Fitting Hierarchical Holographic Modeling into the Theory of Scenario Structuring and a Resulting Refinement to the Quantitative Definition of Risk

Stan Kaplan, Yacov Y. Haimes,* and B. John Garrick

A point of view is suggested from which the Hierarchical Holographic Modeling (HHM) method can be seen as one more method within the Theory of Scenario Structuring (TSS), which is that part of Quantitative Risk Assessment having to do with the task of identifying the set of risk scenarios. Seen in this way, HHM brings strongly to our attention the fact that different methods within TSS can result in different sets of risk scenarios for the same underlying problem. Although this is not a problem practically, it is a bit awkward conceptually from the standpoint of the “set of triplets” definition of risk, in which the scenario set is part of the definition. Accordingly, the present article suggests a refinement to the set of triplets definition, which removes the specific set of scenarios, found by any of the TSS methods, from the definition of risk and casts it, instead, as an approximation to the “true” set of scenarios that is native to the problem at hand and not affected by the TSS method used.

KEY WORDS: Theory of scenario structuring; hierarchical holographic modeling; quantitative risk assessment

1. INTRODUCTION

The different methods, used in risk analysis to identify risk scenarios, have been gradually evolving, clarifying, and organizing themselves into what we have begun to call the “Theory of Scenario Structuring” (TSS). This theory has the objective of providing a unified perspective from which these various methods can be seen as variations on the same rigorous, systematic thought process. The intent is that from this perspective, the multiple methods can be compared, and in that way, better understood. Thus, the risk analyst can be more confident and flexible when choosing, mixing, and designing the method applicable to his or her specific problem.

In the present article we point out how Hierarchical Holographic Modeling (HHM) can be seen as part of the TSS. In so doing, the great generality of the HHM method calls to our attention the fact that different methods of scenario structuring can lead to seemingly different sets of scenarios for the same underlying problem. This fact is a bit awkward from the standpoint of the “set of triplets” definition of risk. To eliminate this awkwardness, it is necessary to go back and refine this definition of risk to make explicit what before was only implicit; namely, that the set of risk scenarios used in a quantitative risk analysis should be (1) complete, (2) finite, and (3) disjoint. These three properties can be achieved by first noting that in realistic problems there is always an underlying continuum of possible scenarios, and then carving up this continuum into a finite set of nonoverlapping....
subsets. Recognizing that each such subset is itself a scenario, we then have our complete, finite, and disjoint set. The mathematical term for this type of carving up is “partitioning.”

The above refinement of the definition gives us a perspective from which to understand the HHM approach. The HHM approach carves up the continuum, but does not necessarily partition it. In other words, it allows the set of subsets to be overlapping, that is, nondisjoint. It argues that disjointness is only required when we are going to quantify the likelihood of the scenarios, and even then, only if we are going to add up these likelihoods (in which case the overlap areas would end up being counted twice.) Thus, if the risk analysis seeks mainly to identify scenarios, rather than to quantify their likelihood, then the requirement of disjointness can be relaxed somewhat, so that it becomes a preference rather than a necessity.

With this understanding, HHM takes its place within the TSS as an extremely general scenario identification process, alongside the other well-known but more specific processes, Failure Modes and Effects Analysis (FMEA), Hazard and Operations Analysis (HAZOP), and Anticipatory Failure Determination (AFD).

2. HISTORICAL REVIEW: THE DEFINITION OF RISK

In the first issue of the journal *Risk Analysis* the following “set of triplets” definition of risk, \( R \), was set forth:

\[
R = \{< S_i, L_i, X_i > \}, \quad (1)
\]

where \( S_i \), here, denotes the \( i \)th “risk scenario,” \( L_i \) denotes the likelihood of that scenario, and \( X_i \) denotes the “damage vector” or resulting consequences. This definition has served the field of risk analysis well since then, and much early debate about how to quantify \( L \) and \( X \), and the meaning of “probability,” “frequency,” and “probability of frequency” in this connection has been thoroughly resolved.\(^{11,12}\)

