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Ergonomic optimization of a spring-loaded bicycle crank

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1. Abstract
This paper describes the optimization of a bicycle crank mechanism equipped with springs. The purpose of the springs is to cause an even torque development over the crank cycle by elimination of the so-called dead centers of the cycle. The technology behind the optimization, musculoskeletal modeling, is described in some detail before the actual case is introduced and subjected to design optimization. It is concluded that this technology holds a large potential as analysis tool in design optimization and opens an entirely new field of practical applications.

2. Keywords: Ergonomic optimization, biomechanics, pedaling, muscles, multibody dynamics.

3. Introduction
The field of virtual prototyping as manifested by technologies such as Computer-Aided Design, Finite Element Methods, and Computational Fluid Dynamics has had a very significant impact on modern product design. Hardly any advanced product today is designed without the use of some sort of computer simulation, and virtually any technical property of products can be analyzed, including strength, vibration, heat conduction, magnetism, flow, acoustics, and light reflection just to mention a few.

However, one prominent property of products has been missing from the range of analysis facilities until recently: The mechanical influence of the product on the human body has not been in the range of analysis. This property – also often called ergonomics – does not seem like a very important addition at first glance. This is because the tradition in many fields of industry has been to regard ergonomics as something that is handled outside the technical realm by specially trained consultants. In other words, the human body is often not regarded as belonging to the technical product design field.

On the other hand, most of us are completely encapsulated in man-made environments for the better part of our lives, and most of the products in these environments have some sort of interface with the human body: the chairs we sit in, the computer keyboards we type at, the coffee cups we drink from, the handles on the doors, the telephone, the stapler, the seat, the steering wheel and gear shift of our cars; the list goes on indefinitely. From this point-of-view it would appear that ergonomics indeed is a very important product property, and that the ability to simulate it holds a very large potential. This paper describes efforts to simulate and subsequently optimize the ergonomic design of products based on a computer model of the human musculoskeletal system exemplified by the optimization of a spring-loaded bicycle crank.

We shall initially introduce the human musculoskeletal system and the mechanical and mathematical challenges it presents. Subsequently we shall introduce the pedaling problem and describe its solution by means of design optimization

Figure 1. A complex musculoskeletal model comprising more than 500 muscles.

4. A musculoskeletal primer
Ergonomic simulation deals with the mechanics of the human body, and as mechanical systems go, this is very complicated indeed. There are more than 200 bones in the human body connected by different and in some cases very complex joints. The bones are

articulated by several hundred muscles. An account of the precise number of muscles in the human body depends on the point-of-view. The traditional anatomical classification of muscles is not adequate mechanically because many of the anatomical muscles span large areas, have fibers going in different directions, and can be activated separately in individual motor units. An estimation of the necessary number of muscle units for a reasonable mechanical modeling of the human body is around 1000, and the most comprehensive models today number roughly half of that (**Error! Reference source not found.**). A muscle's effect depends among other things on its moment arm about the joint(s) it actuates, and correct modeling requires correct moment arms. Many of the body's muscles and in fact most of the shoulder muscles wrap around bones and other tissues on their way from origin to insertion, and this has a profound and complex influence of the moment arms because the contact between muscle and bone changes when the body moves. Muscles can come into contact with bones or release existing contacts, and they may slide along bones as the body moves. This introduces a contact problem into the mechanical modeling, and it causes a significant complication of the mathematics and the numerical procedures. The contact problem is essentially a minimization of the distance from one point to another with a constraint of no penetration of the wrapping surface, and it can be formulated and solved numerically as an optimization problem. Here, sensitivity analysis plays an important role because the derivative of the muscle position is required to form the equilibrium equations of the system via the principle of virtual work.

Another major complication of musculoskeletal analysis is that the system is inherently statically indeterminate because there are usually many more muscles in the system than degrees of freedom. This means that each degree of freedom is carried by several muscles, and there are not enough equilibrium equations available to uniquely distribute the load between the muscles. The usual solution to this problem is to presume that the body in some sense works optimally and let the recruitment of individual muscles depend on an optimality criterion. With 1000 muscles in the system, this creates an optimization problem with 1000 variables that must be solved for each time step of the analysis. The formulation and numerical implementation of these algorithms requires an intricate knowledge of principles of optimality and optimization algorithms.

