Competitive and Cooperative Inventory Policies in a Two-Stage Supply-Chain

(G. P. Cachon and P. H. Zipkin)

Presented by

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Outline

- Introduction
- Model
- Centralized optimization problem
- Inventory games
- Nash equilibrium outcomes vs. centralized solution
- Optimal linear contracts
- Conclusion
- Future scope

Overview

- Two stage serial supply chain
- Stationary stochastic demand
- Fixed transportation time
- Single product
- Inventory holding costs at each stage
- Consumer backorder penalty at each stage

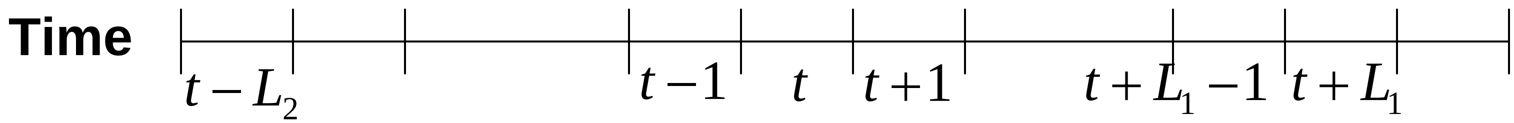
Motivation

- **Retailers**
  - Kroger
  - JCPenny
  - Best Buy
- **Suppliers**
  - Kellogg
  - Nike
  - Apple

Contribution

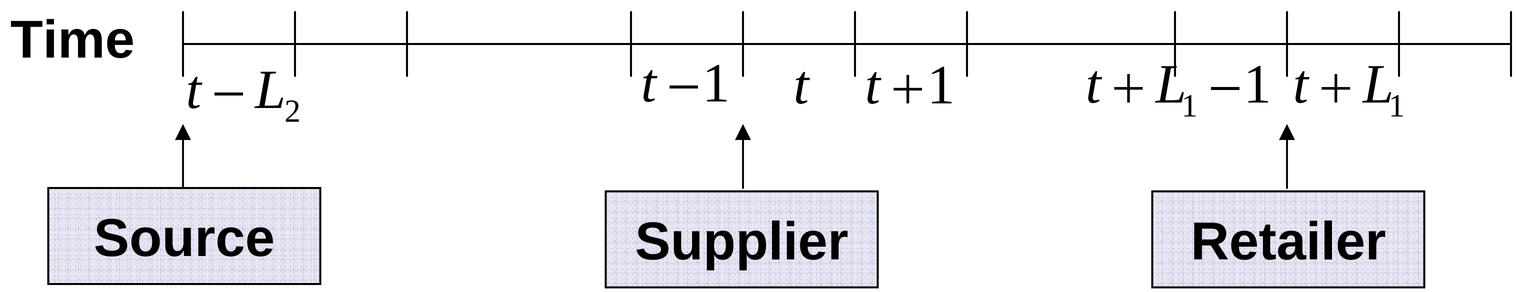
- Models competitive behavior of agents
  - Game theoretic analysis
- Each agent has equal position in the game
  - Analysis of Nash equilibria
- Study of two different games
  - Echelon inventory tracking
  - Local inventory tracking
- Design of linear transfer payments that help minimize system cost at Nash equilibrium