In Kaplan and Garrick,\(^{10}\) the \( S_i \) themselves were defined, somewhat informally, as answers to the question: “What can go wrong?” with the system or process being analyzed. Subsequently, in Kaplan\(^{1,2}\) a subscript “c” was added to the set of triplets:

\[
R = \{< S_i, L_i, X_i >_c \}, \quad (2)
\]

to denote that the set of scenarios, \( \{ S_i \} \), should be “complete,” meaning it should include “all the possible scenarios, or at least all the important ones.”
Also in Kaplan,(1,2) the idea of the “success” or “as planned” scenario, was introduced and denoted by $S_0$. The risk scenarios $S_i$ could then be visualized as deviations from $S_0$. Thus, the idea that the various methods (e.g., FMEA, HAZOP, FT, and ET) used in risk analysis in different industries could be viewed as different systematic ways of identifying and categorizing these deviations, $S_i$, began to jell. With the generalizations of these methods, and with the addition of the Russian method of AFD, this idea matured, in Kaplan et al.,(3) into the subject that we now call the TSS.

3. THE HHM METHOD

At about the same time that the definition of risk article(10) was published, so, too, was the first article on HHM.(8) Central to the HHM method is a particular form of diagram, examples of which are shown in Figs. 1 and 2. This form of diagram is particularly useful for the analysis of systems with multiple, interacting (perhaps overlapping) subsystems such as, for example, a regional transportation or water supply system. The different columns in the diagram reflect different “perspectives” on the overall system.

The HHM methodology recognizes that most organizational as well as technology-based systems are hierarchical in structure, and thus the risk management of such systems must be driven by and responsive to this structure. The risks associated with each subsystem within the hierarchical structure contribute to and ultimately determine the risks of the overall system.

4. FITTING HHM INTO THE TSS

In seeing how to fit HHM into the TSS the key idea is to view the HHM diagram as a depiction of the success scenario $S_0$. Each box in the diagram may
then be viewed as defining a set of actions or results required of the system, as part of the definition of “success.” Conversely then, each box also defines a set of risk scenarios; namely, the set of scenarios in which there is failure to accomplish one or more of the actions or results defined by that box. The union of all these sets of risk scenarios is then “complete” in that it contains all possible risk scenarios.

This completeness is, of course, a very desirable feature. On the other hand, the intersection of two of our risk scenario sets, corresponding to two different HHM boxes, may not be empty. In other words our scenario sets may not be “disjoint.” This is not a desirable feature; however, a modest amount of nondisjointness can easily be tolerated, its major impact being some double counting, and therefore some conservative results when we sum up the likelihoods of the risk scenarios.

5. A REFINEMENT TO THE DEFINITION OF RISK

In Equation (1) the choice of the subscript $i$ on the $S_i$ carries with it, by conventional usage, the implicit assumption that the set of scenarios is denumerable (i.e., countable). Moreover, because Equation (1) is intended to describe the result of an actual risk analysis, there is the further implicit assumption that the number of scenarios in the set $\{S_i\}$ is finite. We wish now to release both these assumptions, and therefore revise Equation (2) to read:

$$R = \{<S_{\alpha}, L_{\alpha}, X_{\alpha} >|, \alpha \in A, \}
\tag{3}$$

where the index $\alpha$ now ranges over a set $A$, which, in general, is nondenumerable. We can thus think of $A$ as the set of points in the interior of Fig. 3.

The set $A$ is therefore infinite and nondenumerable. It has the same order of infinity as the real number continuum. Each point, $\alpha$, in the interior of the figure also represents a scenario (namely $S_{\alpha}$) and, therefore, we can think of the set of interior points as representing the set of all risk scenarios, which we can now designate by $S_A$.

Now we can connect Equations (2) and (3) by recalling the principle that every scenario, $S_i$, that we can describe with a finite number of words is itself a set of scenarios. Thus, each $S_i$ in Equation (2) can be visualized as a subset of $S_A$. For practical purposes we want the set of scenarios in our risk analysis, $\{S_i\}$, to be

1. complete, in the sense that $U(S_i) = S_A$, where $U$ is the set operation “union”;
2. finite; and
3. disjoint, meaning that $S_i \cap S_j = 0$ for all $i \neq j$, where $\cap$ is the set operation “intersection.”

