Computing Global Functions in Asynchronous Distributed Systems with Perfect Failure Detectors

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Abstract—A Global Data is a vector with one entry per process. Each entry must be filled with an appropriate value provided by the corresponding process. Several distributed computing problems amount to compute a function on a global data. This paper proposes a protocol to solve such problems in the context of asynchronous distributed systems where processes may fail by crashing. The main problem that has to be solved lies in computing the global data and in providing each noncrashed process with a copy of it, despite the possible crash of some processes. To be consistent, the global data must contain, at least, all the values provided by the processes that do not crash. This defines the Global Data Computation (GDC) problem. To solve this problem, processes execute a sequence of asynchronous rounds during which they construct, in a decentralized way, the value of the global data and eventually each process gets a copy of it. To cope with process crashes, the protocol uses a perfect failure detector. The proposed protocol has been designed to be time efficient: it allows early decision. Let \( t \) be the maximum number of processes that may crash, \( t < n \) where \( n \) is the total number of processes, and \( f \) be the actual number of process crashes \( (f \leq t) \). In the worst case, the protocol terminates in \( \min(2f + 2, t + 1) \) rounds. Moreover, the protocol does not require processes to exchange information on their perception of crashes. The message size depends only on the size of the global data.

Index Terms—Asynchronous distributed computation, global data, global function computation, perfect failure detector, problem reduction, process crash.

1 INTRODUCTION

In a distributed computation, a Global Data is a vector with one entry per process, each entry being filled with an appropriate value provided by the corresponding process. Global Function computation [11], [15] constitutes a key component for solving many distributed computing problems. It consists of: 1) requiring processes to define a global data; 2) computing a deterministic function of this data; and 3) providing each process with the corresponding result [2], [12]. The Atomic Commitment problem [3] constitutes a relevant example of a global function computation. According to its local computation, each process votes YES or NO. The set of all votes constitutes the global data. The result of the function is COMMIT if all votes are YES, otherwise the result is ABORT. Finally, according to the result (same for all processes), each process commits or invalidates the local computation it has previously done. More generally, some problems require repeated computations of a global function [11]. The Distributed Termination Detection problem is an example of problem that can be solved by two successive global function computations [15], [20], [27].

Computing a global function is relatively easy in reliable asynchronous distributed systems. In this context, two main approaches have been investigated. The centralized approach, also named “asymmetric” approach, consists in the following message exchange pattern: First, a predetermined process \( p \) gathers all the local data and computes the appropriate function of this global data. Then, the process \( p \) disseminates the result to each process. Assuming there is a channel connecting each pair of processes, this approach basically requires the exchange of \( 2(n - 1) \) messages where \( n \) is the number of processes and costs two time units, assuming each message transfer costs one time unit and processing times are negligible. The second approach that has been studied is the distributed, or “symmetric,” approach. Each process sends its data to all processes and, consequently, each process can build a copy of the global data. Then, each process computes the same deterministic function on the same global data and, thus, obtains the same result. With the previous assumptions, this approach costs \( n(n - 1) \) messages and only one time unit.

In this paper, we are interested in investigating the decentralized approach to compute global functions in distributed systems where processes can crash. Let \( t \) be the maximum number of processes that may crash \( (t < n) \) and \( f \) be the number of actual crashes \( (f \leq t) \). When the distributed system is synchronous, it is relatively easy to compute a global function in a decentralized way by requiring processes to execute a sequence of rounds. In that context, it has been shown that \( \min(t + 1, f + 2) \) is a lowest bound on the number of rounds [8]. Asynchronous distributed systems are characterized by the fact that it is impossible to distinguish a very slow process from a crashed process. This makes the computation of a global
function more difficult to solve. One of the main problems created by process crashes concerns the fact that all noncrashed processes must get identical copies of the global data. To illustrate this issue let us consider the following scenario. It involves three processes $p_1, p_2,$ and $p_3$. Processes $p_1$ and $p_2$ do not crash and broadcast their initial data and process $p_3$ sends its data to $p_1$ and crashes before sending it to $p_2$. Eventually, process $p_2$ detects the crash of $p_3$ and considers a global data without an entry from $p_3$. On the other hand, process $p_1$ has received values from $p_2$ and $p_3$, and considers global data with an entry from $p_3$. Thus, $p_1$ and $p_2$ do not have the same view of the global data. So, a crucial issue is to ensure that processes eventually get the same value for each entry of the global data. Let $p$ be one of the processes. If $p$ does not crash, its entry of the global data must be its initial data. If $p$ has crashed before starting the protocol, its initial data is unavailable and, consequently, its entry will contain a default value denoted $\perp$. But if $p$ crashes during the execution of the protocol, as $p_3$ in the previous example, what will be the value of its entry in the global data, the initial value of $p$ or the default value?

To cope with process crashes, this paper follows the approach advocated by Chandra and Toueg [5]. Each process $p$ is equipped with a failure detector module. The module associated with $p$ can be seen as an oracle that provides $p$ with the list of processes it suspects to have crashed. Formally, a failure detector module is defined by two properties, a completeness property and an accuracy property. Of course, the implementation of a failure detector is based on the use of timers and time-out values, but those are implementation mechanisms that are not made visible outside the failure detector module. Hiding all time-dependent aspects in a black box (the failure detector), allows the design of time-free protocols, i.e., protocols in which no statement involves physical time. As a consequence, a protocol based on a failure detector can be used without modification in any system where the assumed failure detector can be implemented. This not only makes easier the portability of the protocol, but also makes its proof independent on any particular timing mechanism.

The problem of computing a global data and providing each noncrashed process with a copy of it, is identified as the Global Data Computation (GDC) problem. In asynchronous distributed systems subject to crash failures and equipped with perfect failure detectors, it is possible to solve the Global Data Computation Problem by executing $n$ parallel instances (one for each process) of the Terminating Reliable Broadcast problem [14]. For each instance of the Terminating Reliable Broadcast problem (an instance is defined with respect to a specific sender process), a value $v$ is delivered to each process. If $v = \perp$, then the sender has necessarily crashed. If $v \neq \perp$, then $v$ is the initial value of the sender process. Solving an instance of the Terminating Reliable Broadcast problem requires $(t + 1)$ rounds [14]. So, this approach for solving the Global Data Computation problem provides a solution requiring at least $(t + 1)$ rounds. In this paper, we are interested in the design of a protocol that allows early decision. This means that the resulting protocol must involve a number of rounds that depend on $f$ (the actual number of crashes), this number never being greater than $(t + 1)$. Based on perfect failure detectors, the proposed protocol achieves the following time performances. If $t = 0$, i.e., when the system is reliable, the protocol requires a single round. When $t > 0$, it terminates in two rounds, in the best case. In the worst case, it terminates in $\min(2f + 2, t + 1)$ rounds. Given that one round of a synchronous system can be simulated by two rounds of an asynchronous system (see the discussion in Section 2.3), this time performance is likely to be optimal. However, the proof of this optimality issue remains an open question. So, to our knowledge, this is the first early decision protocol solving the Global Data Computation problem in asynchronous distributed systems prone to process crash failures. Other interesting features of the protocol are the following:

1. It adopts a “best effort” strategy, doing its best to fill each entry of the global data with the initial value of the corresponding process. This allows for a global data containing as much meaningful values as possible.
2. It does not require processes to exchange information on their perception of crashes. In addition to its identity, a message has only to carry an estimate of the global data value.

