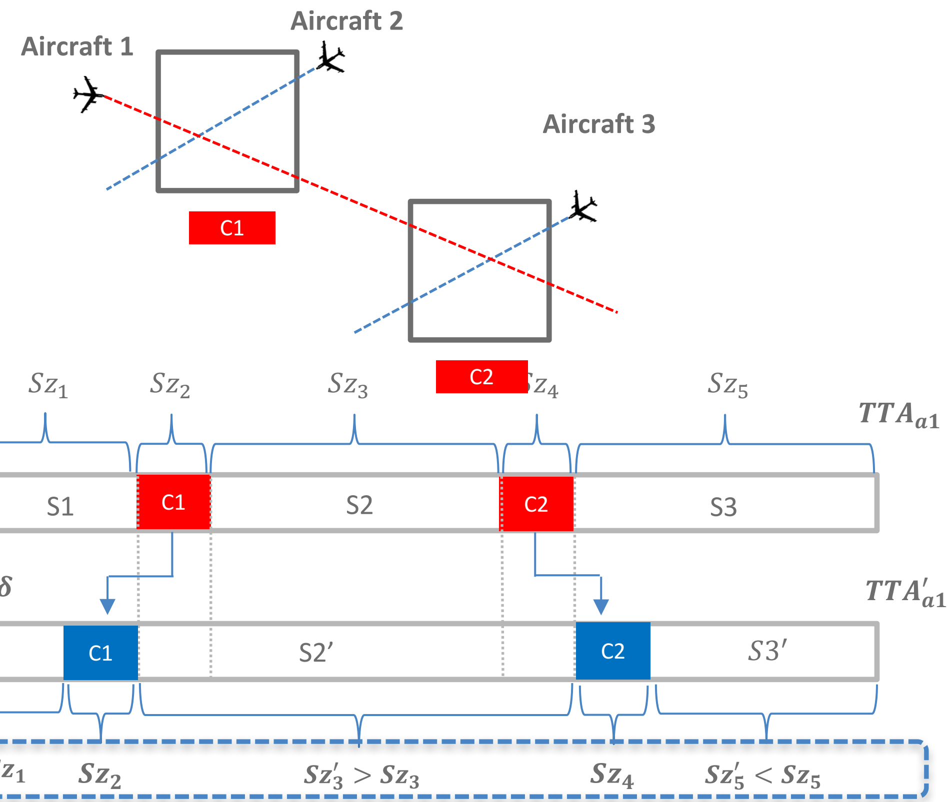
Decision Variables

$$\delta_a \in [-\delta_{min}, \delta_{max}], \forall a \in A$$
$$\forall c_a \in C_a$$
$$P_{ca} = [s_{c_a}, e_{c_a}), \qquad \forall c_a \in C_A$$
$$sz(P_{c_a}) = e_{c_a} - s_{c_a} (= c_a^{ts} - c_a^{te})$$
$$P_{c_a} \in [c_a^{te} - \delta_{min}, c_a^{ts} + \delta_{max}]$$
$$\forall c \in C$$
$$F_c = \{P_{c_a} | c_a \in C_A\}$$
$$\pi: F_c \rightarrow [1, m], m = |F_c|$$
$$\forall P_{c_{ai}}, P_{c_{aj}} \in F_c$$
$$P_{c_{ai}} \neq P_{c_{aj}} \Rightarrow \pi(P_{c_{ai}}) \neq \pi(P_{c_{aj}})$$

Mitigation: tight trajectories resolution



CTOT Shifting and Speed adjustments



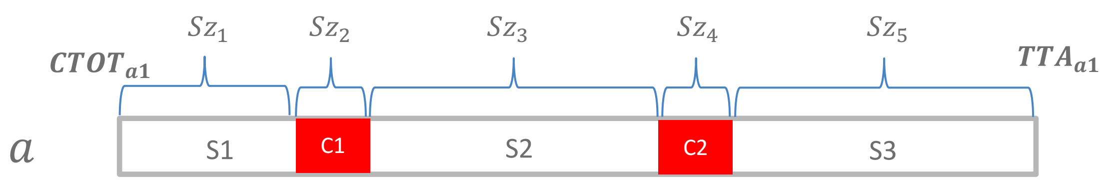
CTOT Shift and Speed adjustments model

Parameters

$RBT_a = \{\hat{g}_i^a \mid \forall a \in A, i = 1..p(a)\}$

$\hat{g}_i^a \equiv$ segment of a trajectory

$p(a) \equiv$ # of required segments



$$RBT_a = \{\hat{g}_1^a, \hat{g}_2^a, \hat{g}_3^a, \hat{g}_4^a, \hat{g}_5^a\} = \{S_1, C_1, S_2, C_2, S_3\}$$

$$p(a) = 5$$

$$s(\hat{g}_i^a) = \text{start time of } \hat{g}_i^a,$$

$$\mathrm{e}(\hat{g}_i^a) = end\ time\ of\ \hat{g}_i^a$$

CTOT Shift and Speed adjustments model

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Parameters

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$\hat{g}_i^a \equiv$ segment of a trajectory