Such a set of subsets of $S_A$ is termed a “partitioning,” $P$, of $S_A$. This type of partitioning can be visualized as the set of boxes in Fig. 4. Thus, we arrive at the point of view that what we want to do in a risk analysis is to identify a partitioning of the underlying risk space $S_A$. The individual sets in this partitioning are the scenarios $S_i$, which are finite in number, disjoint, and together “cover” the underlying space $S_A$. We may then write

$$R_P = \{<S_i, L_i, X_i >|, \}
\tag{4}$$

$R_P$ is thus an approximation to $R$ based on the partition $P$.

$$R_P \approx R. \tag{5}$$

6. COMMENTS ON THE REFINED DEFINITION

The refined definition of risk, Equation (3), gives us the conceptual framework we want. The actual set of risk scenarios is viewed as an infinite and nondenumerable set $S_A$. In any practical, and therefore approximate, quantitative risk assessment the scenarios $S_i$ are viewed as subsets of $S_A$. These subsets are dis-
joint, finite in number, and in aggregate cover \( S_i \). The set of these subsets thus form a partitioning of \( S_i \) and in this sense are “complete.” From the perspective of this framework we can now view the theory of scenario structuring as a study of the various techniques for achieving such a partitioning.

7. IDENTIFYING THE \( S_i \)

Having defined the success scenario \( S_0 \), the process of finding the risk scenarios, \( S_i \), consists of decomposing \( S_0 \) into “parts” or “components.” Then, putting our magnifying glass over each part in turn, we ask, “What could go wrong in this part?” In this way we generate the \( S_i \).

Now we observe that if \( S_i \) is itself decomposed into a complete, finite, and disjoint set of parts, then simply defining \( S_i \) as “something goes wrong with part \( i \)” generates a complete, finite, and disjoint set of \( S_i \). Pushing this idea further, if we have identified a complete, finite, and disjoint subset of risk scenarios originating in each part of \( S_0 \), then the aggregate, that is, the union of those subsets is a complete, finite, and disjoint set of \( S_i \) for the entire problem (subject again, however to the multiple failure comment in footnote 2).

8. THE HHM APPROACH TO DECOMPOSING \( S_0 \)

The HHM diagram may now be viewed as a portrayal of the success scenario \( S_0 \), and a decomposition of that scenario into its various parts and pieces. The decomposition strives to be complete but not necessarily disjoint. Indeed, HHM regards nondisjointness, or “overlapping” of the decomposed parts and pieces as a useful feature, reflecting different “perspectives” on the system. Thus, HHM recognizes that most organizational as well as technology-based systems are not only hierarchical in structure, but are “multiply hierarchical,” in that different, overlapping hierarchical structures can be identified within the system. The risk management of such systems must then be driven by, and responsive to, this structure. The risks associated with each subsystem within the hierarchical structures contribute to, and ultimately determine, the risks of the overall system. The distribution of risks among the subsystems often plays a dominant role in the allocation of resources. This is manifested in the quest to achieve a level of risk that is deemed acceptable in the normal, judgmental, decision-making process, when the trade-offs among all the costs, benefits, and risks are considered.

To say this another way, in the process of modeling large-scale and complex systems, more than one decomposition or conceptual model of \( S_0 \) is likely to emerge. Each of these models may focus on a specific aspect, yet all may be regarded as acceptable representations of the system. This phenomenon is particularly common in hierarchical multilevel modeling, in which more than one decomposition approach may be both feasible and desirable. Consequently, decomposing a system often presents a dilemma concerning the choice, or definition of subsystems. For example, an economic system may be decomposed into geographic regions or activity sectors. An electric power management system may be decomposed according to the various functions of the system (e.g., power generation units, energy storage units, transmission units, etc.) or along geographic or political boundaries. Another choice might be a timewise decomposition into planning periods. If several aspects of the system are involved, such as the geographic regions and activity sectors of an economic system, it could be advantageous to employ several decompositions (this is the meaning of the “holographic” in HHM). For example, four major decomposition structures may be identified for water resources systems: political/geographical, hydrological, temporal, and functional, where each such structure leads to identification of its own set of scenarios. Thus, one of the valuable contributions of the HHM framework for risk assessment and management is its ability to identify risk scenarios that result from and propagate through the multiple overlapping hierarchies in real-life systems. In the planning, design, or operational mode, the ability to model and quantify the risks contributed by each subsystem markedly facilitates understanding, quantifying, and evaluating the risk from the whole system. In particular, the ability to model the intricate relations among the various subsystems and account for all relevant and important elements of risk and uncertainty renders the modeling process more tractable and the risk assessment process more representative and encompassing.