The paper is composed of eight sections. Section 2 presents the distributed system model. Section 3 specifies the Global Data Computation problem. Section 4 presents the protocol that solves this problem: Sections 4.1 and 4.2 describe the protocol and Section 4.4 proves the protocol correctness and determines an upper bound for the number of rounds. Section 5 shows a simple, but interesting theoretical result, namely, the problem of building a perfect failure detector, problem $P$, and the GDC problem are “equivalent” in the sense that any solution to one of them can be transformed to solve the other [5], [10], [14]. Section 6 investigates a consensus-based approach to solve the GDC problem and compares it with the proposed approach. Then, Section 7 uses the protocol to solve two problems in asynchronous distributed systems with process crashes, namely, the Atomic Commit problem and the Distributed Termination Detection problem. Finally, Section 8 concludes the paper.

## 2 Distributed System Model

### 2.1 Asynchronous Distributed System

The system consists of a finite set of $n$ processes, $\Pi = \{p_1, \ldots, p_n\}$. Processes cooperate and synchronize by exchanging messages through a communication network, there is no shared memory. There is a communication channel connecting each pair of processes. Channels are reliable, i.e., no spurious messages, no loss, and no corruption. They are not required to be FIFO. Moreover, process speeds and communication delays are arbitrary. So, the system model perceived by the upper layer applications is the classical time-free asynchronous distributed system model.

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1. The GDC problem is actually similar to the Interactive Consistency problem that has been defined in the context of synchronous systems prone to Byzantine process failures [22]. Concerning this similarity, see also the open problem (related to optimality [8]) stated at the end of the conclusion.
The communication primitives will be denoted in the following way: When executed by \( p_i \), the primitive “send \( m \) to \( p_j \)” entails the sending of the message \( m \) by \( p_i \) to \( p_j \). When \( p_i \) executes “receive(\( m \))”, it is blocked until a message sent to it has arrived; then, the message is deposited in \( m \) and made available to \( p_i \) which continues its execution.

### 2.2 Process Failure Model and Failure Detection

#### Process Failure Model

A process may fail by crashing. It behaves correctly, i.e., it executes its program text, until it possibly crashes. When a process crashes, it definitively stops its activity. So, the failure model we consider is the Crash/no Recovery model. A process that does not crash is called a correct process. Otherwise, it is a faulty process. Recall that \( t < n \) denote the maximum number of processes that may crash and \( (t \leq t) \) denote the actual number of process crashes.

#### Process Crash Detection

Each process \( p_i \) is equipped with a failure detector module \( F_{D_i} \). This module provides \( p_i \) with a set variable \( (\text{suspected}_i) \) that contains the identities of the processes that \( F_{D_i} \) guesses to have crashed. The process \( p_i \) can only read this variable, which is continuously updated by \( F_{D_i} \). According to the quality of guesses made by failure detector modules, several classes of failure detectors can be defined [5]. Here we consider perfect failure detector modules: no guess is mistaken. More precisely, these failure detectors are defined by the following two properties (when \( p_j \in \text{suspected}_i \), we say that “\( p_i \) suspects \( p_j \)”) [5]:

- **Completeness**: Eventually, every process that crashes is suspected by every correct process.
- **Accuracy**: No process is suspected before it crashes.

An important issue is the implementation of such failure detectors modules. Some networks have a notion of privileged, high priority channels satisfying strong timing assumptions [4], [23], [28] (e.g., field buses such as FIP or CAN [25]). Such channels can be dedicated to the implementation of perfect failure detector modules. So, at the application level, the computation model we actually consider is the time-free asynchronous distributed system model augmented with perfect failure detectors [5], [18]. This means that privileged, high priority channels are hidden to system applications. These channels are visible and known only by the underlying layer which implements the failure detector modules. This layer uses “I am alive” messages sent on privileged channels and timeout values to detect process crashes.

#### 2.3 Asynchronous Systems with Perfect Failure Detectors vs. Synchronous Systems

Most agreement protocols that have been designed so far obey a regular communication pattern, based on the notion of round [1], [5], [8], [9], [15], [18], [27]. More precisely, each correct process \( p_i \) owns a variable \( r_i \), whose values are natural integers. As soon as \( r_i = r \), we say that process \( p_i \) reaches round \( r \). Then, until \( r_i = r + 1 \), process \( p_i \) is said to be in round \( r \) [7]. While in its round \( r \), each process executes sequentially the following steps: 1) It sends a round \( r \) message to the other processes, 2) it waits for a round \( r \) message from each process, and 3) it executes local computations.

It is important to remark that, when we consider round-based protocols, synchronous distributed systems are more constrained than asynchronous distributed systems equipped with perfect failure detectors. This is due to the following observation: Let us consider two processes, \( p_i \) and \( p_j \). Assume that, during round \( r \), \( p_i \) has crashed after sending its round \( r \) message, namely \( m \), to \( p_j \).

- In a synchronous distributed system, the synchrony assumption ensures that all messages are received in the round they have been sent. So, \( p_j \) receives \( m \) before ending its round \( r \). Consequently, \( p_j \) cannot detect the crash of \( p_i \) before executing in round \( r + 1 \).
- In an asynchronous distributed system, it is possible that \( p_i \) has started executing round \( r \) while \( p_j \) is executing round \( r - 1 \), \( r \), or \( r + 1 \). So, if \( p_i \) crashes during \( r \), \( p_j \) learns it, due to the underlying perfect failure detector:
  - while it is in round \( r + 1 \) (in that case the last message received by \( p_j \) from \( p_i \) is its round \( r \) message),
  - while it is in round \( r \) (in that case, the last message received by \( p_j \) from \( p_i \) is its round \( r - 1 \) message or its round \( r \) message), or
  - while it is executing round \( r - 1 \) (in that case, the last message received by \( p_j \) from \( p_i \) is its round \( r - 2 \) message or its round \( r - 1 \) message).

Consequently, in such a system, the crash of \( p_i \) during \( r \), is known by \( p_j \) while it is at round \( r - 1 \), \( r \), or \( r + 1 \) (first uncertainty). Moreover, the crash of \( p_i \) can be known by \( p_j \) after or before having received \( p_i \)’s round \( r \) message, or even before having received \( p_i \)’s round \( r - 1 \) message (second uncertainty).

The three situations that give rise to the uncertainty from \( p_j \)’s point of view, on the round number at which \( p_i \) crashed, are described in the Fig. 1 (\( F(i) \) denotes the notification of the failure of \( p_i \)). The situations due to the second uncertainty (has \( p_i \) sent a round \( r \) or \( r - 1 \) message?) are described in Fig. 2. So, for round-based protocols, an

2. Let us note that in other round-based computation models, a process waits for messages from only a majority of processes.
asynchronous distributed system, equipped with a perfect failure detector, is not equivalent to a synchronous distributed system.