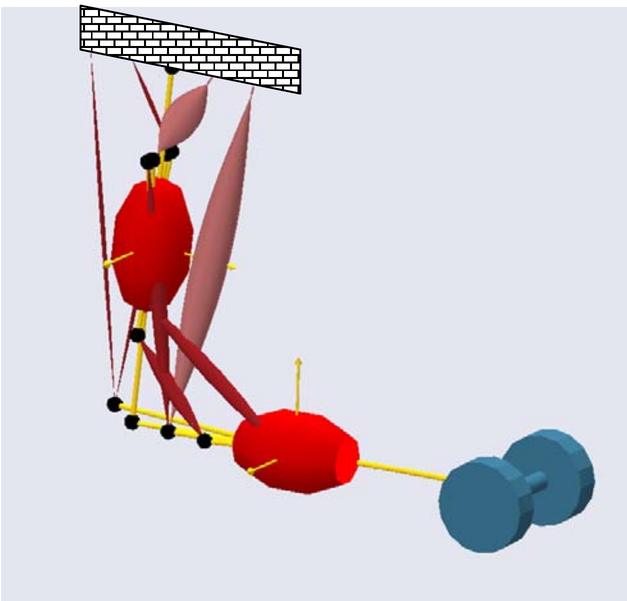

Figure 1. Statical indeterminacy. A very simple model of a human arm doing a dumbbell curl. Five muscles (indicated with bulging) share the task of flexing the two degrees of freedom of the shoulder and elbow joints.

The following sections deal with the mechanical details of the musculoskeletal analysis with a focus on the optimization-related issues.

4.1 The AnyBody modeling system

The techniques and models described in this work have been implemented into a software system named "The AnyBody Modeling System". It is designed for construction of complex models of the human body and for determination of the environment's influence on the body, and it must consequently exhibit a significant computational efficiency. This has led to the computational procedure illustrated in Figure 2.

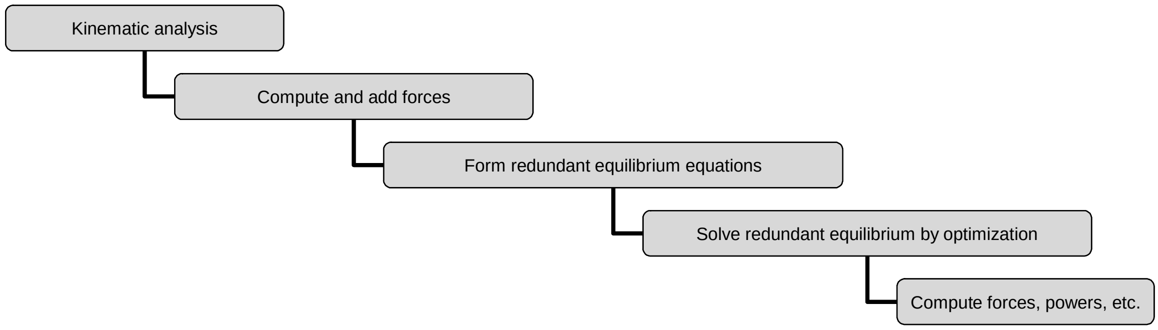

Figure 2. The computational procedure for an analysis of one time step in the AnyBody Modeling System.