9. SHORT DESCRIPTIONS OF METHODS WITHIN THE TSS

9.1. Failure Modes and Effects Analysis

Suppose the system being analyzed is an automobile. This system can be decomposed into a finite number of distinct, that is, nonoverlapping “parts.” This set
of parts is a complete, finite, and disjoint decomposition of the automobile. If we now define $S_i$ as the scenario “part $i$ fails” then we have a finite and disjoint set of scenarios for the whole system. Because the system works, by definition, if all the parts work, this set of $S_i$ is complete except for the possibility that two or more parts could fail essentially simultaneously. If we include one more scenario, representing “multiple independent failures,” then our set of scenarios is also complete. This sketches the FMEA approach to scenario structuring.

A “two-dimensional” version of FMEA is useful in cases in which we wish to distinguish among phases of operation in the system under study. We simply define these phases and then extend our set of scenarios by defining $S_{i,j}$ to mean “part $i$ fails during phase $j$.” This set of scenarios can be conveniently displayed in a two-dimensional chart such as that shown in Fig. 5. Each box in this chart represents a scenario set $S_{i,j}$. We can now write in each box the likelihood, $L_{i,j}$, of the scenario and/or the consequences, $X_{i,j}$. This summarizes the results of the risk assessment very conveniently. Looking at the chart we can readily see where our major risks are and take appropriate actions to reduce or eliminate them.

9.2. Hazard and Operations Analysis

In risk analysis of a chemical processing plant, the widely used HAZOP approach would examine each length of pipe, for example, and ask “What would happen if there was too much flow in this pipe?” (or too little, or flow in the wrong direction, wrong temperature or pressure, wrong substance, etc.). The analysis then goes on to ask: “What are the consequences of such a situation?” and “How could such a situation arise?” This process can be viewed as a way of generating risk scenarios, $S_i$. It is obviously a lot of work, but if done carefully and thoroughly can yield a complete set of $S_i$.

A generalization of this idea comes from observing that in each part of the plant or in each piece of equipment, and during each phase of the operation, certain “functions” must be accomplished (Fig. 6). One could then ask analogous questions in terms of functions, that is, “What happens if we have too much function here? Too little function? Wrong function?” and so forth. In this way we can generate a complete, finite, and disjoint subset of scenarios for each function. The union of all these subsets then gives us a complete, finite, and distinct set of scenarios for the whole system. Once again, the results of the risk assessment could be summarized in a two-dimensional table such as Fig. 7, where now the $i,j$th box represents “failure of the $i$th function during the $j$th phase.”
9.3. Event Trees

The ET approach to finding the risk scenarios also begins, as should all risk assessments, with a careful description of the success scenario $S_0$. This scenario can be represented as a trajectory in the “phase space” of the system (see Fig. 8). Breaking this trajectory into a complete, finite, and distinct set of segments, we can then focus on each segment of $S_0$ and ask “What can go wrong here?” The answer to that question is termed an “initiating event” (IE). Given that the IE occurs, then, depending on what happens next, a set of $S_i$, represented as an ET (Fig. 9), emerges from that IE. Each path through the tree represents an $S_i$. If the set of IEs is complete, finite, and distinct, and the set of paths in each tree is complete, finite, and distinct for that IE, then the set of all paths in all the trees constitutes a complete, finite, and distinct set of scenarios for the entire problem.

Fig. 8. Scenario $S_0$ viewed as a trajectory in the state space of the system.

Fig. 9. Event tree emerging from the initiating event (IE). ES = end state.

In many engineering systems there are “safety” or “backup systems” that are meant to be activated in response to the various IEs. If the safety system works as intended, the IE typically would not cause significant damage. However, if the IE occurs and the corresponding backup system fails, there could be serious consequences. Again, a two-dimensional display, such as that shown in Fig. 10, could show the likelihood and/or the consequences of these combined failures.