It follows that round-based protocols are more difficult to design in asynchronous distributed systems equipped with a perfect failure detector than in synchronous systems.3

3 THE GDC PROBLEM: CONSTRUCTING LOCAL COPIES OF A GLOBAL DATA

3.1 Distributed Computation of a Global Function

Let GD[1..n] be a vector of data with one entry per process, the ith entry being associated with pi, and let F be a deterministic function of GD. Moreover, let vi denote the value provided by pi to fill its entry of the global data.

The computation of the global function is described in Fig. 3. The function Global_data returns the value of GD that is locally saved by pi in its local variable GDi. The local variable termi is a Boolean (initialized to false) that takes the value true when and only when, pi has got its copy of GD. Then, each process can locally compute the value of F(GD).

3.2 The Global Data Computation Problem

As indicated in the Introduction, the crucial issue consists in building GD and providing a copy of it to each process. As defined previously, GD is the local variable of pi intended to contain the local copy of GD. The problem of providing the same copy of GD to each process is formally specified by a set of four properties. These properties have to be satisfied by any protocol that claims to solve the problem. Let ⊥ be a default value that will be used instead of the value vj when the corresponding process pj crashes prematurely. These properties are:

- **Termination.** The Boolean flag termi of every correct process pi eventually becomes true: pi correct ⇒ termi.
- **Validity.** No spurious initial value: ∀i: (termi ⇒ (∀j: GD[j] ∈ {vj, ⊥})).
- **Agreement.** No two processes decide different Global Data: ∀i,j: ((termi ∧ termj) ⇒ (∀k: (GD[k] = GD[k]))).
- **Obligation.** If a process terminates, its initial value belongs to the Global Data: ∀i: (termi ⇒ (GD[i] = vi)).

The Termination property is a Nonblocking property (i.e., a Liveness property). It states that at least all correct processes must terminate despite the crash of other processes. The next three properties are Safety properties. The Validity property defines the value domain of each global data entry. The Agreement property indicates that all the processes that terminate get the same copy of the global data. Finally, the Obligation property states that if pi terminates, then its entry of the global data cannot be the default value.

It is important to note that the Global Data Computation problem is harder than the Consensus problem [5], [9]. In the Consensus problem, processes propose values, and all correct processes have to agree on one of the proposed values. Here, the values “proposed” by, at least, all correct processes have to be pieced together to define the global data: The value “proposed” by a correct process can not be “missed.” This informally explains why perfect failure detectors are needed: A correct process cannot be mistakenly suspected, as this could entail the absence of its value in the global data. Section 5 will provide a more formal treatment of this issue.

4 A PROTOCOL THAT BUILDS CONSISTENT COPIES OF A GLOBAL DATA

As indicated in the introduction, our aim is the design of a distributed round-based protocol solving the GDC problem, that allows early decision. Moreover, differently from other protocols based on perfect failure detectors [18], we are interested in a protocol that does not require processes to exchange lists of suspects. Messages are allowed to carry only a control data (the identity of the message) and an estimate of the value of the global data. Allowing messages to carry lists of suspects would give rise to more costly protocol.

4.1 The Protocol

The protocol that computes the global data and provides processes with the same copy of it is described in Fig. 4. Each process pi calls the function Global_data(vi) which returns a consistent copy of the global data. This function is made of two concurrent tasks T1 and T2.

Task T1. This is the main task: Its aim is to construct consistent local copies of the global data. To attain this goal, each process pi has a local variable gd, that contains its current estimate of the global data. Initially, gd contains

3. When there are no process crashes, it is well-known how to to simulate synchronous systems on top of asynchronous systems: synchronizers [1], [15] perform such simulations.

4. The identity of an estimate message is the pair defined by the identity of its sender plus its sequence number, namely, a round number.
only $v_i$, the value provided by $p_i$ to fill its entry of the global data (line 1). Processes execute a sequence of asynchronous rounds to construct the global data. During round $r$ they exchange their current estimates of the value of the global data, in order to improve their estimate value. The local variable $r_v$, initialized to zero, defines the current round number of $p_i$.

Let $\text{part}(i,j,r)$ denote the predicate: “during its execution of round $r$, $p_i$ has received and taken into account a round $r$ estimate from $p_j$. “ In other words, from $p_i$’s point of view, $p_j$ has participated in round $r$. By convention, $\forall r \leq 0, \forall i, \forall j, \text{part}(i,j,r) = \text{true}$. The local set variables $\text{prev}_\text{expected}_i, \text{cur}_\text{expected}_i$, and $\text{next}_\text{expected}_i$, are used by $p_i$ to keep track of round participants. They have the following meaning at the end of any round $r \geq 1$:

$\text{prev}_\text{expected}_i = \{j | \text{part}(i,j,r-2)\}$. This set contains the processes from which $p_i$ has received a round $r-2$ message during its execution of round $r-2$. Hence, it is the set of processes that were expected by $p_i$ to participate in round $r-1$.

$\text{cur}_\text{expected}_i = \{j | \text{part}(i,j,r-1)\}$. This set contains the processes from which $p_i$ has received a round $r-1$ message during its execution of round $r-1$. Hence, it is the set of processes that are expected by $p_i$ to participate in the current round $r$.

$\text{next}_\text{expected}_i = \{j | \text{part}(i,j,r)\}$. This set contains the processes from which $p_i$ has received a round $r$ message during its execution of round $r$. Hence, it is the set of processes that are expected by $p_i$ to participate in the next round $r+1$. Let $x_i^r$ be the value of the set variable $x_i$ at the end of round $r$. From these definitions, we have:

$\text{prev}_\text{expected}_i \supseteq \text{cur}_\text{expected}_i \supseteq \text{next}_\text{expected}_i$, and:

$(\text{prev}_\text{expected}_i^{+1} = \text{cur}_\text{expected}_i) \land (\text{cur}_\text{expected}_i^{+1} = \text{next}_\text{expected}_i)$.  

A round $r$ is made of two parts: a communication phase followed by a computation phase. More precisely, a process $p_i$ does the following actions:

- **Estimate exchange:**
  - First, $p_i$ starts its participation in the round by incrementing $r_v$, updating $\text{prev}_\text{expected}_i$, and $\text{cur}_\text{expected}_i$, (line 4). It also sends its current estimate of the global data ($gd_i$) to the processes of $\text{cur}_\text{expected}_i$, (line 5). (Note that $p_i$ always belongs to $\text{cur}_\text{expected}_i$.)
  - Then, $p_i$ waits until, for each $p_j \in \text{cur}_\text{expected}_i$, either it receives the round $r$ estimate of $p_j$ ($\text{rec}_\text{gd}_j$), or it suspects $p_j$ to have crashed (line 6).

- **Local computation:**
  - According to the set of estimates it has received, $p_i$ computes $\text{next}_\text{expected}_i$, (line 7) and updates its current estimate $gd_i$, (lines 8-12).
  - Finally, $p_i$ tests line 13, a Termination Condition (see next paragraph), to know whether it has got the final value of the global data. There are two cases. If the answer is negative, $p_i$ proceeds to the next round (line 3). If the answer is positive, then $p_i$ sends its current value of the global data, namely $gd_i$, to all the nonsuspected processes (message $\text{decide}(i,gd_i)$ sent at line 14) and returns $gd_i$, as the result of the call to $\text{Global_data}(i)$ (return statement, line 14).