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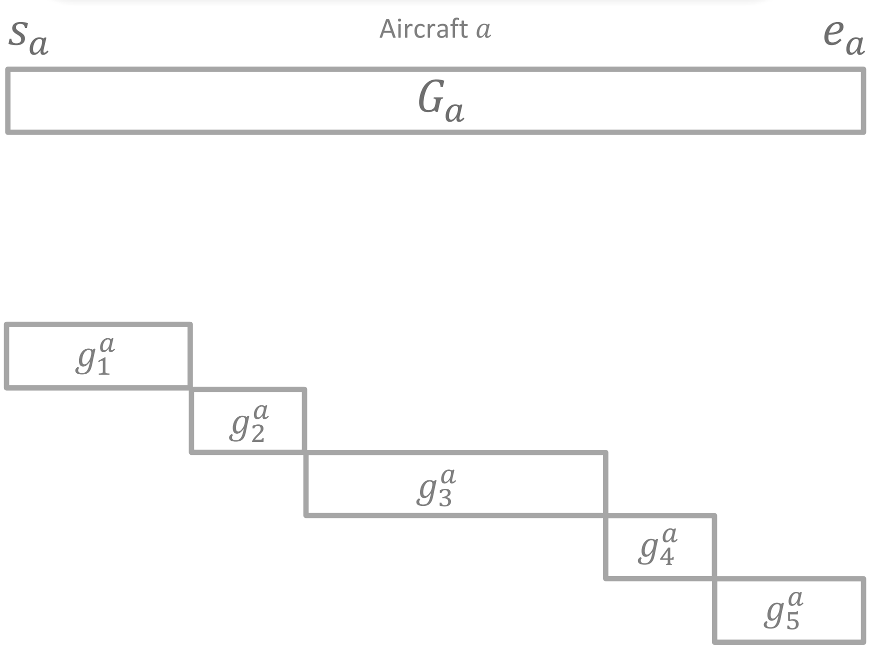

Decision Variables

$G_a = [s_a, e_a)$

$\forall a \in A$

$g_i^a = [s(g_i^a), e(g_i^a)), i = 1..p(a)$

$sz(g_i^a) = e(g_i^a) - s(g_i^a)$

$sz(g_i^a) \in [sz(\hat{g}_i^a) - l(\hat{g}_i^a), sz(\hat{g}_i^a) + l(\hat{g}_i^a)]$

$l(\hat{g}_i^a) = sz(\hat{g}_i^a) \times l, \quad l \in [0,1]$

$\forall a \in A$

$T_a = \{g_i^a \mid \forall a \in A, i \in 1..p(a)\}$

$\pi: T_a \rightarrow [1, p(a)]$

$\forall g_i^a, g_j^a \in T_a$

$g_i^a \neq g_j^a \Rightarrow \pi(g_i^a) \neq \pi(g_j^a)$

CTOT Shift and Speed adjustments model

Parameters

$RBT_a = \{\hat{g}_i^a \mid \forall a \in A, i = 1..p(a)\}$

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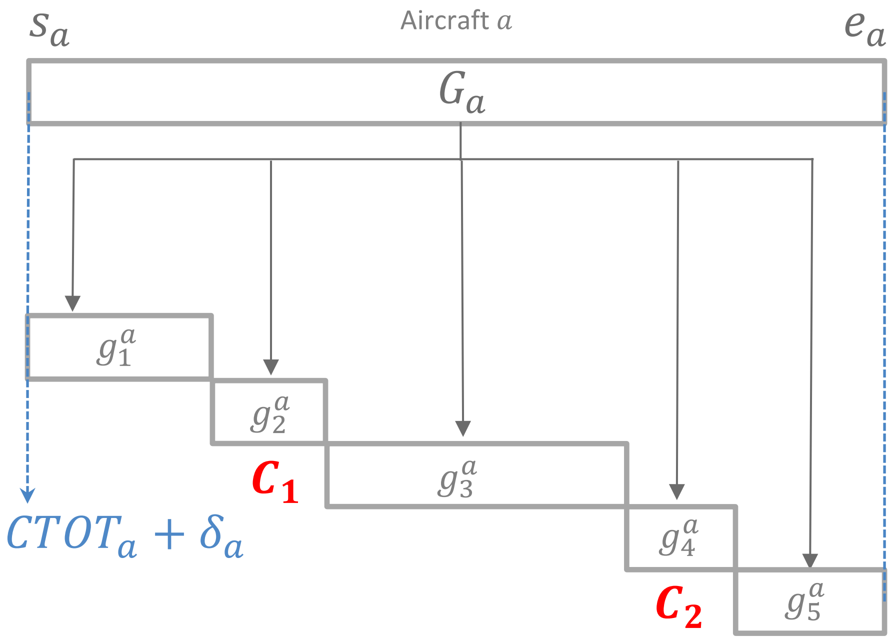


Constraints

$span(G_a, \{g_i^a\}), \forall a \in A, \forall g_i^a \in T_a$

$$\begin{cases} s(G_a) = \min_{i \in [1,p(a)]}(\{s(g_i^a)\}) \\ e(G_a) = \max_{i \in [1,p(a)]}(\{e(g_i^a)\}) \end{cases}$$

$NO(G_a) \Leftrightarrow \pi(g_i^a) < \pi(g_j^a)$

$\Rightarrow e(g_i^a) \leq s(g_j^a), \forall i, j$

$e(g_i^a) \leq s(g_j^a), \forall i, j : i < j$

$e(g_i^a) = s(g_j^a), \forall i, j : j = i + 1$

$s(G_a) = CTOT_a + \delta_a$

$e(G_a) \in [TTA_a - 1, TTA_a + 1]$

$\forall c_a \in c_A$

$$\begin{cases} s(g_i^a) = s(P_{c_a}) \\ e(g_i^a) = e(P_{c_a}) \end{cases} \Leftrightarrow \begin{matrix} s(\hat{g}_i^a) = c_a^{t_e} \\ e(\hat{g}_i^a) = c_a^{t_s} \end{matrix}$$

