hep-th/0602241
MAD-TH-06-2
DESY 06-017

Generalized $N = 1$ Orientifold Compactifications and the Hitchin functionals

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ABSTRACT

The four-dimensional $N = 1$ supergravity theories arising in compactifications of type IIA and type IIB on generalized orientifold backgrounds with background fluxes are discussed. The Kähler potentials are derived for reductions on $SU(3)$ structure orientifolds and shown to consist of the logarithm of the two Hitchin functionals. These are functions of even and odd forms parameterizing the geometry of the internal manifold, the B-field and the dilaton. The superpotentials induced by background fluxes and the non-Calabi-Yau geometry are determined by a reduction of the type IIA and type IIB fermionic actions on $SU(3)$ and generalized $SU(3) \times SU(3)$ manifolds. Mirror spaces of Calabi-Yau orientifolds with electric and part of the magnetic NS-NS fluxes are conjectured to be certain $SU(3)\times SU(3)$ structure manifolds. Evidence for this identification is provided by comparing the generalized type IIA and type IIB superpotentials.

1 Introduction

The construction of semi-realistic type II string vacua for particle physics and cosmology attracted many efforts within the last years [1, 2]. Of particular interest are scenarios with space-time filling D-branes, which can provide for non-Abelian gauge groups on their world-volume. However, demanding the internal manifold to be compact, consistent setups also need to include orientifold planes carrying a negative tension. They arise in string theories modded out by a geometric symmetry of the background in addition to the world-sheet parity operation [3, 4, 5].

From a phenomenological point of view orientifold compactifications resulting in a four-dimensional $N = 1$ supergravity theory are of importance. Prominent examples are type II theories on Calabi-Yau orientifolds, since reductions on Calabi-Yau manifolds yield a four-dimensional $N = 2$ supergravity while the orientifold projection breaks the supersymmetry further down to $N = 1$ [6, 7, 8, 9, 10, 11]. In general these theories admit a large number of moduli fields which are flat directions of the potential and not fixed in the vacuum. A possible mechanism to generate a non-trivial potential for these fields is the inclusion of background fluxes arising as vacuum expectation values of field strengths in the supergravity theory [11]. This potential generically possesses supersymmetric vacua in which a part or all moduli are fixed [12, 7, 13, 14, 15, 16, 17, 18, 19, 20]. In order to study the properties of these vacua it is necessary to know the characteristic data of the corresponding four-dimensional $N = 1$ supergravity theory. Using a Kaluza-Klein reduction the four-dimensional $N = 1$ theories of type II Calabi-Yau orientifolds were determined in refs. [7, 22, 23, 24, 25] and reviewed, for example, in refs. [26, 11, 27, 28].

In this paper we determine the $N = 1$ data for a more general class of compactifications arising if the internal manifold $\mathcal{M}_6$ is no longer restricted to be Calabi-Yau. In order that the resulting four-dimensional theory still admits some supersymmetry $\mathcal{M}_6$ cannot be chosen arbitrarily, but rather has to admit at least one globally defined spinor. In case that $\mathcal{M}_6$ has exactly one globally defined spinor the structure group of the manifold reduces to $SU(3)$ [29, 30, 31]. Equivalently, these manifolds are characterized by the existence of two globally defined forms, a real two-form J and a complex three-form Ω. These forms are in general not closed, which indicates a deviation from the Calabi-Yau case. This difference can also be encoded by specifying a new connection on $\mathcal{M}_6$ with torsion which replaces the ordinary Levi-Cevita connection. The torsion can be interpreted as a background flux of the metric connection. Compactifications on $SU(3)$ structure manifolds were considered in the early works [32, 33, 34] and more recently extended in refs. [35]–[69]. In specific settings these 'metric fluxes' arise as mirrors of Calabi-Yau compactifications with electric NS-NS fluxes [36, 43].

Compactifying type II string theory on an $SU(3)$ structure manifold leads to an effective four-dimensional $N = 2$ supergravity theory with a potential depending on the torsion of $\mathcal{M}_6$. As we will discuss in more detail below, one can still impose an appropriate orientifold projection which truncates this theory to an $N = 1$ supergravity. For specific set-ups this was also argued in the recent works [21, 70, 71, 72]. Supersymmetric orientifold projections yield setups with $O6$ planes in type IIA while for type IIB reductions two setups with $O3$ and $O7$ as well as $O5$ or $O9$ planes are encountered. Our analysis focuses on the effective $N = 1$ four-dimensional supergravity theory for the bulk fields of these configurations, while freezing all moduli arising from the D-brane sector.

In contrast to the standard Calabi-Yau compactifications the reduction on $SU(3)$ structure manifolds is more subtle. This can be traced back to the fact that in these generalized compactifications the distinction between massless or light modes and the massive Kaluza-Klein modes is not anymore straightforward. Recall that in Calabi-Yau compactifications the massless modes are in one-to-one correspondence with the harmonic forms of $\mathcal{M}_6$. Background fluxes generate a potential for these modes and can lift them to an intermediate mass scale. In reductions on $SU(3)$ structure manifolds a potential is induced by the non-trivial torsion of $\mathcal{M}_6$. However, the masses acquired by the four-dimensional fields need not be generated at an intermediate scale. The specification of a distinguished finite set of modes corresponding to the light degrees of freedom is missing so far. It is therefore desirable to avoid a truncation to light modes by working with general forms on the ten-dimensional background $M_{3,1} \times \mathcal{M}_6$. Most of our calculations will be performed within this general approach. We will argue that it remains possible to determine the four-dimensional $N = 1$ spectrum by imposing the orientifold projection. Only in a second step we specify a reduction to a finite set of modes in order to illustrate our results and to discuss mirror symmetry to Calabi-Yau orientifolds with background fluxes.

In this paper we will focus mainly on the chiral field space of the four-dimensional theory. We will determine the local metric on this space and show that it can be derived from a Kähler potential as demanded by $N = 1$ supersymmetry. Since the orientifold projection induces a consistent reduction of a four-dimensional $N = 2$ supergravity theory to $N = 1$ this Kähler manifold is a subspace of the full $N = 2$ scalar field space [73, 74]. Locally it takes the form $\mathcal{M}^{\rm K} \times \mathcal{M}^{\rm Q}$ where $\mathcal{M}^{\rm K}$ and $\mathcal{M}^{\rm Q}$ are the subspaces of the $N = 2$ special Kähler and quaternionic manifolds respectively. $\mathcal{M}^{\rm Q}$ has half the dimension of the quaternionic space. The Kähler potentials for both manifolds are shown to be the logarithms of the Hitchin functionals [75, 76, 77] for specific even and odd forms on $\mathcal{M}_6$.

In compactifications on $SU(3)$ structure manifolds a scalar potential is induced by the torsion as well as possible background fluxes. Due to the $N = 1$ supersymmetry it can be encoded by a holomorphic superpotential and D-terms arising due to non-trivial gaugings. In this work we derive the superpotentials for both type IIA and type IIB orientifold setups by evaluating appropriate fermionic mass terms. This extends and confirms the results already present in the literature [18, 19, 59, 62, 21, 11]. The knowledge of the superpotential together with the Kähler potential is necessary to determine the conditions on four-dimensional supersymmetric vacua. It is readily checked that these conditions evaluated for the orientifold set-ups are in accord with the $N = 1$ conditions on ten-dimensional backgrounds derived in ref. [61, 66].

Recently, it was argued that more general four-dimensional supergravity theories can arise in compactifications of type II string theory on generalized manifolds with $SU(3) \times SU(3)$ structure [32, 47, 48, 61, 62]. The notion of a generalized (complex) manifold was first introduced by Hitchin [76] and Gualtieri [80]. An intensive discussion of $SU(3) \times SU(3)$ structures and their application in $N = 2$ compactifications can be found in the work of Graña, Waldram and Louis [62]. We will make some first steps in exploring $N = 1$ orientifold compactifications on manifolds with $SU(3) \times SU(3)$ structure by extending the orientifold projection to these spaces and deriving the induced super-potential due to the non-Calabi-Yau nature of the internal space. Our aim is to use these extended superpotentials to discuss possible mirror geometries of type II Calabi-Yau compactifications with fluxes.

The completion of mirror symmetry in the presence of NS-NS background fluxes is an area of intense current research [36, 43, 54, 78, 79, 62, 70, 65, 71]. For compactifications with electric NS-NS fluxes it was argued in refs. [36, 43] that the mirror geometry is a set of specific $SU(3)$ structure manifolds known as half-flat manifolds. To extend this conjecture to the magnetic NS-NS fluxes various more drastic deviations from the standard compactifications are expected [78, 79, 62, 70]. We will use our results on the orientifold superpotentials to conjecture a possible mirror geometry of compactifications with part of the electric and magnetic background fluxes. These mirrors are extensions of generalized manifolds with $SU(3) \times SU(3)$ structure.[1] Note however, that in order to accommodate the mirror of the magnetic NS-NS fluxes the mirror metric on the internal space might no longer be well-defined. In our analysis it will be sufficient to characterize these generalized spaces by the existence of special even and odd forms not making use of an associated metric.

This paper is organized as follows. At the end of this introduction we give a short summary of our results. In section 2 we briefly review some mathematical facts about $SU(3)$ structure manifolds and comment on the compactifications of the type II supergravity on these spaces. We immediately turn to the definition of the orientifold projections of the type IIA/B theories in section 3.1. This allows us to determine the $N = 1$ spectrum of the four-dimensional supergravity theories arising in the orientifold compactifications in section 3.2. The Kähler potentials and their relation to the Hitchin functionals are discussed in section 3.3. In section 3.4 we derive the superpotentials of type IIA and type IIB orientifolds induced by the background fluxes and the torsion of $\mathcal{M}_6$. In order to fully identify these superpotentials under mirror symmetry the compactifications need to be performed on a more general class of spaces $\mathcal{M}_{\tilde{Y}}$. In section 4 we use our results on the Kähler and superpotentials to conjecture a possible identification of part of the magnetic NS-NS fluxes with properties of the mirror space $\mathcal{M}_{\tilde{Y}}$.

Summary of results

For the convenience of the reader we will here briefly summarize our results. In type IIA orientifolds with $O6$ planes, the globally defined three-form Ω is combined into a normalized three-form $\Pi^{\rm odd} = C\Omega$, where C is proportional to the dilaton $e^{-\hat{\phi}}$. The real part of this form is complexified with the R-R three-form $\hat{C}_3$ with indices entirely on $\mathcal{M}_6$ into the combination $\Pi^{\rm odd}_c = \hat{C}_3 + i\text{Re}(\Pi^{\rm odd})$. The globally defined two-form J is complexified with the NS-NS field $\hat{B}_2$ as $J_c = -\hat{B}_2 + iJ$. The chiral fields of the four-dimensional theory arise by expanding the complex forms J_c and $\Pi^{\rm odd}_c$ into an appropriate set of real two- and three-forms of $\mathcal{M}_6$. The complex scalar coefficients in this expansion are the bosonic fields in the chiral multiplets. The Kähler potential on the chiral field space is given by

$$K\big[J_c, \Pi^{\rm odd}_c\big] = -\ln\big[-i\int_{\mathcal{M}_6}\big\langle \Pi^{\rm ev}, \bar{\Pi}^{\rm ev}\big\rangle\big] - 2\ln\big[i\int_{\mathcal{M}_6}\big\langle \Pi^{\rm odd}, \bar{\Pi}^{\rm odd}\big\rangle\big] \ , \qquad (1.1)$$

where $\Pi^{\rm ev} = e^{J_c}$. The anti-symmetric pairing $\langle \cdot, \cdot \rangle$ is defined in (3.20) and replaces the wedge product. The Kähler potential can be identified as the logarithm of the

[1] A similar conjecture was mentioned in ref. [62] and we are grateful to Jan Louis for discussions on that point.

Hitchin functionals for two- and three-forms on $\mathcal{M}_6$ [75]. The superpotential for the chiral multiplets is given by [2]

$$W\big[J_c, \Pi_c^{\rm odd}\big] = \int_{\mathcal{M}_6} \big\langle F^{\rm ev} + d_H \Pi_c^{\rm odd}, \Pi^{\rm ev} \big\rangle \tag{1.2}$$

where $F^{\rm ev}$ is the background flux of the even R-R field strengths. The NS-NS background flux H_3 of the NS-NS field strength $d\hat{B}_2$ arises through the exterior derivative $d_H = d - H_3\wedge$. The superpotential is readily shown to be holomorphic in the complex $N = 1$ chiral multiplets. In the expression (1.2) both $d\Pi_c^{\rm odd}$ and $d\Pi^{\rm ev}$ are linear in the complex coordinates and indicate a deviation from the Calabi-Yau compactifications where J and Ω are closed.

In type IIB orientifold compactifications the role of even and odd forms is interchanged. One combines the globally defined two-form J together with the B-fields $\hat{B}_2$ and the dilaton into the complex even form $\Phi^{\rm ev} = e^{-\hat{\phi}} e^{-\hat{B}_2 + iJ}$. In orientifolds with $O3/O7$ planes the real part of this form is complexified with the sum of even R-R potentials while it contains the imaginary part of $\Phi^{\rm ev}$ for $O5/O9$ orientifolds:

$$O3/O7: \quad \Phi_c^{\rm ev} = e^{-\hat{B}_2} \wedge \hat{C}^{\rm ev} + i{\rm Re}(\Phi^{\rm ev})\ , \tag{1.3}$$

$$O5/O9: \quad \Phi_c^{\rm ev} = e^{-\hat{B}_2} \wedge \hat{C}^{\rm ev} + i{\rm Im}(\Phi^{\rm ev})\ , \tag{1.4}$$

where $e^{-\hat{B}_2} \wedge \hat{C}^{\rm ev}$ contains only forms with all indices on the internal manifold. The complex forms $\Phi_c^{\rm ev}$ are expanded into real even forms on the manifold $\mathcal{M}_6$ with complex scalar coefficients in four space-time dimensions. These complex fields are the bosonic components of a set of chiral multiplets. The expansion is chosen in accord with the orientifold projection which differs for $O3/O7$ and $O5/O9$ orientifolds. Additional chiral multiplets are complex scalars z parameterizing independent degrees of freedom of the globally defined three-form $\Phi^{\rm odd} = \Omega$. The Kähler potential for all chiral multiplets is given by

$$K\big[z, \Phi_c^{\rm ev}\big] = -\ln\big[-i\int_{\mathcal{M}_6} \big\langle \Phi^{\rm odd}, \bar{\Phi}^{\rm odd}\big\rangle\big] - 2\ln\big[i\int_{\mathcal{M}_6} \big\langle \Phi^{\rm ev}, \bar{\Phi}^{\rm ev}\big\rangle\big]\ . \tag{1.5}$$

Non-trivial NS-NS and R-R background fluxes H_3 and F_3 as well as the torsion of $\mathcal{M}_6$ induce a superpotential for the chiral fields. It differs for the two type IIB setups and reads [2]

$$W^{O3/O7} = \int_{\mathcal{M}_6} \big\langle F_3 + d_H \Phi_c^{\rm ev}, \Phi^{\rm odd}\big\rangle\ , \qquad W^{O5/O9} = \int_{\mathcal{M}_6} \big\langle F_3 + d\Phi_c^{\rm ev}, \Phi^{\rm odd}\big\rangle\ , \tag{1.6}$$

where $d_H = d - H_3\wedge$. In addition several D-terms arise due to fluxes and torsion, which are more carefully discussed in a separate publication [81]. In both type IIA and type IIB dual linear multiplets can become massive. The scalar function encoding their kinetic terms are the Legendre transforms of the Kähler potentials given above.

Note that in the large volume and large complex structure limit the type II Kähler potentials are formally mirror symmetric under the exchange $\Pi^{\rm ev/odd} \leftrightarrow \Phi^{\rm odd/ev}$. The complex forms $\Pi_c^{\rm odd}$ and $\Phi_c^{\rm ev}$ are linear in the complex fields and identified under the

[2]See also refs. [18, 19, 59, 62, 21, 11].

mirror map. The type IIA and type IIB superpotentials cannot be identified under mirror symmetry. This is due to the fact that the dual of half of the NS-NS flux H_3 has no mirror parter. Choosing a symplectic basis of harmonic three-forms on $\mathcal{M}_6$ electric and magnetic NS-NS fluxes can be distinguished. We propose that the mirror for part of the electric and magnetic NS-NS fluxes H_3^Q arises if one compactifies on a more general class of spaces. In this conjecture one can allow for all but one magnetic and electric flux direction.[3] The dual spaces are extensions of almost generalized complex manifolds with a more generic globally defined odd form. More precisely, in addition to the three-form Ω the globally defined odd forms Π^{odd} and Φ^{odd} locally contain a one- and five-form Ω_1 and Ω_5 as [4]

$$\Pi^{\text{odd}} = e^{-\hat{B}_2} \wedge (C\Omega_1 + C\Omega + C\Omega_5) \ , \qquad \Phi^{\text{odd}} = e^{-\hat{B}_2} \wedge (\Omega_1 + \Omega + \Omega_5) \ . \tag{1.7}$$

For the general odd forms Π^{odd} and Φ^{odd} the Kähler potentials (1.1) and (1.5) are replaced by the extended Hitchin functionals introduced in refs. [76, 77]. Furthermore, using a fermionic reduction the superpotentials (1.2) and (1.6) are shown to naturally generalize to the odd forms Π^{odd} and Φ^{odd}. Also the complex form Π_c^{odd} including the R-R fields is generalized to

$$\Pi_c^{\text{odd}} = e^{-\hat{B}_2} \wedge \hat{C}^{\text{odd}} + i\text{Re}(\Pi^{\text{odd}}) \ , \tag{1.8}$$

where $e^{-\hat{B}_2} \wedge \hat{C}^{\text{odd}}$ contains only forms with all indices on the internal manifold.

In a finite reduction the magnetic fluxes arise as the mirror of the torsion $d\Omega_1$ such that

$$d\text{Re}(\Omega_1 + \Omega + \Omega_5) \quad \leftrightarrow \quad H_3^Q \ , \tag{1.9}$$

where the electric NS-NS fluxes are identified as the mirrors of $d\Omega$ as proposed in ref. [36]. Hence, 'generalized half-flat' manifolds obeying $d\text{Im}(\Omega_1 + \Omega + \Omega_5) = 0$ and generically non-zero $d\text{Re}(\Omega_1 + \Omega + \Omega_5)$ are candidate mirrors of NS-NS flux compactifications. We provide evidence for the identification (1.9) by comparing the holomorphic superpotentials including the corrections due to $d\Omega_1$. For these generalized spaces the role of the tangent bundle $T\mathcal{M}_6$ is taken by the generalized tangent bundle E locally given by $T\mathcal{M}_6 \oplus T^*\mathcal{M}_6$ [77]. Supersymmetry implies that E has a structure group $SU(3) \times SU(3)$ [48, 62]. These generalized geometries might not necessarily descend to standard Riemannian manifolds with metric on $T\mathcal{M}_6$. It is expected that they are more closely resemble the non-geometric compactifications introduced in refs. [79]. The relation to the non-commutative background suggested in refs. [78] has to be clarified.

2 Manifolds with $SU(3)$ structure

It is a well-known fact that type II supergravity compactified on a Calabi-Yau sixfold leads to an $N = 2$ supergravity theory in four space-time dimensions. In the absence of fluxes the effective four-dimensional theory contains no potential for the scalar fields and

[3] Interpreting mirror symmetry as three T-dualities [82], the forbidden magnetic flux is the one having only indices in the T-dualized directions. Setting this flux quantum to zero, the dual space was termed the Q-space in ref. [70]. Hence, the index Q on H_3^Q.

[4] From a mathematical point of view, the forms Π^{odd} and Φ^{odd} are expected to undergo type changes when moving along the internal manifold [80].

all vacua are Minkowski preserving the full supersymmetry. This changes as soon as we include background fluxes and localized sources such as D-branes and orientifold planes. In these situations it is a non-trivial task to perform consistent compactifications such that the four-dimensional effective theory remains supersymmetric. In particular, this is due to the fact, that the inclusion of sources forces the geometry to back-react. For example in orientifolds with $D3$ branes and fluxes the spacetime has to be non-trivially warped over an internal conformally Calabi-Yau manifold [7]. In other situations the internal manifold is no longer directly related to a Calabi-Yau manifold and a more general class of compactification manifolds has to be taken into account [11].

In this section we discuss such a more general set of six-manifolds which yield upon compactification an $N = 2$ supergravity theory in four space-time dimensions (see, for example, [36, 37, 62]). To start with we specify the Kaluza-Klein Ansatz for the metric background. Topologically our ten-dimensional space-time is taken to be a product $M_{3,1} \times \mathcal{M}_6$, where $M_{3,1}$ is a four-dimensional non-compact space and $\mathcal{M}_6$ is a compact six-dimensional manifold. The background metric is block-diagonal and reads

$$ds^2 = e^{2\Delta(y)} g_{\mu\nu}(x) dx^\mu dx^\nu + g_{mn}(y) dy^m dy^n \ , \tag{2.1}$$

where $x^\mu, \mu = 0, \ldots, 3$ are the coordinates on $M_{3,1}$ while $y^m, m = 1, \ldots, 6$ are the coordinates of $\mathcal{M}_6$. Here $g_{\mu\nu}$ is the metric on $M_{3,1}$ and g_{mn} is the metric on the internal manifold $\mathcal{M}_6$. Note that the metric (2.1) generically includes a non-trivial warp factor $\Delta(y)$. However, in the following we restrict our analysis to a large volume regime where Δ is approximately constant.[5]

The amount of supersymmetry preserved by $\mathcal{M}_6$ can be obtained by counting supercharges. Type II theories admit 32 supercharges in $D = 10$ which can be represented by two (Majorana-Weyl) spinors $\epsilon^{(10)}_{1,2}$. In type IIA the two spinors have opposite chirality, while in type IIB they are of the same chirality. Demanding $N = 2$ supersymmetry in four space-time dimensions the internal manifold has to admit one globally defined spinor η.[6] We decompose the ten-dimensional spinors as

$$\begin{aligned} \text{IIA:} \quad \epsilon^{(10)}_1 &= \epsilon_1 \otimes \eta_+ + \bar{\epsilon}_1 \otimes \eta_- & \qquad \text{IIB:} \quad \epsilon^{(10)}_1 &= \epsilon_1 \otimes \eta_+ + \bar{\epsilon}_1 \otimes \eta_- \ , \\ \epsilon^{(10)}_2 &= \epsilon_2 \otimes \eta_- + \bar{\epsilon}_2 \otimes \eta_+ & \epsilon^{(10)}_2 &= \epsilon_2 \otimes \eta_+ + \bar{\epsilon}_2 \otimes \eta_- \ , \end{aligned} \tag{2.2}$$

where $\epsilon_{1,2}$ and $\bar{\epsilon}_{1,2}$ are four-dimensional Weyl spinors which label the preserved $N = 2$ supersymmetry. The spinors are chosen such that $\epsilon_{1,2}$ have positive four-dimensional chiralities and $\bar{\epsilon}_{1,2}$ have negative chiralities. We indicate the six-dimensional chirality of the globally defined spinor η by a subscript $\pm$. These spinors are related by complex conjugation $(\eta_\pm)^* = \eta_\mp$ and normalized as $\eta^\dagger_\pm \eta_\pm = \frac{1}{2}$. We summarize our spinor conventions in appendix A.

The existence of one globally defined spinor η reduces the structure group of the internal manifold from $SO(6)$ to $SU(3)$ [29, 30, 31]. If this spinor is also covariantly constant with respect to the Levi-Civita connection the manifold has $SU(3)$ holonomy and hence satisfies the Calabi Yau conditions. For a general $SU(3)$ structure manifold the

[5] It would be desirable to extend our analysis to a general $\Delta(y)$ along the lines of [83, 84].

[6] Note that in [62] it was argued that $N = 2$ supersymmetry can be obtained by compactifying on a manifold with two globally defined spinors, which may coincide at points in $\mathcal{M}_6$. We will come back to this generalization in section 4.

spinor η is not any more covariantly constant. The failure of the Levi-Civita connection to annihilate the spinor η is measured by the contorsion tensor τ. Using τ one defines a new connection D^T_m such that

$$D^T_m \eta = \left(D^{LC}_m - \tfrac{1}{4}\tau_{mnp}\gamma^{np}\right)\eta = 0 \ , \tag{2.3}$$

where $\gamma^{mn} = \frac{1}{2!}\gamma^{[m}\gamma^{n]}$ is the anti-symmetrized product of six-dimensional gamma matrices. The spinor η is now covariantly constant with respect to the new connection D^T_m, which additionally contains the information about the torsion τ.

