Abstract— In this paper we propose a new model and algorithm for region segmentation and feature extraction from 2D images containing imprecise regions. Region modeling is done in two phases. In the first phase a region is represented as a classical fuzzy set, and in second phase the obtained fuzzy set is approximated by a fuzzy polygon, another fuzzy set whose borders are represented as an array of fuzzy points in linear fuzzy space. Membership functions for the fuzzy set in the first phase are represented by feed forward neural network trained on the set consisting of pixels’ feature vectors. The feature vector model is formed based on 2D wavelet transformation of pixel’s neighborhood. Utilization of the model and algorithm is demonstrated through the example of calculating region diameter in DICOM 2D medical images.

I. INTRODUCTION

Images are fundamental tools in the health care for diagnosis, clinical studies, research and learning. Currently, there are multiple techniques to capture images from patients to help diagnostic tasks such as X-ray images, Computed Tomography (CT), Magnetic Resonance Imaging (MRI), Positron Emission Tomography (PET), and Ultrasonography, see [1], [6]-[13]. The diagnostic task generates a large amount of images which must be archived for future evaluations. Fortunately, most of these techniques produce digital images, which are more efficiently archived and handled, by means of computer systems, than physical ones [1]. Both diagnostic and therapeutic indications for radiologic imaging are expanding rapidly. The rapid expansion is a consequence of the need for more rapid, accurate, cost effective, and less invasive treatment. In many real radiologic practices, automated and intelligent image analysis and understanding are accepted as an essential part or procedure, such as image segmentation, registration, and computer-aided diagnosis and detection. In addition, in the area of cancer prognosis and treatment, automated and intelligent algorithms have a large market and are welcomed broadly, in areas such as radiation therapy planning or automatic identification of imaging biomarkers from radiological images of certain diseases, etc. However, most image segmentation methods deals with imprecise data. In our previous works [2-5] we present simple and yet effective models of imprecise spatial data based on fuzzy sets. Machine learning algorithms underpin the algorithms and software that make computer-aided diagnosis/prognosis/treatment possible. Machine learning identifies complex patterns automatically and helps radiologists make intelligent decisions on radiology data such as conventional radiographs, CT, MRI, and PET images [6].

In many applications, the performance of machine learning-based automatic detection and diagnosis systems has shown to be comparable to that of a well-trained and experienced radiologist.

In this paper we propose a solution for region (in medicine often referred to as “mass”) detecting and segmenting in DICOM images. Masses are characterized by their location, size, shape, margin, and associated findings (i.e. architectural distortion, contrast). These associated properties are examined by radiologists as they are strongly correlated with the classification (benign versus malignant) of the mass. It is generally accepted that mass detection is a more challenging problem than the detection of micro-califications, not only for the large variation in size and shape in which masses can appear but also because masses often exhibit poor image contrast [7]. Based on these variations in shapes and sizes of regions, poor contrasts, and generally imprecise image data, we propose the use of fuzzy spatial objects for the representation of these regions.

The paper consists of six sections. Following this introductory section and an overview of related work in Section II, several definitions and preliminaries related to imprecise point object model are set out in Section III. Mathematical model of the imprecise polygon is set out in Section IV. Section V describes the algorithm and its implementation. Section VI contains one example of practical application of the proposed model and the corresponding algorithm. The final section contains concluding remarks and future research directions.