4.2 Equilibrium

The first step of the analysis of Figure 2 is kinematics, but it is instructive to initially regard the solution of the mechanical equilibrium. In this work we perceive the human body and other mechanical parts connected to it as a multibody system. The muscles play the role of actuators of the system and the individual segments are modeled as rigid bodies. Advanced applications of such a modeling scheme have been demonstrated (for instance by Anderson and Pandy [1], H.v.d.Kooij et al [2] , 2003, D.G.Thelen et al [7]), but the technology has not yet reached a level where it is practically useful to a broad population of users. A brief review of the attempts to simulate the human body as a mechanical system is best initiated with the observation that the methods traditionally fall into one of two categories, inverse dynamics and forward dynamics, which, as the names indicate, are opposite approaches. Forward dynamics leads to an optimum control problem that is very computationally demanding. This work focuses on the opposite approach, namely inverse dynamic

In inverse dynamics, the motion and the external loads on the body are assumed known, and the purpose of the computation is to determine the internal forces. When the “internal forces” are mere joint moments and joint reaction forces, this, in most cases, is a straightforward procedure involving the solution of a system of linear equilibrium equations. However, for the purpose of computing individual muscle forces, inverse dynamics is haunted by the statical indeterminacy described in the preceding section: not enough equilibrium equations are available to determine all the muscle forces. Infinitely many different sets of muscle forces, of which the central nervous system (CNS) instantly chooses one, can therefore produce the identified joint moments. Constructing an algorithm to determine the activation of each muscle therefore entails guessing the motives behind the CNS’s function. We are able to repeat movements with considerable precision, so many researchers believe that the control of muscle forces must be based on some rational criterion. Indeed, it has been stated by Prilutsky et al [4]: "It is not known why in skilled multi-joint tasks such as cycling, the pattern of muscle activity is rather stereotypical at similar cycling conditions, whereas an infinite number of activity patterns can theoretically be used by the CNS to perform the same task -- to produce the same combination of joint moments".

Assuming that muscles are recruited according to an optimality criterion leaves the task of selecting the right one. Let us briefly state the mathematical form of the inverse dynamics problem:

Minimize $$G(\mathbf{f}^{(\mathrm{M})}) \tag{1}$$

Subject to $$\mathbf{C}\mathbf{f} = \mathbf{d} \tag{2}$$

$$f_i^{(\mathrm{M})} \geq 0, \quad i \in \{1,\ldots,n^{(\mathrm{M})}\} \tag{3}$$

where **f** is the vector of $n^{(\mathrm{M})}$ unknown muscle forces, $\mathbf{f}^{(\mathrm{M})}$, and joint reactions, $\mathbf{f}^{(\mathrm{R})}$. **C** is the coefficient matrix, and **d** is the right hand side comprised by external forces, inertia forces, and passive elasticity in the tissues of the body. In the AnyBody Modeling System, a min/max criterion is used for the objective function *G*:

$$G(\mathbf{f}^{(\mathrm{M})}) = \max\left(\frac{f_i^{(\mathrm{M})}}{N_i}\right) \tag{4}$$

where N_i is the momentary strength of muscle *i*. The problem can be converted to a linear form via the so-called bound formulation. For full details please refer to Rasmussen et al [5]. This leads to a linear programming problem with muscle forces and joint reactions as free variables. Since the joint reactions are free in sign and without side constraints, they can conveniently be eliminated from the system, leaving behind a linear program with as many unknowns as there are muscles in the system, i.e. for a detailed full body model approximately 1000. This is a medium to large size problem, and it can be solved by a variety of methods including Simplex and interior point methods. However, it turns out that the min/max problem suffers from an inherent indeterminacy that calls for an iterative solution scheme. The min/max criterion only concerns the muscles that are maximally activated or which help support muscles that are maximally activated. The system may contain muscles that have no influence on the maximum muscle activity in the system, and they are left undetermined by the problem formulation above. This means that the problem must be solved iteratively, where each iteration eliminates muscles that are uniquely determined and removes their contribution to the support of the external load from the right hand side. The next iteration then determines the submaximal muscles. This process continues until there are no muscles left in the system.

Each of these iterations shrinks the system, but if an analysis involves many, say 100, time steps, and each time step involves the determination of 1000 muscle forces in several subsequent LP problems, it is still a very numerically demanding task that requires careful implementation and numerical consideration

4.3 Kinematics

The first step of the computational sequence of Figure 2 is the kinematic analysis, i.e. determination of the position, velocity, and acceleration of each segment in the multibody dynamics model. While kinematics is a well-developed field and advanced computer systems for mechanism analysis are commercially available, the modeling of the human body requires additional and quite special facilities.