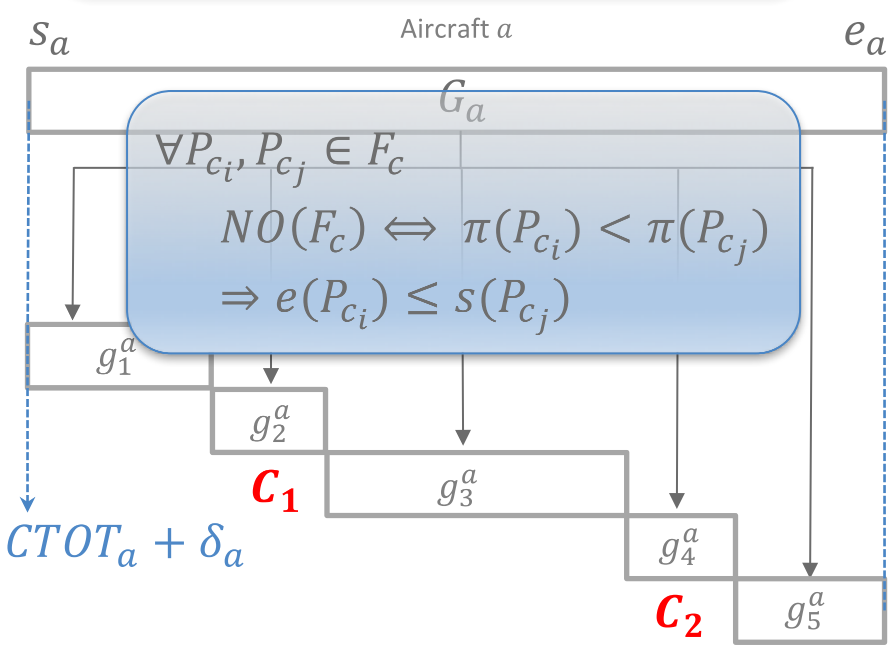
CTOT Shift and Speed adjustments model

Parameters

$RBT_a = \{\hat{g}_i^a \mid \forall a \in A, i = 1..p(a)\}$

$\hat{g}_i^a \equiv$ segment of a trajectory

$p(a) \equiv$ # of required segments



Constraints

$span(G_a, \{g_i^a\}), \forall a \in A, \forall g_i^a \in T_a$

$$\begin{cases} s(G_a) = \min\limits_{i \in [1,p(a)]}(\{s(g_i^a)\}) \\ e(G_a) = \max\limits_{i \in [1,p(a)]}(\{e(g_i^a)\}) \end{cases}$$

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$e(g_i^a) \leq s(g_j^a), \forall i, j: i < j$

$e(g_i^a) = s(g_j^a), \forall i, j: j = i + 1$

$s(G_a) = CTOT_a + \delta_a$

$e(G_a) \in [TTA_a - 1, TTA_a + 1]$

$\forall c_a \in c_A$

$$\begin{cases} s(g_i^a) = s(P_{c_a}) \\ e(g_i^a) = e(P_{c_a}) \end{cases} \Leftrightarrow \begin{matrix} s(\hat{g}_i^a) = c_a^{te} \\ e(\hat{g}_i^a) = c_a^{ts} \end{matrix}$$

CTOT Shift and Speed adjustments model

Parameters

$RBT_a = \{\hat{g}_i^a \mid \forall a \in A, i = 1..p(a)\}$

$\hat{g}_i^a \equiv$ segment of a trajectory

$p(a) \equiv$ # of required segments

Objective function

$$L(G_a) = \begin{cases} 1, & e(G_a) \notin [TTA_a - 1, TTA_a + 1] \\ 0, & \text{otherwise} \end{cases}$$

$$\min_{a \in A} w_1 \sum_{a=1}^{n} |\delta_a| + w_2 \sum_{a=1}^{n} L(G_a)$$

Constraints

$span(G_a, \{g_i^a\}), \forall a \in A, \forall g_i^a \in T_a$

$$\begin{cases} s(G_a) = \min_{i \in [1,p(a)]} (\{s(g_i^a)\}) \\ e(G_a) = \max_{i \in [1,p(a)]} (\{e(g_i^a)\}) \end{cases}$$

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$s(G_a) = CTOT_a + \delta_a$

$e(G_a) \in [TTA_a - 1, TTA_a + 1]$

$\forall c_a \in c_A$

$$\begin{cases} s(g_i^a) = s(P_{c_a}) \\ e(g_i^a) = e(P_{c_a}) \end{cases} \Leftrightarrow \begin{matrix} s(\hat{g}_i^a) = c_a^{te} \\ e(\hat{g}_i^a) = c_a^{ts} \end{matrix}$$

CTOT Shift and Speed adjustments model with along track uncertainty

Parameters

$C_A = \{< c, a > \mid \forall c \in C, \forall a \in A\}$

$c_a^{te} + \rho(t_e) \equiv$ actual entry time

$c_a^{ts} + \rho(t_s) \equiv$ actual exit time

$RBT_a = \{\hat{g}_i^a \mid \forall a \in A, i = 1..p(a)\}$

$\hat{g}_i^a \equiv$ segment of a trajectory

$p(a) \equiv$ # of required segments

Objective function

$$L(G_a) = \begin{cases} 1, & e(G_a) \notin [TTA_a - 1, TTA_a + 1] \\ 0, & \text{otherwise} \end{cases}$$

$$\min_{a \in A} w_1 \sum_{a=1}^{n} |\delta_a| + w_2 \sum_{a=1}^{n} L(G_a)$$

Constraints

$span(G_a, \{g_i^a\}), \forall a \in A, \forall g_i^a \in T_a$

$$\begin{cases} s(G_a) = \min_{i \in [1, p(a)]} (\{s(g_i^a)\}) \\ e(G_a) = \max_{i \in [1, p(a)]} (\{e(g_i^a)\}) \end{cases}$$