Equivalently to the spinor language, $SU(3)$ structure manifolds can be characterized by the existence of two no-where vanishing forms J and ρ_η. The form J is a real two-form while ρ_η is a real three-form on $\mathcal{M}_6$. We denote the space of real n-forms on $\mathcal{M}_6$ by

$$\Lambda^n T^* \equiv \Lambda^n(T^*\mathcal{M}_6) \ , \tag{2.4}$$

such that $J \in \Lambda^2 T^*$ and $\rho_\eta \in \Lambda^3 T^*$. The index η indicates a specific normalization chosen as we define J and ρ_η in terms of the spinor η. Here we first give a characterization independent of η following the definition of Hitchin [75]. In this case one demands that J and ρ_η are stable forms, i.e. are elements of open orbits under the action of general linear transformations $GL(6,\mathbb{R})$ at every point of the tangent bundle $T\mathcal{M}_6$. These forms define a reduction of the structure group from $GL(6,\mathbb{R})$ to $SU(3)$ if they furthermore satisfy

$$J\wedge J\wedge J = \tfrac{3}{2}\rho_\eta \wedge \hat\rho_\eta \ , \qquad J\wedge \rho_\eta = 0 \ , \tag{2.5}$$

where $\hat\rho_\eta = *\rho_\eta$ is shown to be a function of ρ_η only as we review in appendix B.

The spinor and the form descriptions of the $SU(3)$ structure are related by expressing the components of the two-form J and the complex three-form $\Omega_\eta = \rho_\eta + i\hat\rho_\eta$ in terms of the spinor η as

$$J^{mn} = \mp 2i\eta^\dagger_\pm \gamma^{mn}\eta_\pm \ , \qquad \Omega^{mnp}_\eta = 2\eta^\dagger_- \gamma^{mnp}\eta_+ \ , \qquad \bar\Omega^{mnp}_\eta = 2\eta^\dagger_+\gamma^{mnp}\eta_- \ . \tag{2.6}$$

Later on we will relate Ω_η to the three-form Ω used in the compactification by an appropriate rescaling. In the normalization (2.6) one can apply Fierz identities to derive the $SU(3)$ structure constraints equivalent to (2.5),

$$J\wedge J\wedge J = \tfrac{3i}{4}\,\Omega_\eta \wedge \bar\Omega_\eta \ , \qquad J\wedge\Omega_\eta = 0 \ . \tag{2.7}$$

Moreover, defining $I_m{}^n = J_{mp}g^{pn}$ by raising one of the indices on J by the metric g_{mn} one shows that

$$I_p{}^n I_m{}^p = -\delta^n_m \ , \qquad I_n{}^p I_m{}^q\, g_{pq} = g_{mn} \ . \tag{2.8}$$

This implies that I^n_m is an almost complex structure with respect to which the metric g_{mn} is hermitian. The almost complex structure can be used to define a (p,q) grading of forms. Within this decomposition the form J is of type $(1,1)$ while Ω_η is of type $(3,0)$.

The condition (2.3) can be translated to the form language implying that neither J nor Ω_η are closed. The non-closedness is parameterized by the torsion τ which decomposes under $SU(3)$ into irreducible representations. The representations are conveniently encoded by five torsion classes $\mathcal{W}_i$ defined as [29, 30, 37],

$$\begin{aligned} dJ &= -\tfrac{3}{2}\mathrm{Im}(\mathcal{W}_1\overline{\Omega}_\eta) + \mathcal{W}_4\wedge J + \mathcal{W}_3 \\ d\Omega_\eta &= \mathcal{W}_1 J\wedge J + \mathcal{W}_2\wedge J + \overline{\mathcal{W}}_5\wedge\Omega_\eta \ , \end{aligned} \tag{2.9}$$

with constraints $J \wedge J \wedge \mathcal{W}_2 = J \wedge \mathcal{W}_3 = \Omega_\eta \wedge \mathcal{W}_3 = 0$. The pattern of vanishing torsion classes defines the properties of the manifold $\mathcal{M}_6$. For example, $\mathcal{M}_6$ is complex in case $\mathcal{W}_1 = \mathcal{W}_2 = 0$. Of particular interest are half-flat manifolds, since they are believed to arise as mirrors of flux compactifications [36]. These are defined by $\mathcal{W}_4 = \mathcal{W}_5 = 0$ and $\text{Im}\mathcal{W}_1 = \text{Im}\mathcal{W}_2 = 0$. Equivalently, by using (2.9), half-flat manifolds are defined by the two conditions

$$dJ \wedge J = 0 \,, \quad d\text{Im}\Omega_\eta = 0 \,, \tag{2.10}$$

while dJ and $d\text{Re}\Omega_\eta$ are not necessarily vanishing.

As discussed in the beginning of this section, the compactification on $SU(3)$ structure manifolds leads to an $N = 2$ supergravity theory. The supersymmetry is further reduced to $N = 1$ by imposing an appropriate orientifold projection. The aim of the next section is to define this projection and to determine the characteristic data of the four-dimensional supergravity theory obtained by compactification on an $SU(3)$ structure orientifold.

3 Type II $SU(3)$ structure orientifolds

In this section we study compactifications of type IIA and type IIB supergravity on $SU(3)$ structure orientifolds. As reviewed in the previous section compactifications on $SU(3)$ structure manifolds lead to four-dimensional theories with $N = 2$ supersymmetry. The inclusion of D-branes and orientifold planes further reduced the amount of supersymmetry. In order that the four-dimensional effective theory possesses $N = 1$ supersymmetry the D-branes and orientifold planes can not be chosen arbitrarily but rather have to fulfill certain supersymmetry conditions (BPS conditions).[7] In this paper our main focus will be the bulk theory. In section 3.1 we specify the orientifold projections which yield supersymmetric orientifold planes preserving half of the $N = 2$ supersymmetry. We show in section 3.2 that the orientifold invariant spectrum arranges into $N = 1$ supermultiplets. Performing a Kaluza-Klein reduction allows us to determine the Kähler potential of the four-dimensional theory in section 3.3. The discussion of the superpotential induced by the fluxes and torsion will be presented in section 3.4.

3.1 The orientifold projection

In this section we specify the orientifold projections under consideration. We start from type II string theory and compactify on a $SU(3)$ structure manifold $\mathcal{M}_6$. In addition we mod out by orientation reversal of the string world-sheet Ω_p together with an internal symmetry σ which acts solely on $\mathcal{M}_6$ but leaves the $D = 4$ space-time untouched. We will restrict ourselves to involutive symmetries ($\sigma^2 = 1$) of $\mathcal{M}_6$. In a next step we have to specify additional properties of σ in order that it provides a symmetry of the string theory under consideration. The type IIA and type IIB cases are discussed in turn.

Type IIA orientifold projection

The orientifold projection for type IIA $SU(3)$ structure orientifolds can be obtained in

[7] In addition, the configurations of D-branes, orientifold planes and fluxes have to obey consistency conditions such as the cancellation of tadpoles [19, 20, 21, 70, 71, 72].

close analogy to the Calabi-Yau case. Recall that for Calabi-Yau orientifolds the demand for $N = 1$ supersymmetry implies that σ has to be an anti-holomorphic and isometric involution [8, 9, 10]. This fixes the action of σ on the Kähler form J as $\sigma^* J = -J$, where σ^* denotes the pull-back of the map σ. Furthermore, supersymmetry implies that σ acts non-trivially on the holomorphic three-form Ω. This naturally generalizes to the $SU(3)$ structure case, since we can still assign a definite action of σ on the globally defined two-form J and three-form Ω defined in (2.6). Together the orientifold constraints read

$$\sigma^* J \;=\; -J\;, \qquad\qquad \sigma^*\Omega \;=\; e^{2i\theta}\bar{\Omega}\;, \tag{3.1}$$

where $e^{2i\theta}$ is a phase and we included a factor 2 for later convenience. Note that the second condition in (3.1) can be directly inferred from the compatibility of σ with the $SU(3)$ structure condition $\Omega\wedge\bar{\Omega} \propto J\wedge J\wedge J$ given in (2.7). In order that σ is a symmetry of type IIA string theory it is demanded to be an isometry. Hence, the first condition in (3.1) implies that σ yields a minus sign when applied to the almost complex structure $I_n^m = J_{np}g^{pm}$ introduced in the previous section. This reduces to the anti-holomorphicity of σ if I_n^m is integrable as in the Calabi-Yau case. The complete orientifold projection takes the form [8]

$$\mathcal{O} = (-1)^{F_L}\Omega_p\sigma\;, \tag{3.2}$$

where Ω_p is the world-sheet parity and F_L is the space-time fermion number in the left-moving sector.

The orientifold planes arise as the fix-points of σ. Just as in the Calabi-Yau case supersymmetric $SU(3)$ structure orientifolds generically contain $O6$ planes. This is due to the fact, that the fixed point set of σ in $\mathcal{M}_6$ are three-cycles Λ_{O6} supporting the internal part of the orientifold planes. These are calibrated with respect to the real form $\text{Re}(e^{-i\theta}\Omega)$ such that

$$\text{vol}(\Lambda_{\text{O6}}) \propto \text{Re}(\text{e}^{-\text{i}\theta}\Omega)\;, \qquad \text{Im}(\text{e}^{-\text{i}\theta}\Omega)|_{\Lambda_{\text{O6}}} = \text{J}|_{\Lambda_{\text{O6}}} = 0 \tag{3.3}$$

where $\text{vol}(\Lambda_{\text{O6}})$ is the induced volume form on Λ_{O6} and the overall normalization of Ω was left undetermined. The conditions (3.3) also allow us to give a more explicit expression for the phase $e^{i\theta}$ as

$$e^{-2i\theta} = \bar{Z}(\Lambda_{O6})/Z(\Lambda_{O6})\;, \tag{3.4}$$

where $Z(\Lambda_{O6})$ is given by $Z(\Lambda_{O6}) = \int_{\Lambda_{O6}}\Omega$. This expression determines the transformation behavior of θ under complex rescalings of Ω. Later on we include $e^{-i\theta}$ to define a scale invariant three-form $C\Omega$.

Type IIB orientifold projection

Let us turn to type IIB $SU(3)$ structure orientifolds. Recall that for type IIB Calabi-Yau orientifolds consistency requires σ to be a holomorphic and isometric involution of $\mathcal{M}_6$ [8, 10]. A holomorphic isometry leaves both the metric and the complex structure of the Calabi-Yau manifold invariant, such that $\sigma^*J = J$. We generalize this condition to the $SU(3)$ structure case by demanding that the globally defined two-form J defined in (2.6) transforms as

$$\sigma^* J = J\;. \tag{3.5}$$

[8] The factor $(-1)^{F_L}$ is included in $\mathcal{O}$ to ensure that $\mathcal{O}^2 = 1$ on all states.

Once again we impose that σ is an isometry of the manifold $\mathcal{M}_6$, such that (3.5) translates to the invariance of the almost complex structure I_m^n. Due to this fact the involution respects the (p,q)-decomposition of forms. Hence the $(3,0)$ form Ω defined in (2.6) will be mapped to a $(3,0)$ form. Demanding the resulting form to be globally defined we have two possible choices

$$(1) \quad O3/O7: \quad \sigma^*\Omega = -\Omega\ , \qquad\qquad (2) \quad O5/O9: \quad \sigma^*\Omega = +\Omega\ , \tag{3.6}$$

where the dimensionality of the orientifold planes is determined by the dimension of the fix-point set of σ [8]. Correspondingly, depending on the transformation properties of Ω two different symmetry operations are possible [85, 86, 8, 10]

$$\mathcal{O}_{(1)} = (-1)^{F_L}\Omega_p\,\sigma\ , \qquad\qquad \mathcal{O}_{(2)} = \Omega_p\,\sigma \tag{3.7}$$

where Ω_p is the world-sheet parity and F_L is the space-time fermion number in the left-moving sector. The type IIB analog of the calibration condition (3.3) involves a contribution from the NS-NS two-form $\hat{B}_2$. It states that the even cycles of the orientifold planes in $\mathcal{M}_6$ are calibrated with respect to the real or imaginary parts of $e^{-\hat{B}_2+iJ}$. The explicit form of this condition can be found, for example, in refs. [87, 88, 63].

3.2 The orientifold spectrum

Having specified the orientifold projections (3.2) and (3.7) of the type IIA and type IIB orientifolds we can examine the invariant spectrum. Recall that the bosonic NS-NS fields of both type IIA and type IIB supergravity are the scalar dilaton $\hat{\phi}$, the ten-dimensional metric $\hat{G}_{MN}$ and the two-form $\hat{B}_2$.[9] Considering the theory on the product space $M_{3,1} \times \mathcal{M}_6$ these fields decompose into $SU(3)$ representation as summarized in the table 3.1 [62]. We denote the $SU(3)$ representation $\mathbf{R}$ with four-dimensional spin $\mathbf{s}$ by $\mathbf{R_s}$. For example, a triplet under $SU(3)$ yielding a vector in four-dimensions is denoted by $\mathbf{3_1}$. A four-dimensional tensor (or pseudo-scalar) is indicated by an index $\mathbf{T}$.

$\hat{G}$	$g_{\mu\nu}$	$\mathbf{1_2}$
	$g_{\mu m}$	$(\mathbf{3}+\bar{\mathbf{3}})_\mathbf{1}$
	g_{mn}	$\mathbf{1_0} + (\mathbf{6}+\bar{\mathbf{6}})_\mathbf{0} + \mathbf{8_0}$
$\hat{B}_2$	$B_{\mu\nu}$	$\mathbf{1_T}$
	$B_{\mu m}$	$(\mathbf{3}+\bar{\mathbf{3}})_\mathbf{1}$
	B_{mn}	$\mathbf{1_0} + (\mathbf{3}+\bar{\mathbf{3}})_\mathbf{0} + \mathbf{8_0}$
$\hat{\phi}$	ϕ	$\mathbf{1_0}$

Table 3.1: *Decomposition of the NS sector in $SU(3)$ representations*

In the R-R sector type IIA consists of odd forms $\hat{C}_{2n-1}$, while type IIB consists of even forms $\hat{C}_{2n}$. Their decomposition into $SU(3)$ representations is displayed in tables

[9] The hat on the fields indicates ten-dimensional quantities.

3.2 and 3.3 [62]. We list only the decompositions of the $\hat{C}_1$ and $\hat{C}_3$ in type IIA and $\hat{C}_0, \hat{C}_2, \hat{C}_4$ in type IIB. The higher forms are related to these fields via Hodge duality of their field strengths. The form $\hat{C}_4$ has a self-dual field strength and hence only half of its components are physical.

$\hat{C}_1$	C_μ	$\mathbf{1_1}$
	C_m	$(\mathbf{3}+\bar{\mathbf{3}})_\mathbf{0}$
$\hat{C}_3$	$C_{\mu\nu p}$	$(\mathbf{3}+\bar{\mathbf{3}})_\mathbf{T}$
	$C_{\mu np}$	$\mathbf{1_1}+(\mathbf{3}+\bar{\mathbf{3}})_\mathbf{1}+\mathbf{8_1}$
	C_{mnp}	$(\mathbf{1}+\mathbf{1})_\mathbf{0}+(\mathbf{3}+\bar{\mathbf{3}})_\mathbf{0}+(\mathbf{6}+\bar{\mathbf{6}})_\mathbf{0}$

Table 3.2: *Type IIA decomposition of the RR sector in $SU(3)$ representations*

$\hat{C}_0$	C_0	$\mathbf{1_0}$
$\hat{C}_2$	$C_{\mu\nu}$	$\mathbf{1_T}$
	$C_{\mu m}$	$(\mathbf{3}+\bar{\mathbf{3}})_\mathbf{1}$
	C_{mn}	$\mathbf{1_0}+(\mathbf{3}+\bar{\mathbf{3}})_\mathbf{0}+\mathbf{8_0}$
$\hat{C}_4$	$C_{\mu npq}$	$\frac{1}{2}\left[(\mathbf{1}+\mathbf{1})_\mathbf{1}+(\mathbf{3}+\bar{\mathbf{3}})_\mathbf{1}+(\mathbf{6}+\bar{\mathbf{6}})_\mathbf{1}\right]$
	$C_{mnpq}/C_{\mu\nu mn}$	$\mathbf{1_0}+(\mathbf{3}+\bar{\mathbf{3}})_\mathbf{0}+\mathbf{8_0}$

Table 3.3: *Type IIB decomposition of the RR sector in $SU(3)$ representations*

The fields arising in this decomposition can be arranged into one $N=8$ gravitational multiplet. As discussed in ref. [62], a possible reduction to standard $N=2$ supergravity theory with a gravity multiplet as well as some vector, hyper and tensor multiplets is obtained by removing all the triplets from the spectrum. In particular, this amounts to discarding all four-dimensional fields which arise in the expansion of the ten-dimensional fields into one- and five-forms on $\mathcal{M}_6$.

In a second step we impose the orientifold projection to further reduce to an $N=1$ supergravity theory. Independent of the properties of the internal manifold we can give the transformation behavior of all supergravity fields under the world-sheet parity Ω_p and $(-1)^{F_L}$ [4, 5]. Ω_p acts on $\hat{B}_2$ with a minus sign, while leaving the dilaton $\hat{\phi}$ and the ten-dimensional metric $\hat{G}$ invariant. To display the transformation behavior of the R-R fields we introduce the parity operator λ by

$$\lambda(\mathcal{C}_{2n}) = (-1)^n \mathcal{C}_{2n} \ , \qquad \lambda(\mathcal{C}_{2n-1}) = (-1)^n \mathcal{C}_{2n-1} \ , \tag{3.8}$$

where $\mathcal{C}_{2n}$ are even and $\mathcal{C}_{2n-1}$ are odd forms. Evaluated on the R-R forms λ is minus the world-sheet parity operator Ω_p such that

$$\Omega_p \hat{C}_k = -\lambda(\hat{C}_k) \ , \tag{3.9}$$

where k is odd for type IIA and even for type IIB. Finally, $(-1)^{F_L}$ acts on the R-R bosonic fields of the supergravity theories with a minus sign while leaving the NS-NS fields invariant.

The type IIA orientifold spectrum

Let us now determine the invariant spectrum for type IIA orientifolds. It turns out to be convenient to combine the odd R-R forms $\hat{C}_{2n+1}$ as [89]

$$\hat{C}^{\rm odd} = \hat{C}_1 + \hat{C}_3 + \hat{C}_5 + \hat{C}_7 + \hat{C}_9 \ . \tag{3.10}$$

Note that only half of the degrees of freedom in $\hat{C}^{\rm odd}$ are physical, while the other half can be eliminated by a duality constraint [89]. Invariance under the orientifold projection $\mathcal{O}$ implies by using the transformation of the fields under Ω_p and $(-1)^{F_L}$ that the ten-dimensional fields have to transform as

$$\sigma^* \hat{B}_2 = -\hat{B}_2 \ , \qquad \sigma^* \hat{\phi} = \hat{\phi} \ , \qquad \sigma^* \hat{C}^{\rm odd} = \lambda\big(\hat{C}^{\rm odd}\big) \ , \tag{3.11}$$

where the parity operator λ is defined in (3.8) and we used (3.9). It turns out to be convenient to combine the forms Ω and J with the ten-dimensional dilaton $\hat{\phi}$ and $\hat{B}_2$ into new forms $\Pi^{\rm ev/odd}$ as

$$\Pi^{\rm ev} = e^{-\hat{B}_2 + iJ} \ , \qquad \Pi^{\rm odd} = C\Omega \ , \tag{3.12}$$

where

$$C = e^{-\hat{\phi} - i\theta} e^{(K^{cs} - K^{\rm K})/2} \ , \qquad e^{-K^{cs}} = i\Omega \wedge \bar{\Omega} \ , \qquad e^{-K^{\rm K}} = \tfrac{4}{3} J \wedge J \wedge J \ . \tag{3.13}$$

In the expression for C the form contributions precisely cancel such that C is a complex scalar on $\mathcal{M}_6$. It depends on the ten-dimensional dilaton $\hat{\phi}$ and fixes the normalization of Ω such that the combination $C\Omega$ stays invariant under complex rescaling of Ω.[10] The four-dimensional dilaton is defined as

$$e^{-2D} = \tfrac{4}{3} \int_{\mathcal{M}_6} e^{-2\hat{\phi}} J \wedge J \wedge J \ , \tag{3.14}$$

and reduces to the definition $e^{-D} = e^{-\hat{\phi}} \sqrt{\mathrm{Vol}(\mathcal{M}_6)}$ in case $\hat{\phi}$ is constant along $\mathcal{M}_6$. Applied to the forms $\Pi^{\rm ev/odd}$ and $\hat{C}^{\rm odd}$ the orientifold conditions (3.1) and (3.11) are expressed as

$$\sigma^* \Pi^{\rm ev} = \lambda\big(\Pi^{\rm ev}\big) \ , \qquad \sigma^* \Pi^{\rm odd} = \lambda\big(\bar{\Pi}^{\rm odd}\big) \ . \tag{3.15}$$

In order to perform the Kaluza-Klein reduction one needs to specify the modes of the internal manifold $\mathcal{M}_6$ used in the expansion of $\Pi^{\rm ev/odd}$ and $\hat{C}^{\rm odd}$. This implies that one needs to specify a set of forms on $\mathcal{M}_6$ which upon expansion yields the light fields in the spectrum of the four-dimensional theory. In general this issue is very hard to address and one can only hope to find an approximate answer in certain limits where the torsion is 'small'. Most of the difficulty is due to the fact that a non-trivial torsion

[10] Note that also θ depends on the three-form Ω as given in (3.4). Hence, using the scaling behavior of θ and $K^{\rm cs}$ one finds $C \to Ce^{-f}$ as $\Omega \to e^f \Omega$ for every complex function f.

may not generate an additional scale below the Kaluza-Klein scale.[11] Hence, discarding the Kaluza-Klein modes needs some justification. Surprisingly, much of the analysis performed below does not explicitly depend on the basis used in the expansion of $\Pi^{\text{ev/odd}}$ and $\hat{C}^{\text{odd}}$. We therefore only assume that the triplets in the $SU(3)$ decomposition are projected out while otherwise keeping the analysis general [62]. Later on we restrict to a particular finite number of modes.

To implement the orientifold projection we note that the operator $\mathcal{P}_6 = \lambda\sigma^*$ squares to the identity and thus splits the space of two- and three-forms $\Lambda^2 T^*$ and $\Lambda^3 T^*$ on $\mathcal{M}_6$ into two eigenspaces as

$$\Lambda^2 T^* = \Lambda^2_+ T^* \oplus \Lambda^2_- T^* \ , \qquad \Lambda^3 T^* = \Lambda^3_+ T^* \oplus \Lambda^3_- T^* \ , \tag{3.16}$$

where $\Lambda^2_\pm T^*$ contains forms transforming with a $\pm$ sign under $\mathcal{P}_6$.

In performing the Kaluza-Klein reduction one expands the forms $\Pi^{\text{ev/odd}}$ and $\hat{C}^{\text{odd}}$ into the appropriate subset of $\Lambda^2 T^*$ and $\Lambda^3 T^*$ consistent with the orientifold projection. The coefficients arising in these expansions correspond to the fields of the four-dimensional theory. In the case at hand the compactification has to result in an $N = 1$ supergravity theory. The spectrum of this theory consists of a gravity multiplet a number of chiral multiplets and vector multiplets. Note that before the truncation to the light modes the number of multiplets is not necessarily finite, as the Kaluza-Klein tower consist of an infinite number of modes. These modes can acquire a mass via a generalized Higgs mechanism. For example, a two-from can become massive by 'eating' a vector [90]. In the following we will discuss the massless field content before such a Higgsing takes place.

Let us first concentrate on the $N = 1$ chiral multiplets arising in the expansion of the forms Π^{ev}. Due to supersymmetry the bosonic components of these multiplets span a complex Kähler manifold. Its complex structure can be determined by specifying appropriate complex combinations of the forms J and $\hat{B}_2$ which upon expansion into modes of the internal manifold yield the complex chiral coordinates. The globally defined two-form J combines with the B-field into the complex combination [12]

$$J_c \equiv -\hat{B}_2 + iJ \qquad \in \quad \Lambda^2_+ T^*_{\mathbb{C}} \ , \tag{3.17}$$

where J is given in the string frame. The field $\hat{B}_2$ is only extended along $\mathcal{M}_6$, since due to (3.11) the four-dimensional two-form in $\hat{B}_2$ transforms with the wrong sign under the orientifold symmetry σ^* and hence is projected out. In comparison to the general $SU(3)$ decomposition of $\hat{B}_2$ given in table 3.1 we only kept the $\mathbf{1_0} + \mathbf{8_0}$ representations while all other components left the spectrum. The complex form J_c is expanded in real elements of $\Lambda^2_+ T^*$ consistent with the orientifold projection (3.1), (3.11) and the definition of λ given in (3.8).[13] The coefficients of this expansion are complex scalar fields in four space-time

[11]This is in contrast to standard RR and NS fluxes, which correspond to background values of the field strengths of $\hat{C}^{\text{odd}}$ and $\hat{B}_2$. The quantization condition implies that these fluxes can generate an intermediate scale. This allows to keep modes of the order of the flux scale, but discard all massive Kaluza-Klein modes.

