Minimum Distortion Variance
Concatenated Block Codes for
Embedded Source Transmission

Suayb S. ARSLAN
Quantum Corporation

International Conference on Computing,
Networking and Communications (ICNC),
Feb 6, 2014, Honolulu, Hawaii/USA.
Outline

- Source quality assessment basics
- Progressive source compression
- Unequal Protection Schemes:
  - Conventional Schemes.
  - Previous work: Concatenated Block Coding
- Few results and issues about the previous work
- Description of the extension scheme (proposed)
  - Optimization of parameters
- Numerical results
Source quality assessment Basics: Image compression

- Given two images $I$ and $I'$ (original and the noisy version), the distortion will be measured by Mean Square Error (MSE):

$$MSE = \frac{1}{L_x \times L_y} \sum_{y=1}^{L_y} \sum_{x=1}^{L_x} [I(x, y) - I'(x, y)]^2$$

where $L_x$ and $L_y$ are dimensions of the image.

- Peak Signal to Noise Ratio (PSNR in dB) is defined to be

$$PSNR = 10 \times \log_{10} \left( \frac{I_{max}^2}{MSE} \right)$$

where $I_{max}$ is the maximum possible intensity value of the image.

- For monochromatic gray scale image: $I_{max} = 255$

- Lower MSE (larger PSNR) means better image quality.

- “Source rate” means the average number of bits spent per pixel (bpp). For a given PSNR value, the lower the source rate is, the better the compression will be.
Progressive Source Compression

SPIHT Encoded Bit Stream

%4

0.01bpp, PSNR=22.55dB

Ex: SPIHT image compression algorithm [1]. 4% gives you only a brief description of the source.

Progressive Source Compression

SPIHT Encoded Bit Stream

- 0.01bpp, PSNR=22.55dB
- 0.05bpp, PSNR=27.17dB

- 20% is good enough to say what the picture looks like.
Progressive Source Compression

At 40%, it begins to refine the image.

0.01bpp, PSNR=22.55dB
0.05bpp, PSNR=27.17dB
0.1bpp, PSNR=29.81dB
Progressive Source Compression

At 100%, it gives more refinement but no major difference from 40%.

SPIHT Encoded Bit Stream

- 0.01bpp, PSNR=22.55dB
- 0.05bpp, PSNR=27.17dB
- 0.1bpp, PSNR=29.81dB
- 0.25bpp, PSNR=33.68dB
Progressive Source Compression

- We consider progressive type of encoders.
  - Embedded image encoders: EZW, SPIHT, JPEG2000 etc.
  - Image compression using singular value decomposition (SVD).
- Result: Very sensitive to bit errors.
- Protection and performance improvement is achieved by error correction coding.
- Way to go: Unequal error protection (UEP) is beneficial for progressively encoded sources. This can be provided by several known techniques.
- We consider a concatenated coded scheme.
Unequal Error Protection Schemes: REVIEW

- **FixedInfo**, single channel code rate for all the packets.
- **FixedCoded**, single channel code rate for all the packets.
- **FixedInfo & FixedCoded**, different channel code rates for each packet.

**Error Correction Codes include:**
- Conventional Block Codes (BCH, Golay, etc),
- Rate-Compatible Punctured Convolutional (RCPC) Codes,
- Rate-Compatible (RC) Turbo codes, RC-LDPC codes
- Reed Solomon (RS) codes.
Find the number of source blocks $M$, the rate of channel codes based on a bit budget constraint (Transmission rate) and a target error rate using minimum average distortion criterion.
Few results...

Use 512 X 512 Lena Image

RCPC codes with rates:
\[ C = \{8/9, 4/5, 2/3, 4/7, 1/2, 4/9, 2/5, 4/11, 1/3, 4/13, 2/7, 4/15, 1/4\} \]

\[ \varepsilon_0 = 0.1 \] and transmission rate \((r_{tr}) = 0.3\text{bpp}\) \((0.3 \times 512 \times 512 = 79643 \text{ bits})\)

<table>
<thead>
<tr>
<th>(M)</th>
<th>(r_1)</th>
<th>(r_2)</th>
<th>(r_3)</th>
<th>(r_4)</th>
<th>(r_5)</th>
<th>PSNR (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1/4</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>20.44</td>
</tr>
<tr>
<td>2</td>
<td>2/3</td>
<td>1/3</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>28.45</td>
</tr>
<tr>
<td>3</td>
<td>8/9</td>
<td>4/5</td>
<td>4/13</td>
<td>-</td>
<td>-</td>
<td>28.71</td>
</tr>
<tr>
<td>4</td>
<td>8/9</td>
<td>8/9</td>
<td>4/5</td>
<td>1/3</td>
<td>-</td>
<td>28.79</td>
</tr>
<tr>
<td>5</td>
<td>1</td>
<td>8/9</td>
<td>8/9</td>
<td>4/5</td>
<td>1/3</td>
<td>28.75</td>
</tr>
</tbody>
</table>
Observations