$NO(G_a) \Leftrightarrow \pi(g_i^a) < \pi(g_j^a)$

$\Rightarrow e(g_i^a) \leq s(g_j^a), \forall i, j$

$e(g_i^a) \leq s(g_j^a), \forall i, j: i < j$

$e(g_i^a) = s(g_j^a), \forall i, j: j = i + 1$

$s(G_a) = CTOT_a + \delta_a$

$e(G_a) \in [TTA_a - 1, TTA_a + 1]$

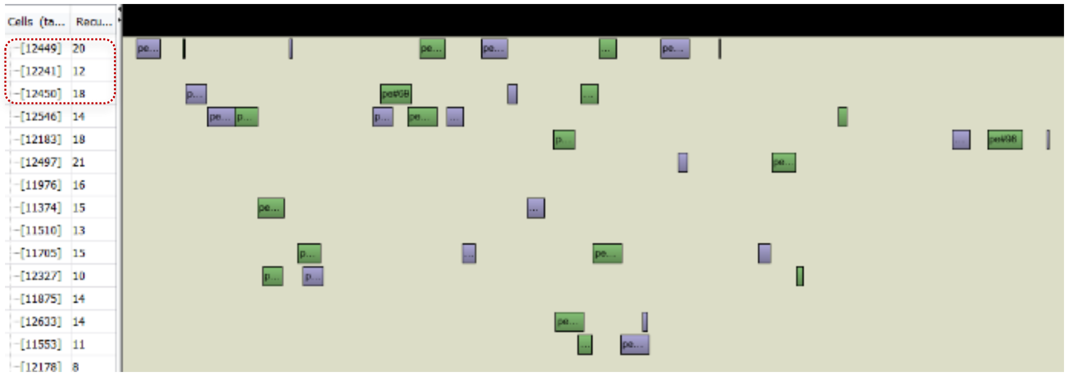
$\forall c_a \in c_A$

$$\begin{cases} s(g_i^a) = s(P_{c_a}) \\ e(g_i^a) = e(P_{c_a}) \end{cases} \Leftrightarrow \begin{matrix} s(\hat{g}_i^a) = c_a^{te} \\ e(\hat{g}_i^a) = c_a^{ts} \end{matrix}$$

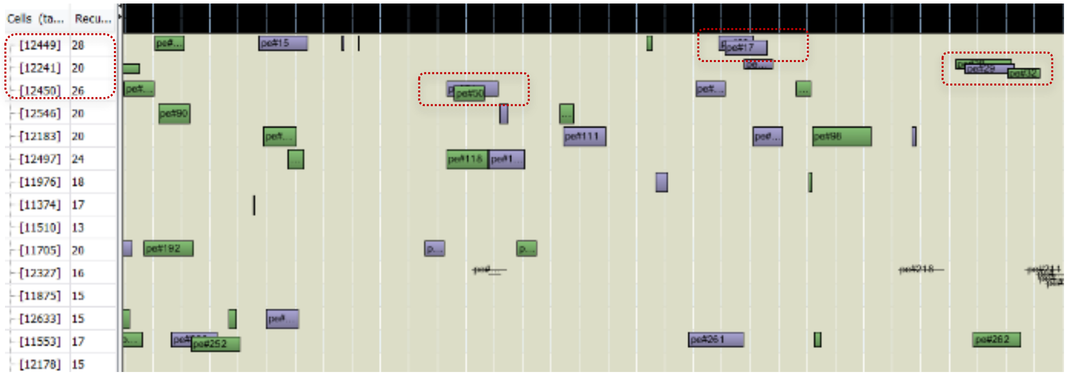
Results

Test scenario

- Over-stressed realistic scenario with 4010 real 4D trajectories in the European airspace for a 2h time frame
- We assumed TBO without uncertainties
- The CP model has been implemented with the ILOG Optimization Suite