II. RELATED RESULTS

In the papers [8] and [9] authors discuss various approaches to automatic classifications, detections and segmentations in mammogram imaging. These approaches include support vector machines (SVM), artificial neural networks (ANN), and also region-based
and contour-based methods. In the paper [10] authors propose an advanced morphological approach for medical image processing applicable to various density types (solid, non-solid, part-solid and solitary types, vascularized and juxtapleural types). First, the algorithm separates lung parenchyma and radiographically denser anatomical structures with coupled competition and diffusion processes. Second, it locates the core of a node in a manner that is applicable to juxtapleural types using a transformation applied on the Euclidean distance transform of the foreground. Third, it detaches the node from attached structures by a region growing on the Euclidean distance map followed by a procedure to delineate the surface of the node based on the patterns of the region growing and distance maps. Finally, convex hull of the node surface intersected with the foreground constitutes the final segmentation. In the paper [11], an algorithm using fuzzy classification and symmetry analysis is proposed for 3D brain tumor segmentation in MRI. First, the brain is segmented using a new approach, robust to the presence of tumors. Then a first tumor detection is performed, based on selecting asymmetric areas with respect to the approximate brain symmetry plane and fuzzy classification. Its result constitutes the initialization of a segmentation method based on a combination of a deformable model and spatial relations, leading to a precise segmentation of the tumors. Imprecision and variability are taken into account at all levels, using appropriate fuzzy models.

In the paper [12] a method is given for detection of neuron membranes in electron microscopy images using a series of artificial neural networks (ANNs) in a framework combined with a feature vector that is composed of image intensities sampled over a stencil neighborhood. Several ANNs are applied in series allowing each ANN to use the classification context provided by the previous network to improve detection accuracy. Authors have developed the method of serial ANNs and they have shown that the learned context does improve detection over traditional ANNs.

In paper [13] authors propose a novel method based on MRF (Markov Random Fields) and a hybrid of social algorithms that contain an ant colony optimization (ACO) and a Gossiping algorithm which can be used for segmenting single and multispectral MRIs in real time environments. Combining ACO with the Gossiping algorithm helps to find a better path using neighborhood information.

Concerning DICOM images, in paper [1] a medical image viewer is implemented in Java. Its innovative features are: capability for visual edition and storage of measurements involved in diagnosis and treatment of scoliosis and performed on digital X-rays; capability for retrieving images in a flexible way from medical image databases on the basis of those measurements.

III. PRELIMINARIES

We shall give in this section some basic facts on the linear fuzzy space introduced in [2, 3], and which is a base for a model of the imprecise polygon introduced in the present paper.

**Definition 2.1** Fuzzy point \( P \in \mathbb{R}^2 \), denoted by \( \tilde{P} \), is defined by its membership function \( \mu_P \in F^2 \), where the set \( F^2 \) contains all membership functions \( u : \mathbb{R}^2 \rightarrow [0,1] \) satisfying the following conditions:

i) \( (\forall u \in F^2) (\exists_{P \in \mathbb{R}^2}) u(P) = 1 \),

ii) \( (\forall X_1, X_2 \in \mathbb{R}^2) (\forall \lambda \in [0,1]) u(\lambda X_1 + (1 - \lambda) X_2) \geq \min \{ u(X_1), u(X_2) \} \),

iii) function \( u \) is upper semi continuous,

iv) \( \alpha \)-cut \( [u^\alpha] = \{ X | X \in \mathbb{R}^2, u(X) \geq \alpha \} \) of function \( u \) is convex.

A point \( P \) from \( \mathbb{R}^2 \), with membership function \( \mu_P(P) = 1 \), will be denoted by \( P \) (the core of the fuzzy point \( \tilde{P} \)), and the membership function of the point \( \tilde{P} \) will be denoted by \( \mu_P \). By \( [P]^\alpha \) we denote the \( \alpha \)-cut of the fuzzy point (set from \( \mathbb{R}^2 \)).

**Definition 2.2** \( \mathbb{R}^2 \) Linear fuzzy space is a set \( \mathcal{H}^2 \subset \mathbb{F}^2 \) of all functions which, in addition to the properties given in Definition 2.1, are:

i) Symmetric with respect to the core \( S \in \mathbb{R}^2 \) \( (\mu(S) = 1) \),

\( \mu(V) = \mu(M) \wedge \mu(M) \neq 0 \Rightarrow d(S,V) = d(S,M) \),

where \( d(S,M) \) is the usually distance in \( \mathbb{R}^2 \).

ii) Inverse-linear decreasing w.r.t. points’ distance from the core according to:

If \( r \neq 0 \), then

\[ \mu_S(V) = \max \left( 0, 1 - \frac{d(S,V)}{|r|} \right), \]

if \( r = 0 \), then

| \( \mu_S(V) = \{ \begin{array}{ll} 1 & \text{ako je} \ S = V, \\
0 & \text{ako je} \ S \neq V, \end{array} \) |

where \( d(S,V) \) is the distance between the point \( V \) and the core \( S \) \( (V,S \in \mathbb{R}^n) \) and \( r \in \mathbb{R} \) is constant.