**Task T2.** This task is associated with the processing of a $\text{decide}(k,\text{rec}_\text{gd}_k)$ message. If, during the execution of $\text{Global_data}(i)$, such a message is received, $p_i$ also forwards the final value of the global data, namely $\text{rec}_\text{gd}_k$, and locally returns it as the result of the call to $\text{Global_data}(i)$ (return statement, line 18). The $\text{decide}$ messages actually implement a Reliable Broadcast [14] that disseminates the termination.

### 4.2 The Termination Condition

The Termination Condition (line 13) constitutes the core of the protocol. During a round, no local variable is updated after line 12. So, as far as the values of variables are concerned, the end of a round occurs at line 13.

A process $p_i$ stops executing rounds when it knows that “its current estimate $gd_i$ contains no more and no less non-nil values than the other estimates.” When this occurs, the estimates of all noncrashed processes are equal and, consequently, $p_i$’s current estimate can no longer be improved; this value, then, defines the global data. Moreover, as the processes have then the same estimate value, if a process $p_i$ terminates during a subsequent round, it will necessarily return the same value of the global data.

The termination condition is the disjunction of two conditions, denoted $C_1$ and $C_2$. The first one, namely $C_1$, is on the number of rounds that have been executed: It actually defines an upper bound on the maximal number of rounds to be executed before reaching agreement. The second, namely $C_2$, allows early termination by detecting a “stable state” reached by the set of processes. This second condition is a conjunction of two predicates; $C_{21}$ and $C_{22}$.

5. Note that some nonsuspected processes can actually be crashed.
4.2.1 Condition $C_1$: $r = t + 1$

At the beginning of each round, every process broadcasts the initial values it has received during the previous rounds (line 5). This behavior is similar to the one of a reliable broadcast protocol [14]. Such a protocol ensures that after $t + 1$ rounds, every initial value is known either by every noncrashed process, or by none of them. This observation will be used to conclude that, after $t + 1$ rounds, the set of noncrashed processes agrees on the global data, and thus can terminate.

4.2.2 Condition $C_2$: $(\forall j \in next \_expected, \exists (gd_i = rec \_gd_j)) 
\land (\forall j \in next \_expected) \exists (gd_i = rec \_gd_j)$.

The aim of this condition is to allow early termination. When this conjunction of predicates, evaluated by a process $p_i$, holds at the end of a round $r$, it guarantees that all processes that have completed round $r - 1$ have the same estimate value. This condition is actually a predicate that detects “a stability property” on the last two rounds.

- **Subcondition $C_{21}$**:
  \[
  prev \_expected = next \_expected
  \]
  This condition can be restated as: $prev \_expected = cur \_expected = next \_expected$. When it is satisfied at the end of $r$, $p_i$ knows that all the processes assumed to participate in round $r - 1$, processes of $prev \_expected$, have actually participated in rounds $r - 1$ and $r$. In other words, $p_i$ knows that each process that started round $r - 1$ has completed this round. If the system was synchronous, we could conclude that all the processes that have terminated round $r - 1$ have the same global data estimate. But, due to the asynchrony in crash suspicion (see the discussion of Section 2.3), there can be some processes that crash while executing $r$ and that are suspected by other processes while those are executing $r - 1$. Because of these suspicions, $C_{21}$ alone cannot allow $p_i$ to safely terminate.

- **Subcondition $C_{22}$**: $\forall j \in next \_expected, \exists (gd_i = rec \_gd_j)$. When this condition is true, all the processes that, to $p_i$’s knowledge, have terminated $r - 1$ have the same estimate, which is equal to its current estimate, namely $gd_i$. But, as before, due to the asynchrony in crash suspicion, there may be processes that have completed round $r - 1$ and, having crashed while executing $r$, do not belong to $next \_expected$. Hence, this condition alone does not allow $p_i$ to conclude that all processes that have completed round $r - 1$ have the same estimate. While each subcondition is not sufficient to allow $p_i$ to safely terminate, it appears (as shown by the proof) that their conjunction, $C_{21} \land C_{22}$, allows $p_i$ to safely terminate: When they are satisfied at the end of $r$, all the estimates were equal at the end of $r - 1$ and, consequently, “$gd_i$ contains no more and no less non⊥ values than the other estimates.”

In the worst case, $C_2$ holds during the round $2f + 2$. Combined with $C_1$, it will allow to conclude that $\min(t + 1, 2f + 2)$ is an upper bound for the number of rounds.

**Remark.** The protocol presented in Fig. 4 has some similarities with the $S$-based consensus protocol introduced in [5] (let CT$_S$ be this protocol). $S$ is a class of failure detectors weaker than the class $P$ of perfect failure detectors. In both the proposed protocol and CT$_S$, each process computes an array; the protocols ensure that eventually these arrays are equal. The main difference between the proposed protocol and CT$_S$ lies in the introduction of the sets $prev \_expected$, $cur \_expected$, and $next \_expected$, and in the associated
early termination predicate. It is important to note that, even when executed with a perfect failure detector, CTs: 1) requires $t + 1$ rounds and 2) does not implement a best effort strategy as the proposed protocol does: When there is a conflict between $\perp$ and $v_x$ for determining the final value of $gd[k]$, the second phase of CTs always imposes the value $\perp$ (let us note that this second phase of CTs is necessary for the processes to have the same final array). Differently, the proposed protocol does its best to have as many entries of $gd$, different from $\perp$ as possible (see line 10). More details on relations between consensus and GDC problems are given in Section 6.

4.3 Improvements

The test $(gd[k] = \perp) \land (rec.gd)[k] \neq \perp$ (line 10) can be simplified into $(rec.gd)[k] \neq \perp$. We have chosen to consider the extended test to clearly show the situation where $p_i$ “learns” new data.

Moreover, the sending of decide messages at line 14 is not necessary in the particular case where $r_1 = t + 1$. In the general case ($r_1 < t + 1$), this statement is required to get the termination property. At the end of a round, the $C_2$ condition, which allows early termination, can be true for a correct process $p_j$, while it is false for another correct process $p_i$. If this occurs, $p_j$ stops executing the protocol, while $p_i$ proceeds to $r + 1$. As $p_j$ does not suspect $p_i$ and $p_j$ does not participate in $r + 1$, it follows that $p_i$ will remain blocked forever at line 6. The decide messages prevent such a blocking.

The above two improvements are not considered in the proof that follows.

4.4 Correctness Proof

Recall that a round is an execution of the loop (lines 3-16). The first two lines of the protocol correspond to a fictitious round $r = 0$. In the rest of the proof, we denote, by $x_i$, the value of a variable $x$ at the end of round $r$ for the process $p_i$.

4.4.1 Synchronization

Lemma 1. When a process completes the execution of the wait statement of a round $r$, any noncrashed process has already completed the execution of the round $r - 1$.