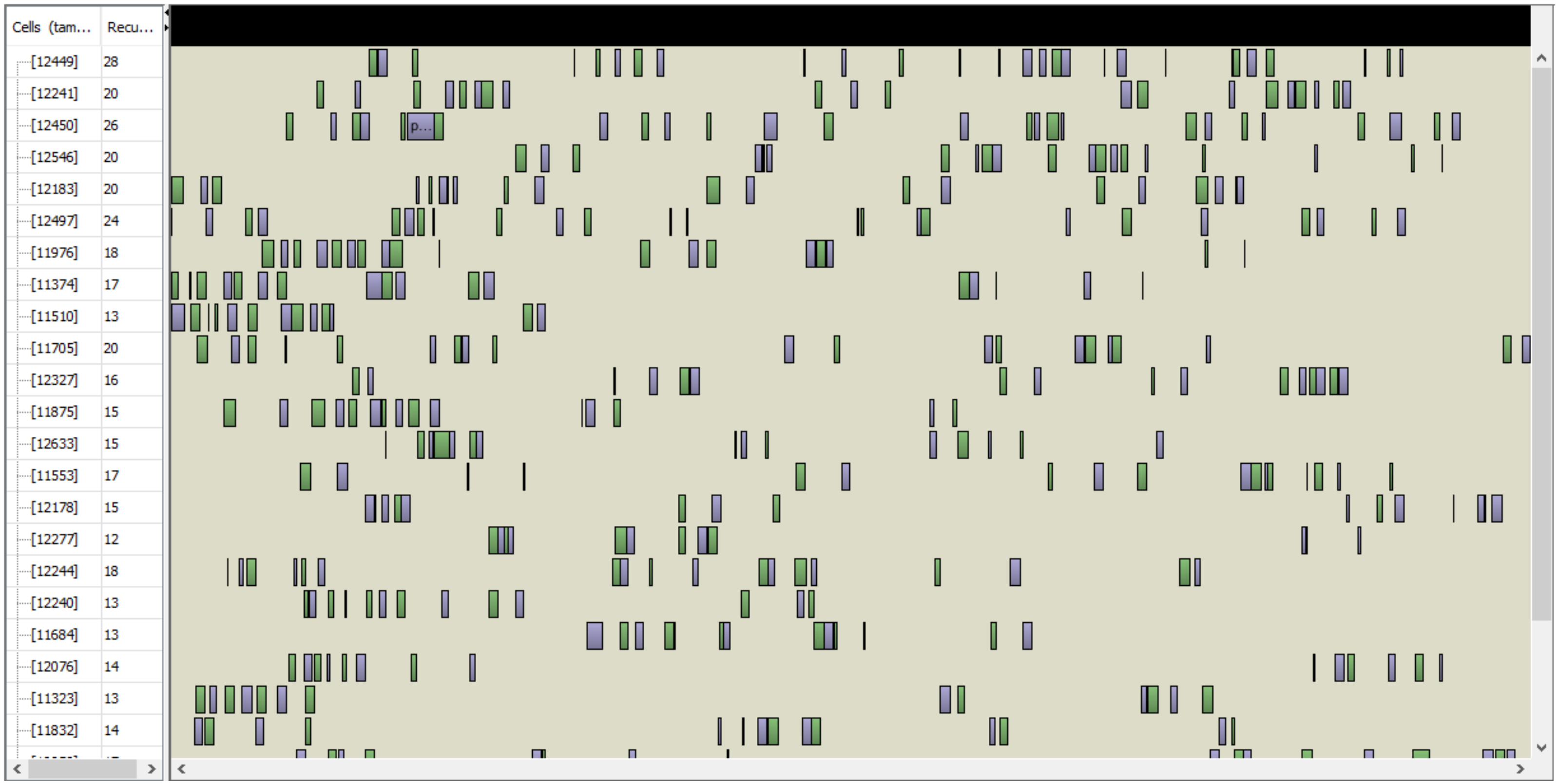
conflict free enroute traffic



emerging conflict when inserting the departing traffic

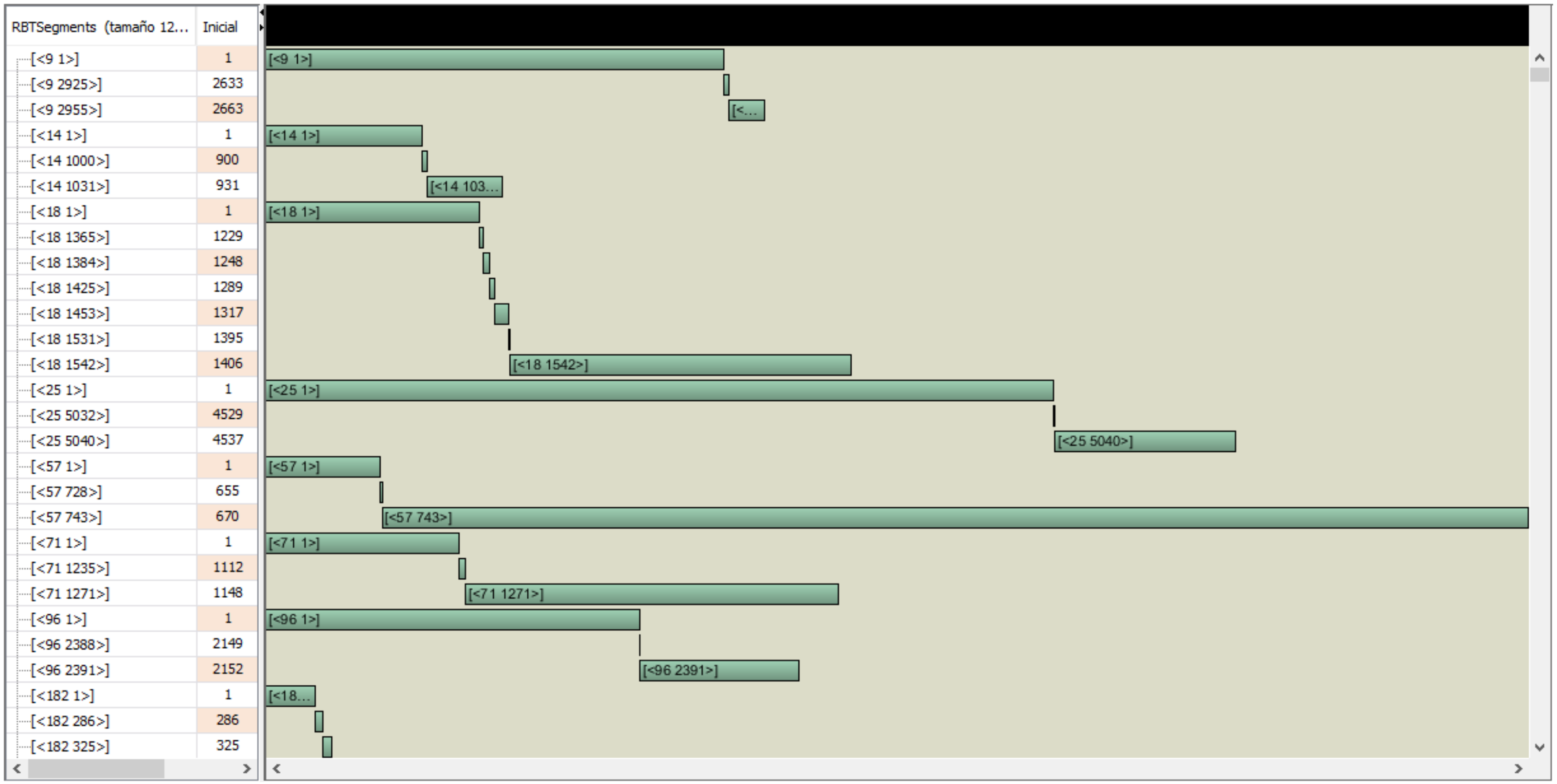
Results

SESAR
JOINT UNDERTAKING



Conflict free solution after applying small adjustments on CTOT and segment' speed

