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Baryon Asymmetry and Neutrino Mixing

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Abstract

In theories where $B-L$ is a spontaneously broken local symmetry, the cosmological baryon asymmetry can be generated by the out-of-equilibrium decay of heavy Majorana neutrinos. We study this mechanism assuming a similar pattern of mixings and masses for leptons and quarks, as suggested by $SO(10)$ unification. This implies that $B-L$ is broken at the unification scale $\Lambda_{\text{GUT}} \sim 10^{16}$ GeV, if $m_{\nu_\mu} \sim 3 \cdot 10^{-3}$ eV as preferred by the MSW explanation of the solar neutrino deficit. The observed value of the baryon asymmetry, $n_B/s \sim 10^{-10}$, is then obtained without any fine tuning of parameters.

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The standard model of electroweak interactions and its unified extensions contain the ingredients which are necessary to explain the observed cosmological baryon asymmetry [1]. However, despite much effort during almost twenty years, the origin of the baryon asymmetry has not yet been unequivocally identified. Unified theories, with or without supersymmetry, offer several interesting scenarios, but it proved difficult to satisfy all constraints imposed by the dynamics of the cosmological expansion, which appears to require an inflationary period. So far no ‘standard model’ of baryogenesis has emerged.

At temperatures above the critical temperature of the electroweak phase transition baryon (B) and lepton (L) number violating processes are in thermal equilibrium [2]. This observation is of crucial importance for the theory of baryogenesis. In principle, it opens the possibility to generate the baryon asymmetry at the electroweak phase transition [3]. However, as a result of detailed studies of the thermodynamics of this transition in recent years, this now appears unlikely, at least within the standard model [4].

At high temperatures, where baryon and lepton number violating processes are in thermal equilibrium, a baryon asymmetry can be generated from a lepton asymmetry. This was suggested by Fukugita and Yanagida [5]. The primordial lepton asymmetry is generated by the out-of-equilibrium decay of heavy Majorana neutrinos in the standard manner. This mechanism has subsequently been studied by several authors [6, 7, 8], and it has been shown that the observed baryon asymmetry,

$$Y_B = \frac{n_B}{s} = (0.6 - 1) \cdot 10^{-10}, \quad (1)$$

can be obtained for a wide range of parameters.

In the high temperature phase of the standard model the asymmetries of baryon number B and of $B - L$ are proportional in thermal equilibrium [9],

$$Y_B = \left(\frac{8N_f + 4N_H}{22N_f + 13N_H} \right) Y_{B-L}. \quad (2)$$

Here N_f is the number of quark-lepton families and N_H is the number of Higgs doublets. In the standard model, as well as its unified extension based on the group $SU(5)$, $B - L$ is conserved. Hence, no asymmetry in $B - L$ can be generated, and Y_B vanishes. Furthermore, as mentioned above, baryogenesis at the electroweak phase transition appears unlikely. As a consequence, the non-vanishing of the baryon asymmetry is a strong argument for lepton number violation. This is naturally realized by adding right-handed Majorana neutrinos to the standard model. This extension of the standard model can be embedded into grand unified theories with gauge groups containing $SO(10)$ [10]. Heavy right-handed Majorana neutrinos can also explain the smallness of the light neutrino masses via the see-saw mechanism [11].

In unified theories with right-handed neutrinos $B - L$ is spontaneously broken. In this paper we study the implications of baryogenesis on the scale of $B - L$ breaking and on CP violating leptonic interactions. Adding right-handed neutrinos to the standard model introduces many new parameters. We shall restrict this freedom by assuming a similar pattern of mixings and masses for leptons and quarks, which is natural in SO(10) unification.