The analysis in the AnyBody Modeling System proceeds through a sequence of time steps defined by the user. A static problem has only one time step, and a dynamic problem has the time span of the analysis divided into steps of equal length. One of the advantages of inverse dynamic analysis is that time steps can be considered independent and without much bearing on the numerical convergence of the analysis. The purpose of the kinematic analysis is to identify each segment's[1] position, velocity and acceleration in each time step. The AnyBody Modeling System uses the Cartesian formulation of the kinematic problem, in which each segment has six independent degrees of freedom, and constraints corresponding to joints are imposed on the full size system of equations.

All segments of the mechanical system are modeled as rigid bodies, neglecting effects such as the wobbly masses of soft tissues. We more or less adopt the formulation of Nikravesh [3]. The position of the i'th segment is described by the coordinates $\mathbf{q}_i=[\mathbf{r}_i^{\mathrm{T}}\ \mathbf{p}_i^{\mathrm{T}}]^{\mathrm{T}}$, where $\mathbf{r}_i$ is the global position vector of the center of mass and $\mathbf{p}_i$ is a vector of four Euler parameters. The velocities of the segments is defined as $\mathbf{v}_i=[\dot{\mathbf{r}}_i^T\ \ \omega_i'^{\mathrm{T}}]^{\mathrm{T}}$, and the accelerations as time-derivatives, i.e., $\dot{\mathbf{v}}$. The vector ω_i' is the angular velocity of the segment, where the apostrophe indicates that it refers to the segment-fixed reference frame.

The kinematic analysis is carried out in terms of all the Cartesian coordinates, i.e., we assemble the coordinate vectors for all n segments of the system, including both human segments and machine parts. This provides the system coordinate vectors $\mathbf{q}=[\mathbf{q}_1^{\mathrm{T}} .. \mathbf{q}_n^{\mathrm{T}}]^{\mathrm{T}}$ and $\mathbf{v}=[\mathbf{v}_1^{\mathrm{T}} .. \mathbf{v}_n^{\mathrm{T}}]^{\mathrm{T}}$. Furthermore, we assemble all kinematic constraint equations associated with joints, drivers, and the constraints on the Euler parameter stating that all $\mathbf{p}_i$ are unit vectors. This provides $7n$ independent non-linear equations.

$$\Phi(\mathbf{q}, t) = \mathbf{0} \tag{5}$$

The number of equations matches the number of unknown coordinates in $\mathbf{q}$ if the problem is kinematically determinate. The position analysis is carried out by solving the equations with a suitable numerical method, for instance Newton-Raphson iteration using the constraint Jacobian, $\Phi_{\mathbf{q}}$. Velocity and acceleration analysis is carried out by solving the linear equations arising from time-derivation of (5) and transformation into the basis of $\mathbf{v}$.

$$\Phi_{\mathbf{q}*}\mathbf{v} = -\Phi_t \tag{6}$$

$$\Phi_{\mathbf{q}*}\dot{\mathbf{v}} = \gamma(\mathbf{q}, \mathbf{v}, t) \tag{7}$$

In Eqs. (6) and (7), the use of * indicates that it is the transformed Jacobian, and not $\Phi_{\mathbf{q}}$. For simplicity, $\Phi_{\mathbf{q}*}$ can be derived directly from the constraints without actually using a transformation between $\dot{\mathbf{q}}$ and $\mathbf{v}$. This is carried out by differentiation with respect to time in terms of $\mathbf{v}$. Moreover, the Euler parameter constraints can be left out of Eqs. (6) and (7), as they are redundant in terms of angular velocity. The equation solver applied to the solution of the equations handles other redundant constraints.

Having solved Eqs. (5)-(7), we know the motion completely if the system is properly specified, and we can generate the input to the muscle recruitment problem, Eqs. (1)–(3), and solve to find the muscle and joint forces of the system.