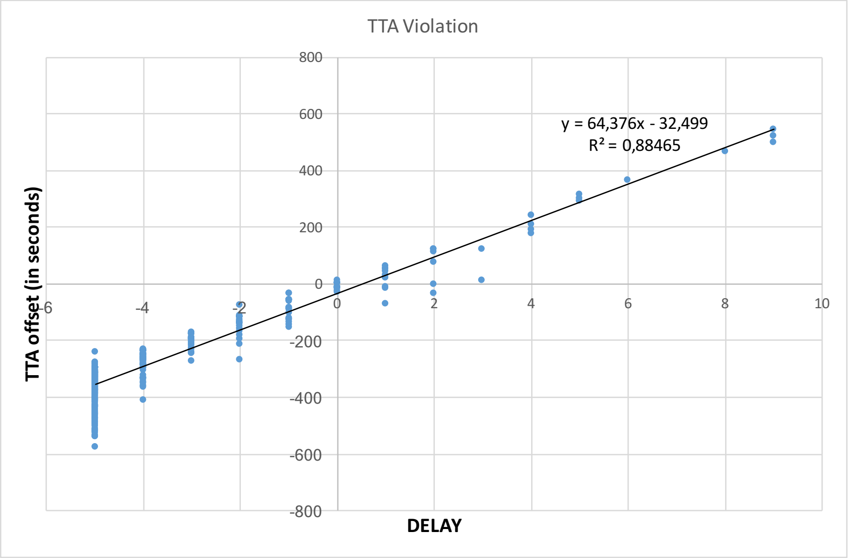
Results



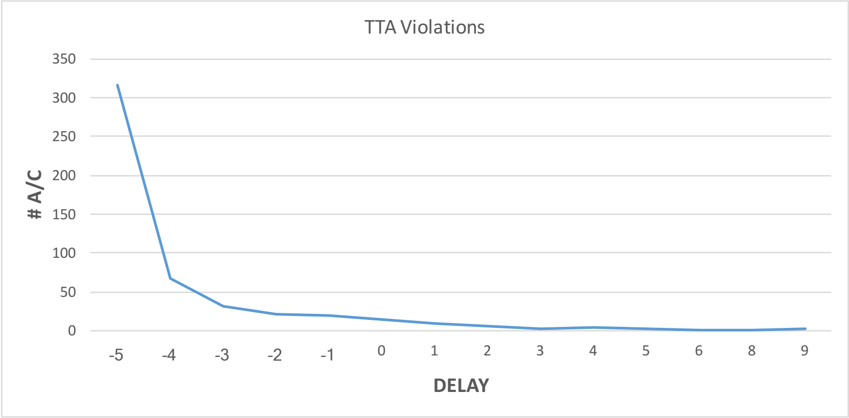
Flight segments obtained for each aircraft

Results

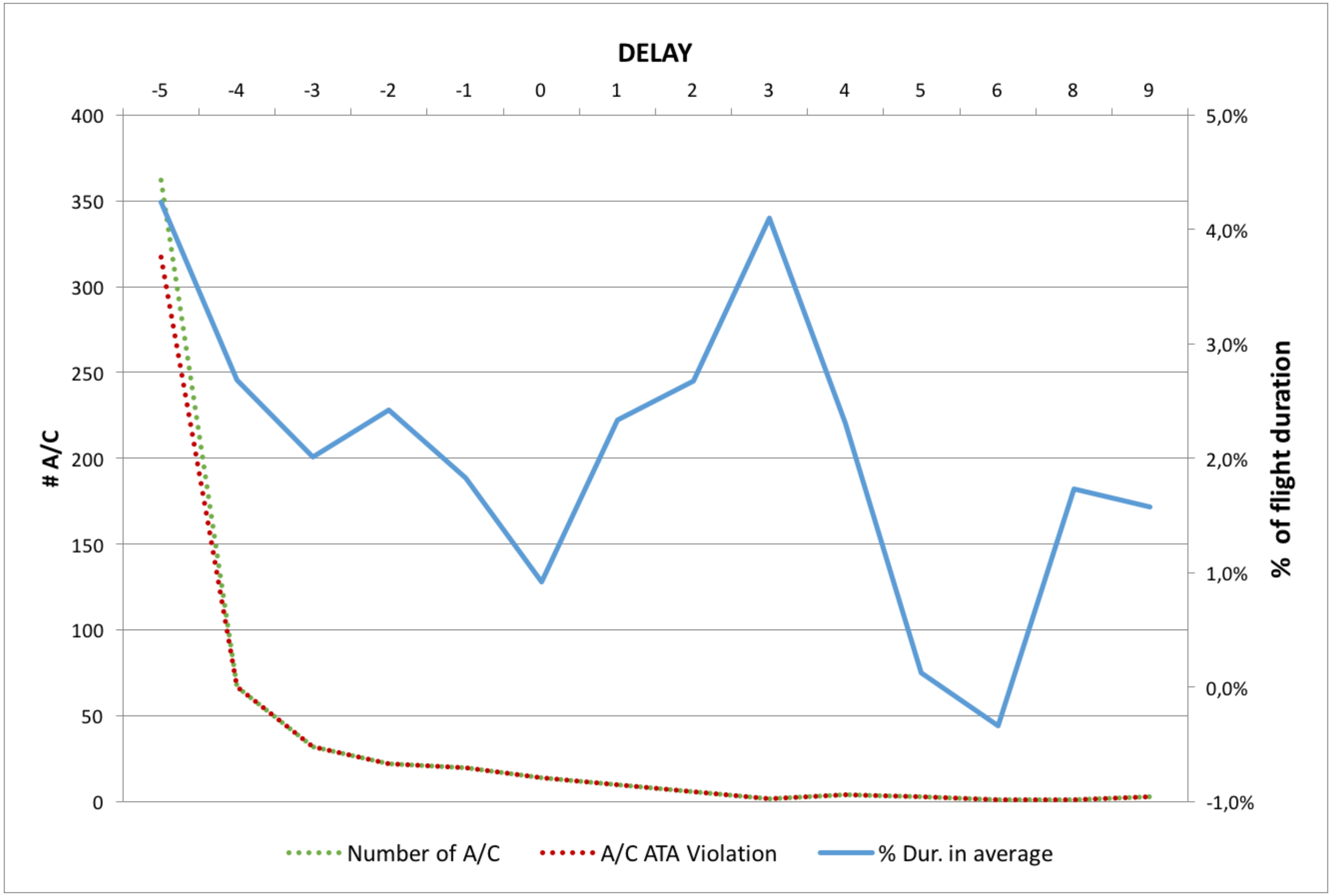
Correlation between TTA violation and the delays applied to the aircraft takeoff times