The Model



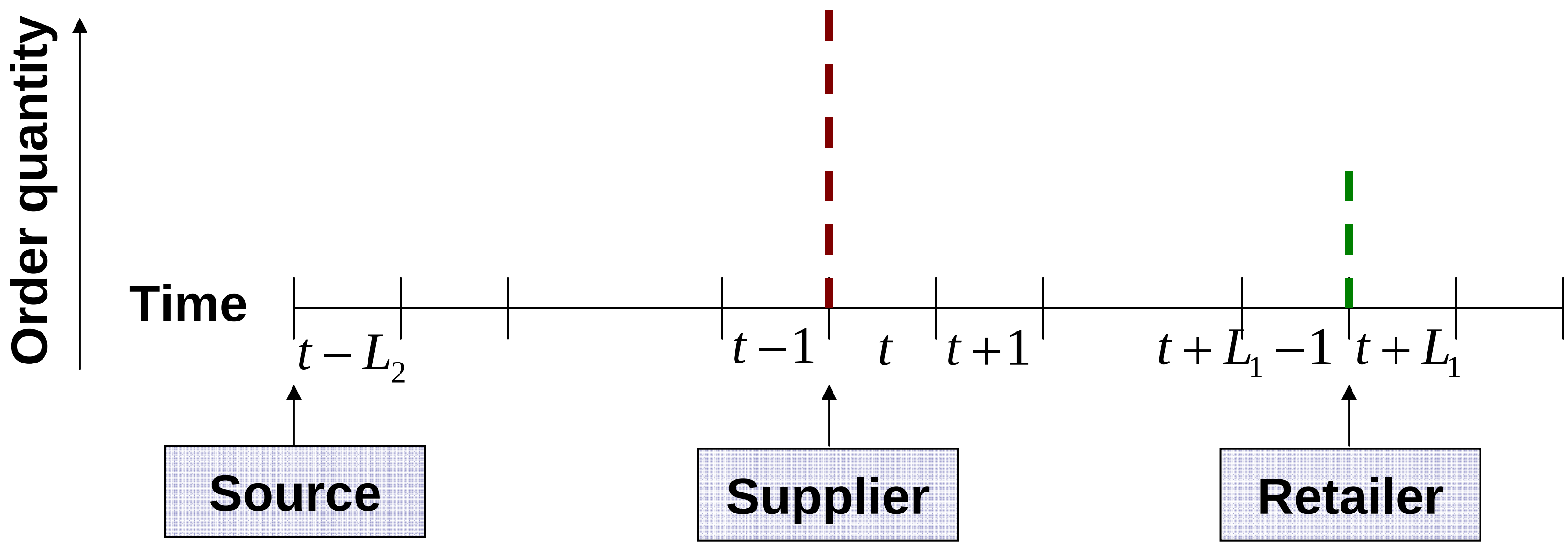
- Time is slotted

The Model



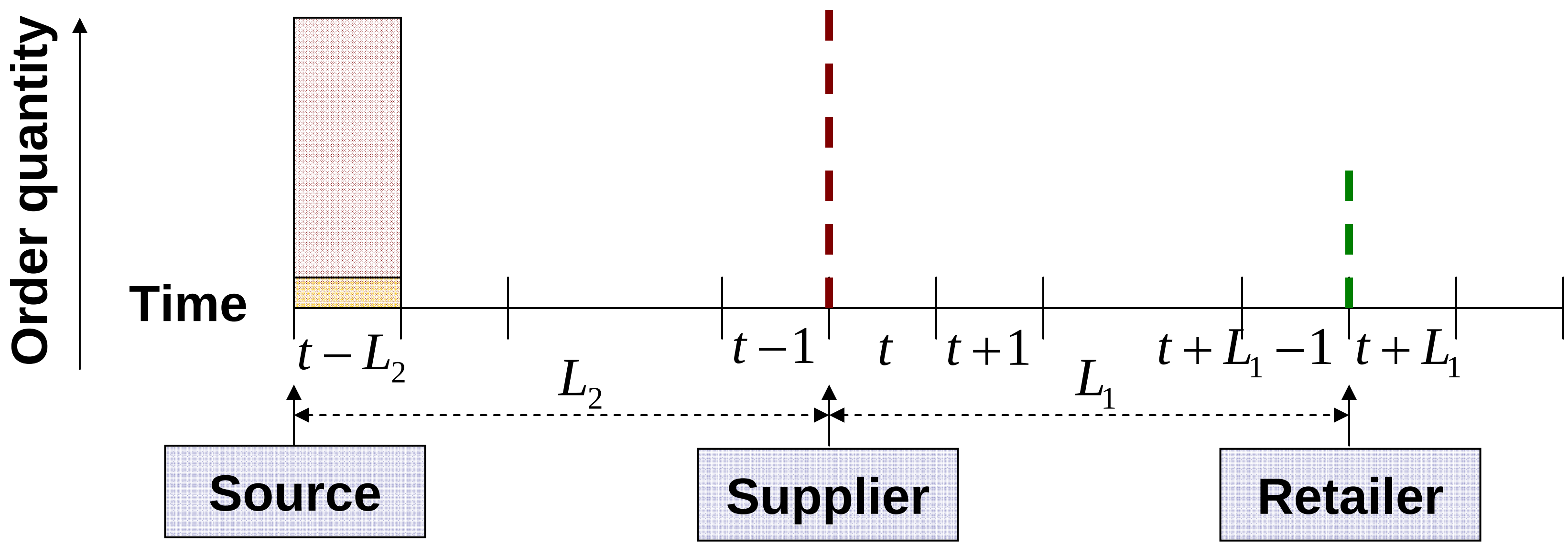
- Time is slotted
- Flow of product: Source $\longrightarrow$ Supplier $\longrightarrow$ Retailer