As mentioned, the use of Cartesian coordinates and the Newton-Euler equations for each rigid segment has been adopted here in the view of simplicity of implementation. To enable general application of the method to systems including the human body as well as mechanical artifacts, a general model formulation is needed, and it is easily obtained by this approach. A drawback may be low efficiency for large mechanical systems, or the need for efficient sparse system handling, compared to models using fewer (relative) coordinates. Another issue is the large number of parameters to be estimated, hereunder geometric, inertia, and muscle data. Such data can be difficult to estimate and the chosen approach does not provide the best basis for such parameter estimation. However, it must be emphasized that the solution to the muscle recruitment problem does not depend on the actual formulation of the equations of motion. In other words, (2) can be obtained by any other, more suitable approach depending on the actual application and the data at hand.

The generality of the Cartesian method facilitates the implementation of useful features in the kinematic system. This is a major advantage in musculoskeletal modeling, where the complications of the human body kinematics and interfaces to the experimental techniques used in the field call for special considerations in the software architecture. The following section describes two such useful features.

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[1] The term ”segment” is used for a rigid body in musculoskeletal analysis because the term “body” can be misinterpreted when the context is physiology.

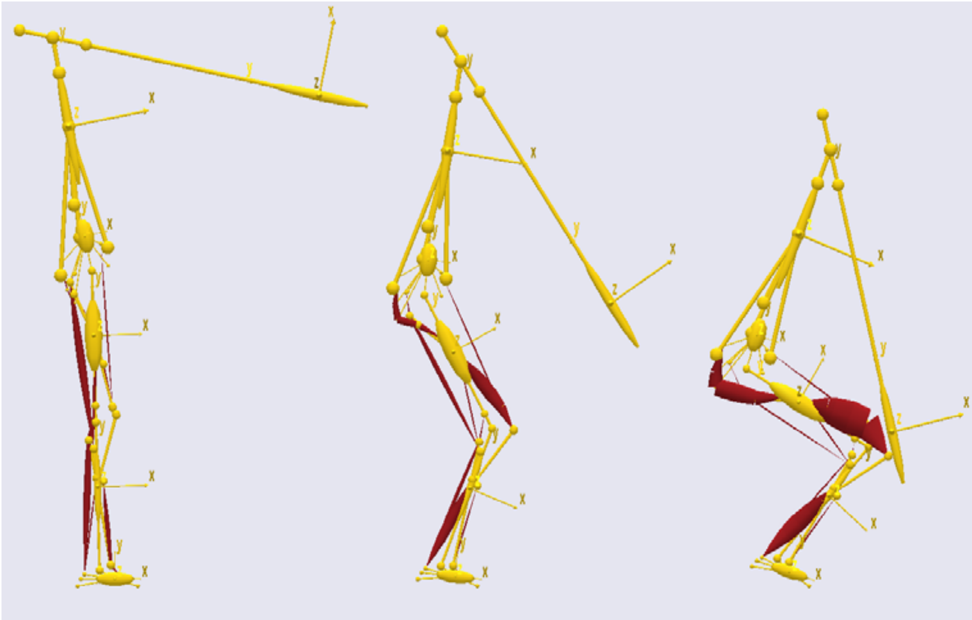

Figure 3. The control of arm positions in a squat model by means of the collective center of mass. The arm moves automatically to maintain the collective center of mass above the ball of the foot. The bulging of the muscles reflects the simulated muscle forces.

4.4 Kinematic measures

It is tempting to interpret the degrees of freedom of a human body model physiologically, for instance as the flexion of a knee or the twist of a forearm. However, binding the system to express the degrees of freedom in such physiological terms would seriously deplete the system's applicability for studies of humans in free movement and in connection with various types of equipment, and the general statement of the kinematic problem, (5), in fact allows for more flexible approaches.

In gait analysis, for instance, which is a major field of clinical diagnostics and research, movement is typically recorded by video tracking of optical markers attached to the body. The kinematics of the human body is sufficiently complex to make the conversion of marker positions to physiological joint angles a challenging task, and it would be desirable to be able to use the marker positions directly to drive the model. Another example is when the posture or movement of the human body is defined by its interaction with an artifact such as a bicycle, a hand tool, a chair, or a workplace. In the case of the bicycle, the movement of the feet is defined by the pedal cycle rather than the anatomical joint angles.