A/C not meeting their TTA with respect to the applied CTOT delay



Results

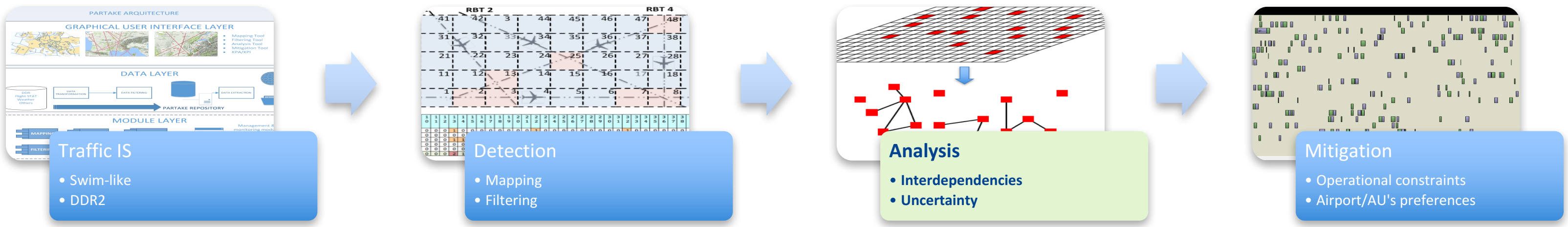


DELAY
-5
-4
-3
-2
-1
0
1
2
3
4
5
6
8
9
400
350
300
250
200
150
100
50
0
A/C
5,0%
4,0%
3,0%
2,0%
1,0%
0,0%
-1,0%
% of flight duration
Number of A/C
A/C ATA Violation
% Dur. in average

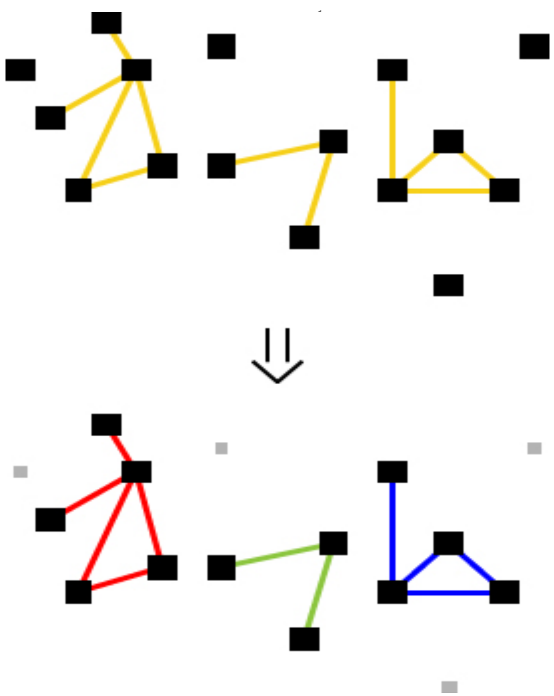
Conclusions

- In this work a CP model is presented for solving the concurrence events that might happen when the departure traffic is inserted into the enroute traffic.
- The CP model has been proved in a realistic and overstressed scenario and it has been able to find suboptimal solutions in a timeframe of 180 seconds for all the performed experiments
- The model constraints ensure that all the proximate events are resolved by introducing small time adjustment both on the CTOT and relevant TTO’s while maximizing the adherence to the RBT’s
- Preserving the TTA has been relaxed and the objective function penalizes the TTA violation, since there is a limit of the trajectory elasticity and speed adjustments are bounded to a percentage of the total RBT duration
- Although the model is not able to ensure that the ATM concept of preserving the TTA in a strict time frame is met, the CP solver can find solutions that remove all the conflicts reducing the number of potential ATC interventions.

Next steps: graph modeling and analysis



Graph representation of coupled concurrence



Graph theory and algorithms advantages

1. **Reducing problem size**: Since interdependencies are the most critical aspect for achieving good enough resolutions, extracting the connected components of G(V,E) **ensures a reduction** of the problem size **maintaining all interdependencies**, which enable **parallelization** during mitigation phase.
2. **Deadlock detection**: Analyzing the connected components of G in terms of the nature of its vertices and cycles, the analysis tool can anticipate and remove interdependencies leading to **deadlock** configurations or to a degradation in the solutions.

Graph theory has shown very promising results for finding independent clusters of trajectories

First experiments are ongoing to validate the adopted time uncertainty approach

Consortium partners:

UAB Universitat Autònoma de Barcelona

ENAC

Cranfield UNIVERSITY

ASLOGIC

A Constraint Programming Model with Time Uncertainty for Cooperative Flight Departures