An element of that space is represented as ordered pair \( S = (S, r_S) \), where \( S \in \mathbb{R}^2 \) is the core of \( \tilde{S} \), and \( r_S \in \mathbb{R} \) is the distance from the core for which the function value becomes 0; in the sequel parameter \( r_S \) will be called fuzzy support radius.

**Definition 2.4** Let \( \mathcal{H}^2 \) be a linear fuzzy space and a function \( f \) be a linear combination of fuzzy points \( \tilde{A} \) and \( \tilde{B} \). Then the fuzzy set \( \tilde{A}B \) is a fuzzy line if the following holds

\[ \tilde{A}B = \bigcup_{u \in [0,1]} f(\tilde{A}, \tilde{B}, u). \]

Analogously to fuzzy point, fuzzy lines can be represented as a pair of two fuzzy points. A fuzzy line is a
minimal extension of a precise line defined by two discrete points.

IV. FUZZY POLYGON IN $\mathbb{R}^2$ LINEAR FUZZY SPACE

One of the main contributions of this paper is a model of the imprecise polygon. It is based on special class of the fuzzy sets introduced in [2] and [3].

**Definition 3.1** Let $\mathcal{H}^2$ be a linear fuzzy space and $\mathcal{A} = \{\mathcal{A}_1, \ldots, \mathcal{A}_n\}$ be the ordered set of the fuzzy points $\mathcal{A}_i \in \mathcal{H}^2$. Then linear fuzzy path $s(\mathcal{A})$ is given by

$$s(\mathcal{A}) = \bigcup_{i=1}^{n-1} \mathcal{A}_i \mathcal{A}_{i+1}.$$  

**Remark.** If $X \in \mathbb{R}^2$, then the membership function $\mu_{s(\mathcal{A})}$ of the linear fuzzy path $s(\mathcal{A})$ is given by

$$\mu_{s(\mathcal{A})}(X) = \max_{i=1}^{n-1} \mu_{\mathcal{A}_{i+1}}(X).$$

**Definition 3.2** Let $\mathcal{H}^2$ be a linear fuzzy space and $\mathcal{A} = \{\mathcal{A}_1, \ldots, \mathcal{A}_n\}$ be the ordered set of the fuzzy points $\mathcal{A}_i \in \mathcal{H}^2$. Then closed linear fuzzy path $c(\mathcal{A})$ is given by

$$c(\mathcal{A}) = s(\mathcal{A}) \bigcup \mathcal{A}_n \mathcal{A}_1.$$  

**Remark.** If $X \in \mathbb{R}^2$, then the membership function $\mu_{c(\mathcal{A})}$ of the closed linear fuzzy path $c(\mathcal{A})$ is given by

$$\mu_{c(\mathcal{A})}(X) = \max(\mu_{s(\mathcal{A})}(X), \mu_{\mathcal{A}_n \mathcal{A}_1}(X)).$$

**Definition 3.3** Let $\mathcal{H}^2$ be a linear fuzzy space and $\mathcal{A} = \{\mathcal{A}_1, \ldots, \mathcal{A}_n\}$ be the ordered set of the fuzzy points $\mathcal{A}_i \in \mathcal{H}^2$. Then linear fuzzy polygon $p(\mathcal{A})$ is given by

$$\mu_{p(\mathcal{A})}(X) = \begin{cases} 1 & \text{if } X \text{ inside polygon } c(\mathcal{A})^0 \\ \mu_{c(\mathcal{A})}(X) & \text{otherwise}, \end{cases}$$

where $c(\mathcal{A})^0$ is the core of the fuzzy set $c(\mathcal{A})$.