[12]Note that the complex combination (3.17) precisely gives the correct coupling to the string world-sheet wrapped around supersymmetric two-cycles in $\mathcal{M}_6$.

[13]Note that the eigenspaces $\Lambda^2_\pm T^*$ are obtained from the operator $\mathcal{P}_6 = \lambda\sigma^*$ and hence differ by a minus sign from the eigenspaces of σ^*.

dimensions parameterizing a manifold $\mathcal{M}^{\mathrm{K}}$ and provide the bosonic components of chiral multiplets.

Turning to the expansion of the R-R forms $\hat{C}^{\mathrm{odd}}$ we first note that $\hat{C}_1$ (and hence $\hat{C}_7$) are completely projected out from the spectrum. The four-dimensional part of $\hat{C}_1$ is incompatible with the orientifold symmetry as seen in (3.11). On the other hand the internal part of $\hat{C}_1$ is a triplet under $SU(3)$ and hence discarded following the assumptions made above. In contrast the expansion of $\hat{C}_3$ yields four-dimensional scalars, vectors and three-forms. Therefore, we decompose

$$\hat{C}_3 = C_3^{(0)} + C_3^{(1)} + C_3^{(3)} \ , \tag{3.18}$$

where $C_3^{(n)}$ are n-forms in $M_{3,1}$ times $(3-n)$-forms in $\mathcal{M}_6$. More precisely, in order to fulfill the orientifold condition (3.11) the components $C_3^{(0)}$, $C_3^{(1)}$ and $C_3^{(3)}$ are expanded in forms $\Lambda^3_+ T^*$, $\Lambda^2_- T^*$ and $\Lambda^0 T^*$ of $\mathcal{M}_6$ respectively. The coefficients in this expansion correspond to four-dimensional real scalars, vectors and three-forms. Let us note that we project out fields which arise in the expansion into one-forms on $\mathcal{M}_6$ as well as all other triplets. In summary the components kept, are the $\mathbf{1_1} + \mathbf{8_1}$ and $(\mathbf{1}+\mathbf{1})_\mathbf{0} + (\mathbf{6}+\bar{\mathbf{6}})_\mathbf{0}$ while all other representations in table 3.2 have left the spectrum.

The four-dimensional real scalars in $C_3^{(0)}$ need to combine with scalars arising in the expansion of Π^{odd} to form the components of chiral multiplets. The complex structure on the corresponding Kähler field space is defined through the complex form [14]

$$\Pi^{\mathrm{odd}}_c \equiv C_3^{(0)} + i\mathrm{Re}(\Pi^{\mathrm{odd}}) \qquad \in \quad \Lambda^3_+ T^*_{\mathbb{C}} \ . \tag{3.19}$$

where we used that $\mathrm{Re}(\Pi^{\mathrm{odd}})$ transforms with a plus sign as seen from eqn. (3.15). The complex coefficients of Π^{odd}_c expanded in real forms $\Lambda^3_+ T^*$ are the bosonic components of chiral multiplets. Note that in the massless case theses chiral multiplets can be dualized to linear multiplets containing a scalar from $\mathrm{Re}(\Pi^{\mathrm{odd}})$ and a two-form dual to the scalar in $C_3^{(0)}$ [91]. Due to the generality of our discussion both chiral and linear multiplets can become massive. The full $N=1$ spectrum for type IIA orientifold is summarized in table 3.4.

multiplet	bosonic fields	$\mathcal{M}_6$-forms
gravity multiplet	$g_{\mu\nu}$	
chiral multiplets	J_c	$\Lambda^2_+ T^*$
chiral/linear multiplets	Π^{odd}_c	$\Lambda^3_+ T^*$
vector multiplets	$C_3^{(1)}$	$\Lambda^2_- T^*$

Table 3.4: $N=1$ *spectrum of type IIA orientifolds*

The analysis so far was not restricted to a finite set of fields. Even though most of the calculations can be performed in this more general setting we will also give a reduction

[14] Note that the complex combination (3.19) precisely gives the correct coupling to D-branes wrapped around supersymmetric cycles in $\mathcal{M}_6$ [87, 88, 63].

to a four-dimensional theory with finite number of fields. This is particularly useful in the discussion of mirror symmetry between $SU(3)$ structure orientifolds and Calabi-Yau orientifolds with background fluxes. A finite reduction is achieved by picking a finite basis of forms Δ_{finite} on the $SU(3)$ structure manifold slightly extending the Calabi-Yau reductions [36, 62]. The explicit construction of such a finite set of forms is difficult, however, we can specify its properties. Before turning to the orientifold constraints let us briefly recall the construction of ref. [62].

To define the properties of Δ_{finite} we first need to introduce an additional structure on $\Lambda^{\text{ev,odd}}T^*$ known as Mukai pairings. These anti-symmetric forms are defined by

$$\big\langle \varphi, \psi \big\rangle = \big[\lambda(\varphi)\wedge\psi\big]_6 = \begin{cases} \varphi_0\wedge\psi_6 - \varphi_2\wedge\psi_4 + \varphi_4\wedge\psi_2 - \varphi_6\wedge\psi_0 \ , \\ -\varphi_1\wedge\psi_5 + \varphi_3\wedge\psi_3 - \varphi_5\wedge\psi_1 \ , \end{cases} \tag{3.20}$$

where λ is given in eqn. (3.8) and $[\ldots]_6$ denotes the forms of degree 6. Clearly $\langle\varphi,\psi\rangle$ is proportional to a volume form on $\mathcal{M}_6$ and can be integrated over the manifold $\mathcal{M}_6$. Demanding this integrated Mukai pairings to be non-degenerate on Δ_{finite} puts a first constraint on the possible set of forms. To make this more precise, let us denote the finite set of forms in $\Lambda^n T^*$ by Δ^n, with dimensions $\dim\Delta^n$. As a first condition we demand that $\dim\Delta^0 = \dim\Delta^6 = 1$ and assume that Δ^0 consists of the constant functions while Δ^6 contains volume forms $\epsilon \propto J\wedge J\wedge J$. Moreover, demanding non-degeneracy of the integrated Mukai pairings on Δ^{ev} one defines a (canonical) symplectic basis on this space. Denoting a basis of $\Delta^0\oplus\Delta^2$ by $\omega_{\hat A} = (1, \omega_A)$ one defines its dual basis $\tilde\omega^{\hat A} = (\tilde\omega^A, \epsilon)$ of $\Delta^4\oplus\Delta^6$ by

$$\int_{\mathcal{M}_6} \big\langle \omega_{\hat A}, \tilde\omega^{\hat B}\big\rangle = \delta_{\hat A}^{\hat B} \ , \qquad \hat A, \hat B = 0, \ldots, \dim\Delta^2 \ , \tag{3.21}$$

with all other intersections vanishing. Turning to the odd forms Δ^{odd} we follow a similar strategy to define a symplectic basis. However, in accord with our assumption above, we will set $\dim\Delta^1 = \dim\Delta^5 = 0$ such that no one- or five-forms are used in the expansion of the fields.[15] Hence, non-degeneracy of the integrated Mukai pairings implies that a symplectic basis $(\alpha_{\hat K}, \beta^{\hat K})$ of Δ^3 can be defined as

$$\int_{\mathcal{M}_6} \big\langle \alpha_{\hat L}, \beta^{\hat K}\big\rangle = \delta_{\hat L}^{\hat K} \ , \qquad \hat K, \hat L = 1, \ldots, \tfrac{1}{2}\dim\Delta^3 \ , \tag{3.22}$$

with all other intersections vanishing. Note that the non-degeneracy of the integrated Mukai pairings implies that $\Delta^{\text{ev/odd}}$ contains the same number of exact and non-closed forms. We will come back to this issue later on when we introduce torsion fluxes.

After this brief review let us now specify how the orientifold symmetry acts on Δ_{finite}. Under the operator $\mathcal{P}_6 = \lambda\sigma^*$ the forms Δ^n decompose into eigenspaces as

$$\Delta^n = \Delta^n_+ \oplus \Delta^n_- \ . \tag{3.23}$$

Using the properties (3.1) and (3.8) one infers $\dim\Delta^0_- = \dim\Delta^6_- = 0$. Furthermore, under the split (3.23) the basis $(\omega_{\hat A}, \tilde\omega^{\hat A})$ introduced in (3.21) decomposes as

$$(\omega_{\hat A}, \tilde\omega^{\hat A}) \quad \to \quad (1, \omega_a, \tilde\omega^\alpha, \epsilon) \in \Delta^{\text{ev}}_+ \ , \quad (\omega_\alpha, \tilde\omega^a) \in \Delta^{\text{ev}}_- \ , \tag{3.24}$$

[15] In section 4 we discuss a possible way to weaken this condition.

where $\alpha = 1, \ldots, \dim \Delta^2_-$ while $a = 1, \ldots, \dim \Delta^2_+$. Using the intersections (3.21) one infers that $\dim \Delta^2_\pm = \dim \Delta^4_\mp$. Turning to the odd forms consistency requires that

$$\int_{\mathcal{M}_6} \langle \Delta^3_\pm, \Delta^3_\pm \rangle = 0 \ , \qquad *\Delta^3_\pm = \Delta^3_\mp \ , \tag{3.25}$$

where in the second equality we used the fact that σ is an orientation-reversing isometry. The first condition is a consequence of the fact that $\Delta^3_\pm \wedge \Delta^3_\pm$ transforms with a minus sign under $\mathcal{P}_6$ and hence is a subset of Δ^6_- up to an exact form. The equations (3.25) imply that $\Delta^3_\pm$ are Lagrangian subspaces of Δ^3 with respect to the integrated Mukai parings. Hence, also the symplectic basis $(\alpha_{\hat{K}}, \beta^{\hat{K}})$ introduced in (3.22) splits as

$$(\alpha_{\hat{K}}, \beta^{\hat{K}}) \quad \to \quad (\alpha_k, \beta^\lambda) \in \Delta^3_+ \ , \qquad (\alpha_\lambda, \beta^k) \in \Delta^3_- \ , \tag{3.26}$$

where the numbers of α_k and β^λ in Δ^3_+ equal to the numbers of β^k and α_λ in Δ^3_- respectively. This is in accord with equation (3.22).

We are now in the position to give an explicit expansion of the fields into the finite form basis of Δ_{finite}. As discussed in the general case above the four-dimensional complex chiral fields arise in the expansion of the forms J_c and Π^{odd}_c introduced in eqn. (3.17) and (3.19). Restricted to Δ^2_+, Δ^3_+ and Δ^2_- one has

$$J_c = t^a \omega_a \ , \qquad \Pi^{\text{odd}}_c = N^k \alpha_k + T_\lambda \beta^\lambda \ , \qquad C^{(1)}_3 = A^\alpha \omega_\alpha \ , \tag{3.27}$$

where the basis decompositions (3.24) and (3.26) were used. Hence, in the finite reduction the $N = 1$ spectrum consists of $\dim \Delta^2_+$ chiral multiplets t^a and $\frac{1}{2} \dim \Delta^3$ chiral multiplets N^k, T_λ. In addition one finds $\dim \Delta^2_-$ vector multiplets, which arise in the expansion of $\hat{C}_3$. Moreover, one four-dimensional massless three-form arises in the expansion of $C^{(3)}_3$ into the form $1 \in \Delta^0_+$. It carries no degrees of freedom and corresponds to an additional flux parameter.

The type IIB orientifold spectrum

Let us next turn to the spectrum of type IIB $SU(3)$ structure orientifolds. To identify the invariant spectrum we first analyze the transformation properties of the ten-dimensional fields. In contrast to type IIA supergravity the type IIB theory consists of even forms $\hat{C}_{2n}$ in the R-R sector, which we conveniently combine as [89]

$$\hat{C}^{\text{ev}} = \hat{C}_0 + \hat{C}_2 + \hat{C}_4 + \hat{C}_6 + \hat{C}_8 \ . \tag{3.28}$$

Only half of the degrees of freedom in $\hat{C}^{\text{ev}}$ are physical and related to the second half by a duality constraint [89]. Using the transformation properties of the fields under Ω_p and $(-1)^{F_L}$ the invariance under the orientifold projections $\mathcal{O}_{(i)}$ implies that the ten-dimensional fields have to transform as [16]

$$\sigma^* \hat{B}_2 = -\hat{B}_2 \ , \qquad \sigma^* \hat{\phi} = \hat{\phi} \ , \qquad \sigma^* \hat{C}^{\text{ev}} = \pm \lambda \big(\hat{C}^{\text{ev}} \big) \ , \tag{3.29}$$

[16]The transformation behavior of the R-R forms under the world-sheet parity operator Ω_p was given in eqn. (3.9).

where the plus sign holds for orientifolds with $O3/O7$ planes, while the minus sign holds for $O5/O9$ orientifolds. The parity operator λ was introduced in eqn. (3.8). We combine the globally defined forms J and Ω with the fields $\hat{B}_2$, $\hat{\phi}$ and $\hat{C}^{\text{ev}}$ as

$$\Phi^{\text{odd}} = \Omega \ , \qquad \Phi^{\text{ev}} = e^{-\hat{\phi}} e^{-\hat{B}_2 + iJ} \ , \qquad \hat{A}^{\text{ev}} = e^{-\hat{B}_2} \wedge \hat{C}^{\text{ev}} \ . \tag{3.30}$$

where in comparison to (3.12) one finds that Φ^{odd} takes the role of Π^{ev} and Φ^{ev} replaces Π^{odd}. Applied to these forms the orientifold conditions (3.5), (3.6) and (3.29) read

$$\sigma^* \Phi^{\text{odd}} = \mp \lambda(\Phi^{\text{odd}}) \ , \qquad \sigma^* \Phi^{\text{ev}} = \lambda(\bar{\Phi}^{\text{ev}}) \ , \qquad \sigma^* \hat{A}^{\text{ev}} = \pm \lambda(\hat{A}^{\text{ev}}) \ , \tag{3.31}$$

where the upper sign corresponds to $O3/O7$ and the lower sign to $O5/O9$ orientifolds.

In a next step we have to specify the basis of forms on $\mathcal{M}_6$ used in the Kaluza-Klein reduction. In doing so we will face similar problems like in the type IIA case. Following the strategy advanced above we first briefly discuss the general case and later simplify the reduction to the finite set of forms Δ_{finite}. The decomposition of the ten-dimensional fields into $SU(3)$ representations is given in tables 3.1 and 3.3. Also in the type IIB case we will remove all triplets of $SU(3)$ from the spectrum [62].

In order to perform the reduction we first investigate the splitting of the spaces of forms on $\mathcal{M}_6$ under the operator $\mathcal{P}_6 = \lambda \sigma^*$. Since $\mathcal{P}_6$ squares to the identity operator it splits the forms as in eqn. (3.16). More generally, we will need the decomposition of all even forms as

$$\Lambda^{\text{ev}} T^* = \Lambda^{\text{ev}}_+ T^* \oplus \Lambda^{\text{ev}}_- T^* \ . \tag{3.32}$$

The four-dimensional fields arising as the coefficients of $\Phi^{\text{ev/odd}}$ and $\hat{A}^{\text{ev}}$ expanded on $\Lambda^3_\pm T^*$ and $\Lambda^{\text{ev}}_\pm T^*$ fit into $N = 1$ supermultiplets.

Firstly, we decompose the odd form Φ^{odd} into the eigenspaces of $\mathcal{P}_6$. In accord with the orientifold constraint (3.31) we find

$$O3/O7: \quad \Phi^{\text{odd}} \ \in \ \Lambda^3_- T^*_{\mathbb{C}} \ , \qquad O5/O9: \quad \Phi^{\text{odd}} \ \in \ \Lambda^3_+ T^*_{\mathbb{C}} \ . \tag{3.33}$$

Note that the actual degrees of freedom of $\Phi^{\text{odd}} = \Omega$ are reduced by several constraints. More precisely, one has to specify forms Φ^{odd} which are associated to different reductions of the $\mathcal{M}_6$ structure group to $SU(3)$. As already discussed in section 2 those reductions can be parameterized by real three-forms $\rho = \text{Re}(\Phi^{\text{odd}})$ which are in addition stable. The imaginary part $\text{Im}(\Phi^{\text{odd}})$ can be expressed as a function of $\text{Re}(\Phi^{\text{odd}})$ such that only half of the degrees of freedom in Φ^{odd} are independent [75]. Moreover, as in the case of a Calabi-Yau manifold, different complex normalizations of Φ^{odd} correspond to the same $SU(3)$ structure of $\mathcal{M}_6$. Therefore, one additional complex degree of freedom in Φ^{odd} is unphysical and has to be removed from the $D = 4$ spectrum.

In the reduction also the ten-dimensional form $\hat{A}^{\text{ev}}$ is expanded into a basis of forms on $\mathcal{M}_6$ while additionally satisfying the orientifold condition (3.31). In analogy to (3.18) we decompose

$$\hat{A}^{\text{ev}} = A^{\text{ev}}_{(0)} + A^{\text{ev}}_{(1)} + A^{\text{ev}}_{(2)} + A^{\text{ev}}_{(3)} \ , \tag{3.34}$$

where the subscript (n) indicates the form degree in four dimensions. Note that in a general expansion of $\hat{A}^{\text{ev}}$ in odd and even forms of $\mathcal{M}_6$ as $\hat{A}^{\text{ev}} = \text{ev}|_4 \times \text{ev}|_6 + \text{odd}|_4 \times \text{odd}|_6$ it would be impossible to assign a four-dimensional form degree as done in eqn. (3.34).

This is due to the fact that such a decomposition only allows to distinguish even and odd forms in four dimensions. However, the orientifold imposes the constraint (3.31) which introduces an additional splitting within the even and odd four-dimensional forms. Let us first make this more precise in the case of $O3/O7$ orientifolds where $\hat{A}^{\text{ev}}$ transforms as $\sigma^* \hat{A}^{\text{ev}} = \lambda(\hat{A}^{\text{ev}})$. Using the properties of the parity operator λ one finds that the scalars in $A^{\text{ev}}_{(0)}$ arise as coefficients of forms in $\Lambda^{\text{ev}}_+ T^*$ while the two-forms in $A^{\text{ev}}_{(2)}$ arise as coefficients of forms in $\Lambda^{\text{ev}}_- T^*$. Similarly, one obtains the four-dimensional vectors in $A^{\text{ev}}_{(1)}$ as coefficients of $\Lambda^3_+ T^*$ and the three-forms in $A^{\text{ev}}_{(3)}$ as coefficients of $\Lambda^3_- T^*$. In the case of $O5/O9$ orientifolds the ten-dimensional form $\hat{A}^{\text{ev}}$ transforms as $\sigma^* \hat{A}^{\text{ev}} = -\lambda(\hat{A}^{\text{ev}})$ and all signs in the $O3/O7$ expansions above are exchanged. For both cases the decomposition (3.34) is well defined and we can analyze the multiplet structure of the four-dimensional theory.

In four dimensions massless scalars are dual to massless two-forms, while massless vectors are dual to vectors. Using the duality condition on the field strengths of the even forms $\hat{A}^{\text{ev}}$ one eliminates half of its degrees of freedom. Indeed it can be shown that in the massless case the scalars in $A^{\text{ev}}_{(0)}$ are dual to the two-forms $A^{\text{ev}}_{(2)}$. However, due to the generality of our discussion also massive scalars, vectors, two-form and three-forms can arise in the expansion (3.34). In these cases the duality constraint gives a complicated relation between these fields. In the following we will first restrict our attention to the massless case and eliminate the two-forms in $A^{\text{ev}}_{(2)}$ in favor of the scalars in $A^{\text{ev}}_{(0)}$.

Let us start with the chiral multiplets. As the bosonic components these multiplets contain the real scalars in $A^{\text{ev}}_{(0)}$ which are complexified by the real scalars arising in the expansion of $\text{Re}(\Phi^{\text{ev}})$ or $\text{Im}(\Phi^{\text{ev}})$. From the orientifold constraint (3.31) on infers that $\text{Re}(\Phi^{\text{ev}})$ is expanded in forms of $\Lambda^{\text{ev}}_+ T^*$ while $\text{Im}(\Phi^{\text{ev}})$ is expanded in forms of $\Lambda^{\text{ev}}_- T^*$. Therefore, one finds the complex forms [17]

$$O3/O7: \quad \Phi^{\text{ev}}_c = A^{\text{ev}}_{(0)} + i\text{Re}(\Phi^{\text{ev}}) \ , \qquad\qquad O5/O9: \quad \Phi^{\text{ev}}_c = A^{\text{ev}}_{(0)} + i\text{Im}(\Phi^{\text{ev}}) \ . \tag{3.35}$$

The complex scalars arising in the expansion of the forms Φ^{ev}_c span a complex manifold $\mathcal{M}^{\text{Q}}$. This manifold is Kähler as discussed in the next section. Here we conclude our general analysis of the spectrum of type IIB $SU(3)$ structure orientifolds by summarizing the four-dimensional multiplets in table 3.5.

To end this section let us give a truncation to a finite number of the four-dimensional fields. As we have argued in the previous section this is achieved by expanding the ten-dimensional fields on the finite set of forms on $\mathcal{M}_6$ denoted by Δ_{finite}. This is done in accord with the orientifold constraints for $O3/O7$ and $O5/O9$ orientifolds. Once again, the n-forms Δ^n split as $\Delta^n = \Delta^n_+ \oplus \Delta^n_-$, where $\Delta^n_\pm$ are the eigenspaces of the operator $\mathcal{P}_6 = \lambda\sigma^*$. However, since Δ^6 contains forms proportional to $J \wedge J \wedge J$ one infers from condition (3.5) that $\dim \Delta^6_+ = 0$. Clearly, one has $\dim \Delta^0_- = 0$ since Δ^0 contains constant scalars which are invariant under $\mathcal{P}_6$. A further investigation of the even forms in Δ^2 and Δ^4 shows that the basis introduced in eqn. (3.21) decomposes as

$$(\omega_{\hat{A}}, \tilde{\omega}^{\hat{A}}) \quad \to \quad (1, \omega_a, \tilde{\omega}^\alpha) \in \Delta^{\text{ev}}_+ \ , \quad (\epsilon, \omega_\alpha, \tilde{\omega}^a) \in \Delta^{\text{ev}}_- \ , \tag{3.36}$$

where $\alpha = 1, \ldots, \dim \Delta^2_-$ and $a = 1, \ldots, \dim \Delta^2_+$. Using $J \wedge J \wedge J \in \Delta^6_-$ and eqn. (3.21) one finds that $\Delta^2_\pm = \Delta^4_\mp$. Together with the fact that $\int \left\langle \Delta^{\text{ev}}_\pm, \Delta^{\text{ev}}_\pm \right\rangle = 0$ one concludes

[17] Note that also in the type IIB cases the complex forms Φ^{ev}_c encode the correct couplings to D-branes fully wrapped on supersymmetric cycles in $\mathcal{M}_6$ [87, 88, 63].

multiplet	bosonic fields	$\mathcal{M}_6$-forms	
		$O3/O7$	$O5/O9$
gravity multiplet	$g_{\mu\nu}$		
chiral multiplets	$\Phi^{\rm odd}$	$\Lambda^3_- T^*$	$\Lambda^3_+ T^*$
chiral/linear multiplets	$\Phi^{\rm ev}_c$	$\Lambda^{\rm ev}_+ T^*$	$\Lambda^{\rm ev}_- T^*$
vector multiplets	$A^{\rm ev}_{(1)}$	$\Lambda^3_+ T^*$	$\Lambda^3_- T^*$

Table 3.5: $N = 1$ *spectrum of type IIB orientifolds*

that $\Delta^{\rm ev}_\pm$ are Lagrangian subspaces of $\Delta^{\rm ev}$. This is the analog of the Lagrangian condition (3.25) found for the odd forms in type IIA. Let us turn to the odd forms $\Delta^3 = \Delta^3_+ \oplus \Delta^3_-$. Due to the condition (3.6) the three-form Ω is an element of Δ^3_- for $O3/O7$ orientifolds, while it is an element of Δ^3_+ for $O5/O9$ orientifolds. Note that in contrast to the even forms $\Delta^{\rm ev}_\pm$ the spaces Δ^3_- and Δ^3_+ have generically different dimensions. The basis of three-forms introduced in (3.22) splits under the action of $\mathcal{P}_6$ as

$$(\alpha_{\hat K}, \beta^{\hat K}) \quad \rightarrow \quad (\alpha_\lambda, \beta^\lambda) \in \Delta^3_+ \ , \quad (\alpha_k, \beta^k) \in \Delta^3_- \ , \tag{3.37}$$

where $\lambda = 1, ..., \frac{1}{2} \dim \Delta^3_+$, $k = 1, ..., \frac{1}{2}\Delta^3_-$.