The Model



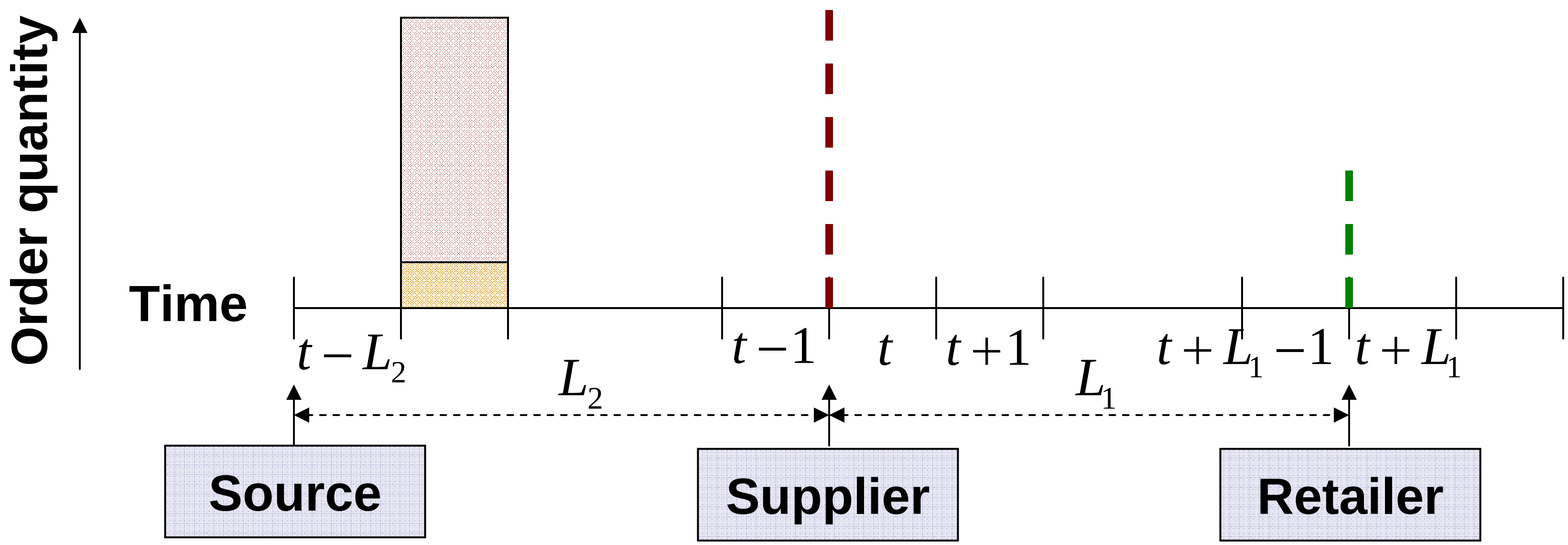
- Time is slotted
- Flow of product: Source $\longrightarrow$ Supplier $\longrightarrow$ Retailer
- Supplier and retailer submit the orders

The Model



- Time is slotted
- Flow of product: Source $\longrightarrow$ Supplier $\longrightarrow$ Retailer
- Supplier and retailer submit the orders
- Shipments are immediately released
- Lead time: Source to Supplier (L_2), Supplier to Retailer (L_1)