Masses and couplings of charged leptons and neutrinos are given by the lagrangian

$$\mathcal{L}_Y = -\bar{l}_L \tilde{\phi} g_l e_R - \bar{l}_L \phi g_\nu \nu_R - \frac{1}{2} \bar{\nu}_R^C M \nu_R + \text{h.c.} , \quad (3)$$

where $l_L = (\nu_L, e_L)$ is the left-handed lepton doublet and $\phi = (\varphi^0, \varphi^-)$ is the standard model Higgs doublet. The vacuum expectation value of the Higgs field $\langle \phi \rangle = v \neq 0$ generates Dirac masses m_l and m_D for charged leptons and neutrinos,

$$m_l = g_l v \quad \text{and} \quad m_D = g_\nu v , \quad (4)$$

which are assumed to be much smaller than the Majorana masses M . Therefore, we have light and heavy neutrinos

$$\nu \simeq K^\dagger \nu_L + \nu_L^C K \quad , \quad N \simeq \nu_R + \nu_R^C , \quad (5)$$

with masses

$$m_\nu \simeq -K^\dagger m_D \frac{1}{M} m_D^T K^* \quad , \quad m_N \simeq M , \quad (6)$$

as mass eigenstates. Here K is a unitary matrix which relates weak and mass eigenstates. Since the heavy neutrinos N_i are Majorana fermions, they violate lepton number if they decay to lepton and Higgs scalar. In the rest system the decay width of N_i reads at tree level,

$$\Gamma_{Di} := \Gamma_{rs} (N^i \rightarrow \phi^\dagger + l) + \Gamma_{rs} (N^i \rightarrow \phi + \bar{l}) = \frac{M_i (m_D^\dagger m_D)_{ii}}{8\pi v^2} . \quad (7)$$

Interference between tree level and one-loop amplitudes yields the CP asymmetry [8]

$$\epsilon_i = \frac{1}{8\pi v^2 (m_D^\dagger m_D)_{ii}} \sum_j \text{Im} \left[(m_D^\dagger m_D)_{ij}^2 \right] f \left(\frac{M_j^2}{M_i^2} \right) \quad (8)$$

$$\text{with} \quad f(x) = \sqrt{x} \left[1 - (1+x) \ln \left(\frac{1+x}{x} \right) \right] .$$

In a quantitative analysis of this mechanism one has to take into account several other processes as well, especially the lepton number violating scatterings mediated by a massive neutrino N_i . In the following we shall take all three heavy neutrino families into account as intermediate states, but we shall only calculate the asymmetry generated

by the lightest of the right-handed neutrinos, since the asymmetries generated by the heavier neutrinos are washed out.

Neutrino masses and mixings

In this paper we make the ansatz of a similar pattern of mixings and mass ratios for leptons and quarks, which is natural in SO(10) unification. Such an ansatz is most transparent in a basis where all mass matrices are maximally diagonal. In addition to real mass eigenvalues two mixing matrices appear. We can always choose a basis for the lepton fields such that the mass matrices m_l for the charged leptons and M for the heavy Majorana neutrinos N_i are diagonal with real and positive eigenvalues,

$$m_l = \begin{pmatrix} m_e & 0 & 0 \\ 0 & m_\mu & 0 \\ 0 & 0 & m_\tau \end{pmatrix} \quad M = \begin{pmatrix} M_1 & 0 & 0 \\ 0 & M_2 & 0 \\ 0 & 0 & M_3 \end{pmatrix}. \quad (9)$$

In this basis m_D is a general complex matrix, which can be diagonalized by a biunitary transformation. Therefore, we can write m_D in the form

$$m_D = V \begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix} U^\dagger, \quad (10)$$

where V and U are unitary matrices and the m_i are real and positive. In the absence of a Majorana mass term V and U would correspond to Kobayashi-Maskawa type mixing matrices of left- and right-handed charged currents, respectively.

According to eqs. (7) and (8) the CP asymmetry is determined by the mixings and phases present in the product $m_D^\dagger m_D$, where the matrix V drops out. Therefore, to leading order, the mixings and phases which are responsible for baryogenesis are entirely determined by the matrix U . Correspondingly, the mixing matrix K in the leptonic charged current, which determines CP violation and mixings of the light leptons, depends on mass ratios and mixing angles and phases of U and V . Hence, there is no direct connection between the CP violation and generation mixing at high and low energies.