Fig. 10. Frequency of occurrence of various initiating events (IEs) and subsequent failure of backup systems.

9.4. Fault Trees

If, in connection with $S_0$, we identify a complete set of possible “end states” (ESs) we can ask with respect to each, “How could this ES come about?” It could occur if and only if certain finite combinations of other events, which we call “mid-states” (MSs), occurs. These MSs could occur if and only if certain other states occur. Working backward in this way (see Fig. 11) we can identify scenarios that start with IEs and lead to the postulated ES. If this is done carefully, we can arrive at a complete, finite, and distinct set of scenarios leading to each ES of interest. This is a lot of work, but so are all other methods of finding complete sets of $S_i$. The FT method is especially useful when there are particular ESs to be avoided, such as the release of a health- or life-threatening substance.

Two-dimensional displays could also be useful in this case, for example to display the likelihood of each ES and the magnitude of the various types of damage that could result (see Fig. 12).
9.5. Anticipatory Failure Determination

For the problem of finding risk scenarios, AFD may be regarded as an application of TRIZ, the Russian theory of inventive problem-solving. Like FMEA, HAZOP, and the ET methods, AFD also begins with a decomposition of $S_0$ into a complete, finite, and distinct set of parts, components, segments, and/or phases. However, where the other methods ask, “What can go wrong with this part, segment or component?” AFD asks instead, “If I wanted to, how could I make something go wrong with this part?”

This form of the question can stimulate our creative mind to think of scenarios that have not been identified before. It helps overcome the mindset that resists the idea that anything could go wrong with “our” system. Also, this form of the question is better suited to a class of scenarios of increasing importance currently, namely, those resulting from sabotage, terrorism, or some form of conflict.

9.6. Hierarchical Holographic Modeling

A fundamental attribute of large-scale systems is their inescapably multifarious nature, hierarchical noncommensurable objectives, multiple decision makers, multiple transcending aspects, and elements of risk and uncertainty. In part, this may be a natural consequence of the fact that most large-scale systems respond to a variety of needs that are basically marred with uncertainty; are noncommensurable; and may, under some circumstances, become openly conflicting. It is impracticable to represent within a single model all the aspects of a truly large-scale system that may be of interest at any given time to its management, government regulators, students, or any other group.

Central to holographic modeling (from the perspective of theoretical constructs) is the overlapping among various holographic models. In this context, holographic modeling may be viewed as the generalization of hierarchical overlapping coordination, where two or more decompositions partially overlap. In the abstract, a mathematical model may be viewed as a limited, one-sided image of the real system that it portrays. With single-model analysis and interpretation, it is quite impossible to clarify and document the sources of risk associated with not only the multiple components, objectives, and constraints of a system, but also with its welter of societal aspects (e.g., functional, temporal, geographical, economic, political, legal, environmental, sectoral, and institutional). Given this multifarious characteristic of real systems and the notion that even present integrated models cannot adequately cover all systemic aspects, the concept of HHM constitutes a comprehensive theoretical framework for systems modeling and identification of real or perceived sources of risk.

The impact of HHM in the planning phase may be most profound in the way that risks and uncertainties can be identified and integrated into the analysis. From the planning perspective, two major types of risks and uncertainties can be identified. The first type is concerned with the impact of exogenous events on the proposed plan, such as natural disasters or new legislation. The second is concerned with the impact of endogenous events that affect the execution of the plan, such as hardware, software, or organizational or human failures. Because the basic philosophy of HHM
is to build a family of models that address different aspects of the system, this is a natural setting in which the impact of both types of risks and uncertainties can be studied in a unified way.

10. HHM FOR RISK IDENTIFICATION

It may be helpful to share a few examples and case studies\(^{(9)}\) in which HHM serves as a very general approach to generating risk scenarios. HHM is particularly suited to large-scale, hierarchical, multilevel systems in which it attempts to deal with the inescapably multifarious nature of such systems.