Given the basis decompositions (3.36) and (3.37) we can explicitly determine the finite four-dimensional spectrum of the type IIB orientifold theories. For orientifolds with $O3/O7$ planes one expands $\Phi^{\rm ev}_c$ and $A^{\rm ev}_{(1)}$ into $\Delta^{\rm ev}_+$ and Δ^3_+ as

$$\Phi^{\rm ev}_c = \tau + G^a \omega_a + T_\alpha \tilde\omega^\alpha \ , \qquad A^{\rm ev}_{(1)} = A^\lambda \alpha_\lambda \ , \tag{3.38}$$

where τ, G^a, T_α are complex scalars in four dimensions. The vector coefficients of the forms β^λ in the expansion of $A^{\rm ev}_{(1)}$ are eliminated by the duality constraint on the field strength of $\hat A^{\rm ev}$. In addition we find that $\Phi^{\rm odd}$ depends on $\frac{1}{2}(\dim \Delta^3_- - 2)$ complex deformations z^k. Therefore the full $N = 1$ spectrum consists of $\frac{1}{2}(\dim \Delta^3_- - 2)$ chiral multiplets z^k as well as $\dim \Delta^2 + 1$ chiral multiplets τ, G^a, T_α. Moreover, we find $\frac{1}{2} \dim \Delta^3_+$ vector multiplets A^λ.

The story slightly changes for orientifolds with $O5/O9$ planes. In this case the chiral coordinates are obtained by expanding

$$\Phi^{\rm ev}_c = t^\alpha \omega_\alpha + u_b \tilde\omega^b + S\, \epsilon \ , \qquad A^{\rm ev}_{(1)} = A^k \alpha_k \ , \tag{3.39}$$

where t^α, u_b, S are complex four-dimensional scalars and the volume form ϵ is normalized as $\int_{\mathcal{M}_6} \epsilon = 1$. Moreover, the form $\Phi^{\rm odd}$ depends on $\frac{1}{2}(\dim \Delta^3_+ - 2)$ complex deformations z^λ. In summary the complete $N = 1$ spectrum consists of $\frac{1}{2}(\dim \Delta^3_+ - 2)$ chiral multiplets z^λ as well as $\dim \Delta^2 + 1$ chiral multiplets t^α, u_b, S. Finally, the expansion of $A^{\rm ev}_{(1)}$ yields $\frac{1}{2} \dim \Delta^3_-$ independent vector multiplets A^k.

3.3 The Kähler potential

In this section we determine the Kähler potential encoding the kinetic terms of the chiral or dual linear multiplets. Recall that the standard bosonic action for chiral multiplets with bosonic components M^I contains the kinetic terms [92]

$$S_{\text{chiral}} = \int_{M_{3,1}} G_{I\bar{J}}\, \mathbf{d}M^I \wedge *_4\, \mathbf{d}\bar{M}^J \ , \tag{3.40}$$

where $\mathbf{d}$ and $*_4$ are the exterior derivative and the Hodge-star on $M_{3,1}$. The metric $G_{I\bar{J}} = \partial_{M^I}\partial_{\bar{M}^J}K$ is Kähler and locally given as the second derivative of a real Kähler potential $K(M, \bar{M})$. In other words, the function K determines the dynamics of the system of chiral multiplets. Similarly, one can derive the kinetic terms for a set of linear multiplets from a real function, the kinetic potential $\tilde{K}$. Since in the massless case the linear multiplets are dual to chiral multiplets one can always translate $\tilde{K}$ into an associated K via a Legendre transformation [91].[18] It therefore suffices to derive the Kähler potential. In the massive case the duality between chiral and linear multiplets is no longer valid, however, the function $\tilde{K}$ can still be formally related to a Kähler potential K. In the following we will determine the Kähler potential K for type IIA and type IIB orientifolds in turn.

The IIA Kähler potential and the Kähler metric

Let us start by discussing the type IIA Kähler potential first. As in section 3.2 we will keep our analysis general and only later specify a finite reduction. We found in the previous section that the complex scalars in the chiral multiplets are obtained by expanding the complex forms Π^{ev} and Π^{odd}_c into appropriate forms on $\mathcal{M}_6$. Locally, the field space takes the form

$$\mathcal{M}^{\text{K}} \times \mathcal{M}^{\text{Q}} \ , \tag{3.41}$$

where $\mathcal{M}^{\text{K}}$ and $\mathcal{M}^{\text{Q}}$ are spanned by the complex scalars arising in the expansion of Π^{ev} and Π^{odd}_c respectively. $N = 1$ supersymmetry demands that both manifolds in (3.41) are Kähler with metrics locally encoded by Kähler potentials K^{K} and K^{Q}. From the point of view of an $N = 2$ to $N = 1$ reduction, the manifold $\mathcal{M}^{\text{K}}$ is a complex submanifold of the $N = 2$ special Kähler manifold spanned by the complex scalars in the vector multiplets. As we will discuss momentarily the manifold $\mathcal{M}^{\text{K}}$ directly inherits its Kähler structure from the underlying $N = 2$ theory. On the other hand, $\mathcal{M}^{\text{Q}}$ is a submanifold of the quaternionic space spanned by the hyper multiplets and has half its dimension. It is a non-trivial result that $\mathcal{M}^{\text{Q}}$ is a Kähler manifold since the underlying quaternionic manifold is not necessarily Kähler.

We analyze first the structure of the field space $\mathcal{M}^{\text{K}}$ spanned by the complex fields arising in the expansion of $J_c = -\hat{B}_2 + iJ$ into forms $\Lambda^2_+ T^*$. Note that as in the original $N = 2$ theory not all forms J are allowed and one restricts to the cases where J, $J \wedge J$ and $J \wedge J \wedge J$ measure positive volumes of two-, four and six-cycles [93]. We abbreviate this condition by writing $J \geq 0$. Hence, the coefficients of J_c define the complex cone

$$\mathcal{M}^{\text{K}} = \left\{ J_c \in \Lambda^2_+ T^*_{\mathbb{C}} : J \wedge J \wedge J \neq 0 \text{ and } J \geq 0 \right\} \ . \tag{3.42}$$

[18]For a brief review, see also section 4 of ref. [28].

This manifold has the same complex structure as the underlying $N = 2$ special Kähler manifold. It also inherits its Kähler structure with a Kähler potential given by [93, 62]

$$K^{\rm K}(J_c) = -\ln\big[-i\int_{\mathcal{M}_6}\big\langle\Pi^{\rm ev},\bar{\Pi}^{\rm ev}\big\rangle\big] = -\ln\big[\tfrac{4}{3}\int_{\mathcal{M}_6} J\wedge J\wedge J\big]\ , \qquad (3.43)$$

where $\Pi^{\rm ev} = e^{J_c}$ is introduced in (3.12) and the pairing $\langle\cdot,\cdot\rangle$ is defined in (3.20).[19] The Kähler metric is obtained as the second derivative of $K^{\rm K}$ given in (3.43) with respect to J_c and $\bar{J}_c$. More precisely, one finds

$$G^{\rm K}(\omega,\omega') = \big[\partial_{J_c}\partial_{\bar{J}_c}K^{\rm K}\big](\omega,\omega') = -2e^{K^{\rm K}}\int_{\mathcal{M}_6}\big\langle\omega, *_6\,\omega'\big\rangle\ , \qquad (3.44)$$

where $*_6$ is the six-dimensional Hodge-star and ω, ω' are two-forms in $\Lambda^2_+T^*$. Note that in this general approach the derivatives are taken with respect to two-forms on $\mathcal{M}_6$ containing the $D = 4$ scalars such that the result needs to be evaluated on elements of $\Lambda^2_+T^*$. The four-dimensional kinetic terms (3.40) read [20]

$$S_{\Pi^{\rm ev}} = \int_{M_{3,1}} G^{\rm K}(\mathbf{d}J_c, *_4\,\mathbf{d}\bar{J}_c) \qquad (3.45)$$

with four-dimensional derivative $\mathbf{d}$. From (3.44) one concludes that the metric $G^{\rm K}$ only depends on $\Pi^{\rm ev}$. It is straight forward to evaluate (3.45) in the finite basis $\omega_a \in \Delta^2_+$ introduced in equation (3.24). On this basis the complex form J_c decomposes as $J_c = t^a\omega_a$ and one finds

$$S_{\Pi^{\rm ev}} = \int_{M_{3,1}} G^{\rm K}_{a\bar{b}}\ \mathbf{d}t^a\wedge *_4\,\mathbf{d}\bar{t}^b\ , \qquad G^{\rm K}_{a\bar{b}} = 2e^{K^{\rm K}}\int \omega_a\wedge *\omega_b\ . \qquad (3.46)$$

In the finite basis the metric G^K takes a form similar to the case where $\mathcal{M}_6$ is a Calabi-Yau orientifold [24]. However, since the forms ω_a are not necessarily harmonic a potential for the fields t^a is introduced as we will discuss in section 3.4.

Let us now turn to the second factor in (3.41) and investigate the Kähler structure of the manifold $\mathcal{M}^{\rm Q}$. As introduced in section 3.2 the complex coordinates on this space are obtained by expanding the form $\Pi^{\rm odd}_c$ into elements of $\Lambda^3_+T^*$. The metric on the field space $\mathcal{M}^{\rm Q}$ is derived by inserting the expansion of $\Pi^{\rm odd}_c$ in the ten-dimensional effective action of type IIA supergravity. For the form $C_3^{(0)}$ the reduction of the R-R sector yields the term

$$S_{C_3^{(0)}} = \int_{M_{3,1}} G^{\rm Q}(\mathbf{d}C_3^{(0)}, *_4\,\mathbf{d}C_3^{(0)})\ , \qquad (3.47)$$

where the metric $G^{\rm Q}$ is defined as

$$G^{\rm Q}(\alpha,\alpha') = 2e^{2D}\int_{\mathcal{M}_6}\big\langle\alpha, *_6\,\alpha'\big\rangle\ , \qquad (3.48)$$

[19] Note that in contrast to ref. [62] we included an integration in the definition of $K^{\rm K}$ such that it is independent of the coordinates on $\mathcal{M}_6$. This implies that four-dimensional supergravity theory takes the standard $N = 1$ form. However, this also implies that we have to exclude modes which correspond the rescalings of J by a function $\mathcal{M}_6$ (see also appendix B.3). We will come back to this issue in a separate publication [81].

[20] The action is given in the four-dimensional Einstein frame where the kinetic term for the metric takes the form $\frac{1}{2}R$.

with $\alpha, \alpha' \in \Lambda^3_+ T^*$. The four-dimensional dilaton D was defined in eqn. (3.14) and arises in (3.47) due to a Weyl rescaling to the four-dimensional Einstein frame. The R-R field $C_3^{(0)}$ is complexified by $\mathrm{Re}(\Pi^{\mathrm{odd}})$ as given in eqn. (3.19). Therefore, the full kinetic terms for the complex scalars in Π_c^{odd} are given by

$$S_{\Pi_c^{\mathrm{odd}}} = \int_{M_{3,1}} G^{\mathrm{Q}}(\mathbf{d}\Pi_c^{\mathrm{odd}}, *_4\, \mathbf{d}\bar{\Pi}_c^{\mathrm{odd}}) \ . \tag{3.49}$$

The metric G^{Q} is Kähler on the manifold $\mathcal{M}^{\mathrm{Q}}$ if we carefully specify the forms used in the expansion of Π_c^{odd}. As already explained in section 2, the real three-forms $\rho = \mathrm{Re}(\Pi^{\mathrm{odd}})$ defining an $SU(3)$ structure manifold have to be 'stable'.[21] We denote all stable forms in $\Lambda^3_+ T^*$ by U^3_+. Using this definition the field space $\mathcal{M}^{\mathrm{Q}}$ spanned by the complex scalars in Π_c^{odd} is locally of the form

$$\mathcal{M}^{\mathrm{Q}} = \big\{\mathrm{Re}(\Pi^{\mathrm{odd}}) \in U^3_+\big\} \times \Lambda^3_+ T^* \ , \tag{3.50}$$

where $\Lambda^3_+ T^*$ is parameterized by the real scalars in the R-R field $C_3^{(0)}$.

In appendix B.3 we show that the metric G^{Q} can be obtained as the second derivative of a Kähler potential. Note however, that we have to impose an additional constraint on the forms in $\mathcal{M}^{\mathrm{Q}}$ in order to obtain a Kähler potential independent of the coordinates on $\mathcal{M}_6$. More precisely, we demand that all $(3,0)+(0,3)$ forms in $\mathcal{M}^{\mathrm{Q}}$ are proportional to ρ with a coefficient constant on $\mathcal{M}_6$.[22] On this the set of stable forms one shows that the metric G^{Q} is Kähler with a Kähler potential given by

$$K^{\mathrm{Q}}(\Pi_c^{\mathrm{odd}}) = -2\ln\big[i\int_{\mathcal{M}_6} \langle \Pi^{\mathrm{odd}}, \bar{\Pi}^{\mathrm{odd}}\rangle\big] = -\ln\big[e^{-4D}\big] \ , \tag{3.51}$$

where in the second equality we have used the definition of $\Pi^{\mathrm{odd}} = C\Omega$ given in equations (3.12) and (3.13) to express K^{Q} in terms of the four-dimensional dilaton e^D defined in eqn. (3.14). The functional appearing in the logarithm of the Kähler potential,

$$H\big[\mathrm{Re}(\Pi^{\mathrm{odd}})\big] = i\int \big\langle \Pi^{\mathrm{odd}}, \bar{\Pi}^{\mathrm{odd}}\big\rangle \ , \tag{3.52}$$

was first introduced by Hitchin in refs. [75]. A more explicit definition of H as a functional of $\mathrm{Re}(\Pi^{\mathrm{odd}})$ can be found in appendix B. The metric G^{Q} defined in (3.48) is obtained by the second derivative

$$G^{\mathrm{Q}}(\alpha, \alpha') = \big[\partial_{\Pi_c^{\mathrm{odd}}}\, \partial_{\bar{\Pi}_c^{\mathrm{odd}}} K^{\mathrm{Q}}\big](\alpha, \alpha') \ . \tag{3.53}$$

Note that K^{Q} is a function of $\mathrm{Re}(\Pi^{\mathrm{odd}})$ and does not depend on the R-R fields $\mathrm{Re}(\Pi_c^{\mathrm{odd}}) = C_3^{(0)}$. Hence, the metric G^{Q} possesses various shift symmetries and the second factor in (3.50) is a vector space.

Finally, we will restrict the results obtained for $\mathcal{M}^{\mathrm{Q}}$ to the finite basis Δ_{finite}. In order to do so, one expands the complex form Π_c^{odd} in the real basis $\alpha_k, \beta^\lambda \in \Delta^3_+$ as given in

[21] The detailed definition of stable forms is given in appendix B.

[22] This condition can be weakened in case the Kähler potential is defined as a logarithm of a function varying along $\mathcal{M}_6$ as we also discuss in appendix B.3. In this case, the orientifold theory is an $N=1$ reformulation of the ten-dimensional supergravity theory [81].

eqn. (3.27). The coefficients of this expansion are complex scalars N^k, T_λ. The Kähler metric is the second derivative of $K^{\rm Q}$ given in eqn. (3.51) with respect to these complex fields. Explicitly, it takes the form

$$\begin{aligned}\partial_{N^k}\partial_{\bar N^l} K &= 2e^{2D}\int_{\mathcal{M}_6} \alpha_k \wedge *_6\, \alpha_l\ , \qquad \partial_{N^k}\partial_{\bar T_\kappa} K = 2e^{2D}\int_{\mathcal{M}_6} \alpha_k \wedge *_6\, \beta^\kappa\ , \\ \partial_{T_\kappa}\partial_{\bar T_\lambda} K &= 2e^{2D}\int_{\mathcal{M}_6} \beta^\kappa \wedge *_6\, \beta^\lambda\ .\end{aligned} \tag{3.54}$$

This ends our discussion of the Kähler metric on the type IIA field spaces $\mathcal{M}^{\rm K}\times\mathcal{M}^{\rm Q}$. We found that the Kähler potentials are the two Hitchin functionals depending on real two- and three-forms on $\mathcal{M}_6$. A similar result with odd and even forms exchanged is found for type IIB orientifolds to which we turn now.

The IIB Kähler potential and the Kähler metric

In the following we investigate the Kähler structure of the scalar field space in type IIB orientifolds. The complex scalars in the chiral multiplets are obtained by expanding $\Phi^{\rm odd}$ and $\Phi^{\rm ev}_c$ into appropriate forms on $\mathcal{M}_6$ as introduced in eqns. (3.33) and (3.35). These complex scalars locally span the product manifold $\mathcal{M}^{\rm K}\times\mathcal{M}^{\rm Q}$, where $\mathcal{M}^{\rm K}$ contains the independent scalars in $\Phi^{\rm odd}$ while $\mathcal{M}^{\rm Q}$ contains the scalars in $\Phi^{\rm ev}_c$. Note that we are now dealing with two type IIB setups corresponding to two truncations of the original $N=2$ theory.

As in the type IIA orientifolds the complex and Kähler structure of $\mathcal{M}^{\rm K}$ is directly inherited from the underlying $N=2$ theory. Independent reductions of the structure group of $\mathcal{M}_6$ are parameterized by a set of real stable forms $\rho = {\rm Re}(\Phi^{\rm odd})$ denoted by U^3. In order to satisfy the orientifold constraints (3.31) this field space is reduced to U^3_- for $O3/O7$ orientifolds and to U^3_+ for $O5/O9$ orientifolds. Furthermore, complex rescalings of the complex three-form $\Phi^{\rm odd}$ are unphysical. Hence, the moduli space encoded by $\Phi^{\rm odd}$ is obtained by dividing $U^3_\pm$ by reparameterizations $\Phi^{\rm odd}\rightarrow c\,\Phi^{\rm odd}$ for complex non-zero $c\in\mathbb{C}^*$. The field space $\mathcal{M}^{\rm K}$ is then defined as

$$\mathcal{M}^{\rm K} = \big\{{\rm Re}(\Phi^{\rm odd})\in U^3_\mp\big\}/\mathbb{C}^*\ , \tag{3.55}$$

where the minus sign stands for $O3/O7$ and the plus sign for $O5/O9$ orientifolds. The field space $\mathcal{M}^{\rm K}$ is a complex Kähler manifold. This is shown in analogy to the $N=2$ case discussed in refs. [75, 62], since the orientifold projections preserve the complex structure and only reduce the dimension of $\mathcal{M}^{\rm K}$. We denote the complex scalars parameterizing $\mathcal{M}^{\rm K}$ by z's. The Kähler potential as a function of these fields and their complex conjugates is given by

$$K^{\rm K}(z,\bar z) = -\ln\big[-i\int_{\mathcal{M}_6}\big\langle\Phi^{\rm odd},\bar\Phi^{\rm odd}\big\rangle\big] = -\ln\big[-i\int_{\mathcal{M}_6}\Omega\wedge\bar\Omega\big]\ , \tag{3.56}$$

where in the second equality we used the definitions (3.30) and (3.20) of $\Phi^{\rm odd}$ and the pairings $\langle\cdot,\cdot\rangle$. The manifold $\mathcal{M}^{\rm K}$ possesses a special geometry completely analogous to the $N=2$ case, such that in particular the three-form $\Omega(z)$ is a holomorphic function in the complex coordinates z on $\mathcal{M}^{\rm K}$. This special geometry was used in ref. [62] to derive the Kähler metric corresponding to $K^{\rm K}$. We will not review the result here, but rather immediately turn to the field space $\mathcal{M}^{\rm Q}$ which is a Kähler field space in the $N=1$ theory.

Let us now determine the Kähler potential encoding the metric on the field space $\mathcal{M}^{\rm Q}$. As discussed in section 3.2 the complex coordinates spanning $\mathcal{M}^{\rm Q}$ are obtained by expanding $\Phi^{\rm ev}_c$ into elements of $\Lambda^{\rm ev}_{\pm}$ depending on whether we are dealing with $O3/O7$ or $O5/O9$ orientifolds. The precise definition of $\Phi^{\rm ev}_c$ was given in eqn. (3.35). Note that not every form in $\Lambda^{\rm ev}_{\pm}$ corresponds to a reduction of the structure group of $\mathcal{M}_6$ to $SU(3)$ and we have additionally to impose constraints on $\mathrm{Im}(\Phi^{\rm ev}_c)$ analog to the stability condition discussed above. Recall that in the $O3/O7$ case $\mathrm{Im}(\Phi^{\rm ev}_c) = \mathrm{Re}(e^{-\hat\phi}e^{-\hat B_2+iJ})$ and in the $O5/O9$ case $\mathrm{Im}(\Phi^{\rm ev}_c) = \mathrm{Im}(e^{-\hat\phi}e^{-\hat B_2+iJ})$ as given in eqns. (3.30) and (3.35). In these definitions the real two-from J has to satisfy $J\wedge J\wedge J\neq 0$ and $J\geq 0$ as in (3.42). Altogether the field space $\mathcal{M}^{\rm Q}$ locally takes the form

$$\mathcal{M}^{\rm Q} = \big\{\mathrm{Im}(\Phi^{\rm ev}_c)\in\Lambda^{\rm ev}_{\pm} : J\wedge J\wedge J\neq 0 \text{ and } J\geq 0\big\}\times \Lambda^{\rm ev}_{\pm}T^* \ , \tag{3.57}$$

where the vector space $\Lambda^{\rm ev}_{\pm}T^*$ is spanned by the fields $A^{\rm ev}_{(0)}$. The plus sign in the expression (3.57) corresponds to orientifolds with $O3/O7$ planes while the minus sign stands for the $O5/O9$ orientifolds. The metric on the manifold $\mathcal{M}^{\rm Q}$ is obtained by inserting the expansion of the R-R form $A^{\rm ev}_{(0)}$ into the ten-dimensional action of type IIB supergravity. Performing a Weyl rescaling to the four-dimensional Einstein frame one finds

$$S_{A^{\rm ev}_{(0)}} = \int_{M_{3,1}} G^{\rm Q}(\mathbf{d}A^{\rm ev}_{(0)}, *_4\, \mathbf{d}A^{\rm ev}_{(0)}) \ , \tag{3.58}$$

where $G^{\rm Q}$ is defined on forms $\nu,\nu'\in\Lambda^{\rm ev}_{\pm}T^*$ as

$$G^{\rm Q}(\nu,\nu') = 2e^{2D}\int_{\mathcal{M}_6}\big\langle \nu, *_B\,\nu'\big\rangle \ . \tag{3.59}$$

The reason for this simple form is that we have replaced the ordinary Hodge-star by the B-twisted Hodge star $*_B$ acting on an even form ν as (see, for example, ref. [94])

$$*_B\,\nu = e^{\hat B_2}\wedge *\lambda(e^{-\hat B_2}\wedge\nu) \ , \tag{3.60}$$

where λ is the parity operator introduced in (3.8). In equation (3.59) the four-dimensional dilaton D is defined as in the type IIA case (3.14). Including the reduction of $\mathrm{Im}(\Phi^{\rm ev}_c)$ the action (3.58) is completed as

$$S_{\Phi^{\rm ev}_c} = \int_{M_{3,1}} G^{\rm Q}(\mathbf{d}\Phi^{\rm ev}_c, *_4\, \mathbf{d}\bar\Phi^{\rm ev}_c) \ . \tag{3.61}$$

The metric $G^{\rm Q}$ is shown to be the second derivative of the Kähler potential [23]

$$K^{\rm Q}(\Phi^{\rm ev}_c) = -2\ln\big[i\int_{\mathcal{M}_6}\big\langle\Phi^{\rm ev},\bar\Phi^{\rm ev}\big\rangle\big] = -\ln\big[e^{-4D}\big] \ , \tag{3.62}$$

[23]As in the type IIA case we discard the non-trivial modes proportional to $\mathrm{Im}(\Phi^{\rm ev}_c)$. These can be included if the Kähler potential is the logarithm of a volume form varying along $\mathcal{M}_6$.

where in the second equality we have used the definition of $\Phi^{\rm ev}$ as given in (3.30). Note that $K^{\rm Q}$ is a function of $\mathrm{Im}(\Phi^{\rm ev}_c)$ only, such that it depends on $\mathrm{Re}(\Phi^{\rm ev})$ in $O3/O7$ orientifolds while it depends on $\mathrm{Im}(\Phi^{\rm ev})$ in $O5/O9$ orientifolds. The functionals appearing in the logarithm are the Hitchin functionals (see also appendix B) [76]

$$H[\mathrm{Re}(\Phi^{\rm ev})] = i\int_{\mathcal{M}_6} \langle \Phi^{\rm ev}, \bar{\Phi}^{\rm ev}\rangle\ , \qquad H[\mathrm{Im}(\Phi^{\rm ev})] = i\int_{\mathcal{M}_6} \langle \Phi^{\rm ev}, \bar{\Phi}^{\rm ev}\rangle\ , \tag{3.63}$$

depending on whether we are dealing with $O3/O7$ and $O5/O9$ orientifolds.[24] The metric $G^{\rm Q}$ given in eqn. (3.59) is obtained by taking the second derivative of $K^{\rm Q}$ as

$$G^{\rm Q}(\nu,\nu') = \big[\partial_{\Phi^{\rm ev}_c}\partial_{\bar{\Phi}^{\rm ev}_c} K^{\rm Q}\big](\nu,\nu')\ . \tag{3.64}$$

Due to the independence of $K^{\rm Q}$ of the R-R scalars in $A^{\rm ev}_{(0)}$ the metric $G^{\rm Q}$ possesses shift symmetries.