V. SEGMENTATION AND FEATURE EXTRACTION ALGORITHM

The creation of the feature vector of every point (pixel) in the image consists of four steps as shown by the activity diagram from Figure 1: forming a square matrix of pixel neighborhood, 2D wavelet transformation, zig-zag algorithm for transforming matrix to vector and finally feature vector normalization.

The main parameter for forming a pixel model is the matrix dimension. Most often it is a 3x3 matrix. Figure 2 shows an example of the 3x3 matrix which represents pixel (marked as x) and its neighborhood pixels (marked as o).

The second step in forming a feature vector of a pixel is 2D wavelet transformation. In this case, the Haar wavelets are applied [14].

Then, using the zig-zag algorithm, the obtained 2D matrix is transformed to a vector. Figure 3 shows an example of the 3x3 matrix and its corresponding vector.

Following this transformation, vector values are normalized to represent the pixel.
Once each pixel is represented by its feature vector, segmentation and feature extraction can take place. An activity diagram showing principal steps of the segmentation and feature extraction algorithm is given in Figure 4.

The first phase of modeling a region with imprecise border is creation of the fuzzy set with membership function mapping a set of pixels feature vectors to a [0,1] interval. For that purpose, a two layer perception is used (sigmoid activation function).

The number of input neurons corresponds to a number of feature vector elements \( f_0 \ldots f_8 \) and number of output neurons corresponds to a number of different fuzzy regions.

The training set for the neural network contains the pixels that strongly belong to the particular fuzzy region (marked with corresponding rectangle) and pixels that strongly do not belong to this particular fuzzy region (pixels that surely belong to other fuzzy region).

In Figure 6 an example of medical image with two rectangles is shown, each representing a basis for two fuzzy regions.

Figure 4. Segmentation and feature extraction algorithm

The next step in image segmentation is extracting the core and support border points. Each core border point is a center of the closed linear fuzzy path. Corresponding support radius is calculated as maximal number such that support of the fuzzy points is completely inside support of the original fuzzy set.

Figure 7 shows an image that is created using output from the first output neuron. It corresponds to the fuzzy set of the region 1. Here, white color represents pixels that surely belong to the region 1, opposing to black color which corresponds to pixels that surely do not belong to the region 1. Gray color indicates partial membership.
The last step is a linear fuzzy polygon approximation. The procedure performing this step is given by the pseudo code in Listing 1.

```plaintext
foreach P in core border point set do
    calculate maximal radius R that does not intersect with support border
    create Fuzzy point with center in point P and radius R
```

Listing 1. The linear fuzzy polygon approximation procedure

Figure 9 shows region 1 fuzzy set approximation with linear fuzzy polygon.

VI. DETERMINING POLYGON DIAMETER

For the purpose of diagnostics and therapy results evaluation it is often necessary to determine some geometrical features of the extracted regions, among which the diameter is a very important one.

In our previous work [2] we have proposed an algorithm for calculating the diameter of the convex fuzzy hull. In this paper we have applied this algorithm to calculate the diameter of the fuzzy polygons extracted from the 2D image.

Figure 10 shows diameters of the two fuzzy polygons extracted from the 2D medical image shown in Figure 6.

Figure 10. Diameters of the two linear fuzzy polygons

CONCLUSION

In this paper we propose the model of imprecise region based on the notion of the linear fuzzy space [3]. Also, we propose the novel algorithm for the medical image segmentation and feature extraction in medical 2D images.

Imprecise region is modeled through two approximations. Firstly, it is modeled as a fuzzy set with feed forward neural network representing its membership function. After this first approximation is done, we apply second approximation by linear fuzzy polygon with notion of the fuzzy linear polygon introduced in this paper. According to our previous works [2, 4, 5] such models are suitable for efficient manipulation like diameter calculation.

In order to deal with 3D images, modeling of the complex 3D fuzzy mesh based on linear fuzzy space is one of the main future research directions we have in mind. In addition to this main direction, there are several topology relations between fuzzy polygons that should be properly modeled in order to provide means for efficient medical images manipulation for diagnosis, clinical studies, research and learning purposes.
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