Consider a project to organize a chamber music concert. A portrayal of the project is shown in Fig. 13, where six major heading topics (i.e., aspects, perspectives, or sub-HHMs) are identified: Musicians, Electrical, Building/Location, Audience, Organizational, and Miscellaneous. The top row in this figure may be thought of as a decomposition of the “as planned scenario,” \(S_0\). If the musicians and audience arrive as planned, the electrical system works, and so forth, then the concert will be a success. Under each such component is a list of the \(S_i\), that is, the things that could go wrong in connection with that component. Thus, in this example the HHM directly identifies a set of risk scenarios. On close inspection one

---

**Fig. 13.** Hierarchical Holographic Modeling for chamber music example.
can see that the $S_i$ thus defined are not all disjoint. This causes us no problem, however, as long as we don’t seek to quantify and add up the likelihoods of these scenarios.

Given the complexity of the commonly interconnected and interdependent components (subsystems) of most real-world systems, capturing the plethora of risk scenarios through HHM would necessarily lead to some overlapping (primarily at the subtopic level). A distinction must be made, however, between identifying scenarios and quantifying them. For purposes of quantification, the scenarios should be constructed as disjoint, but the scenarios identified via HHM may overlap, as suggested in Fig. 14.

As another example, consider a software acquisition process. In Fig. 15 the $S_i$ for this process is decomposed into components and subcomponents reflecting the multiple aspects of such a project. Again, the $S_i$ are found by placing a magnifying glass over each box and asking “What can go wrong here?” or “What could make the process fail?”

![Fig. 14. Overlapping or Nondisjoint scenarios.](image)

![Fig. 15. Hierarchical Holographic Modeling for software acquisition. RFP = request for proposal.](image)
Another way to identify and categorize risk scenarios, $S_i$, is to group them according to the “target” or aspect of $S_0$ that is damaged by the $S_i$. (This is similar in spirit to the FT process.) A case study is given in Haimes\(^9\) in connection with the objective of hardening water supply systems against accident or terrorist attack. In this case, the $S_i$ is decomposed into various aspects of the system that might be targeted or affected by accidents or attacks (see Fig. 16). The $S_i$ are then generated by asking “How could I attack
As a final example, in connection with the goals of sustainable development, environmental protection, and quality of life, the HHM approach decomposes these concepts into the various aspects shown in Fig. 17. This breakdown then forms a basis for identifying and categorizing risk scenarios that could adversely affect one or more such aspects.

11. CONCLUSION

Within the subject of risk analysis, the evolving TSS aspires to be a comprehensive treatment of the process of finding, organizing, and categorizing the set of risk scenarios. As such, it should include within itself the well-known standard methods of scenario identification such as FT, ET, FMEA, and HAZOP. It should also include the more recent AFD method and the HHM method. The purpose of this article is primarily to demonstrate this last inclusion.

Along the way to showing this inclusiveness, our attention is drawn to the fact that the set of risk scenarios, $S_i$, developed by the different methods for the same problem, could well be different. This is a bit awkward conceptually. Accordingly, we found it desirable to back up and refine the original “set of triplets” definition of risk so that it did not assume or imply, as part of the definition itself, that the set of risk scenarios is finite or denumerable. Rather, this refined definition allows the set of risk scenarios to be a continuum, that is, nondenumerable. This continuous set of scenarios constitutes the “true” risk, and is independent of the method used to identify them.

For practical, computational purposes, this “true” scenario set is then partitioned into a finite, disjoint, and complete set of subsets. That is what the various risk scenario identification methods accomplish. Each such subset then “is” a risk scenario, $S_j$, which then makes it perfectly acceptable for the different methods to arrive at different partitionings. Moreover, if...
the scenarios are not going to be quantified, it is also acceptable if they are not disjoint.

Thus this refinement takes the finite set of $S_i$ out of the definition of risk, and casts it more properly as an approximation to the true, underlying, nondenumerable set of risk scenarios. Different sets of $S_i$, arrived at by different methods, are thus seen as just different approximations to the same underlying truth. This is a much more satisfactory viewpoint conceptually. Practically, it also suggests that the risk analyst would do well to apply more than one of the methods to a specific problem, to gain more insight and more confidence that all the important scenarios have been brought to light.

ACKNOWLEDGMENT

We wish to thank Grace Zisk for her editorial assistance and Della Dirickson for her administrative assistance.

REFERENCES