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SESAR JOINT UNDERTAKING

Founding Members

EUROPEAN UNION EUROCONTROL

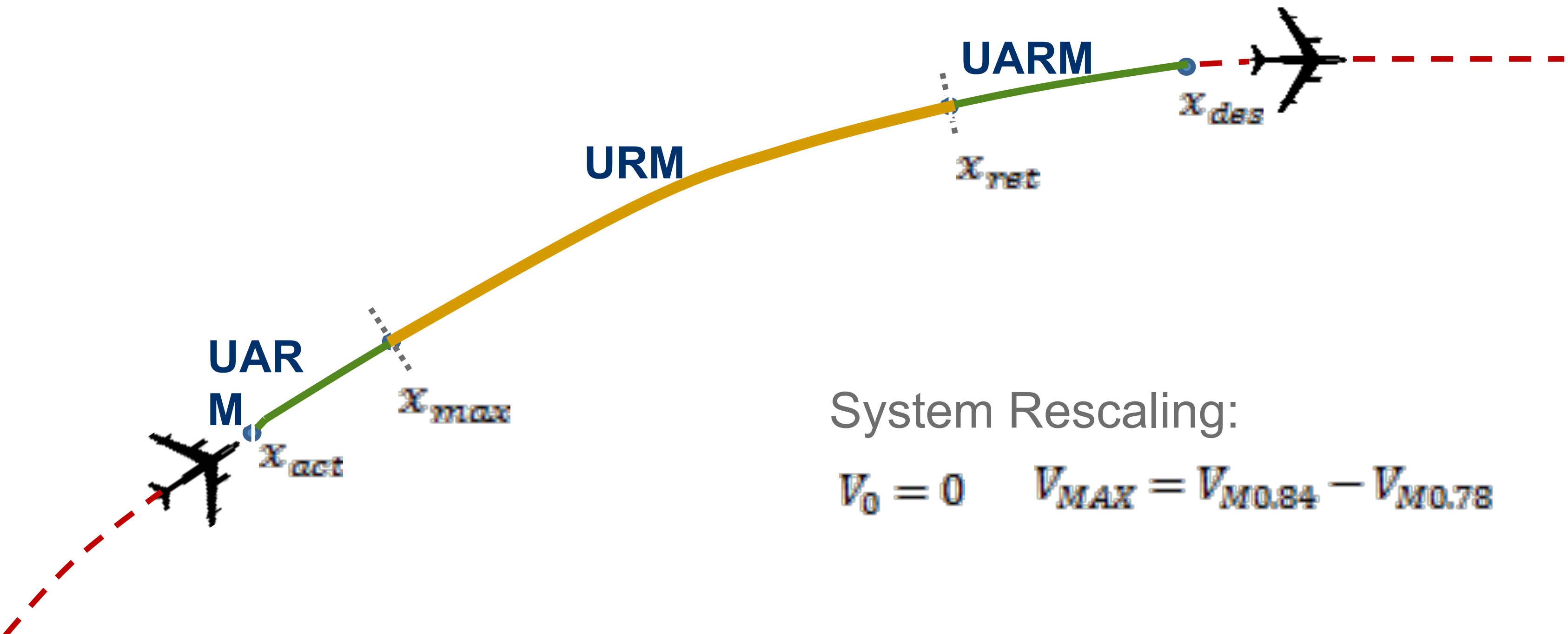
Theoretical concepts

- Distance
  Haversine formula
- Bearing
  Initial bearing
- Wind
  Wind effect
- Atmosphere
  formulas

Model assumptions

- Initial time diversion
- FMS changes the flight profile in order to remove the diversion as soon as possible
- Constant True Airspeed
- Aircraft will go over the rhumb line
- Constant instant accelerations

FMS interventions



System Rescaling:

$$V_0 = 0 \qquad V_{MAX} = V_{M0.84} - V_{M0.78}$$

PHASE	ECUATION TO SOLVE
UARM1	$(V_{MAX} - W)^2 = 2 \times a \times \Delta_{UARM1}x \qquad t_{UARM1} = \frac{V_{MAX} - W}{a}$
UARM2	$(V_{MAX} - W)^2 = -2 \times \Delta_{UARM2}x \qquad t_{UARM2} = V_{MAX} - W$
URM	$\Delta x - \Delta_{UARM1}x - \Delta_{UARM2}x = (V_{MAX} - W) \times t_{URM} \qquad t_{URM} = \frac{\Delta_{URM}x}{V_{MAX} - W}$

Distance:

$$\mathrm{hav}\left(\frac{\mathrm{d}}{\mathrm{r}}\right) = \mathrm{hav}(\phi_2 - \phi_1) + \cos(\phi_1)\cos(\phi_2)\mathrm{hav}(\lambda_2 - \lambda_1)$$

Bearing:

$$\tan^{-1}\frac{\sin(\lambda_1 - \lambda_2)\cos(\phi_2)}{\cos(\phi_1)\sin(\phi_2) - \sin(\phi_1)\cos(\phi_2)\cos(\lambda_1 - \lambda_2)}$$

Wind:

$$\mathrm{W} = \mathrm{V_W} \times \cos(|\mathrm{B} - \mathrm{H_W} \pm \delta|)$$

Atmosphere (BADA):

$$a = \sqrt{\kappa R T}$$

$$V_{TAS} = M\sqrt{\kappa R T}$$

$$R = 287.05287\ \mathrm{m}^2/(\mathrm{K} \cdot \mathrm{s}^2)$$

$$\kappa = 1.4$$

$$V_{MAX} = V_{M0.84} - V_{M0.78}$$

$$V_0 = 0$$

$$x_{act}$$

$$x_{max}$$

$$x_{ret}$$

$$x_{des}$$

$$(\mathrm{V_{MAX}} - \mathrm{W})^2 = 2 \times \mathrm{a} \times \Delta_{\mathrm{UARM1}}\mathrm{x}$$

$$(\mathrm{V_{MAX}} - \mathrm{W})^2 = -2 \times \Delta_{\mathrm{UARM2}}\mathrm{x}$$

$$\Delta\mathrm{x} - \Delta_{\mathrm{UARM1}}\mathrm{x} - \Delta_{\mathrm{UARM2}}\mathrm{x} = (\mathrm{V_{MAX}} - \mathrm{W}) \times \mathrm{t_{URM}}$$

$$\mathrm{t_{UARM1}} = \frac{\mathrm{V_{MAX}} - \mathrm{W}}{\mathrm{a}}$$

$$\mathrm{t_{UARM2}} = \mathrm{V_{MAX}} - \mathrm{W}$$

$$\mathrm{t_{URM}} = \frac{\Delta_{\mathrm{URM}}\mathrm{x}}{\mathrm{V_{MAX}} - \mathrm{W}}$$