We now concentrate on the mixing matrix U . One can factor out five phases, which yields

$$U = e^{i\gamma} e^{i\lambda_3\alpha} e^{i\lambda_8\beta} U_1 e^{i\lambda_3\sigma} e^{i\lambda_8\tau}, \quad (11)$$

where the λ_i are the Gell-Mann matrices. The remaining matrix U_1 depends on three mixing angles and one phase, like the Kobayashi-Maskawa matrix for quarks. In analogy to the quark mixing matrix we choose the Wolfenstein parametrization [12] as ansatz for

U_1 ,

$$U_1 = \begin{pmatrix} 1 - \frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix}, \quad (12)$$

where A and $|\rho + i\eta|$ are of order one, while the mixing parameter λ is assumed to be small. For the masses m_i and M_i we assume a hierarchy like for up-type quarks,

$$m_1 = b\lambda^4 m_3 \quad m_2 = c\lambda^2 m_3 \quad b, c = \mathcal{O}(1) \quad (13)$$

$$M_1 = B\lambda^4 M_3 \quad M_2 = C\lambda^2 M_3 \quad B, C = \mathcal{O}(1). \quad (14)$$

For the eigenvalues m_i of the Dirac mass matrix this choice is motivated by SO(10) unification. The masses M_i cannot be degenerate, because in this case there exists a basis for ν_R such that $U = 1$, which implies that no baryon asymmetry is generated. For simplicity we therefore assume that the masses M_i scale like the Dirac neutrino masses.

The light neutrino masses are given by the seesaw formula (6). The matrix K , which diagonalises the neutrino mass matrix, can be evaluated in powers of λ . A straightforward calculation gives the following masses for the light neutrino mass eigenstates

$$m_{\nu_e} = \frac{b^2}{|C + e^{4i\alpha} B|} \lambda^4 m_{\nu_\tau} + \mathcal{O}(\lambda^6) \quad (15)$$

$$m_{\nu_\mu} = \frac{c^2 |C + e^{4i\alpha} B|}{BC} \lambda^2 m_{\nu_\tau} + \mathcal{O}(\lambda^4) \quad (16)$$

$$m_{\nu_\tau} = \frac{m_3^2}{M_3} + \mathcal{O}(\lambda^4). \quad (17)$$

The CP -asymmetry in the decay of the lightest right-handed neutrino N_1 is easily obtained from eqs. (8) and (12)-(14),

$$\epsilon_1 = -\frac{1}{16\pi} \frac{B A^2}{c^2 + A^2 |\rho + i\eta|^2} \lambda^4 \frac{m_3^2}{v^2} \text{Im} [(\rho - i\eta)^2 e^{i2(\alpha + \sqrt{3}\beta)}] + \mathcal{O}(\lambda^6). \quad (18)$$

This yields for the magnitude of the CP asymmetry,

$$|\epsilon_1| \leq \frac{1}{16\pi} \frac{B A^2 |\rho + i\eta|^2}{c^2 + A^2 |\rho + i\eta|^2} \lambda^4 \frac{m_3^2}{v^2} + \mathcal{O}(\lambda^6). \quad (19)$$

How close the value of $|\epsilon_1|$ is to this upper bound depends on the phases α , β and $\arg(\rho + i\eta)$. Since $\epsilon_1 \propto m_3^2/v^2$, we can already conclude that a large value of the Yukawa coupling m_3/v will be preferred by this mechanism of baryogenesis. This holds irrespective of the neutrino mixings.

Numerical results

To obtain a numerical value for the produced baryon asymmetry, we have to specify the free parameters in our ansatz (12)-(14). In the following we will always use as a constraint the value for the ν_μ -mass which is preferred by the MSW explanation [13] of the solar neutrino deficit (cf. [14]),

$$m_{\nu_\mu} \simeq 3 \cdot 10^{-3} \text{ eV} . \quad (20)$$

A generic choice for the free parameters is to take all $\mathcal{O}(1)$ parameters equal to one and to fix λ to a value which is of the same order as the λ parameter of the quark mixing matrix,

$$A = B = C = b = c = |\rho + i\eta| \simeq 1 , \quad \lambda \simeq 0.1 . \quad (21)$$