It is straight forward to evaluate the Kähler metric $G^{\rm Q}$ for the finite basis of $\Delta^{\rm ev}_{\pm}$ introduced in (3.36). The coefficients are complex fields $M^{\hat{A}} = (\tau, G^a, T_\alpha)$ for $O3/O7$ orientifolds and $M^{\hat{A}} = (S, t^\alpha, A_a)$ for $O5/O9$ orientifolds as seen in eqns. (3.38) and (3.39). Explicitly the metric $G^{\rm Q}$ is given by

$$\partial_{M^{\hat{A}}}\partial_{\bar{M}^{\hat{B}}} K = e^{2D}\int \langle \nu_{\hat{A}}, *_B\, \nu_{\hat{B}}\rangle\ , \tag{3.65}$$

where $\nu_{\hat{A}} = (1, \omega_a, \tilde{\omega}^\alpha)$ for $O3/O7$ orientifolds while $\nu_{\hat{A}} = (\epsilon, \omega_\alpha, \tilde{\omega}^a)$ for $O5/O9$ orientifolds. These metrics are identical to the ones derived for type IIB Calabi-Yau orientifolds [23] if the finite basis $\Delta_{\rm finite}$ is consisting of harmonic forms only. Compared to the expression given in ref. [23] we simplified the result considerably by introducing the B-twisted Hodge-star $*_B$.

To summarize we found that also in the type IIB setups the field space $\mathcal{M}^{\rm K}\times\mathcal{M}^{\rm Q}$ is a Kähler manifold with Kähler potentials given by the logarithm of the Hitchin functionals. This fixes the kinetic terms of the chiral or dual linear multiplets. Surprisingly, our analysis can be performed in a rather general setting without specifying a finite reduction. To illustrate the results we nevertheless gave the reduction to the finite basis $\Delta_{\rm finite}$. We will now turn to the analysis of the superpotential terms induced on $SU(3)$ structure orientifolds.

3.4 The Superpotential of type II $SU(3)$ structure orientifolds

In this section we derive the superpotentials for type IIA and type IIB $SU(3)$ structure orientifolds in presence of fluxes and torsion. The calculation is most easily done on the level of the fermionic effective action. This is due to the fact that the superpotential W appears linearly in a four-dimensional $N = 1$ supergravity theory as the mass of the gravitino ψ_μ. The corresponding mass term reads

$$S_{\rm mass} = -\int_{M_{3,1}} e^{K/2}\big(W\bar{\psi}_\mu\bar{\sigma}^{\mu\nu}\bar{\psi}_\nu + \bar{W}\psi_\mu\sigma^{\mu\nu}\psi_\nu\big) *_4 \mathbf{1}\ , \tag{3.66}$$

[24] In the first case one obtains the functional dependence of H by evaluating $\mathrm{Im}(\Phi^{\rm ev})$ as a function of the real part $\mathrm{Re}(\Phi^{\rm ev})$, while in the second case one needs do find $\mathrm{Re}(\Phi^{\rm ev})\big[\mathrm{Im}(\Phi^{\rm ev})\big]$.

where $*_4\mathbf{1} = \sqrt{-g_4}\, d^4x$ is the four-dimensional volume element and K is the Kähler potential on the chiral field space. To determine (3.66) for the orientifold setups one dimensionally reduces the fermionic part of the type IIA and type IIB actions. As in the bosonic part, the orientifold projections ensure that the resulting four-dimensional theories possess $N = 1$ supersymmetry.

Let us start by recalling the relevant fermionic terms for our discussion in the ten-dimensional type IIA and type IIB supergravity theories. We conveniently combine the two gravitinos into a two-vector $\hat{\psi}_N = (\hat{\psi}^1_N, \hat{\psi}^2_N)$. The effective action for the gravitinos in string frame takes the form [25]

$$S_\psi = -\int_{\mathcal{M}_{10}} \left[e^{-2\hat{\phi}} \hat{\bar{\psi}}_M \Gamma^{MNP} D_N \hat{\psi}_P \ *\mathbf{1} + \tfrac{1}{4} e^{-2\hat{\phi}} \hat{H}_3 \wedge *\Psi + \tfrac{1}{8} \sum_n \hat{F}_n \wedge *\Psi_n\right] , \qquad (3.67)$$

where we are using the democratic formulation of ref. [89]. The R-R field strengths $\hat{F}_n$ are defined as

$$\hat{F}_n = d\hat{C}_{n-1} - \hat{H}_3 \wedge \hat{C}_{n-3} , \qquad *\hat{F}_n = \lambda(\hat{F}_{10-n}) , \qquad (3.68)$$

where n runs from 0 to 8 for type IIA and from 1 to 9 for type IIB and we set $\hat{H}_3 = d\hat{B}_2$. The self-duality condition in eqn. (3.68) implies that half of the R-R fields in $\hat{C}^{\mathrm{ev/odd}}$ carry no extra degrees of freedom. Furthermore, Ψ and Ψ_n are ten-dimensional three- and n-forms which are bilinear in $\hat{\psi}_M$ and have components

$$\begin{aligned} (\Psi)_{M_1 M_2 M_3} &= \hat{\bar{\psi}}_M \Gamma^{[M} \Gamma_{M_1 M_2 M_3} \Gamma^{N]} \mathcal{P} \hat{\psi}_N , \\ (\Psi_n)_{M_1 \ldots M_n} &= e^{-\hat{\phi}} \hat{\bar{\psi}}_M \Gamma^{[M} \Gamma_{M_1 \ldots M_n} \Gamma^{N]} \mathcal{P}_n \hat{\psi}_N , \end{aligned} \qquad (3.69)$$

where $\mathcal{P} = \Gamma_{11}$, $\mathcal{P}_n = (\Gamma_{11})^n$ for type IIA while for type IIB one has $\mathcal{P} = -\sigma^3$, $\mathcal{P}_n = \sigma^1$ for $\frac{n+1}{2}$ even and $\mathcal{P}_n = i\sigma^2$ for $\frac{n+1}{2}$ odd.

In a next step we dimensionally reduce the action (3.67) on the manifold $M_{3,1} \times \mathcal{M}_6$ focusing on the derivation of four-dimensional mass terms of the form (3.66). In order to do that we decompose the ten-dimensional gravitinos $\hat{\psi}_M$ into four-dimensional spinors on $M_{3,1}$ times six-dimensional spinors on the $SU(3)$ structure manifold $\mathcal{M}_6$. Of particular interest is the reduction of $\hat{\psi}_\mu$ where μ labels the four space-time directions on $M_{3,1}$. In type IIB both ten-dimensional gravitinos have the same chirality and split as

$$\hat{\psi}^A_\mu = \psi^A_\mu \otimes \eta_- + \bar{\psi}^A_\mu \otimes \eta_+ \qquad A = 1, 2 , \qquad (3.70)$$

where η denotes the globally defined spinor introduced in eqn. (2.2) with six-dimensional chirality $\pm$. The four-dimensional spinors $\psi^{1,2}_\mu$ and $\bar{\psi}^{1,2}_\mu$ are Weyl spinors with positive and negative chiralities respectively. In type IIA supergravity the gravitinos have different chiralities and hence decompose as

$$\hat{\psi}^1_\mu = \psi^1_\mu \otimes \eta_+ + \bar{\psi}^1_\mu \otimes \eta_- , \qquad \hat{\psi}^2_\mu = \psi^2_\mu \otimes \eta_- + \bar{\psi}^2_\mu \otimes \eta_+ . \qquad (3.71)$$

[25] We only display terms which are quadratic in the gravitinos $\hat{\psi}_N$ since we aim to calculate terms of the form (3.66). Moreover, note that the ten-dimensional fermions are Majorana-Weyl spinors and the conjugate spinor $\hat{\bar{\psi}}_M = \psi^\dagger_M \Gamma^0$ is obtained by hermitian conjugation and multiplication with the ten-dimensional gamma-matrix Γ^0.

The spinor $\psi_\mu^{1,2}$ appearing in (3.70) and (3.71) yield the four-dimensional gravitinos when appropriately combined with four-dimensional spinors arising in the expansion of $\hat{\psi}_m,\ m = 1,\dots,6$. However, since they are combined linearly the mass terms of $\psi_\mu^{1,2}$ take the same form as the one for the four-dimensional gravitinos which label the $N = 2$ supersymmetry.

The orientifold projections reduce the four-dimensional theory to an $N = 1$ supergravity. Hence, the two four-dimensional gravitinos as well as the spinors $\psi_\mu^{1,2}$ are not independent, but rather combine into one four-dimensional spinor ψ_μ which parameterizes the $N = 1$ supersymmetry. This spinor is chosen in such a way that its ten-dimensional extension $\hat{\psi}_M$ is invariant under the projections $\mathcal{O}$ and $\mathcal{O}_{(1,2)}$ given in eqns. (3.2) and (3.7) respectively. To investigate the transformation behavior of ten-dimensional spinors, recall that the world-sheet parity Ω_p exchanges $\hat{\psi}_M^1$ and $\hat{\psi}_M^2$. If the orientifold projection contains the operator $(-1)^{F_L}$ one finds an additional minus sign when applied to $\hat{\psi}_M^2$. In this we asserted that $\hat{\psi}_M^2$ is in the NS-R sector while $\hat{\psi}_M^1$ is in the R-NS sector. The geometric symmetry σ acts only on the internal space $\mathcal{M}_6$ which translates to a non-trivial transformation of the globally defined spinor η. The precise action of σ^* is different for type IIA and type IIB orientifolds. In the following we will discuss the reduction of both ten-dimensional type II theories in turn and determine the induced superpotentials.

The type IIA superpotential

Let us first determine the superpotential for type IIA orientifolds induced by non-trivial background fluxes and torsion. Background fluxes are vacuum expectation values for the R-R and the NS-NS field strengths. We denote the background flux of $d\hat{B}_2$ by H_3 while the fluxes of the R-R forms $d\hat{C}_n$ are denoted by F_{n+1}. In order that the four-dimensional background $M_{3,1}$ is maximally symmetric the fluxes have to be extended in the internal manifold $\mathcal{M}_6$ or correspond to a four-form on $M_{3,1}$. In type IIA supergravity we additionally allow for a scalar parameter F_0, which corresponds to the mass in the massive type IIA theory introduced by Romans [95]. In order that the background fluxes respect the orientifold condition (3.11) they have to obey

$$\sigma^* H_3 = -H_3 \ , \qquad\qquad \sigma^* F_n = \lambda(F_n) \ . \tag{3.72}$$

It is convenient to combine the R-R background fluxes into an even form F^{ev} on $\mathcal{M}_6$ as

$$F^{\text{ev}} = F_0 + F_2 + F_4 + F_6 \ . \tag{3.73}$$

In addition to the background fluxes also a non-vanishing intrinsic torsion of the $SU(3)$ structure manifold will induce terms contributing to the $N = 1$ superpotential. These arise due to the non-closedness of the globally defined two-form J and three-form Ω_η and can be parameterized as given in eqn. (2.9).

In order to actually perform the reduction we need to specify the action of the orientifold projection $\mathcal{O} = (-1)^{F_L}\Omega_p\sigma^*$ on the spinors $\hat{\psi}_\mu^1$ and $\hat{\psi}_\mu^2$. The transformation behavior of the ten-dimensional gravitinos under $(-1)^{F_L}\Omega_p$ was already discussed above. We supplement this by the action of σ^* on the globally defined spinor η. In accord with condition (3.1) one has

$$\sigma^*\eta_+ = e^{i\theta}\eta_- \ , \qquad\qquad \sigma^*\eta_- = e^{-i\theta}\eta_+ \ , \tag{3.74}$$

where θ is the phase introduced in eqn. (3.1). Therefore, the invariant combination of the four-dimensional spinors is given by $\psi_\mu = \frac{1}{2}(e^{i\theta/2}\,\psi^1_\mu - e^{-i\theta/2}\,\psi^2_\mu)$ with a similar expression for the Weyl spinors $\bar\psi^{1,2}_\mu$. In order to ensure the correct form of the four-dimensional kinetic terms for ψ_μ we restrict to the specific choice

$$\psi_\mu = e^{i\theta/2}\,\psi^1_\mu = -e^{-i\theta/2}\,\psi^2_\mu\ , \qquad\qquad \bar\psi_\mu = e^{-i\theta/2}\,\bar\psi^1_\mu = -e^{i\theta/2}\,\bar\psi^2_\mu\ . \tag{3.75}$$

These conditions define a reduction of a four-dimensional $N = 2$ to an $N = 1$ supergravity theory [73, 74, 62]. Hence, the mass terms of the spinors ψ_μ take the standard $N = 1$ form given in eqn. (3.66).

Now we turn to the explicit reduction of the ten-dimensional effective action (3.67) focusing on the mass terms of ψ_μ induced by the background fluxes H_3 and F_n and the torsion of $\mathcal{M}_6$. We use the decomposition (3.71) together with (3.75) and the gamma-matrix conventions summarized in appendix A to derive

$$\begin{aligned} S_\psi \;=\; & -\int_{M_{3,1}} e^{\frac{K}{2}}\,\bar\psi_\mu\bar\sigma^{\mu\nu}\bar\psi_\nu *_4 \mathbf{1}\int_{\mathcal{M}_6}\Big[4e^{-\hat\phi+i\theta}\eta^\dagger_+\gamma^m D_m\eta_- + 4e^{-\hat\phi-i\theta}\eta^\dagger_-\gamma^m D_m\eta_+ \\ & + \tfrac{1}{3!}\,e^{-\hat\phi+i\theta}\,(\hat H_3)_{mnp}\,\eta^\dagger_+\gamma^{mnp}\eta_- \;-\; \tfrac{1}{3!}\,e^{-\hat\phi-i\theta}\,(\hat H_3)_{mnp}\,\eta^\dagger_-\gamma^{mnp}\eta_+ \\ & + \tfrac{1}{2}\sum_{k\ \text{even}} \tfrac{1}{k!}\big((\lambda\hat F_k)_{m_1\ldots m_k}\eta^\dagger_+\gamma^{m_1\ldots m_k}\eta_+ + (\hat F_k)_{m_1\ldots m_k}\eta^\dagger_-\gamma^{m_1\ldots m_k}\eta_-\big)\Big] *_6 \mathbf{1} + \ldots\ , \end{aligned} \tag{3.76}$$

where $e^{K/2} = e^{2D}e^{K^{\rm K}/2}$ with $K^{\rm K}$ as defined in eqn. (3.43). The four-dimensional dilaton e^D is introduced in (3.14). Note that after the reduction of the $D = 10$ string frame action to four space-time dimensions we performed a Weyl-rescaling to obtain a standard Einstein-Hilbert term. More precisely, in the derivation of (3.76) we made the rescaling

$$g_{\mu\nu} \to e^{2D}g_{\mu\nu}\ , \qquad \sigma^\mu \to e^{-D}\sigma^\mu\ , \qquad \psi_\mu \to e^{D/2}\psi_\mu\ . \tag{3.77}$$

The rescaling of ψ_μ ensures that the four-dimensional theory has a standard kinetic term for the gravitino. The superpotential can be obtained by comparing the action (3.76) with the standard $N = 1$ mass term (3.66). We will discuss the arising terms in turn and rewrite them into the form language used in the previous sections.

Let us next express the result (3.76) in terms of the globally defined two-form J and Ω_η defined in (2.6). First note that Ω_η is related to the Ω used in analysis of the bosonic terms (sections 3.2 and 3.3) by a rescaling

$$\Omega_\eta = e^{(K^{\rm cs}-K^{\rm K})/2}\Omega\ , \tag{3.78}$$

where $e^{-K^{\rm cs}} = i\int\Omega\wedge\bar\Omega$ and $K^{\rm K}$ is defined in (3.43). The three-form Ω_η is defined in such a way, that it satisfies automatically the first condition in (2.7) when integrated over $\mathcal{M}_6$. The quantities in the first line of (3.76) are expressed in terms of the forms Ω_η and J by using the identities [26]

$$\int_{\mathcal{M}_6}\eta^\dagger_-\gamma^m D_m\eta_+ *_6 \mathbf{1} = -\tfrac{1}{8}\int_{\mathcal{M}_6}\Omega_\eta\wedge dJ\ , \qquad \int_{\mathcal{M}_6}\eta^\dagger_+\gamma^m D_m\eta_- *_6 \mathbf{1} = -\tfrac{1}{8}\int_{\mathcal{M}_6}\bar\Omega_\eta\wedge dJ\ , \tag{3.79}$$

[26] The expression (3.79) can be shown by using the Fierz identity (A.13) and expression (A.14) for $\eta^1 = \eta^2 = \eta$.

where d is the six-dimensional exterior derivative. Using these integrals as well as (3.78) and the definition of $\Pi^{\rm odd} = C\Omega$ displayed in (3.12), (3.13) one finds

$$4\int_{\mathcal{M}_6} e^{-\hat\phi}\Big[e^{i\theta}\eta_+^\dagger\gamma^m D_m\eta_- + e^{-i\theta}\eta_-^\dagger\gamma^m D_m\eta_+\Big] *_6 \mathbf{1} = -\int_{\mathcal{M}_6}\big\langle d{\rm Re}(\Pi^{\rm odd}), J\big\rangle\ . \qquad (3.80)$$

Similarly, one expresses the remaining terms in the action (3.76) using the three-from $\Pi^{\rm odd}$ and the two-form J. More precisely, the terms in the second line of eqn. (3.76) are rewritten by applying eqns. (2.6), (3.78), (3.12) and $*\Omega = -i\Omega$ as

$$\begin{aligned}\tfrac{1}{3!}\int_{\mathcal{M}_6}&\big[e^{-\hat\phi+i\theta}(\hat H_3)_{mnp}\eta_+^\dagger\gamma^{mnp}\eta_- - e^{-\hat\phi-i\theta}(\hat H_3)_{mnp}\eta_-^\dagger\gamma^{mnp}\eta_+\big] *_6 \mathbf{1} \qquad (3.81)\\ &= -i\int_{\mathcal{M}_6}\Big[\big\langle H_3\wedge{\rm Re}(\Pi^{\rm odd}),1\big\rangle + \big\langle d{\rm Re}(\Pi^{\rm odd}),\hat B_2\big\rangle\Big]\ ,\end{aligned}$$

where we have used that $\hat H_3 = d\hat B_2 + H_3$ with H_3 being the background flux. Finally, we apply gamma-matrix identities and the definition (2.6) of J to rewrite the terms appearing in the last line of (3.76) as

$$\begin{aligned}\tfrac12\sum_{k\ {\rm even}}\tfrac{1}{k!}\int_{\mathcal{M}_6}&\Big[(\lambda\hat F_k)_{m_1\ldots m_k}\eta_+^\dagger\gamma^{m_1\ldots m_k}\eta_+ + (\hat F_k)_{m_1\ldots m_k}\eta_-^\dagger\gamma^{m_1\ldots m_k}\eta_-\Big] *_6\mathbf{1} \qquad (3.82)\\ &= \int_{\mathcal{M}_6}\Big[\big\langle F^{\rm ev}, e^{-\hat B_2+iJ}\big\rangle - \big\langle H_3\wedge C_3^{(0)},1\big\rangle - \big\langle dC_3^{(0)},\hat B_2\big\rangle + i\big\langle dC_3^{(0)},J\big\rangle\Big]\ ,\end{aligned}$$

where $C_3^{(0)}$ is defined in (3.18) as the part of $\hat C_3$ being a three-form on $\mathcal{M}_6$ yielding scalar fields in $M_{3,1}$. In deriving this identity one uses the definition of $\hat F_k$ given in eqn. (3.68) while eliminating half of the R-R fields by the duality condition (3.68) .

In summary one can now read off the complete type IIA superpotential induced by background fluxes and torsion. Introducing the differential operator $d_H = d - H_3\wedge$ one finds (see also refs. [62, 11])

$$W^{O6} = \int_{\mathcal{M}_6}\big\langle F^{\rm ev} + d_H\Pi_c^{\rm odd}, e^{J_c}\big\rangle\ , \qquad (3.83)$$

where we used the definitions of $J_c = -\hat B_2 + iJ$ and $\Pi_c^{\rm odd} = C_3^{(0)} + i{\rm Re}(\Pi^{\rm odd})$ given in eqns. (3.17) and (3.19). The superpotential extends the results of refs. [18, 19, 59, 62, 21, 11] and together with the discussions above it is readily checked to be holomorphic in the $N = 1$ coordinates. As discussed in section 3.2 the complex forms J_c and $\Pi_c^{\rm ev}$ are linear in the complex $N = 1$ coordinates. This is also the case for their derivatives dJ_c and $d\Pi_c^{\rm ev}$, where d is the exterior derivative along $\mathcal{M}_6$. Therefore we deduce that W is a polynomial of cubic order in J_c times a linear polynomial in $\Pi_c^{\rm odd}$. Let us now determine W for the type IIB orientifold compactifications.

The type IIB superpotential

In the following we will determine the superpotential of the type IIB orientifolds induced by the background fluxes and torsion. In the type IIB theory we allow for a non-trivial NS-NS flux H_3 as well as odd R-R fluxes. Due to the fact that we do not expand in one- or five-forms on $\mathcal{M}_6$ the only non-vanishing R-R is the three-form F_3. The equation (3.29) implies that H_3, F_3 transform under the orientifold projection as

$$\sigma^* H_3 = -H_3 \ , \qquad\qquad \sigma^* F_3 = \mp F_3 \ , \tag{3.84}$$

where the minus sign in the second condition applies to type IIB orientifolds with $O3/O7$ planes while the plus sign is chosen for $O5/O9$ orientifolds. Since, there are some qualitative differences between both cases we will discuss them in the following separately.

$\underline{O3/O7}$: Our analysis starts with the $O3/O7$ orientifolds. As in the type IIA case we need to specify the spinor invariant under the orientifold projections $\mathcal{O}_{(1)}$ defined in eqn. (3.7). We already gave the transformation of the ten-dimensional spinor under the world-sheet parity Ω_p and $(-1)^{F_L}$. It remains to specify how σ^* acts on the internal spinor $\eta_\pm$. Using eqns. (3.5) and (3.6) one infers [96]

$$\sigma^* \eta_+ = i\eta_+ \ , \qquad\qquad \sigma^* \eta_- = -i\eta_- \ , \tag{3.85}$$

such that $(\sigma^*)^2\eta_\pm = -\eta_\pm$ consistent with the fact the $(-1)^{F_L}\Omega_p$ squares to -1 on the ten-dimensional gravitinos. With these identities at hand one defines the four-dimensional linear combinations $\psi_\mu = \frac{1}{2}(\psi^1_\mu + i\psi^2_\mu)$ together with the conjugate expression for $\bar\psi_\mu$. Combining $\psi_\mu, \bar\psi_\mu$ into a ten-dimensional spinor $\hat\psi_\mu$ by multiplication with η_- and η_+ respectively it is readily shown that $\hat\psi_\mu$ is invariant under $\mathcal{O}_{(1)}$. It turns out to be sufficient to determine W for a more simple choice of the four-dimensional spinor ψ_μ given by

$$\psi_\mu = \psi^1_\mu = -i\psi^2_\mu \ , \qquad \bar\psi_\mu = \bar\psi^1_\mu = i\bar\psi^2_\mu \ . \tag{3.86}$$

These conditions define the reduction of the $N = 2$ theory to $N = 1$ induced by the orientifold projection. Inserting the decompositions (3.70) together with (3.86) into the ten-dimensional action (3.67) one determines the ψ_μ mass terms

$$S_\psi = -\int_{M_{3,1}} e^{\frac{K}{2}}\bar\psi_\mu\bar\sigma^{\mu\nu}\bar\psi_\nu *_4 \mathbf{1}\int_{\mathcal{M}_6}\tfrac{1}{3!}\Big[(e^{-\hat\phi}(\hat H_3)_{mnp} + i(\hat F_3)_{mnp})\Omega^{mnp}\Big] *_6 \mathbf{1} + \ldots, \tag{3.87}$$

where $e^{K/2} = e^{2D}e^{K^{\rm cs}/2}$ with $K^{\rm cs}$ as defined in eqn. (3.56). In order to derive this four-dimensional action we performed the Weyl-rescaling (3.77) to obtain a standard Einstein-Hilbert term. Moreover, we used the identities (2.6) and (3.78) to replace the gamma-matrix expressions $\eta^\dagger_-\gamma^{mnp}\eta_+$ with the complex three-form Ω^{mnp} and absorbed a factor arising due to the Weyl-rescaling (3.77) into $e^{K/2}$. It is interesting to note that there is no contribution from the reduction of the ten-dimensional kinetic term in the action (3.67). This can be traced back to the fact that in type IIB orientifolds with $O3/O7$ planes the globally defined three- and two-forms Ω and J transform with opposite signs under the map σ^*. However, since the volume form is positive under the orientation preserving map σ the integral over terms like $d\Omega\wedge J$ vanishes. The non-closed forms dJ and $d\Omega$ nevertheless yield a potential for the four-dimensional scalars which is encoded by non-trivial D-terms.