To enable definition of kinematics in terms of non-anatomical parameters, the AnyBody Modeling System has been equipped with an abstract concept named "kinematic measures". A kinematic measure is just about any dimension that can be measured on the body model. Typical examples could be the distance between two points, the coordinates of a point (such as a video tracking marker) in space, the length of a muscle, or a joint angle. The concept of kinematic measures thereby encapsulates also the anatomical postural dimensions such as joint angles.

As an example of the generality of kinematic measures we can consider the problem of modeling a human body in an unsupported slow squatting movement (Figure 3). When performing such a task, care must be taken to maintain the position of the collective center of mass vertically above the convex hull of the two feet's contact with the ground, lest the model would fall over due to lack of support. Since a squat involves individual movements of arms, trunk, thighs, shanks, and feet, it would be a very challenging task to specify a set of anatomical joint movements that would constrain the collective center of gravity. In the concept of kinematic measures, the collective center of mass is simply a point in the model, and it is consequently possible to drive it by inserting the specifications of its position into the position analysis, (5). For instance the consequence is that a driver on the arm position can be neglected from the model, and the arms of the model will automatically attain the position necessary to balance the model in each stage of the movement, exactly as a human reaches out in front of him during the squat to avoid falling backwards.

5. A spring-enhanced bicycle crank

Bicycling is an interesting example for biomechanical simulation for a variety of reasons:

1. It is a very controlled movement. Regarding only the legs, knowledge of the crank angle will leave just two degrees of freedom in the system, namely the two ankle angles, and their variation is very similar between individuals and known from experiments.
2. It is a very well-investigated movement.

3. Its metabolic efficiency can be shown experimentally to be close to the theoretical maximum of the body. Therefore it would appear that pedaling is almost optimal from a metabolic point-of-view.

A pedaling cycle of 360 degrees has two so-called dead centers, which on an ordinary bicycle fall when the pedal arms are close to vertical. In these positions it is difficult for the human body to produce much crank torque because the tangential pedal force direction is perpendicular to the preferred force direction of the legs. Much effort has been invested into mechanisms that reduce or eliminate these dead centers. One of the best known initiatives was the oval chain wheel marketed by Shimano in the 1980'ies. Rasmussen et al. [6] demonstrated an intricate mechanism for paraplegics' bicycles that helps overcoming the dead center by making sure that it does not occur simultaneously for the two legs.

Figure 4. Two springs suspended between the frame and the chain crank mechanism.

Another possible solution would be to allow the mechanism to store energy when the leg has maximum leverage and subsequently release the energy when the dead center is approached. A spring arrangement as depicted in Figure 4 may serve this purpose. The two springs are stretched and compressed as the crank revolves, and careful positioning may allow the legs to store elastic energy that can subsequently help overcome the dead centers of the pedal cycle. On the other hand, it is almost universally accepted that storage of elastic energy due to flexibility of the bicycle frame is a disadvantage in pedaling. Furthermore, the symmetrical arrangement of the two springs may lead to the suspicion that they will cancel out whatever beneficial effect they may have individually.

A musculoskeletal model can clarify the situation considerably. To focus the attention on the springs' effect on the prevalence of dead centers, the two legs are artificially required to produce a constant crank torque of 30 Nm at a cadence of 80 rpm corresponding to a power output of 251 W. This is a fairly high power output, and in a real situation it would be produced by an almost sinusoidally varying crank torque causing some amount of speed variation over the cycle. For slow speeds as when riding up hill, the variation may be significant, and cycling coaches recommend riders to make their tread as even as possible. Disregarding the springs and requiring the model to produce a completely even crank torque produces the variation of maximum muscle activation over the cycle shown in Figure 5.