Proof. Let us consider a process $p_i$ which completes the execution of the wait statement of a round $r$. Due to the Accuracy property of the failure detector, $p_i$ cannot suspect a process that is not crashed. Thus, $p_i$ has necessarily received an estimate message timestamped with the round number $r$ from all processes that have not crashed. Those messages have been sent at the beginning of the round $r$. Consequently, any noncrashed process previously has completed the execution of the round $r - 1$.

Corollary 1. At any time, any two noncrashed processes execute either the same round or consecutive rounds.

4.4.2 Agreement Property

Lemma 2. If all the noncrashed processes complete a round $r$ with the same global data value, then any process $p_i$ that completes a round $\geq r$ also has the same global data value.

Proof. Obviously, Lemma 2 is satisfied when $r = r'$. Let us assume that Lemma 2 is satisfied for a round $r' \geq r$. In other words, all the processes which complete the round $r'$ have a global data equal to $gd$ at the end of $r'$. We now demonstrate, by contradiction, that a process $p_i$, which ends the round $r'$, also has a global data equal to $gd$ at the end of this round. At the beginning of round $r'$, the value of $gd$ is equal to $gd$ by assumption. Thus, if this is not the case at the end of the round, it means that $p_i$ has received a message from a process $p_j$ which contained a global data $rec.gd$, not equal to $gd$ (the test of line 10 has hold at least once.). Yet, as the value $rec.gd$ has also been computed at the end of the round $r'$, this value cannot be different from $gd$.

Lemma 3. Let $r$ be a round such that some processes learn new initial values during this round. In particular, let $p_i$ be such a process and $v_x$, the initial value of process $p_i$, be one of the values learned by $p_i$. Then, there exists a sequence of $r + 1$ processes $(p_{0}(0) = p_k, \ldots, p_{r}(r) = p_s)$, such that:

1. $\forall l \geq 1 \ (gd_{p_i(l)}^l = v_x) \land (gd_{p_i(l-1)}^{l-1} = \perp)$,
2. The first $r - 1$ processes are faulty and crash before round $r + 1$.

Proof. The proof is by induction on the number of rounds.

1. Base case $r = 1$. The first part of the lemma holds with $p_{0}(0) = p_k$ and $p_{1}(1) = p_l$. The second part of the lemma trivially holds.
2. Induction. Suppose that the lemma holds for $r - 1, (r \geq 2)$.

   a. First part. Let $p_i = p_{r-1}(r-1)$ be a process learning $v_x$ during round $r$, i.e., $gd_{p_i(r)}^r = v_x$ and $gd_{p_i(r-1)}^{r-1} = \perp$. From line 10, there exists at least one process, let us denote it $p_{r-1}(r-1)$, such that: $part(\sigma(r), \sigma(r-1), r) \land (gd_{p_{r-1}(r-1)}^r = v_x)$. We now show (by contradiction) that $p_{r-1}(r-1)$ has learned the value $v_x$ during round $r - 1$. If it was not case, one would have: 1) $gd_{p_{r-1}(r-1)}^{r-1} = v_x$. Since $part(\sigma(r), \sigma(r - 1), r)$, then $p_{r-1}(r-1)$ was not crashed at the beginning of round $r - 1$. Thus, 2) $part(\sigma(r), \sigma(r - 1), r - 1)$, 1) and 2) implies that: $gd_{p_{r-1}(r-1)}^{r-1} = v_x$, a contradiction. Thus, $p_{r-1}(r-1)$ satisfies the assumptions of the lemma for $r - 1$, from which we conclude the first part of the result for round $r$.

   b. Second part. Let us consider three consecutive processes in this sequence, namely $p_{r-2}(r-2), p_{r-1}(r-1)$ and $p_{r}(0)$. From $gd_{p_{r-2}(r-2)}^r = v_k$ and $gd_{p_{r-1}(r-1)}^{r-2} = \perp$, one concludes $-part(\sigma(l), \sigma(l - 2), l - 1)$ and, thus, $p_{r}(0)$ has suspected $p_{r-2}(r-2)$ before round $l$. Thus, by Lemma 1, $p_{r-2}(r-2)$ has crashed during round $l - 1$, or round $l$. Consequently, the first $r - 1$ processes of the
Lemma 4. The processes that terminate the round \( t + 1 \) share the same global data.

Proof. Let \( r \) be a round during which a process learns a new initial value \( v_t \). We show that \( r \leq t + 1 \). From Lemma 3, there exists a sequence of \( r + 1 \) processes \( \{p_{r}(0), \ldots, p_{r}(r)\} \), such that the first \( r - 1 \) processes are faulty and crash before round \( r + 1 \). Two cases have to be considered.

1. \( t = n - 1 \). In the worst case, all the processes of the system are included in the sequence. Hence, \( r + 1 = n \). This means that \( r + 1 = n = t + 1 \), i.e., \( r = t \). The sequence becomes \( \{p_{r}(0), \ldots, p_{r}(r)\} = \{p_{r}(0), \ldots, p_{r}(n-2), p_{r}(n-1)\} \). From Lemma 3, only \( p_{r}(n-2) \) and \( p_{r}(n-1) \) are noncrashed during round \( n = r + 1 \) and both have learned \( v_t \) by the end of round \( n - 1 = t \). It follows that every noncrashed process has learned \( v_t \) by the end of round \( t \).

2. \( t < n - 1 \). Since there are at most \( t \) crashed, we have \( r - 1 \leq t \), i.e., \( r \leq t + 1 \). In the worst case, \( r = t + 1 \) and the first \( t \) processes are faulty. Thus, \( p_{r}(t) \), which is necessarily a correct process, has learned \( v_t \) during round \( t \) and has broadcasted it during round \( t + 1 \). It follows that every correct processes has the same global data at the end of round \( t + 1 \). \( \square \)

Lemma 5. Let \( r \) be the first round, during which at least one process \( \{p_j\} \) decides at line 14. Then, all the processes that have completed the round \( r - 1 \) belong to the set \( \text{prev\_expected}_r \).

Proof. The proof is a case analysis.

- \( r = 1 \). \( \text{prev\_expected}_1 = \text{cur\_expected}_1 = \text{II} \) (line 2)
- \( r \geq 2 \). Any process \( p_k \) that terminates the round \( r - 1 \) has obviously completed the \text{wait} statement of round \( r - 1 \). At this time denoted \( t \), due to Lemma 1, any noncrashed process, in particular \( p_k \), has also completed the round \( r - 2 \). Moreover, at time \( t \), \( p_k \) is not crashed and, thus, no process can have suspected \( p_k \) before \( t \). Consequently, \( p_k \) has not suspected \( p_k \) during round \( \leq r - 2 \).