Let us now express the action (3.87) in terms of the globally defined three-form Ω and the form $\Phi^{\rm ev}$. Using the definition (3.30) of $\Phi^{\rm ev}$ one infers

$$\begin{aligned}\tfrac{1}{3!}\int_{\mathcal{M}_6}\big[e^{-\hat\phi-i\theta}(\hat H_3)_{mnp}\Omega^{mnp}\big] *_6 \mathbf{1} &= -i\int_{\mathcal{M}_6} e^{-\hat\phi}\big[\langle H_3,\Omega\rangle + \langle d\hat B_2,\Omega\rangle\big] \\ &= -i\int_{\mathcal{M}_6}\big[\langle H_3\wedge \mathrm{Re}(\Phi^{\rm ev}),\Omega\rangle - \langle d\mathrm{Re}(\Phi^{\rm ev}),\Omega\rangle\big]\ ,\end{aligned} \tag{3.88}$$

where we have used $\mathrm{Re}(\Phi^{\mathrm{ev}})_0 = e^{-\hat{\phi}}$ and $\mathrm{Re}(\Phi^{\mathrm{ev}})_2 = -e^{-\hat{\phi}}\hat{B}_2$ as simply deduced from the definition (3.30). For the R-R term in (3.87) one derives

$$\tfrac{i}{3!}\int_{\mathcal{M}_6} (\hat{F}_3)_{mnp}\Omega^{mnp} *_6 \mathbf{1} = \int_{\mathcal{M}_6} \big[\langle F_3, \Omega\rangle + \langle dA_2^{(0)}, \Omega\rangle - \langle H_3 \wedge A_0^{(0)}, \Omega\rangle\big] \ , \qquad (3.89)$$

where $A_2^{(0)}$ and $A_0^{(0)}$ denote the two- and zero- forms in $A^{\mathrm{ev}}_{(0)}$ defined in (3.34).[27] Together the two terms (3.88) and (3.89) combine into the superpotential

$$W^{O3/O7} = \int_{\mathcal{M}_6} \langle F_3 + d_H \Phi_c^{\mathrm{ev}}, \Omega\rangle \qquad (3.90)$$

where $d_H = d - H_3\wedge$ and Φ^{ev}_c is defined in eqn. (3.35). This superpotential contains the well-known Gukov-Vafa-Witten superpotential [97, 98] as well as contributions due to non-closed two-forms $\hat{B}_2$ and $\hat{C}_2$. Also in this type IIB case it is straight forward to check the holomorphicity of W. As mentioned in section 3.3 the form $\Omega(z)$ is in general a complicated holomorphic function of the chiral coordinates z. On the other hand Φ^{ev}_c as well as $d\Phi^{\mathrm{ev}}_c$ depends linearly on the $N = 1$ chiral coordinates. Hence, the superpotential $W^{O3/O7}$ is a linear function in Φ^{ev}_c times a holomorphic function in the fields z and contains no conjugate fields. Let us complete the discussion of the type IIB orientifolds by determining the $O5/O9$ superpotential.

<u>$O5/O9$:</u> To derive the superpotential for the $O5/O9$ orientifolds we first specify the combination of the two ten-dimensional gravitinos invariant under $\mathcal{O}_{(2)}$ defined in eqn. (3.7). We deduce the action of σ^* on the globally defined spinor η by examining the expressions (3.5) and (3.6), which yield

$$\sigma^*\eta_+ = \eta_+ \ , \qquad \sigma^*\eta_- = \eta_- \ . \qquad (3.91)$$

The invariant combination of the four-dimensional spinors is given by $\psi_\mu = \frac{1}{2}(\psi^1_\mu + \psi^2_\mu)$ with a similar relation for $\bar{\psi}_\mu$. As a specific choice for this combination we simplify to

$$\psi_\mu = \psi^1_\mu = \psi^2_\mu \ , \qquad \bar{\psi}_\mu = \bar{\psi}^1_\mu = \bar{\psi}^2_\mu \ . \qquad (3.92)$$

Together with the decomposition (3.70) we reduce the action (3.67) to determine the mass term of ψ_μ as

$$S_\psi = -\int_{M_{3,1}} e^{\frac{K}{2}}\bar{\psi}_\mu \bar{\sigma}^{\mu\nu}\bar{\psi}_\nu *_4 \mathbf{1} \int_{\mathcal{M}_6} \langle -i\hat{F}_3 + d(e^{-\hat{\phi}}J), \Omega\rangle *_6 \mathbf{1} + \dots, \qquad (3.93)$$

where we have applied (3.79) and performed the Weyl rescaling (3.77). Note that the term involving the NS-NS fluxes vanishes in the case of $O5/O9$ orientifolds since Ω and $\hat{H}_3$ transform with an opposite sign under the symmetry σ^* as can be deduced from eqns. (3.6) and (3.29). Inserting the definition (3.68) of $\hat{F}_3$ into (3.93) one obtains the superpotential [11]

$$W^{O5/O9} = -i\int_{\mathcal{M}_6} \langle F_3 + d\Phi^{\mathrm{ev}}_c, \Omega\rangle \ , \qquad (3.94)$$

where we have used $\mathrm{Im}(\Phi^{\mathrm{ev}})_2 = e^{-\hat{\phi}}J$ and the definition (3.35) of Φ^{ev}_c. The superpotential $W^{O5/O9}$ is a linear function in the $N = 1$ fields encoded by Φ^{ev}_c times a holomorphic function of the fields z. $W^{O5/O9}$ is independent of the NS-NS flux H_3 which was shown in ref. [23] to contribute a D-term potential to the four-dimensional theory.

[27] Expanding $A^{\mathrm{ev}}_{(0)}$ in (3.34) one finds $A_2^{(0)} = \hat{C}_2 - \hat{C}_0\hat{B}_2$ and $A_0^{(0)} = \hat{C}_0$.

4 Generalized orientifolds and mirror symmetry

In this section we discuss $SU(3)\times SU(3)$ structure orientifolds and investigate mirror symmetry of the type IIA and type IIB setups. More precisely, we aim to specify setups dual to an orientifold compactification on a Calabi-Yau manifold Y with background fluxes. In doing that our main focus will be the identification of the $N=1$ superpotentials. The superpotentials are holomorphic functions of the chiral fields of the four-dimensional theory and do not receive perturbative corrections. Hence, they yield a good testing ground for the mirror relations we will propose below. Note however, that the potential for Calabi-Yau orientifolds with $O5$ planes contains in addition to a superpotential contribution also a D-term potential which arises due to the presents of a gauged linear multiplet [23, 90]. We will therefore focus on the mirror identifications between the type IIA orientifolds and the type IIB orientifolds with $O3/O7$ planes. In general Calabi-Yau orientifolds with $O3/O7$ or $O6$ planes the potential induced by non-trivial NS-NS and R-R background fluxes is entirely encoded by a superpotential and the Kähler potential [23, 24, 11]. We propose a possible mirror space $\mathcal{M}_{\tilde{Y}}$ which possesses a geometry dual to part of the electric and magnetic NS-NS fluxes. In other words, we identify a space $\mathcal{M}_{\tilde{Y}}$ such that

$$\text{Typ IIB}_{O3/7}/Y \text{ with } H_3^Q \qquad \overset{\text{mirror}}{\longleftrightarrow} \qquad \text{Typ IIA}_{O(\text{even})}/\mathcal{M}_{\tilde{Y}} \ . \tag{4.1}$$

where the precise definition of the NS-NS flux H_3^Q will be given shortly and the manifold $\mathcal{M}_{\tilde{Y}}$ is specified in section 4.1. The evidence for the identification (4.1) is discussed in section 4.2, where we also check consistency by analyzing the mirror relation

$$\text{Typ IIA}_{O6}/Y \text{ with } H_3^Q \qquad \overset{\text{mirror}}{\longleftrightarrow} \qquad \text{Typ IIB}_{O(\text{odd})}/\mathcal{M}_{\tilde{Y}} \ , \tag{4.2}$$

In both cases we concentrate on the superpotentials induced by the NS-NS flux H_3^Q only.

In order to make the mirror conjectures (4.1) and (4.2) more precise we have to define the background flux $H_3^{\rm Q}$ as well as the properties of $\mathcal{M}_{\tilde{Y}}$. Let us start with $H_3^{\rm Q}$. Recall that the background fluxes in Calabi-Yau compactifications are demanded to be harmonic forms in order to obey the equations of motion and Bianchi identities. This implies that before imposing the orientifold projections the general expansion of the NS-NS flux H_3 reads

$$H_3 = m^{\hat{K}}\alpha_{\hat{K}} - e_{\hat{K}}\beta^{\hat{K}} \ , \qquad \hat{K}=0,\ldots,\dim H^{(2,1)} \ , \tag{4.3}$$

where $(\alpha_{\hat{K}},\beta^{\hat{K}})$ is a real symplectic basis of $H^3(Y)$ satisfying (3.22). We denoted the magnetic and electric flux quanta of H_3 by $(m^{\hat{K}},e_{\hat{K}})$. Different choices of the symplectic basis $(\alpha_{\hat{K}},\beta^{\hat{K}})$ are related by a symplectic rotation which also acts on the vector of flux quanta guaranteeing invariance of H_3. Note however, that due to the fact the supergravity reduction is only valid in the large volume limit the mirror symmetric theory has to be evaluated in the 'large complex structure limit'. Around this point of the moduli space the holomorphic three-form Ω on Y admits a simple dependence on the complex structure moduli $z^K,\ K=1,\ldots,\dim H^{(2,1)}$ explicitly given by [99]

$$\Omega(z) = \alpha_0 + z^K\alpha_K + \tfrac{1}{2!}z^K z^L\kappa_{KLM}\beta^M - \tfrac{1}{3!}z^K z^L z^M\kappa_{KLM}\beta^0 \ , \tag{4.4}$$

where κ_{KLM} are intersection numbers on Y defined, for example, in ref. [93]. The expression (4.4) specifies a certain basis $(\alpha_{\hat{K}},\beta^{\hat{K}})$ of $H^{(3)}(Y)$. In particular it singles out the elements α_0 and β^0 with coefficients constant and cubic in the complex fields z^K. Using this specification we are now in the position to define the NS-NS flux H_3^Q by demanding that the flux quanta e_0, m^0 along β^0, α_0 vanish. In other words we set

$$H_3^Q = m^K \alpha_K - e_K \beta^K \ , \qquad\qquad e_0 = 0 \ , \quad m^0 = 0 \ , \tag{4.5}$$

where the index K runs from $K = 1, \ldots, \dim H^{(2,1)}$. An equivalent definition of H_3^Q can be given by interpreting mirror symmetry as T-duality along three directions of Y [82]. One demands that the components of the NS-NS flux $(H_3^Q)_{mnp}$ have never zero or three indices in the T-dualized directions which corresponds to $e_0 = m^0 = 0$.

4.1 Generalized half-flat manifolds

Let us now turn to the definition of the manifold $\mathcal{M}_{\tilde{Y}}$. In reference [36, 43] it was argued that type II compactifications on Calabi-Yau manifolds with electric NS-NS fluxes are the mirror symmetric duals of compactifications on half-flat manifolds (2.10). In order to also include magnetic fluxes into this mirror identification it is inevitable to generalize away from the $SU(3)$ structure compactifications [78, 79, 62, 70]. This might also lead to the application of the generalized manifolds $\mathcal{M}_{\tilde{Y}}$ with $SU(3) \times SU(3)$ structure [62]. In the remainder of this section we discuss some of the properties of the spaces $\mathcal{M}_{\tilde{Y}}$, which might be mirror dual to Calabi-Yau manifolds with NS-NS fluxes H_3^Q. We term these spaces 'generalized half-flat manifolds'. Some evidence for the mirror identifications (4.1) and (4.2) will be provided in section 4.2.

To start with let us recall the definition of a manifold with $SU(3) \times SU(3)$ structure [76, 80]. Clearly, the group $SU(3) \times SU(3)$ cannot act on the tangent bundle alone and one has to introduce a generalized tangent bundle E. Following the work of Hitchin [76, 77], the generalized tangent bundle E is given by

$$E_p \cong T_p\mathcal{M}_6 \oplus T_p^*\mathcal{M}_6 \ , \qquad\qquad p \in \mathcal{M}_{\tilde{Y}} \ , \tag{4.6}$$

where E is locally identified with the sum of the tangent and cotangent space. Its global definition is more involved, since the spaces E_p might be glued together non-trivially along $\mathcal{M}_{\tilde{Y}}$ [77]. To make this more precise one introduces a natural $O(6,6)$ metric on E_p defined by $(v + \xi, u + \zeta) = \frac{1}{2}(\xi(u) + \zeta(v))$, for $v, u \in T_p$ and $\xi, \zeta \in T_p^*$. Restricting further to transformations preserving the (natural) orientation of E reduces the group down to $SO(6,6)$. A global definition can then be given by specifying elements of this group serving as transition function on overlapping patches on $\mathcal{M}_{\tilde{Y}}$. We are now in the position to define an $SU(3) \times SU(3)$ structure manifold by demanding the structure group of the bundle E to reduce to $SU(3) \times SU(3) \subset SO(6,6)$. As in the case of $SU(3)$ structure manifolds discussed in section 2, this reduction can be specified in terms of two globally defined forms or two globally defined spinors on $\mathcal{M}_{\tilde{Y}}$. We comment on the spinor picture in section 4.2, where it will also become clear that the structure group $SU(3) \times SU(3)$ is dictated by demanding that type II compactifications on $\mathcal{M}_{\tilde{Y}}$ yield four-dimensional $N = 2$ supergravity theories. Let us analyze here the characterization in terms of globally defined forms [76, 80, 62].

Note that the group $SO(6,6)$ naturally admits spin representations on even and odd forms of $\mathcal{M}_{\tilde{Y}}$. More precisely, one finds two irreducible Majorana-Weyl representations

$S^{\rm ev}$ and $S^{\rm odd}$ given by

$$S^{\rm ev} \cong \Lambda^{\rm ev} T^* \otimes |\det T|^{1/2}\ , \qquad S^{\rm odd} \cong \Lambda^{\rm odd} T^* \otimes |\det T|^{1/2}\ , \tag{4.7}$$

where $\det T \cong \Lambda^6 T$ is fixed once a particular volume form is chosen. On elements $\Phi \in S^{\rm ev/odd}$ the group $SO(6,6)$ acts with the Clifford multiplication

$$(v+\xi)\cdot \Pi = v \lrcorner \Pi + \xi \wedge \Pi\ , \tag{4.8}$$

where $v\lrcorner$ indicates insertion of the vector $v \in T$ and $\xi \in T^*$ is a one-form. Using these definitions an $SU(3)\times SU(3)$ structure on $\mathcal{M}_{\tilde Y}$ is specified by two complex globally defined even and odd forms $\Pi'^{\rm ev}$ and $\Pi'^{\rm odd}$ which are annihilated by half of the elements in E.[28] Furthermore, in order to ensure the reduction of $SO(6,6)$ to the direct product $SU(3)\times SU(3)$ the globally defined forms also have to obey [80, 62]

$$\langle \Pi'^{\rm ev}, \bar\Pi'^{\rm ev}\rangle = \tfrac{3}{4}\langle \Pi'^{\rm odd}, \bar \Pi'^{\rm odd}\rangle\ , \qquad \langle \Pi'^{\rm ev}, (v+\xi)\cdot \Pi'^{\rm odd}\rangle = 0\ , \tag{4.9}$$

for all elements $v+\xi \in E$. The pairing $\langle \cdot,\cdot\rangle$ appearing in this expression is defined in (3.20). These conditions reduce to the standard $SU(3)$ structure conditions (2.7) in case we identify

$$\Pi'^{\rm ev} = e^{-\hat\phi} e^{iJ}\ , \qquad \Pi'^{\rm odd} = C\Omega\ , \tag{4.10}$$

where J and Ω are the globally defined two- and three-form. In this expressions the additional degree of freedom in $|\det T|^{1/2}$ is labeled by the ten-dimensional dilaton $e^{-\hat\phi}$ also linearly appearing in the definition (3.13) of C. Note however, that in the general $SU(3)\times SU(3)$ structure case the odd form $\Pi^{\rm odd}$ also contains a one- and five-form contribution such that $\Pi'^{\rm odd} = \Pi'_1 + \Pi'_3 + \Pi'_5$. It was shown in ref. [80] that each of these forms locally admits the expression

$$\Pi^{\rm odd} = e^{-\hat B_2} \wedge \Pi'^{\rm odd} = e^{-\hat B_2 + iJ} \wedge C\Omega_k\ , \tag{4.11}$$

where J is a real two-form and we also included a possible B-field on the internal manifold $\mathcal{M}_{\tilde Y}$. The index k is the degree of the complex form Ω_k. In the special case that $k=3$ on all of $\mathcal{M}_{\tilde Y}$ the form $\Pi'^{\rm odd}$ descends to the form (4.10). However, the degree of Ω_k can change when moving along $\mathcal{M}_{\tilde Y}$ [80].[29] In other words, the form $\Pi^{\rm odd}$ can locally contain a one-form $C\Omega_1$. The presents of this one-form in the expansion (4.11) will be the key to encode the mirror of the magnetic fluxes in H_3^Q given in (4.5).

To make this more precise, one notes that the globally defined forms $\Pi'^{\rm ev}$ and $\Pi'^{\rm odd}$ are not necessarily closed. This is already the case for $SU(3)$ structure manifolds which are half-flat and hence obey (2.10). For these manifolds the special forms (4.10) are no longer closed, since $d{\rm Re}(e^{i\theta}\Pi'^{\rm odd})$ and $d{\rm Im}(\Pi'^{\rm ev})$ are non-vanishing. This obstruction of the internal manifold $\mathcal{M}_{\tilde Y}$ to be Calabi-Yau is interpreted as mirror dual of the electric NS-NS fluxes e_K appearing in the expansion (4.5) of H_3 [36].[30] In order to also encode dual magnetic fluxes we generalize the half-flat conditions to the general odd form $\Pi^{\rm odd}$

[28] More precisely, each form $\Pi'^{\rm ev}$ and $\Pi'^{\rm odd}$ is demanded to be annihilated by a maximally isotropic subspace $E^{\rm ev}$ and $E^{\rm odd}$ of E. Isotropy implies that elements $v+\xi, u+\zeta \in E^{\rm ev/odd}$ obey $(v+\xi, u+\zeta)=0$, while maximality corresponds to $\dim E^{\rm ev/odd} = 6$.

[29] Interesting examples of manifolds allowing such transitions were recently constructed in ref. [100].

[30] The remaining flux parameter e_0 in eqn. (4.5) induces a non-trivial H_3 flux on the mirror $\mathcal{M}_{\tilde Y}$.

given in eqn. (4.11). These generalized half-flat manifolds are $SU(3) \times SU(3)$ structure manifolds for which

$$d\mathrm{Im}(e^{i\theta}\Pi'^{\mathrm{odd}}) = 0 \tag{4.12}$$

where as above $e^{-i\theta}$ is the phase of C. The real part of $e^{i\theta}\Pi'^{\mathrm{odd}}$ and the form Π'^{ev} are non-closed. We conjecture that in a finite reduction the differentials $d\mathrm{Re}(e^{i\theta}\Pi'^{\mathrm{odd}})$ and $d\Pi'^{\mathrm{ev}}$ are identified under mirror symmetry with the NS-NS fluxes H_3^Q.

Let us now make the mirror map between the type II theories on a manifold $\mathcal{M}_6^Y$ and the Calabi-Yau compactifications with NS-NS fluxes explicit. In order to do that, we perform a finite reduction by specifying a set of forms Δ_{finite}. In contrast to the $SU(3)$ case discussed in section 3.2, the forms in the set Δ_{finite} cannot anymore be distinguished by their degree. In the generalized manifolds only a distinction of even and odd forms is possible, such that

$$\Delta_{\mathrm{finite}} = \Delta^{\mathrm{odd}} \oplus \Delta^{\mathrm{ev}} \ , \tag{4.13}$$

where Δ^{odd} now contains forms of all odd degrees. In particular, the one-, three and five-form components of the form Π^{odd} given in eqn. (4.11) can mix once one moves along $\mathcal{M}_{\tilde{Y}}$. Nevertheless, we are able to specify a basis of Δ_{finite} such that a Kaluza-Klein reduction on these forms precisely yields the mirror theory obtained by a Calabi-Yau reduction with NS-NS fluxes.

To make this more precise, we first specify a finite real symplectic basis of Δ^{odd}. We demand that it contains the non-trivial odd forms $(\gamma_{\hat{K}}, \tilde{\gamma}^{\hat{K}})$ defined as

$$\gamma_{\hat{K}} = \big((\alpha_0 + \alpha_{(1)}), \alpha_K\big) \ , \qquad \tilde{\gamma}^{\hat{K}} = \big((\beta^0 + \beta^{(5)}), \beta^K\big) \ , \qquad \int_{\mathcal{M}_{\tilde{Y}}} \langle \gamma_{\hat{K}}, \tilde{\gamma}^{\hat{L}} \rangle = \delta^{\hat{L}}_{\hat{K}} \ , \tag{4.14}$$

where $\alpha_{(1)}$ and $\beta^{(5)}$ are a one-form and five-form respectively. Note that as remarked above, the basis elements $(\gamma_{\hat{K}}, \tilde{\gamma}^{\hat{K}})$ carry no definite form degree since γ_0 and $\tilde{\gamma}^0$ consist of a sum of one-, three- and five-forms. Using this basis the odd form Π^{odd} admits the expansion [31]

$$\Pi^{\mathrm{odd}} = C\big(\gamma_0 + z^K\gamma_K + \tfrac{1}{2!}z^K z^L \kappa_{KLM}\tilde{\gamma}^M - \tfrac{1}{3!}z^K z^L z^M \kappa_{KLM}\tilde{\gamma}^0\big) \ , \tag{4.15}$$

which generalizes the expansion (4.4) for the three-form Ω. In order to identify the fields z^K under the mirror map with the complexified Kähler structure deformations of Y one has $K = 1, \ldots, \dim H^{(1,1)}(Y)$, while $\hat{K}$ takes an additional value 0. We also introduce a basis Δ^{ev} of even forms on $\mathcal{M}_{\tilde{Y}}$ denoted by $\omega_{\hat{A}} = (1, \omega_A)$ and $\tilde{\omega}^{\hat{A}} = (\tilde{\omega}^A, \epsilon)$, with intersections as in equation (3.21). Mirror symmetry imposes that $A = 1, \ldots, \dim H^{(2,1)}(Y)$. Note that due to the fact that $\Pi^{\mathrm{odd}}, \Pi'^{\mathrm{ev}} \in \Delta_{\mathrm{finite}}$ are no longer closed not all basis elements $(\gamma_{\hat{K}}, \tilde{\gamma}^{\hat{K}})$ and $(\omega_{\hat{A}}, \tilde{\omega}^{\hat{A}})$ are annihilated by the exterior differential. More precisely, we assign that

$$d\gamma_0 = -m^A\omega_A - e_A\tilde{\omega}^A \ , \qquad d\omega_A = -e_A\beta^0 \ , \qquad d\tilde{\omega}^A = -m^A\beta^{(5)} \ , \tag{4.16}$$

which is in accord with the non-vanishing intersections (3.21) and (4.14). It is now clear from eqn. (4.16) that the existence of one- and five forms in $\gamma_0, \tilde{\gamma}^0$ is essential to encode

[31] Note that the precise moduli dependence of the expansion (4.15) will be not relevant in the following. The essential part is that Π^{odd} contains a part $C\gamma^0$ which is linear in C. This can be always achieved by an appropriate rescaling of C.

non-vanishing magnetic fluxes. In the case, one evaluates (4.16) for $\alpha_{(1)} = \beta^{(5)} = 0$ one encounters set-ups with dual electric fluxes only [36].