From eqs. (15)-(17), (20) and (21) one now obtains,

$$m_{\nu_e} \simeq 8 \cdot 10^{-6} \text{ eV} , \quad m_{\nu_\tau} \simeq 0.15 \text{ eV} . \quad (22)$$

Finally, a second mass scale has to be specified. In unified theories based on SO(10) the Dirac neutrino mass m_3 is naturally equal to the top-quark mass. Hence, we choose

$$m_3 = m_t \simeq 174 \text{ GeV} . \quad (23)$$

This determines the masses of the heavy Majorana neutrinos N_i ,

$$M_3 \simeq 2 \cdot 10^{14} \text{ GeV} , \quad (24)$$

and, consequently, $M_1 \simeq 2 \cdot 10^{10} \text{ GeV}$ and $M_2 \simeq 2 \cdot 10^{12} \text{ GeV}$. From eq. (19) one obtains the CP asymmetry $|\epsilon_1| \simeq 10^{-6}$, where we have assumed maximal phases. The solution of the set of Boltzmann equations described in [8] now yields the $B - L$ asymmetry (see fig. 1a),

$$Y_{B-L} \simeq 3 \cdot 10^{-10} , \quad (25)$$

which is indeed the correct order of magnitude. The precise value depends on unknown phases.

The large mass M_3 of the heavy Majorana neutrino N_3 (cf. (24)), suggests that $B - L$ is already broken at the unification scale $\Lambda_{\text{GUT}} \sim 10^{16} \text{ GeV}$, without any intermediate scale of symmetry breaking. The large value of M_3 is a consequence of our choice (23), $m_3 \simeq m_t$. To test the sensitivity of our result for Y_{B-L} on this assumption, consider the alternative choice,

$$m_3 = m_b \simeq 4.5 \text{ GeV} , \quad (26)$$

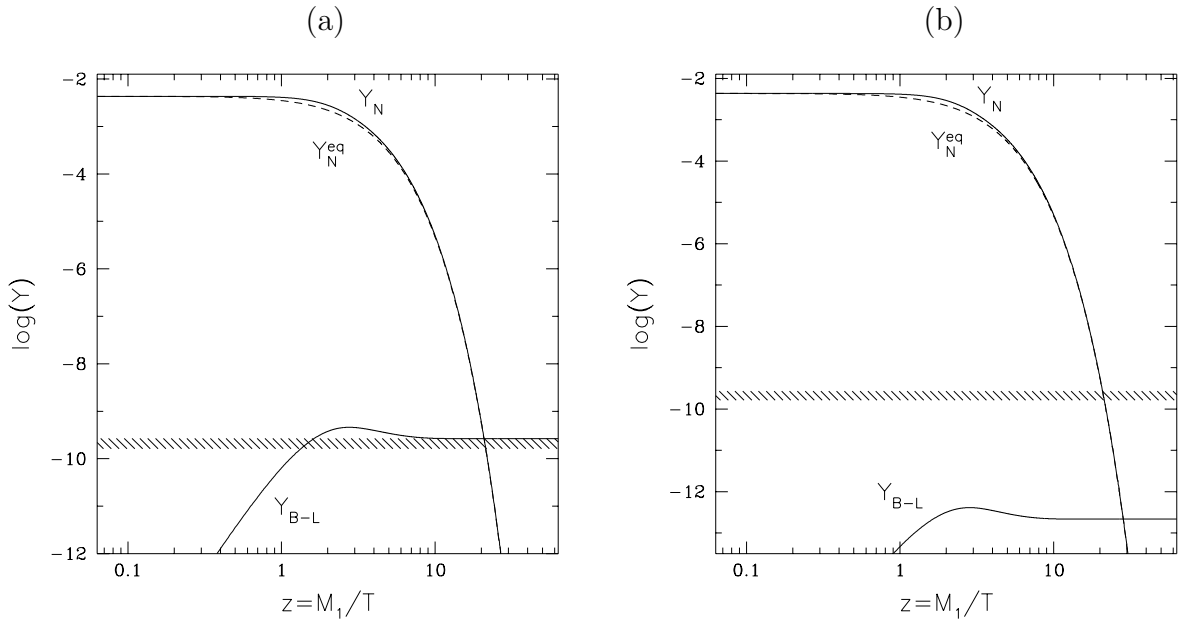


Figure 1: *Time evolution of the neutrino number density and the $B - L$ asymmetry for $\lambda = 0.1$ and for $m_3 = m_t$ (a) or $m_3 = m_b$ (b). The equilibrium distribution for N_1 is represented by a dashed line, while the hatched area shows the measured value for the asymmetry.*