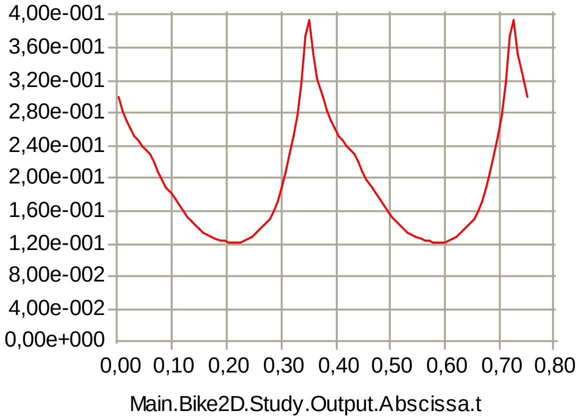

Figure 5. Maximum muscle activity over a pedal cycle at 80 rpm and constant crank moment of 30 Nm.

The dead centers are clearly visible as time in the cycle where the muscles are required to work relatively much to produce the required crank torque. The difference in muscle activity over the cycle is more than three times, signifying that torque generation at the dead centers is much more difficult than at the points of maximum leverage.

Figure 6. Placement of springs after parameter variation.

When springs are added to the model, parameters such as spring stiffness, slack length, fixation point on the frame, and fixation points on the crank can be varied and indeed optimized. The actual optimization calls for a specially tailored algorithm because the multiple iterative numerical solution techniques involved in the analysis generates some amount of noise in the solution result. Furthermore, the min/max muscle recruitment introduces macroscopic discontinuities in the design space. Finally, we have no efficient analytical or semi-analytical sensitivity analysis method available, and hence sensitivity analysis must be accomplished by finite difference, the cost of which is at least the same as an ordinary analysis.

This indicates that an optimization method with a minimum dependency on gradient information is preferable. We choose to conduct a golden section search cyclically along the design variables in the problem. This zeroth order method is insensitive to discontinuities of the objective function as long as the problem is convex. The latter condition is not necessarily fulfilled, so we cannot guarantee more than a local minimum. In site of the primitive optimization strategy, the entire optimization takes mo more than a few minutes on an ordinary PC thanks to the efficient analysis procedure.

Having decided on a procedure that is insensitive to non-smoothness of the objective function we can elect to define the objective function as the maximum value of the graph of Figure 6 regardless of the fact that this introduces further non-smoothness to the problem. The parameters of the problem are spring stiffness, spring slack length, attachment radius on the crank, and attachment point on the frame, totaling 5 independent design variables.

The result of a systematic parameter variation is depicted in Figure 6 with the springs visualized as cylinders. They each have a slack length of 240 mm and a spring stiffness of 16.7 N/mm. Figure 7 shows the variation of maximum muscle activity over the cycle under these circumstances. The springs make the effort over the cycle almost uniform, allowing the rider to produce an even crank torque. Furthermore, the maximum muscle activity is less than half of the case with no springs mounted.

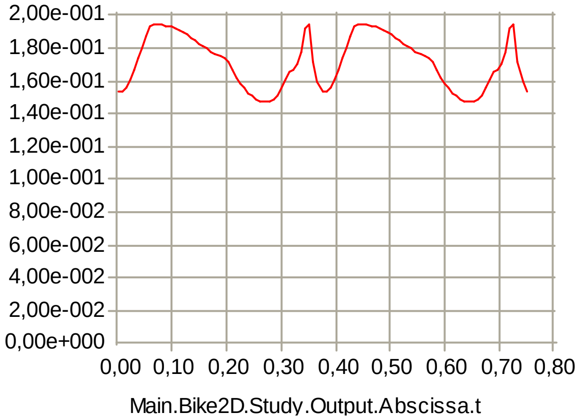

Figure 7. Maximum muscle activity over a cycle with springs added to help overcome the dead centers.

6. Conclusions

This paper has demonstrated one application of ergonomic optimization. A superficial investigation of the products that surround us reveals that a very large number of them have some interface to the human body. Regardless of this fact, no technology has been available for quantification of their ergonomic quality until now. At a time where virtually any technical property of products can be analyzed by means of CAE tools, the ergonomic qualities remain subject to qualitative estimations.

The ergonomic analysis techniques described in this paper have the potential to revolutionize the design of products in contact with the human body, and the core technology behind this opportunity is optimization and mathematical programming.

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