In the finite reduction on $\mathcal{M}_{\tilde{Y}}$ the equation (4.16) parameterizes the deviation of $\mathcal{M}_{\tilde{Y}}$ to be Calabi-Yau. Using the expansion (4.15) of the globally defined forms $\Pi^{\rm odd}$ one easily applies (4.16) to derive

$$d\mathrm{Re}(e^{i\theta}\Pi^{\rm odd}) = -|C|(m^A\omega_A + e_A\tilde{\omega}^A)\ , \qquad d\mathrm{Im}(e^{i\theta}\Pi^{\rm odd}) = 0\ . \tag{4.17}$$

In order that the low energy theories of compactifications on $\mathcal{M}_{\tilde{Y}}$ coincide with the mirror reductions on the Calabi-Yau space Y with fluxes the scale of torsion on $\mathcal{M}_{\tilde{Y}}$ has to be below the Kaluza-Klein scale. In other words, the generalized half-flat manifold should be understood as a 'small' deviation from the Calabi-Yau space $\tilde{Y}$ which is the mirror of Y in the absence of fluxes. Note however, that the topology of $\mathcal{M}_{\tilde{Y}}$ differs from the one of the Calabi-Yau space $\tilde{Y}$ since $\Delta_{\rm finite}$ contains various non-harmonic forms. This suggests that an explicit construction of $\mathcal{M}_{\tilde{Y}}$ might involve the shrinking of cycles in homology, which are later resolved with a non-trivial deformation [41]. Unfortunately, an explicit construction of the manifolds $\mathcal{M}_{\tilde{Y}}$ is still missing. Moreover, it remains challenging to investigate the geometric structure of these manifolds in more detail. Despite of the fact that $\mathcal{M}_{\tilde{Y}}$ possesses two globally defined forms $\Pi^{\rm odd}$ and $\Pi'^{\rm ev}$ it remains to be investigated if this allows to define the mirror of the Riemannian metric. Note however, that from a four-dimensional point of view the globally defined even and odd forms are sufficient to encode the $N = 2$ or $N = 1$ characteristic data.

In the final section we provide some evidence for the conjecture that the generalized half-flat manifolds are the mirrors of Calabi-Yau compactifications with NS-NS fluxes. We do this by deriving the superpotentials induced by the general odd forms $\Pi^{\rm odd}$. The type IIA and type IIB cases will be analyzed in turn.

4.2 The mirrors of type II Calabi-Yau orientifolds with fluxes

In this section we dimensionally reduce the fermionic action (3.67) on a generalized $SU(3) \times SU(3)$ structure manifold $\mathcal{M}_{\tilde{Y}}$. In addition we will impose the orientifold projections ensuring that the four-dimensional theory is an $N = 1$ supergravity. This will allow us to derive the superpotentials arising due to the non-closed forms $\Pi^{\rm odd}$ and $\Pi^{\rm ev}$ and the background fluxes. These can be evaluated for the generalized half-flat manifolds introduced in the previous section. We use the finite expansion (4.17) to compare the superpotentials depending on $d\Pi^{\rm odd}$ to their mirror partners arising due to NS-NS flux.

In order to perform the dimensional reduction of the fermionic action (3.67) the two ten-dimensional gravitinos $\psi^{1,2}_M$ are decomposed on the background $M_{3,1} \times \mathcal{M}_{\tilde{Y}}$. Hence, we are looking for a generalization of the decompositions (3.70) and (3.71). Note however, that the internal manifold $\mathcal{M}_{\tilde{Y}}$ possesses an $SU(3) \times SU(3)$ structure implying that one generically finds two globally defined spinors η^1 and η^2 on this space [80, 62]. In terms of these two globally defined spinors $\eta^{1,2}$ the globally defined forms $\Pi'^{\rm ev}$ and $\Pi'^{\rm odd}$ are expressed as

$$\Pi'^{\rm ev} = 2e^{-\hat{\phi}} \sum_{n\ \rm even} \tfrac{1}{n!}\eta^{\dagger\,2}_+\gamma_{p_1\ldots p_n}\eta^1_+ e^{p_n\ldots p_1}, \qquad \Pi'^{\rm odd} = -2C\sum_{n\ \rm odd} \tfrac{1}{n!}\eta^{\dagger\,2}_-\gamma_{p_1\ldots p_n}\eta^1_+ e^{p_n\ldots p_1}, \tag{4.18}$$

where $e^{p_1 \cdots p_n} = e^{p_1} \wedge \ldots \wedge e^{p_n}$ is a basis of n-forms $\Lambda^n T^*$ on the manifold $\mathcal{M}_{\tilde{Y}}$. The presence of the two spinors $\eta^{1,2}$ ensures that the four-dimensional theory obtained by compactifying on the space $\mathcal{M}_{\tilde{Y}}$ possesses $N = 2$ supersymmetry. More precisely, the type IIB ten-dimensional gravitinos decompose on $M_{3,1} \times \mathcal{M}_{\tilde{Y}}$ as

$$\hat{\psi}^A_\mu = \psi^A_\mu \otimes \eta^A_- + \bar{\psi}^A_\mu \otimes \eta^A_+ \qquad A = 1, 2 \ , \tag{4.19}$$

while the type IIA decomposition is given by

$$\hat{\psi}^1_\mu = \psi^1_\mu \otimes \eta^1_+ + \bar{\psi}^1_\mu \otimes \eta^1_- \ , \qquad\qquad \hat{\psi}^2_\mu = \psi^2_\mu \otimes \eta^2_- + \bar{\psi}^2_\mu \otimes \eta^2_+ \ . \tag{4.20}$$

As in section 3.4, the Weyl spinors $\psi^{1,2}_\mu$ and $\bar{\psi}^{1,2}_\mu$ yield the four-dimensional gravitinos parameterizing the $N = 2$ supersymmetry of the theory. Clearly, the decompositions (4.19), (4.20) reduce on an $SU(3)$ structure manifold to the expressions (3.70), (3.71), if $\eta = \eta^1 = \eta^2$ is the only globally defined spinor. In general η^1 and η^2 are not necessarily parallel along all of $\mathcal{M}_{\tilde{Y}}$. It is precisely this deviation which allows the general odd forms (4.11) to locally contain a one-form component $\eta^{1\,\dagger}\gamma_m \eta^2$.

A dimensional reduction on backgrounds $\mathcal{M}_{\tilde{Y}}$ with $SU(3) \times SU(3)$ structure yields a four-dimensional $N = 2$ supergravity [62]. The number of supersymmetries is further reduced to $N = 1$ by imposing appropriate orientifold projections. To perform the four-dimensional $N = 1$ reductions we discuss the type IIA and type IIB cases in turn.

The type IIA mirror of type IIB Calabi-Yau orientifolds with NS-NS flux

Let us first derive the four-dimensional superpotential for type IIA orientifolds on $\mathcal{M}_{\tilde{Y}}$. It is most conveniently read off from the mass term (3.66) arising in the reduction of the fermionic action (3.67). In this derivation we have to impose the type IIA orientifold projection (3.2). It is straight forward to extend the conditions (3.15) to the more general odd and even forms

$$\Pi^{\rm odd} = e^{-\hat{B}_2} \wedge \Pi'^{\rm odd} \ , \qquad\qquad \Pi^{\rm ev} = e^{\hat{\phi}} e^{-\hat{B}_2} \wedge \Pi'^{\rm ev} \ , \tag{4.21}$$

where $\Pi'^{\rm odd}$ and $\Pi'^{\rm ev}$ are given in expression (4.18). One has

$$\sigma^* \Pi^{\rm odd} = \lambda(\bar{\Pi}^{\rm odd}) \ , \qquad\qquad \sigma^* \Pi^{\rm ev} = \lambda(\Pi^{\rm ev}) \ . \tag{4.22}$$

In complete analogy to section 3.4 the transformations (4.22) impose constraints on the spinors $\eta^{1,2}$ appearing in the component expansion (4.18). Eventually, this implies that the two four-dimensional spinors ψ^1_μ and ψ^2_μ are related as in eqn. (3.75).

We are now in the position to perform the reduction of the action (3.67). The ten-dimensional terms only depending on NS-NS fields reduce using (4.20) and (3.75) as

$$\begin{aligned} S_{\psi-\rm NS} &= -\int_{M_{3,1}} e^{\frac{K}{2}} \bar{\psi}_\mu \bar{\sigma}^{\mu\nu} \bar{\psi}_\nu *_4 \mathbf{1} \int_{\mathcal{M}_6} \big\langle d{\rm Re}(\Pi'^{\rm odd}) - \hat{H}_3 \wedge {\rm Re}(\Pi'^{\rm odd}), \Pi'^{\rm ev} \big\rangle + \ldots \\ &= -\int_{M_{3,1}} e^{\frac{K}{2}} \bar{\psi}_\mu \bar{\sigma}^{\mu\nu} \bar{\psi}_\nu *_4 \mathbf{1} \int_{\mathcal{M}_6} \big\langle d_H {\rm Re}(\Pi^{\rm odd}), \Pi^{\rm ev} \big\rangle + \ldots \ , \end{aligned} \tag{4.23}$$

where $\hat{H}_3 = d\hat{B}_2 + H_3$ and the dots indicate terms depending on the R-R fields or not contributing to the mass term (3.66). The expression (4.23) is a generalization of eqn. (3.80) and (3.81) for the globally defined forms (4.18). However, the derivation of (4.23) is slightly more involved, since terms proportional to $\eta^{A\dagger}\gamma^m D_m \eta^A$ or $H_{mnp}\eta^{A\dagger}\gamma^{mnp}\eta^A$ for $A = 1,2$ need to be converted to the sum of forms (4.18). In order to do that one repeatedly uses the Fierz identities (A.12) and (A.13) [101]. Furthermore, the derivatives on the spinors $\eta^{1,2}$ translate to differentials on the forms $\Pi^{\rm ev/odd}$ defined in eqn. (4.18) by using the identity (A.14). The Kähler potential K appearing in the action (4.23) takes the same form as the one for $SU(3)$ structure manifolds (1.1) if one substitutes the general odd and even forms $\Pi^{\rm ev}$ and $\Pi^{\rm odd}$. More precisely, the Kähler potential K consists of the logarithms of the extended Hitchin functionals introduced in ref. [76]. A brief review of the relevant mathematical definitions can be found in appendix B. As in section 3.4, the factor $e^{K/2}$ in the expression (4.23) arises after a four-dimensional Weyl rescaling (3.77).

It is straight forward to include the R-R fields into the fermionic reduction in full anology to section 3.4. Together with the terms (4.23) the four-dimensional superpotential takes the form

$$W = \int_{\mathcal{M}_6} \left\langle F^{\rm ev} + d_H \Pi_c^{\rm odd}, \Pi^{\rm ev} \right\rangle , \qquad (4.24)$$

where

$$\Pi_c^{\rm odd} = A_{(0)}^{\rm odd} + i\mathrm{Re}(\Pi^{\rm odd}) , \qquad \hat{A}^{\rm odd} = e^{-\hat{B}_2} \wedge \hat{C}^{\rm odd} . \qquad (4.25)$$

The complex form $\Pi_c^{\rm odd}$ can locally contain a one- and five-form contribution. The compex chiral multiplets parametrized by $\Pi_c^{\rm odd}$ arise as complex coeficients of an expansion into real forms $\Lambda_+^{\rm odd}$.

In order to compare this result to the mirror result for type IIB Calabi-Yau orientifolds with $O3/O7$ planes we perform the finite reduction discussed in the previous section. Note that we also have to impose the orientifold condition, such that the expansion of $\Pi^{\rm ev/odd}$ in performed into the appropriate subset of $\Delta_{\rm finite} = \Delta_+ \oplus \Delta_-$. As in eqn. (3.23) this splitting is with respect to the geometric symmetry $\mathcal{P}_6 = \sigma^*\lambda$. Using the finite basis $(\gamma_{\hat{K}}, \gamma^{\hat{K}})$ introduced in eqn. (4.14) one expands

$$\Pi_c^{\rm odd} = N^k \gamma_k + T_\lambda \tilde{\gamma}^\lambda , \qquad (\gamma_k, \tilde{\gamma}^\lambda) \in \Delta_+^{\rm odd} . \qquad (4.26)$$

It is an important requirement that the form γ_0 is an element of $\Delta_+^{\rm odd}$ in order that the type IIA setups are mirror dual to type IIB setups with $O3/O7$ planes [24]. The even form $\Pi^{\rm ev}$ is expanded in a basis $(1, \omega_a, \tilde{\omega}^b, \epsilon)$ of $\Delta_+^{\rm ev}$ as

$$\Pi^{\rm ev} = 1 + t^a \omega_a + \tfrac{1}{2!} t^a t^b \mathcal{K}_{abc} \tilde{\omega}^c + \tfrac{1}{3!} t^a t^b t^c \mathcal{K}_{abc} \epsilon \qquad (4.27)$$

where the $\mathcal{K}_{abc} = \int \omega_a \wedge \omega_b \wedge \omega_c$ are the intersection numbers on Δ_-^2. On the type IIA side the NS-NS and R-R fluxes are set to be zero. Inserting the expressions (4.26), (4.27) and (4.16) into the superpotential (4.24) one finds

$$W = -N^0 (e_b t^b + \tfrac{1}{2!} t^a t^b m^c \mathcal{K}_{abc}) . \qquad (4.28)$$

The superpotential depends on the 'electric' flux parameters e_0, e_a as well as the 'magnetic' fluxes m^a. Under the mirror map these parameters are identified with the NS-NS flux quanta in H_3^Q.

It is not hard to see, that the superpotential (4.28) is precisely the mirror superpotential to the well-known Gukov-Vafa-Witten superpotential for type IIB Calabi-Yau orientifolds with $O3/O7$ planes [97, 98]. Denoting by τ, the type IIB dilaton-axion the Gukov-Vafa-Witten superpotential for vanishing R-R fluxes reads [32]

$$W = -\tau \int_Y H_3^Q \wedge \Omega = -\tau\big(e_k z^k + \tfrac{1}{2!} m^k z^l z^m \kappa_{klm}\big) \ , \tag{4.29}$$

where the NS-NS flux H_3^Q is given in eqn. (4.5) and Ω takes the form (4.4) in the large complex structure limit. Note that we additionally imposed the orientifold projection on the type IIB Calabi-Yau compactification, such that $H_3^Q \in H^3_-(Y)$ contains flux quanta (e_k, m^k), while $\Omega \in H^3_-(Y)$ is parameterized by fields z^k only. It is now straight forward to identify the superpotentials (4.28) with (4.29) by applying the mirror map $t^a \cong z^k$, $N^0 \cong \tau$ and $\mathcal{K}_{abc} \cong \kappa_{klm}$. The fluxes are identified as

$$d\Pi_c^{\text{odd}} \ \cong \ -\tau H_Q^3 \ . \tag{4.30}$$

The fact that the two superpotentials can be identified gives some evidence for the chosen mirror geometry $\mathcal{M}_{\tilde{Y}}$. Next, we will perform a similar analysis for the type IIB theories on $\mathcal{M}_{\tilde{Y}}$ and check the consistency of our assertions.

The type IIB mirror of type IIA Calabi-Yau orientifolds with NS-NS flux

Let us now give a brief check of the second mirror identification displayed in eqn. (4.2) by comparing the induced superpotentials. In order to do that we perform a four-dimensional reduction of the type IIB effective action (3.67) on the generalized manifolds $\mathcal{M}_{\tilde{Y}}$. In addition we impose the orientifold projection $\mathcal{O}_{(1)}$ given in eqn. (3.7) such that the four-dimensional theory has $N = 1$ supersymmetry. We define the forms

$$\Phi^{\text{ev}} = e^{-\hat{B}_2} \wedge \Pi'^{\text{ev}} \ , \qquad \Phi^{\text{odd}} = C^{-1} e^{-\hat{B}_2} \wedge \Pi'^{\text{odd}} \ , \tag{4.31}$$

where Π'^{ev} and Π'^{odd} are defined in eqn. (4.18). The orientifold symmetry σ^* acts on these forms as

$$\sigma^* \Phi^{\text{odd}} = -\lambda(\Phi^{\text{odd}}) \ , \qquad \sigma^* \Phi^{\text{ev}} = \lambda(\bar{\Phi}^{\text{ev}}) \ , \tag{4.32}$$

generalizing the conditions (3.31). These constraints translate into conditions on the spinors $\eta^{1,2}$. It is then consistent to identify the four-dimensional gravitinos parameterizing the original $N = 2$ supersymmetry as in eqn. (3.86). The single spinor ψ_μ acquires a mass term due to background fluxes and the non-closedness of the forms Φ^{ev} and Φ^{odd}.

To derive the mass term (3.66) for the spinor ψ_μ we dimensionally reduce the fermionic action (3.67) for type IIB supergravity. Using the Fierz identities (A.12), (A.13) and the expression (A.14) one derives the superpotential

$$W = \int_{\mathcal{M}_{\tilde{Y}}} \big\langle F^{\text{odd}} + d_H \Phi_c^{\text{ev}}, \Phi^{\text{odd}} \big\rangle \ , \tag{4.33}$$

where the even form $\Phi_c^{\text{ev}} = A_{(0)}^{\text{ev}} + i\text{Re}(\Phi^{\text{ev}})$ is defined as in eqn. (3.35). The odd form Φ^{odd} generically contains a one- and five-form.

This superpotential can be compared with the NS-NS superpotential arising in type IIA Calabi-Yau orientifolds when performing a finite reduction outlined in section 4.1.

[32] The superpotential (4.29) is a special case of the superpotential (1.6) derived in section 3.4.

However, to also incorporate the orientifold constraints (4.32) the expansion of $\Phi_c^{\rm ev}$ and $\Phi^{\rm odd}$ is in forms of the appropriate eigenspace of $\Delta_{\rm finite} = \Delta_+ \oplus \Delta_-$. More precisely, we have

$$\Phi_c^{\rm ev} = \tau + G^a \omega_a + T_\alpha \tilde\omega^\alpha \ , \tag{4.34}$$

where $(1, \omega_a, \tilde\omega^\alpha)$ is a basis of $\Delta_+^{\rm ev}$. The expansion of $\Phi^{\rm odd}$ is given in eqn. (4.15) and reduces under the orientifold projection to

$$\Phi^{\rm odd} = \gamma_0 + z^k \gamma_k + \tfrac{1}{2!} z^k z^l \kappa_{klm} \tilde\gamma^m - \tfrac{1}{3!} z^k z^l z^m \kappa_{klm} \tilde\gamma^0 \ , \tag{4.35}$$

where $(\gamma_0, \gamma_k, \gamma^k, \gamma^0)$ is a basis of $\Delta_-^{\rm odd}$. It is now straight forward to evaluate the general superpotential (4.33) for the even and odd forms (4.34) and (4.35). Setting $F^{\rm odd} = 0$ and $H_3 = 0$ and using the expression (4.16) we find

$$W^{\rm B}_{\mathcal{M}_{\tilde Y}} = -G^a e_a - T_\alpha m^\alpha \ . \tag{4.36}$$

Let us now recall the superpotential for type IIA Calabi-Yau orientifolds with NS-NS background flux $H_3^{\rm Q}$. It was shown in refs. [18, 24] that $W^{\rm A}_{H_3}$ takes the form

$$W^{\rm A}_{H_3} = -\int H_3^Q \wedge \Pi_c^{\rm odd} = -N^k e_k - T_\lambda m^\lambda \ , \tag{4.37}$$

where the expansion of $\Pi_c^{\rm odd} = N^k \alpha_k + T_\lambda \beta^\lambda$ is in harmonic three-forms $(\alpha_k, \beta^\lambda) \in H^3_+(Y)$. The decomposition of H_3^Q is given in eqn. (4.5) and we appropriately imposed the orientifold constraint $H_3^Q \in H^3_-(Y)$. The two superpotentials (4.36) and (4.37) coincide if applies the mirror map $G^a \cong N^k$ and $T_\alpha \cong T_\lambda$.

In summary, we conclude that the mirror identifications (4.1) and (4.2) might be correct for the special generalized half flat manifolds with finite reduction (4.16). Clearly, this is only a first step and more involved checks are necessary to make the identifications (4.1) and (4.2) more precise. Moreover, it is a challenging task to explore more general orientifold compactifications on non-trivial $SU(3) \times SU(3)$ manifolds. Work along these lines is in progress.

5 Conclusions and Discussion

In this paper we discussed the four-dimensional $N = 1$ supergravity theories arising in generalized orientifold compactifications of type IIA and type IIB supergravities. After defining the orientifold projection the $N = 1$ spectrum of the four-dimensional theory was determined. As we have argued, this can be done before specifying a particular finite reduction. The degrees of freedom of the bosonic NS-NS fields encoded by the ten-dimensional metric, the B-field and the dilaton, decompose on $M_{3,1} \times \mathcal{M}_6$ into a four-dimensional metric g_4 and two complex forms on $\mathcal{M}_6$,

$$\underline{\text{Type IIA:}} \quad \Pi^{\rm ev}\ ,\ \Pi^{\rm odd}\ , \qquad\qquad \underline{\text{Type IIB:}} \quad \Phi^{\rm ev}\ ,\ \Phi^{\rm odd}\ . \tag{5.1}$$

The normalization of $\Pi^{\rm odd}$ and $\Phi^{\rm ev}$ is set by the ten-dimensional dilaton, while the normalization of $\Pi^{\rm ev}$, $\Phi^{\rm odd}$ is a unphysical scaling freedom. The forms $\Pi^{\rm ev/odd}$ as well as $\Phi^{\rm ev/odd}$ obey various compatibility conditions ensuring that the four-dimensional theory is supersymmetric. Moreover, the real and imaginary parts of these forms are not independent such that, at least formally, the real part can be expressed as a function of the imaginary part and vice versa. In case $\mathcal{M}_6$ is an $SU(3)$ structure manifold the odd forms $\Pi^{\rm odd}$, $\Phi^{\rm odd}$ only contain a three-form contribution, while the forms $\Pi^{\rm ev}$, $\Phi^{\rm ev}$ are of general even degree.

From a four-dimensional point of view, the introduction of the odd and even forms (5.1) is appropriate to encode the bosonic degrees of freedom in the NS-NS sector. The bosonic fields in the R-R sector are captured by the ten-dimensional forms $\hat{A}^{\rm odd/ev} = e^{-\hat{B}_2} \wedge \hat{C}^{\rm odd/ev}$ for type IIA and type IIB respectively. Once again, not all degrees of freedom in these forms are independent since the duality condition (3.68) on the field strengths of $\hat{A}^{\rm odd/ev}$ needs to be imposed. The four-dimensional spectrum arises by expanding these ten-dimensional fields into forms on the internal manifold $\mathcal{M}_6$. Despite the fact that forms on $\mathcal{M}_6$ might only possess a grading into odd and even forms the orientifold projection allows to distinguish four-dimensional scalars and two-forms as well as vectors and three-forms. Altogether, the fields arrange into $N = 1$ supermultiplets.

In determining the kinetic terms of the four-dimensional supergravity theory we focused on the metric on the chiral field space. Supersymmetry implies that this metric is Kähler and we argued that the Kähler potential consists of the two Hitchin functionals on $\mathcal{M}_6$. These are functions of the odd and even forms listed in eqn. (5.1) respectively. The Kähler potentials are independent of the R-R fields which are protected by continuous shift symmetries. This will no longer be the case when D-instanton corrections are included. Given the Kähler potentials in the chiral description, the kinetic potentials for the dual linear multiplets are determined by a Legendre transform [91]. In general, the theory consists of a set of (possibly massive) chiral and linear multiplets. In this work we did not analyze the vector sector and three-form sector of the four-dimensional theory. In order to gain a full picture of possible supergravity theories in four-dimensions it will be necessary to carefully include these fields.

Due to background fluxes and torsion the chiral multiplets can acquire a scalar potential. This scalar potential consists of an F-term contribution encoded by a holomorphic superpotential and possible D-term contributions due to non-trivial gaugings. Using a fermionic reduction we derived the general form of the superpotential on $SU(3)$ structure manifolds. Together with the Kähler potential this allows to determine the chiral supersymmetry conditions on four-dimensional vacua and their cosmological constant $\Lambda = -3e^K|W|^2$. In order to derive these data and to study moduli stabilization the explicitly construction of non-Calabi-Yau backgrounds is essential. Moreover, the inclusion of matter and moduli fields due to space-time filling D-branes will be needed in attempts to construct specific models for particle physics and cosmology.

We also presented some first results on type II compactifications on $SU(3) \times SU(3)$ structure orientifolds. Even though many of the $SU(3)$ structure results naturally generalize to the $SU(3)\times SU(3)$ structure case the consequences of this extension are enormous. The even and odd forms listed in eqn. (5.1) are in these generalized settings of generic even and odd degree. Moreover, the notion of a specific form degree is not anymore well defined and can change on different patches of $\mathcal{M}_6$. This can be traced back to the fact that the tangent and cotangent bundles T, T^* are no longer the central geometric objects, but rather get replaced with the generalized tangent bundle E locally given by $T \oplus T^*$. A non-closed NS-NS B-field has a natural interpretation in this formalism as a twisting of the forms $\Pi^{\rm ev/odd}$ and $\Phi^{\rm ev/odd}$ with a gerbe [76, 80, 77]. At least from a four-dimensional point of view one may attempt to formulate the supergravity in this generalized language providing a natural unification of all NS-NS fields. However, it should be clear that the generalized set-ups are not anymore 'geometric' in the standard Riemannian sense. The metric g_6 on the tangent bundle is replaced by a metric on the extended tangent bundle E, which supports higher symmetry group then diffeomorphisms of $\mathcal{M}_6$. In general, this might also imply that the metric g_6 and the B-field mix as one moves along the internal manifold.