- In an optimal setting, this coding scheme results in four or five source blocks.
- Number of reconstruction levels is five or six.
- **Result:** User dissatisfaction due to large quality fluctuations.
- We consider a broadcast scenario.
  - One server, multiple receivers with varying channel conditions.
- Minimum average distortion.
  - Sufficient for point-to-point communication.
  - Minimum average does not imply minimum variance.
- **Result:** User dissatisfaction due to unfair service quality.
Extension System and Optimization

- **M** codewords. Each information block is chopped.
- **Number of reconstruction levels:** \( \sum_{l=1}^{M} m_l + 1 \)
- **This extensions increases the redundancy due to CRC.**
  - Less space for source bits:

\[
\sum_{l=1}^{M} I_l - (m_l - 1) N_r \leq \sum_{l=1}^{M} I_l
\]
Extension System and Optimization

- **Original Problem:** A code allocation policy $\pi$ allocates the channel code $c^{(i)}_\pi \in C$ to be used in the $i$-th stage of the algorithm.

- Let $\overline{D}_\pi(n)$ denote the $n$-th moment of the distortion at the receiver using policy $\pi$. Let $N_s$ be the number of source samples and $B$ is the bit budget.

- **Minimum Average Distortion Problem:**

\[
\min_{\pi, \xi, \nu} \overline{D}_\pi(1) \text{ such that } r_{tr} = \frac{1}{N_s} \sum_{i=1}^{M} \frac{m_i \nu}{\prod_{j=i}^{M} r_{\pi}^{(j)}} \leq B
\]

$\xi = \{m_1, \ldots, m_M\}$
Constrained Minimization of Distortion Variance:

\[
\min_{\pi, \xi, \nu} \sigma^2_{\pi} \text{ such that } r_{tr} = \frac{1}{N_s} \sum_{i=1}^{M} \frac{m_i \nu}{\prod_{j=i}^{M} r_{\pi}^{(j)}} \leq B, \overline{D}_{\pi}(1) \leq \gamma_D
\]

\[
\sigma^2_{\pi} = \overline{D}_{\pi}(2) - \overline{D}_{\pi}^2(1)
\]

Assume: \(\sigma^2_{\pi}\) is a non-increasing function of \(\overline{D}_{\pi}(1)\) using policy \(\pi\)

Minimization of Second moment of Distortion: Set \(\overline{D}_{\pi}(1) = \gamma_D\)

\[
\min_{\pi} \overline{D}_{\pi}(2) \text{ subject to } r_{tr} = \frac{1}{N_s} \sum_{i=1}^{M} \frac{m_i \nu}{\prod_{j=i}^{M} r_{\pi}^{(j)}} \leq B
\]
Numerical Results

- We compare the following systems:
  - \textbf{ConMinAve}: Concatenated block coding with minimum average distortion criterion. Let \( d^* \) be the minimum distortion. (Original System [1])
  - \textbf{ConChopMinAve}: Extension scheme with minimum average distortion criterion.
  - \textbf{ConChopMinAve}: Extension scheme with minimum distortion variance criterion subject to a minimum average distortion constraint \( \gamma D \leq d^* \)

- We use a 512 X 512 monochromatic images Lena and Goldhill using SPIHT and JPEG2000 compression algorithms.
- Let us set \( \nu = 850 \), \( M = 2 \), and use RCPC codes [1].
- A BSC with crossover probability \( \varepsilon_0 = 0.05 \).
- Our distortion metric is MSE and we present the mean MSE and MSE variance for all three systems.

Numerical Results
Numerical Results

- Let us vary $\nu$, to increase/decrease the number of reconstruction levels.
- Set $M = 2$. 

![Graph showing numerical results with data points and curves]

- $41.79$ reduction in MSE for the set $\{4/5, 4/9\}$.
- $22.65$ in the set $\{8/9, 4/11\}$.
- $58\%$ reduction in MSE for the set $\{4/5, 4/9\}$.
- MSE in numerics: $9.53$.
- 126 reconstruction levels, block size of 340 bits.
Dramatic improvements can be obtained while maintaining the good mean distortion characteristics.

Similar results can be observed using RC-LDPC codes.
References