with all other parameters remaining unchanged. In this case one obtains $M_3 = 10^{11}$ GeV and $|\epsilon_1| = 5 \cdot 10^{-10}$ for the mass of N_3 and the CP asymmetry, respectively. Since the maximal $B - L$ asymmetry is $-\epsilon_1/g^*$, where g^* is the number of relativistic degrees of freedom (cf. [1]), it is clear that the generated asymmetry will be too small. The solutions of the Boltzmann equations are shown in fig. 1b. The generated asymmetry,

$$Y_{B-L} \simeq 2 \cdot 10^{-13}, \quad (27)$$

is too small by more than two orders of magnitude. We can conclude that high values for both masses m_3 and M_3 are preferred, which is natural in $SO(10)$ unification.

Models for dark matter involving massive neutrinos favour a τ -neutrino mass [15],

$$m_{\nu_\tau} \simeq 5 \text{ eV}, \quad (28)$$

which is significantly larger than the value given in (22). The large value (28) for the τ -neutrino mass does not correspond to the simplest choice of parameters within our ansatz. However, it can be accomodated for the following set of parameters: $b = |\rho + i\eta| \simeq 1$, $A = c \simeq 1/3$, $B = C \simeq 3$, $\lambda \simeq 0.09$, $m_3 \simeq m_t$. In this case one obtains $M_3 \simeq 6 \cdot 10^{12}$ GeV, $m_{\nu_e} \simeq 6 \cdot 10^{-5}$ eV, $|\epsilon_1| \simeq 2 \cdot 10^{-6}$. Integration of the Boltzmann equations yields the $B - L$ asymmetry (see fig 2),

$$Y_{B-L} \simeq 6 \cdot 10^{-10}, \quad (29)$$

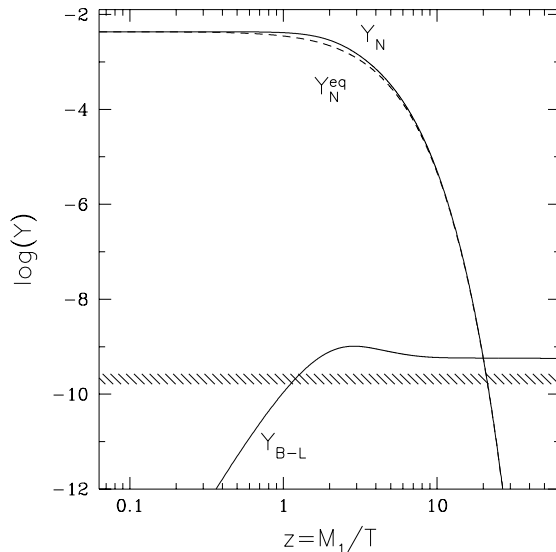


Figure 2: *Solutions of the Boltzmann equations for $m_{\nu\tau} = 5 \text{ eV}$ and $m_3 = m_t$.*

where again maximal phases have been assumed.

We conclude that in unified theories based on the group $\text{SO}(10)$, the out-of-equilibrium decay of heavy Majorana neutrinos naturally explains the cosmological baryon asymmetry. Assuming a similar pattern of mixings and masses for leptons and quarks the observed value of the baryon asymmetry is obtained without any fine tuning of parameters. To leading order in gauge and Yukawa couplings the CP violating phases, which are relevant at high and low energies, decouple. $B - L$ is broken at the unification scale.

Without an intermediate scale of symmetry breaking, the unification of gauge couplings appears to require low-energy supersymmetry. This provides further sources for generating a $B - L$ asymmetry [16], whose size depends on additional assumptions. In this case, especially constraints on the reheating temperature [1] and the possible role of preheating [17] require further studies.

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