In this work we explored an interesting application of the generalized geometries as mirrors of Calabi-Yau compactifications with NS-NS fluxes. We characterized properties of generalized half-flat manifolds which might serve as a mirror of NS-NS fluxes H^Q. The superpotentials of $SU(3)\times SU(3)$ structure compactifications were derived from the reduction of the fermionic type IIA and type IIB actions. In a specific finite truncation the mirror fluxes can be identified as contributions from non-closed one- and three-forms in $\Pi^{\rm odd}$ and $\Phi^{\rm odd}$. Clearly, this is only a first step in the study of compactifications on generalized manifolds with $SU(3)\times SU(3)$ structure. It remains a challenging task to explore the pattern of fluxes supported in generalized compactifications and to determine the complete classical four-dimensional gauged supergravity.

Acknowledgments

We would like to thank Dmitriy Belov, Frederik Denef, Ian Ellwood, Olaf Hohm, Hans Jockers, Albrecht Klemm, Yi Li, Jan Louis and Washington Taylor for valuable discussions and useful comments. TWG also likes to thank Michael Douglas and the Rutgers University for hospitality and financial support. TWG was supported in part by NSF CAREER Award No. PHY-0348093, DOE grant DE-FG-02-95ER40896, a Research Innovation Award and a Cottrell Scholar Award from Research Corporation. IB likes to thank the DFG – The German Science Foundation, European RTN Program MRTN-CT-2004-503369 and the DAAD – the German Academic Exchange Service.

Appendix

A The Clifford Algebra in 4 and 6 dimensions

In this appendix we assemble the spinor conventions used throughout the paper.

In $D=10$ the Γ-matrices are hermitian and satisfy the Clifford algebra

$$\{\Gamma^M,\Gamma^N\}=2g^{MN}\ ,\qquad M,N=0,\ldots,9\ . \tag{A.1}$$

One defines [4]

$$\Gamma^{11}=\Gamma^0\ldots\Gamma^9\ , \tag{A.2}$$

which has the properties

$$(\Gamma^{11})^2 = 1 \ , \quad \{\Gamma^{11}, \Gamma^M\} = 0 \ . \tag{A.3}$$

This implies that the Dirac representation can be split into two Weyl representations

$$\mathbf{32}_{Dirac} = \mathbf{16} + \mathbf{16}' \tag{A.4}$$

with eigenvalue +1 and −1 under Γ^{11}.

In backgrounds of the form (2.1) the 10-dimensional Lorentz group decomposes as

$$SO(9,1) \to SO(3,1) \times SO(6) \ , \tag{A.5}$$

implying a decomposition of the spinor representations as

$$\mathbf{16} = (\mathbf{2}, \mathbf{4}) + (\bar{\mathbf{2}}, \bar{\mathbf{4}}) \ . \tag{A.6}$$

Here $\mathbf{2}, \mathbf{4}$ are the Weyl representations of $SO(3,1)$ and $SO(6)$ respectively.

In the background (2.1) the ten-dimensional Γ-matrices can be chosen block-diagonal as

$$\Gamma^M = (\gamma^\mu \otimes \mathbf{1}, \gamma^5 \otimes \gamma^m), \quad \mu = 0, \ldots, 3, \ m = 1, \ldots, 6 \ , \tag{A.7}$$

where γ^5 defines the Weyl representation in $D = 4$. In this basis Γ^{11} splits as [4]

$$\Gamma^{11} = -\gamma^5 \otimes \gamma^7 \ , \tag{A.8}$$

where γ^7 defines the Weyl representations in $D = 6$.

Let us now turn to our spinor convention in $D = 6$ and $D = 4$ respectively.

A.1 Clifford algebra in 6 dimensions

In $D = 6$ the gamma matrices are chosen hermitian $\gamma^{m\dagger} = \gamma^m$ and they obey the Clifford algebra

$$\{\gamma^m, \gamma^n\} = 2g^{mn} \ , \quad m, n = 1, \ldots, 6 \ . \tag{A.9}$$

The Majorana condition on a spinor η reads

$$\eta^\dagger = \eta^T C \ , \tag{A.10}$$

where C is the charge conjugation matrix

$$C^T = C \ , \quad \gamma_m^T = -C\gamma_m C^{-1} \ . \tag{A.11}$$

The following Fierz identity holds for spinors on $\mathcal{M}_6$ [101]

$$M = \tfrac{1}{8} \sum_{k=0}^{6} \tfrac{1}{k!} \gamma_{p_1 \ldots p_k} \mathrm{Tr}(\gamma^{p_k \cdots p_1} M) \ , \tag{A.12}$$

where M is an arbitrary matrix in spinor space. Relevant examples used in the calculation of (4.23) are $M = \eta_1 \otimes \eta_2^\dagger$, $M = (\gamma^m D_m \eta_1) \otimes \eta_2^\dagger$, etc. Using eqn. (A.12) it is not hard to show that

$$\begin{aligned} \eta^{1\,\dagger}\gamma^m D_m \eta^1 &= \tfrac{1}{8}\sum_{n=0}^{6} \tfrac{1}{n!}\eta^{2\,\dagger}\gamma_{p_1\ldots p_n}\gamma^m D_m\eta^1 \; \eta^{1\,\dagger}\gamma^{p_n\ldots p_1}\eta^2 \;, \\ \eta^{1\,\dagger}\gamma^{mnp}\eta^1 &= \tfrac{1}{8}\sum_{n=0}^{6} \tfrac{1}{n!}\eta^{2\,\dagger}\gamma_{p_1\ldots p_n}\gamma^{mnp}\eta^1 \; \eta^{1\,\dagger}\gamma^{p_n\ldots p_1}\eta^2 \;, \end{aligned} \tag{A.13}$$

with similar expressions for η^2. A second important identity encodes how derivatives on spinors translate into exterior derivatives on forms. Explicitly one has (see for example [66])

$$\sum_n \tfrac{1}{n!}\eta^{2\,\dagger}\{\gamma_{p_1\ldots p_n},\gamma^m\}D_m\eta^1 e^{p_n\ldots p_1} = (d+d^*)\sum_n \tfrac{1}{n!}\eta^{2\,\dagger}\gamma_{p_1\ldots p_n}\eta^1 e^{p_n\ldots p_1} \tag{A.14}$$

where $d^* = -*_6\, d\, *_6$ is the formal adjoint of d, with $*_6$ being the six-dimensional Hodge-star.

A.2 Clifford algebra in 4 dimensions

In $D = 4$ we adopt the conventions of [92] and choose

$$\gamma^\mu = -i\begin{pmatrix} 0 & \sigma^\mu \\ \bar\sigma^\mu & 0 \end{pmatrix}, \qquad \gamma^5 = \begin{pmatrix} \mathbf{1} & 0 \\ 0 & -\mathbf{1} \end{pmatrix} \tag{A.15}$$

where the σ^μ are the 2×2 Pauli matrices

$$\sigma^0 = \begin{pmatrix} -i & 0 \\ 0 & -i \end{pmatrix}, \quad \sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \tag{A.16}$$

and $\bar\sigma^0 = \sigma^0,\ \bar\sigma^{1,2,3} = -\sigma^{1,2,3}$. We define

$$\sigma^{\mu\nu} = \tfrac14(\sigma^\mu\bar\sigma^\nu - \sigma^\nu\bar\sigma^\mu)\;, \qquad \bar\sigma^{\mu\nu} = \tfrac14(\bar\sigma^\mu\sigma^\nu - \bar\sigma^\nu\sigma^\mu)\;. \tag{A.17}$$

B Stable forms and the Hitchin functional

In this appendix we collect some basic facts about the geometry of stable even and odd forms on a six-dimensional manifold $\mathcal{M}_6$. The definition of the Hitchin functionals will be recalled. The case of stable three-forms and the general definition of stable odd and even forms will be reviewed in turn. A more exhaustive discussion of these issues can be found in refs. [75, 76, 80, 77]. We also comment on the derivation of the expression (3.53).

B.1 Stable three-forms and the standard Hitchin functional

Let us first consider a six-dimensional manifold $\mathcal{M}_6$ with a real globally defined three-form $\rho \in \Lambda^3 T^*$. A natural notion of non-degeneracy is that the form ρ is stable. From an abstract point of view a stable form ρ is defined by demanding that the natural action of $GL(6)$ on ρ spans an open orbit in $\Lambda^3_p T^*$ at each point p of $\mathcal{M}_6$. This condition can also be formulated in terms of the map $q: \ \Lambda^3 T^* \ \rightarrow \ \Lambda^6 T^* \otimes \Lambda^6 T^*$ defined as [75]

$$q(\rho) = \big\langle e^m \wedge f_n \lrcorner \rho, \rho \big\rangle \big\langle e^n \wedge f_m \lrcorner \rho \wedge \rho \big\rangle \ , \tag{B.1}$$

where e^m is a basis of $T^*\mathcal{M}_6$ and f_m is a basis of $T\mathcal{M}_6$. The set of stable three-forms on $\mathcal{M}_6$ is then shown to be

$$U^3 = \big\{ \rho \in \Lambda^3 T^* : q(\rho) < 0 \big\} \ , \tag{B.2}$$

where $q(\rho) < 0$ holds if $q(\rho) = -s \otimes s$ for some $s \in \Lambda^6 T^*$. Clearly, since $\Lambda^6 T^* \cong \mathbb{R}$ this means that the product of the coefficients of the volume forms in (B.1) is negative.

It was shown in ref. [75] that each real stable form $\rho \in U^3$ is written as

$$\rho = \tfrac{1}{2}(\Omega + \bar{\Omega}) \ , \tag{B.3}$$

where Ω is a complex three-form satisfying $\big\langle \Omega, \bar{\Omega} \big\rangle \neq 0$. The imaginary part of Ω is unique up to ordering and we denote it by $\hat{\rho} = \mathrm{Im}(\Omega)$. The real three-forms $\hat{\rho}(\rho)$ can also be defined by using the map q introduced in eqn. (B.1). On forms $\rho \in U^3$ one defines the Hitchin function

$$\mathcal{H}(\rho) := \sqrt{-\tfrac{1}{3} q(\rho)} \quad \in \ \Lambda^6 TY \ , \tag{B.4}$$

which is well defined since $q(\chi) < 0$. The form $\hat{\rho}$ is then defined to be the Hamiltonian vector field on $TU^3 \cong \Lambda^3 T^*$ [33]

$$4 \big\langle \hat{\rho}, \alpha \big\rangle = -\mathcal{D}\mathcal{H}(\alpha) \ , \qquad \forall \alpha \ \in \ \Lambda^3 T^* \ , \tag{B.5}$$

where $\mathcal{D}$ is the differential on TU^3. Note that $H(\rho)$ can be rewritten as $\mathcal{H}(\rho) = i \big\langle \Omega, \bar{\Omega} \big\rangle$.

In this paper we mostly use the integrated version of the Hitchin function $\mathcal{H}(\rho)$. Since $\mathcal{H}(\rho)$ is a volume form it is natural to define the Hitchin functional

$$H[\rho] = \int_{\mathcal{M}_6} \mathcal{H}(\rho) = i \int_{\mathcal{M}_6} \big\langle \Omega, \bar{\Omega} \big\rangle \ . \tag{B.6}$$

Its first (variational) derivative is precisely the form $\hat{\rho}$ such that

$$\partial_\rho H = -4 \hat{\rho} \ , \qquad \qquad \partial_\rho H(\alpha) = -4 \int_{\mathcal{M}_6} \big\langle \hat{\rho}, \alpha \big\rangle \ . \tag{B.7}$$

Here we also displayed how $\partial_\rho H$ is evaluated on some real form $\alpha \in \Lambda^3 T^*$. The second derivative of $H[\rho]$ is given by

$$\partial_\rho \partial_\rho H = -4 \mathcal{I} \ , \qquad \partial_\rho \partial_\rho H(\alpha, \beta) = -4 \int_{\mathcal{M}_6} \big\langle \alpha, \mathcal{I} \beta \big\rangle \ . \tag{B.8}$$

[33] The factor 4 is not present in the corresponding expression in ref. [75]. It arises due to the fact that we have set $\rho = \mathrm{Re}(\Omega)$ and not $\rho_{\mathrm{Hitchin}} = 2\mathrm{Re}(\Omega)$ as in ref. [75]

The map $\mathcal{I} : \Lambda^3 T^* \to \Lambda^3 T^*$ is shown to be an almost complex structure on U^3. It is used to prove that U^3 is actually a rigid special Kähler manifold [75]. The real form ρ can be also used to define an almost complex structure I_ρ on $\mathcal{M}_6$ itself by setting (see also the discussion in section 2)

$$(I_\rho)^m_n = \frac{1}{\mathcal{H}(\rho)} (e^m \wedge f_n \lrcorner \rho \wedge \rho) \ , \qquad \text{(B.9)}$$

where $\mathcal{H}(\rho)$ is defined in eqn. (B.4). With respect to I_ρ one decomposes complex three-forms as

$$\Lambda^3 T^*_{\mathbb{C}} = \Lambda^{(3,0)} \oplus \Lambda^{(2,1)} \oplus \Lambda^{(1,2)} \oplus \Lambda^{(0,3)} \ . \qquad \text{(B.10)}$$

Using this decomposition the complex structure $\mathcal{I}$ on U^3 is evaluated to be $\mathcal{I} = i$ on $\Lambda^{(3,0)} \oplus \Lambda^{(2,1)}$ and $\mathcal{I} = -i$ on $\Lambda^{(1,2)} \oplus \Lambda^{(0,3)}$. Furthermore, assuming that $\mathcal{M}_6$ possesses a metric hermitian with respect to I_ρ the six-dimensional Hodge-star obeys $*_6 = i$ on $\Lambda^{(0,3)} \oplus \Lambda^{(2,1)}$, while $*_6 = -i$ on $\Lambda^{(3,0)} \oplus \Lambda^{(1,2)}$. This implies the identifications

$$\mathcal{I} = *_6 \text{ on } \Lambda^{(2,1)} \oplus \Lambda^{(1,2)} \ , \qquad \mathcal{I} = - *_6 \text{ on } \Lambda^{(3,0)} \oplus \Lambda^{(0,3)} \ . \qquad \text{(B.11)}$$

The identity (B.11) is essential to show eqn. (3.53) as we will see in appendix B.3.

B.2 Stable odd/even forms and the extended Hitchin functional

Let us now briefly review the definition of general odd and even stable forms and their associated Hitchin functional. Many of the identities for stable three-forms naturally generalize to the more generic case. We consider real odd or even forms $\rho^{\text{ev/odd}} \in S^{ev/odd}$, where $S^{\text{ev/odd}} = \Lambda^{\text{ev/odd}} T^* \otimes |\det T|^{1/2}$ was already defined in equation (4.7). In most of the discussion a distinction between the odd and even case is not needed and we simplify our notation by writing $\rho^{\cdot} \in S^{\cdot}$, where $\cdot = $ ev or $\cdot = $ odd. As in section 4.1 the generalized tangent bundle is denoted by $E = T \oplus T^*$ (cf. equation (4.6)). A natural Clifford action of elements of E on the forms $\rho^{\cdot}$ is defined in eqn. (4.8). In this sense the elements $S^{\cdot}$ are spinors of the group $SO(6,6)$. In analogy to the definition (B.1) one introduces

$$q(\rho^{\cdot}) = \big\langle e^m \wedge f_n \lrcorner \rho^{\cdot}, \rho^{\cdot} \big\rangle \big\langle e^n \wedge f_m \lrcorner \rho^{\cdot}, \rho^{\cdot} \big\rangle \ , \qquad \text{(B.12)}$$

where e^m is a basis of T^* and f_m is a basis of T. The anti-symmetric Mukai pairing $\langle \cdot, \cdot \rangle$ is defined in eqn. (3.20). The map $q(\rho^{\cdot})$ can be evaluated for elements $\rho^{\cdot} \in S^{\cdot}$ yielding a number. The set of stable spinors $\rho^{\cdot}$ is then defined as

$$U^{\cdot} = \big\{ \rho^{\cdot} \in S^{\cdot} : q(\rho^{\cdot}) < 0 \big\} \qquad \text{(B.13)}$$

All spinors in $U^{\cdot}$ define a reduction of the structure group $SO(6,6)$ of E to $U(3,3)$. Furthermore, the elements of $U^{\cdot}$ can be decomposed as

$$\rho^{\cdot} = \tfrac{1}{2} \big(\Pi^{\cdot} + \bar{\Pi}^{\cdot} \big) \ , \qquad \text{(B.14)}$$

where as above the spinor $\hat{\rho}^{\cdot} = \text{Im}(\Pi^{\cdot})$ is unique up to ordering. It was shown that the complex spinors $\Pi^{\cdot}$ are eliminated by half of the elements in E via the Clifford action (4.8). Such spinors are called pure spinors.

In order to define the Hitchin functional we un-twist $S^{\cdot} \to \Lambda^{\cdot} T^*$ and consider $q(\rho^{\cdot})$ on forms. In this case $\sqrt{-q(\rho^{\cdot})}$ is a volume form and we define the extended Hitchin functional [76]

$$H[\rho^{\cdot}] = \int_{\mathcal{M}_6} \sqrt{-\tfrac{1}{3} q(\rho^{\cdot})} = i \int_{\mathcal{M}_6} \langle \Pi^{\cdot}, \bar{\Pi}^{\cdot} \rangle \ . \tag{B.15}$$

As in the three-form case the first (variational) derivative is precisely the form $\hat{\rho}^{\cdot}$ such that

$$\partial_{\rho^{\cdot}} H = -4 \hat{\rho}^{\cdot} \ , \qquad \partial_{\rho^{\cdot}} H(\alpha) = -4 \int_{\mathcal{M}_6} \langle \hat{\rho}^{\cdot}, \alpha \rangle \ , \tag{B.16}$$

where $\alpha \in \Lambda^{\cdot} T^*$. The second derivative of H is shown to define a complex structure on the space $U^{\cdot}$. Moreover, the space of stable spinors $U^{\cdot}$ naturally admits a rigid special Kähler structure.

B.3 Derivation of the Kähler metric

In this appendix we give more details on the derivation of the expression (3.53) and briefly discuss its generalizations. We first show that

$$\big[\partial_{\Pi_c^{\rm odd}} \partial_{\bar{\Pi}_c^{\rm odd}} K^{\rm Q}\big](\alpha', \alpha) = 2 e^{2D} \int_{\mathcal{M}_6} \langle \alpha', *_6 \alpha \rangle \ , \tag{B.17}$$

where $\alpha, \alpha' \in \Lambda^3_+ T^*$ are real three-forms obeying an additional condition and $K^{\rm Q}$ is given in eqn. (3.51). To begin with, notice that the Kähler potential $K^{\rm Q}$ is independent of the real part of $\Pi_c^{\rm odd}$ and only depends on $\rho = {\rm Re}(\Pi^{\rm odd}) = {\rm Im}(\Pi_c^{\rm odd})$. Hence, one infers $\partial_{\Pi_c^{\rm odd}} \partial_{\bar{\Pi}_c^{\rm odd}} K^{\rm Q} = \frac{1}{4} \partial_\rho \partial_\rho K^{\rm Q}$. Using the expressions (B.7), (B.8) and (3.51) it is straight forward to derive

$$\tfrac{1}{4} \big[\partial_\rho \partial_\rho K^{\rm Q}\big](\alpha', \alpha) = 2 e^{2D} \int_{\mathcal{M}_6} \langle \alpha', \mathcal{I} \alpha \rangle + 8 e^{4D} \int_{\mathcal{M}_6} \langle \hat{\rho}, \alpha' \rangle \int_{\mathcal{M}_6} \langle \hat{\rho}, \alpha \rangle \ , \tag{B.18}$$

where $\hat{\rho} = {\rm Im}(\Pi^{\rm odd})$. Since $\mathcal{M}_6$ admits an almost complex structure (B.9) associated to ρ, each form α can be decomposed as

$$\alpha = \alpha^{(3,0)+(0,3)} + \alpha^{(2,1)+(1,2)} \qquad \in \Lambda^3_+ T^* \ . \tag{B.19}$$

Using the fact that $\hat{\rho}$ is a $(3,0) + (0,3)$-form one has $\langle \hat{\rho}, \alpha^{(2,1)+(1,2)} \rangle = 0$. Therefore it is an immediate consequence of the identifications (B.11) that (B.17) holds on $\alpha^{(2,1)+(1,2)}$. It remains to show that it is also true for $\alpha^{(3,0)+(0,3)}$. Since we demand $\alpha^{(3,0)+(0,3)} \in \Lambda^3_+ T^*$ one has $\alpha^{(3,0)+(0,3)} = f \rho$ for some function f on $\mathcal{M}_6/\sigma$. Note that to show eqn. (B.17) we need to pull f through the integral and hence demand that f is actually constant. With this restriction it is straight forward to use $\int_{\mathcal{M}_6} \langle \hat{\rho}, \rho \rangle = \frac{1}{2} e^{-2D}$ to show that equation (B.17) holds on general $(2,1) + (1,2)$-forms and forms $\alpha^{(3,0)+(0,3)} \propto \rho$.[34]

Let us also briefly comment on the general case. As we have just argued, the identity (B.17) is easily shown on $(2,1) + (1,2)$ forms, while general $(3,0) + (0,3)$ forms are

[34] An alternative derivation of the condition (B.17) might be performed by using decompositions into $SU(3)$ representations, as done for the G_2 analog of (B.17), for example, in ref. [102, 103].

problematic. To also include the generic $(3,0)+(0,3)$ case one defines the 'Kähler potential'

$$\check{K} = -2\ln\left[i\langle\Pi^{\text{odd}},\bar{\Pi}^{\text{odd}}\rangle\right] , \tag{B.20}$$

where now $\Pi^{\text{odd}} = (\rho + i\hat{\rho})\otimes\epsilon^{-1/2}$ is an element of $S^3 = \Lambda^3T^*\otimes|\det T|^{1/2}$ and ϵ is the volume form

$$i\langle\Pi^{\text{odd}},\bar{\Pi}^{\text{odd}}\rangle\epsilon = 2\langle\rho,\hat{\rho}\rangle . \tag{B.21}$$

Note that the product $\langle\Pi^{\text{odd}},\bar{\Pi}^{\text{odd}}\rangle$ yields a number while $\langle\rho,\hat{\rho}\rangle$ is a volume form. Both are depending on the coordinates of $\mathcal{M}_6$. Furthermore, in contrast to the Kähler potential (3.51) there is no integration in the functional (B.20) [62]. The first derivative of $\check{K}$ is obtained from eqn. (B.7) to be

$$\tfrac{1}{2}\left[\partial_{\text{Re}(\Pi^{\text{odd}})}\check{K}\right](\alpha) = 4e^{\check{K}/2}\langle\hat{\rho},\alpha\rangle\otimes\epsilon^{-1/2} , \tag{B.22}$$

for a three-form $\alpha\in\Lambda^3T^*$. Using eqn. (B.8) the second derivative reads

$$\tfrac{1}{4}\left[\partial_{\text{Re}(\Pi^{\text{odd}})}\partial_{\text{Re}(\Pi^{\text{odd}})}\check{K}\right](\alpha,\alpha') = 2e^{\check{K}/2}\langle\alpha,\mathcal{I}\alpha'\rangle + 8e^{\check{K}}\langle\hat{\rho},\alpha\rangle\langle\hat{\rho},\alpha\rangle\epsilon^{-1} , \tag{B.23}$$

for a three-forms $\alpha,\alpha'\in\Lambda^3T^*$. This is precisely a volume form and integration over $\mathcal{M}_6$ yields a metric on three-forms α,α'. Following the same reasoning as above, it is now straight forward to show

$$\int_{\mathcal{M}_6}\left[\partial_{\Pi_c^{\text{odd}}}\partial_{\bar{\Pi}_c^{\text{odd}}}\check{K}\right](\alpha,\alpha') = 2\int_{\mathcal{M}_6} e^{2D}\langle\alpha,*_6\alpha'\rangle , \tag{B.24}$$

on all elements $\alpha,\alpha'\in\Lambda^3T^*_+$. We have used that $\Pi_c^{\text{odd}} = C_3^{(0)} + i\text{Re}(\Pi^{\text{odd}})$ and that $\check{K}$ is independent of the R-R fields $C_3^{(0)}$. In this general case, the four-dimensional dilaton D is defined as $e^{-2D} = i\langle\Pi^{\text{odd}},\bar{\Pi}^{\text{odd}}\rangle$ and can vary along $\mathcal{M}_6$. The equation (B.24) is the generalization of the identity (B.17).

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