1. \( r = 2 \). One has \( \text{prev\_expected}_2 = \text{cur\_expected}_2 = \text{next\_expected}_2 = \text{II} - \text{suspected}_2 \). As \( p_i \) has not suspected \( p_k \) during round \( 0 \), \( p_k \not\in \text{prev\_expected}_2 \).
2. \( r > 2 \). As process \( p_i \) has reached round \( r \), it has obviously received a message send by \( p_k \) during round \( r - 2 \). In other words \( \text{part}([i, k, r - 2]) \) and, thus,

\[
p_k \in \text{next\_expected}_{r-2} = \text{cur\_expected}_{r-2} = \text{prev\_expected}_r \text{ (line 4)}.
\]

Lemma 6. Let \( p_i \) and \( p_j \) be any two processes that have terminated the execution of \text{Global\_data}. We have

\[
GD_i = GD_j \quad \text{(where } GD_i\text{-resp. } GD_j\text{- is the last value of } gd_i\text{-resp. } gd_j)\text{)}.
\]

Proof. If a process \( p_i \) has returned from the execution of \text{Global\_data}, it has either executed line 14 or line 18. In the first case, the decision value is the value of the local global data \( gd_i \). In the last case, it decides on the value carried by a \text{decide} message. This message has been sent by a process, either at line 14 or at line 18, to forward a decision value (this value, launched by a process deciding at line 14, has been possibly relayed by several processes before arriving at \( p_i \)). Thus, if processes decide on different values for the global data, it means that at least two processes have decided different values at line 14.\(^8\) We prove that this scenario is impossible. Let \( r \) be the first round during which a process decides at line 14 and let \( p_i \) be a process that decides during this round. At the end of \( r \), the termination condition is satisfied for \( p_i \). Two cases have to be considered:

- \( C_1(r = t + 1) \) is satisfied. Due to Lemma 4, all processes share the same global data at the end of round \( r \).
- \( C_2 \) is satisfied. In that case, let \( X = \text{prev\_expected}_r = \text{next\_expected}_r \). During \( r \), \( p_i \) has received a message from each process \( p_j \in X \). From Lemma 1 and Lemma 5, we conclude that \( X \) contains all the processes that were not crashed at the end of \( r - 1 \). The second part of the test guarantees that all these processes have the same global data value at the end of \( r - 1 \). Due to Lemma 2, any process that completes a round \( \geq r \) has also the same global data value. In particular, this global data value is decided by any process which executes line 14 during any round \( \geq r \). This ensures there is a single decided global data value. \( \square \)

4.4.3 Obligation Property

Theorem 2. If a process \( p_i \) completes the execution of \text{Global\_data}, then the value of the returned global data contains its own initial value (i.e., \( gd_i[i] = v_i \)).

Proof. Due to the initialization (line 1), \( gd_i[i] \) is equal to \( v_i \) at the beginning of round one. Moreover, \( gd_i[i] \) is never subsequently updated, because the test of line 10 never holds \( (gd_i[i] \neq \perp) \). Let us consider the two following cases:

- \( p_i \) decides at line 14. Its global data necessarily contains its own value.
- \( p_i \) decides at line 18. As already indicated, in the proof of Theorem 1, the global data decided by \( p_i \) has been previously decided by a process \( p_j \) at line 14. Let us assume that \( p_i \) started the execution of line 14 at time \( t \). Obviously, \( p_i \) was not crashed at that time, because \( p_i \) has later received the global data value forwarded by \( p_j \). Moreover, \( p_i \) has executed at least one round; so, it has completed, at some time \( t' \leq t \), the wait statement.

\(^8\) These processes possibly decide at different rounds.
of its first round. Hence, it is impossible for \( p_i \) to have suspected \( p_i \) before \( t \). Consequently, \( p_j \) has received the value \( \perp, \ldots, v_i, \ldots, \perp \) sent by \( p_i \) during the first round and it has updated \( gd_i[j] \) accordingly during its first round (line 10). This update has occurred before \( t \). As the value of \( gd_i[j] \) remains equal to \( v_i \) (later, the test concerning \( gd_i[j] \) at line 10 never holds), it follows that the global data value decided by \( p_j \) and later by \( p_i \), contains the value \( v_i \).

\[ \square \]

### 4.4.4 Validity Property

**Theorem 3.** If a process \( p_i \) completes the execution of the \text{Global-data} function, then \( \forall j, gd[j] \) contains \( v_j \) or \( \perp \).

**Proof.** The proof follows directly from the initialization (line 1), the update of the \( gd[j] \) (line 10), and the channel reliability (no message alteration, no spurious message).

\[ \square \]

### 4.4.5 Termination Property

**Theorem 4.** All correct processes decide.

**Proof.** We first show that at least one correct process decides. Indeed, let us assume that no correct process decides. Due to the termination condition, this occurs if no correct process ever reaches the end of round \( t + 1 \). The proof is by contradiction. Let \( r < t + 1 \) be the smallest round in which some correct process \( p_i \) remains blocked forever at the \text{wait} statement. As far as correct processes are concerned, by assumption, none of them can remain blocked in a previous round, thus, each of them will broadcast a message to \( p_i \) at the beginning of round \( r \). As channels are reliable, \( p_i \) will receive all these messages. As far as faulty processes are concerned, there exists a time \( t \) after which every faulty process has actually crashed and due to the completeness property of the underlying failure detector, they will be suspected by \( p_i \). This shows that \( p_i \) eventually completes round \( r \), a contradiction. Moreover, if at least one process decides (at line 14 or line 18), it must have sent a \text{decide} message to all processes. Due to the channel reliability assumption and to task T2, the theorem follows.

\[ \square \]

### 4.4.6 Upper Bound on the Number of Rounds

**Theorem 5.** In the worst case, the protocol converges in \( \min(2f + 2, t + 1) \) rounds.

**Proof.** Let \( m_1, m_2, \) and \( m_2 \) be the maximal number of rounds for conditions \( C_1, C_2, \) and \( C_3 \) to be satisfied, respectively. Since the termination condition is \( C_1 \lor (C_2 \land C_3) \), one knows that the protocol converges in \( \min(m_1, \max(m_2, m_2)) \). Trivially, \( m_1 = t + 1 \).

Let us now consider \( C_2 \). If at least one process learns a new initial value during round \( r \), Lemma 3 tells us that at least \( r - 1 \) crashes actually occur before \( r + 1 \). Since there are exactly \( f \) crashes, one knows that, in the worst case, the round \( f + 1 \) is the last round during which a process can learn a new initial value. It follows that all the processes will have the same global data value at the round of \( f + 1 \), even if they are not aware of this fact.

Due to Lemma 2, all values exchanged in rounds \( > r \) are equal. Consequently, the condition \( C_2 \) holds at the end of any round \( r \geq f + 2 \). Hence, \( m_2 = f + 2 \).

The condition \( C_3 \) is a local test, for each process \( p_i \), about the absence of new suspicions over two consecutive rounds. In the worst case, this property can not be satisfied as long as exactly one new suspicion is learned by \( p_i \) during every pair of consecutive rounds. Since there are at most \( f \) crashes, this situation cannot last more than \( 2f \) rounds. Thus, the condition \( C_3 \) holds after at most \( 2f + 2 \) rounds.

It follows that the protocol terminates after, at most, \( \min(t + 1, \max(2f + 2, f + 2)) \) rounds.

\[ \square \]

## 5 Hardness of the GDC Problem

### 5.1 Problem Reduction

This section considers the hardness of a problem with respect to the difficulty of solving it in the presence of process crashes: A problem is easy if it can be solved despite many arbitrary process crashes, it is hard if it requires additional assumptions to be solved in the presence of process crashes [10], [14]. To address this issue, Chandra, Hadzilacos, and Toueg [5], [14] have extended the notion of problem reduction to asynchronous distributed systems with process crashes.

A problem \( P_2 \) reduces to a problem \( P_1 \) (denoted \( P_2 \geq P_1 \)) if there exists a protocol \( P_2 \) that transforms any protocol solving \( P_1 \) into a protocol solving \( P_2 \). \( P_1 \geq P_2 \) means that \( P_1 \) is at least as hard to solve as \( P_2 \): All the assumptions, and maybe more, required to solve \( P_2 \) are necessary to solve \( P_1 \); the situation can even be worse—it is possible that \( P_2 \) can be solved while \( P_1 \) cannot. If \( P_1 \geq P_2 \) and \( P_2 \geq P_1 \), then \( P_1 \) and \( P_2 \) are equivalent. This is denoted \( P_1 \simeq P_2 \).

### 5.2 A New Reduction

\( P \geq \text{GDC} \). Let \( P \) be the problem of constructing a perfect failure detector (i.e., a failure detector satisfying the Completeness and Accuracy properties defined in Section 2.2).

Assuming a solution to \( P \), the protocol designed in Section 4.1 solves \( \text{GCD} \). Consequently, this protocol is a reduction transforming any protocol solving \( P \) into a protocol solving \( \text{GCD} \). Hence, we have \( P \geq \text{GDC} \).

\( \text{GDC} \geq P \). We give here a simple protocol that, given a solution to the \( \text{GDC} \) problem, solves \( P \). This protocol is described in Fig. 5. For each process \( p_i \), the protocol is made of two tasks.

---

**Fig. 5.** GDC \( \geq P \).
The task T2 realizes the interface with the upper layer. The variable suspected, contains the set of processes currently suspected by p; So, when the upper layer calls QUERY_P, the task T2 returns the current value of this set.

The task T1 manages the variable suspected. It is made of an infinite loop. Each iteration is a call to the function Global_data, where the value provided by p; is distinct from ⊥. Then, when T1 has got the result of the current call to Global_data, it computes the new value of suspected. This value includes all processes p; that were crashed before this global data computation (note that suspected, can also include processes that have crashed during this computation).

It is relatively easy to show that this protocol constructs a perfect failure detector. Here we sketch such a proof. First of all, a correct process p; participates in all instances of the GDC problem (successive calls to Global_data) and never provides v; = ⊥. Due to the properties of the GDC problem, it follows that, ∀j and for all instances of the GDC problem, p; always gets GD,j[i] = ⊥. So, if p; is correct, it is never suspected. More generally, if p; correct or not, has not crashed when the result of the kth instance of the GDC problem is delivered to p; then GD,j[i] = v; ≠ ⊥ and p; is not added to suspected, during the kth instance of the GDC problem. If p; has crashed before the kth instance of the GDC problem, then it cannot provide a value v; and, consequently, p; will get GD,j[i] = ⊥ and will add p; to suspected,. So, crashed processes are eventually suspected (Completeness) and no process is suspected before it crashes (Accuracy property).

### 5.3 GDC is a Hard Problem

Let us consider the following two distributed computing problems: NBAC, the Nonblocking Atomic Commitment problem [3] and, TRB, the Terminating Reliable Broadcast problem [14]. Combined with previous results on the classification of problems in asynchronous systems prone to process crashes [5], [10], [13], [14], we have the following problem equivalence:

\[ P \simeq NBAC \simeq TRB \simeq GDC. \]

So, according to the problem classes defined in [10], GDC belongs to the class, called NFC, which contains the hardest problems to solve in presence of process crashes.

### 6 A Consensus-Based Approach

This section first presents a consensus-based approach to solve the GDC problem. Then, it compares this approach with the approach developed in Section 4.

#### 6.1 A Consensus-Based GDC Protocol

The consensus problem can be defined informally in the following way. Each noncrashed process proposes a value and all correct processes have to decide a value (termination property) such that: 1) no two correct processes decide different value (agreement property) and 2) the decided value is a proposed value (validity property). (See [5], [9], [10] for a formal introduction to the consensus problem).

The protocol described in Fig. 6 provides a consensus-based solution to the GDC problem. It assumes a perfect failure detector and works as follows: First, each process sends its initial data to the other processes (line 1). Then, each process constructs a local view of the global data (lines 2-3). Finally, the processes use the consensus routine to impose one of these local views as the decided global data (line 4). Due to the consensus termination property, every correct process decides. Due to the consensus agreement, all correct processes decide the same array and due to the validity property, they decide a global data proposed by a process. Moreover, due to the use of a perfect failure detector at line 2, the view built by a process at line 3 includes the initial value of all the correct processes.

#### 6.2 Discussion

The previous protocol uses a perfect failure detector (line 2) in order that the local view of a process does not miss the initial values of correct processes. Its cost is one communication round, to construct the local views, plus the cost of the underlying consensus routine.

Any consensus protocol that uses a failure detector belonging to a failure detector class defined in [5] works when the underlying failure detector is perfect, but the consensus problem does not require a perfect failure detector. It can be solved with failure detectors of weaker classes [5], [6]. When equipped with a perfect failure detector, the most efficient in terms of communication steps, failure detector-based consensus protocols currently known [5], [16], [21], [26] require at least 2(f + 1) communication steps. Hence, the consensus-based protocol described in Fig. 6 is less efficient than the protocol described in Section 4.

This is not counterintuitive. It has been shown in Section 5 that \( P \simeq NBAC \simeq TRB \simeq GDC \). Let CONS denote the consensus problem. It has been shown [14] that TRB > CONS in asynchronous distributed systems. Hence, we have GDC > CONS. This means that, although the consensus problem can be solved with a perfect failure detector, the consensus alone is not sufficient to solve the GDC problem. Intuitively, this means that the lines 1-2 of Fig. 6 cannot be suppressed: Whatever the underlying consensus protocol, a perfect failure detector is required to construct consistent local views of the global data, thereby
ensuring that the decided view satisfies the Obligation property defined in Section 3.2.

7 Examples of Global Function Computation

The solution to several distributed computing problems amounts to compute a global function [12], [15]. This section shows how the previous framework allows solving those problems in the context of asynchronous distributed systems with process crashes. To this end, two problems are briefly examined.

7.1 Nonblocking Atomic Commitment

This well-known problem is mainly encountered in data management systems [3]. It has been sketched in the introduction.

The local variable \( v_i \) contains the YES or NO vote of \( p_i \). After its computation, the global data contains the votes of, at least, all correct processes. Some of its entries can contain the \( \perp \) value: if \( GD_1[j] = \perp \), then \( p_j \) has crashed before the end of the global data computation, but an entry associated with a process that has crashed can contain \( \perp \), or its vote, depending on the crash pattern and on the estimate exchange pattern. Finally, the function \( F \) delivers the result COMMIT if and only if all the entries of the global data contain a YES vote, this is the all-or-nothing Atomic Commitment problem. We can see that a \( \perp \) value in the global data is implicitly interpreted as a NO vote.

It is important to note that the assumption used by the protocol is consistent with Guerraoui’s theoretical result, namely, “The Atomic Commitment problem can be solved in asynchronous distributed systems prone to process crashes, only if the underlying failure detector modules are perfect” [13]. It is also important to note that the NBAC protocols described in [13], [17], [24] solve a weak version of the NBAC problem. In this weaker version, the “process crash” notion is replaced by a weaker notion, namely the “process crash suspicion” notion. See [13], [24] for more details.

7.2 Distributed Termination Detection

This problem is a well-known paradigm of reliable distributed computing. We assume the reader is familiar with it. An application is terminated when it has entered a state from which its processes will remain passive forever (this requires there is no message in transit between processes). A lot of protocols have been proposed to detect distributed termination [15], [20], [27].

In our context a distributed computation is terminated if it has entered a state from which each of its noncrashed processes remains passive forever or crashes. A way to cope with arbitrary transfer delays of messages sent by crashed processes, they sent them before crashing, consists in allowing a process \( p_i \) to stop receiving messages from processes it perceives crashed. This can be done by providing processes with a variable \( NC \) containing the set of processes perceived as noncrashed. Its value is obtained as the result of a global function computation (i.e., all the processes have the same value for \( NC \)). Initially, \( NC = \perp \).

Here we extend a variant of a protocol, denoted \( M \), proposed by Mattern [20] for crash-free systems. \( M \) requires each process \( p_i \) to maintain the following arrays of control variables: \( sent_i[k] \) which counts the number of messages that \( p_i \) has sent to \( p_k \) and \( rec_i[k] \) which counts the number of messages that \( p_i \) has received from \( p_k \). As in \( M \), the proposed extension requires a process be passive in order to participate in a global data computation; so, if it is active, it waits until it becomes passive. It also requires two consecutive global data computations, called \( GD_1 \) and \( GD_2 \), respectively. They are built from the following initial values:

- For \( GD_1 \): \( v_i = (rec_i, suspected_i) \).
- For \( GD_2 \): \( v_i = (sent_i, suspected_i) \).

So, when a global data \( GD_1 \) (resp. \( GD_2 \)) has been computed, some of its entries contain a counter array plus a set of processes, while the others contain the default value \( \perp \). Note, that if a process has crashed before the computation of \( GD_1 \) (resp. \( GD_2 \)), then its corresponding entry in \( GD_1 \) (resp. \( GD_2 \)) is equal to \( \perp \). After having obtained the values of \( GD_1 \) (resp. \( GD_2 \)), a process \( p_i \) computes \( NC_1 = \perp - crashed_1 \) (resp. \( NC_2 = \perp - crashed_2 \)), where \( crashed_1 \) (resp. \( crashed_2 \)) is the union of the sets \( suspected_i \) that are in \( GD_1 \) (resp. \( GD_2 \)). Moreover, \( NC \) is updated to \( NC_1 \) (resp. \( NC_2 \)). Then, \( p_i \) compares \( NC_1 \) and \( NC_2 \) (note that \( NC_1 \supseteq NC_2 \)).

- If \( NC_1 \neq NC_2 \), \( p_i \) cannot claim termination. It starts new successive global data computations to get two new global data \( GD_1 \) and \( GD_2 \).
- If \( NC_1 = NC_2 \), then process \( p_i \) computes:

\[
R_1 = \sum_{j \in \perp \cap NC \cap NC_2} rec_{GD_1}[k].
\]

This value represents the number of messages received by processes in \( NC_2 \) from processes in \( NC \), as perceived by the first computed global data \( GD_1 \).

\[
S_2 = \sum_{j \in \perp \cap NC_2} sent_{GD_2}[k].
\]

This value represents the number of messages sent by processes in \( NC_2 \) to processes in \( NC_2 \), as perceived by the second computed global data \( GD_2 \).

If \( R_1 = S_2 \), then, with respect to the set of processes perceived as noncrashed, the number of messages counted as received by \( GD_1 \) is equal to the number of messages counted as sent by \( GD_2 \). If this is true, \( p_i \) claims termination. Otherwise it starts a new computation to get two new global data \( GD_1 \) and \( GD_2 \).

This extended protocol allows process crashes to be cope with. It is possible to show that if the application terminates, then the protocol will eventually claim it (liveness of the detection); if the protocol claims termination, then the application has actually terminated (safety of the detection). Informally, from \( NC_1 = NC_2 \) we can conclude the following two points: 1) Any process \( p_i \) that is perceived crashed by \( GD_1 \) cannot belong to \( NC_2 \). Consequently, messages sent or received by this process \( p_i \) are counted neither in \( R_1 \) nor in \( S_2 \), and 2) If a process \( p_i \) crashes during \( GD_2 \), it does belong to \( NC_1 \) and, hence, to
NCz. So, messages sent or received by such a process \( p_i \) appear in R1 and S2.

8 Conclusion

This paper has addressed the computation of a global data function in asynchronous distributed systems where processes may fail by crashing. A Global Data is an array of values with one entry per process. A particular global function is defined by the type of the value each process has to provide and by the function that has to be computed. This paper has proposed a protocol to solve such problems in the context of asynchronous distributed systems where processes may fail by crashing.

The main problem that has to be solved lies in computing the global data and providing each noncrashed process with a copy of it, despite the possible crash of some processes. To be consistent, the global data must contain, at least, all the values provided by the processes that do not crash: This is the Global Data Computation (GDC) problem. It has been shown that GDC belongs to the class of problems that are the hardest to solve, with respect to the assumptions they require, in the presence of process crashes (GDC is actually equivalent to the problem of building a perfect failure detector). To solve the GDC problem, the proposed protocol requires processes to execute a sequence of asynchronous rounds during which they construct, in a decentralized way, the value of the global data, in such a way that eventually each process gets a copy of it.

The proposed protocol was designed to allow early decision. In the best case, it terminates in two rounds, when \( t > 1 \). Moreover, processes never exchange information on crashes. The message size depends only on the round number and on the size of the global data: In addition to its identity, a message carries only an estimate of the global data value. Let us also note that, when communication channels are FIFO, round numbers can be implemented \( \text{mod} 2 \). In that case, messages carry only bounded values.

In the worst case, the proposed protocol requires \( \min(2f + 2, t + 1) \) rounds. There is a problem \( (8), \) Section 5, p. 740, similar to the Global Data Computation problem, that can be solved in a synchronous distributed system in \( \min(2f + 2, t + 1) \), which has been shown to be lower bound \( 8 \) in such systems. So, an interesting open question is the following: “In asynchronous distributed systems equipped with perfect failure detectors, is \( \min(2f + 2t + 1) \) a lower bound for the maximal number of rounds of any protocol, in which processes do not exchange lists of suspects, solving the Global Data Computation problem?”

Another interesting open problem is to design a reliable synchronizer able to interpret synchronous protocols on top of an asynchronous system, where processes can crash, equipped with a perfect failure detector.

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References


[11] V.K. Garg and J. Ghosh, “Repeated Computation of Global Data Value. Let us also note that, when communication identity, a message carries only an estimate of the global number and on the size of the global data: In addition to its identity, a message carries only an estimate of the global data value. Let us also note that, when communication channels are FIFO, round numbers can be implemented \( \text{mod} 2 \). In that case, messages carry only bounded values.

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References


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