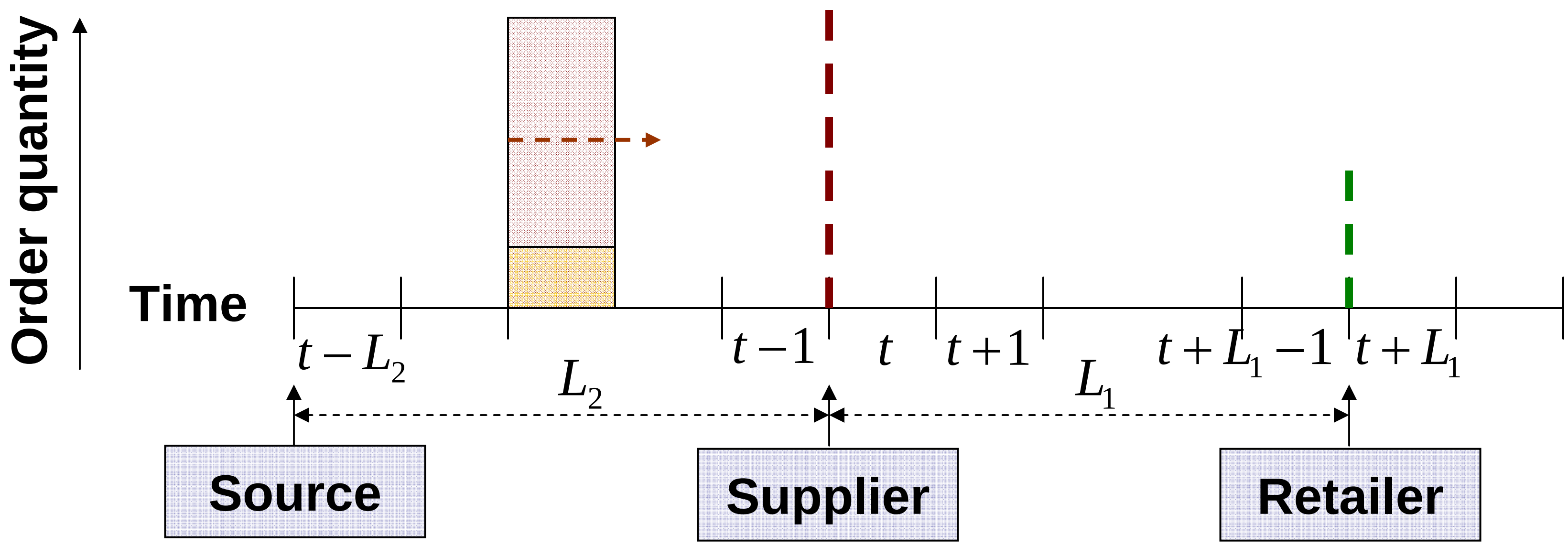
The Model



Demand:

- D^τ random total demand over τ periods
- Stationary distribution: density ϕ^τ, distribution Φ^τ
- Demand is a continuous random variable
- Positive demand occurs in each period

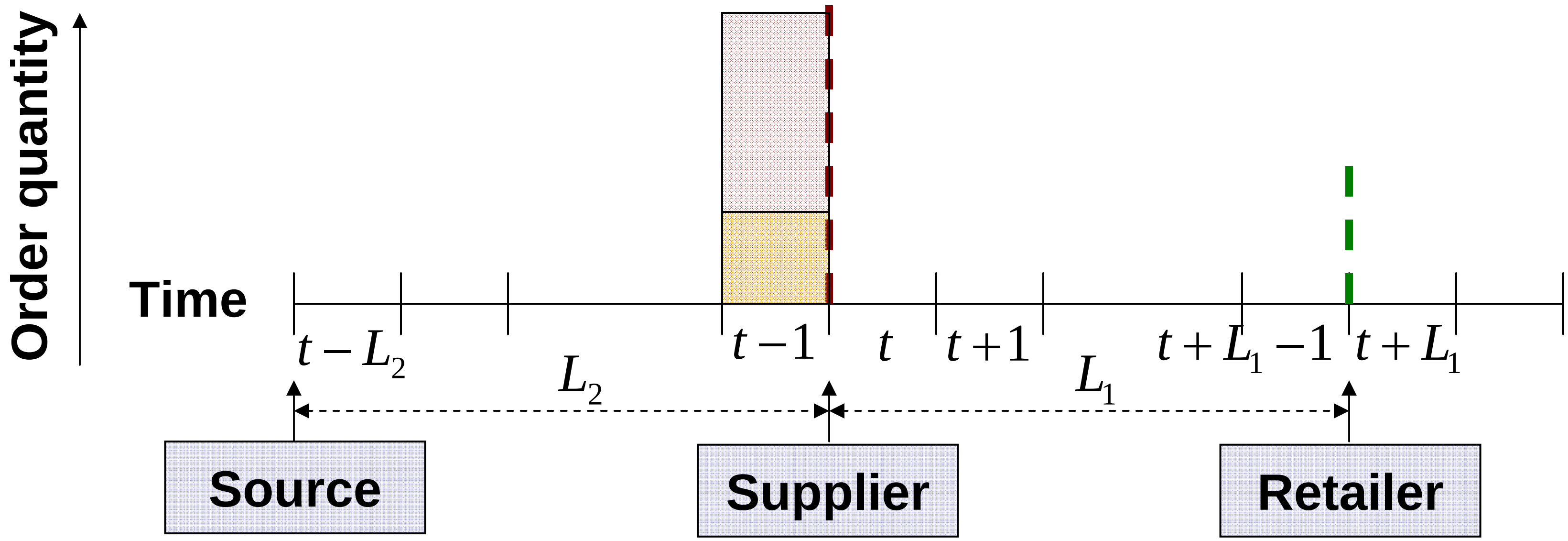
The Model



Inventory levels of interest:

- In transit inventory: IT_{it}, Supplier $(i=2)$, Retailer $(i=1)$
- Echelon inventory level: IL_{it}, all inventory at stage *i* or lower in the system minus consumer backorders
- Local inventory level: $\overline{IL}_{it}$, inventory at stage *i* minus backorders at stage *i*

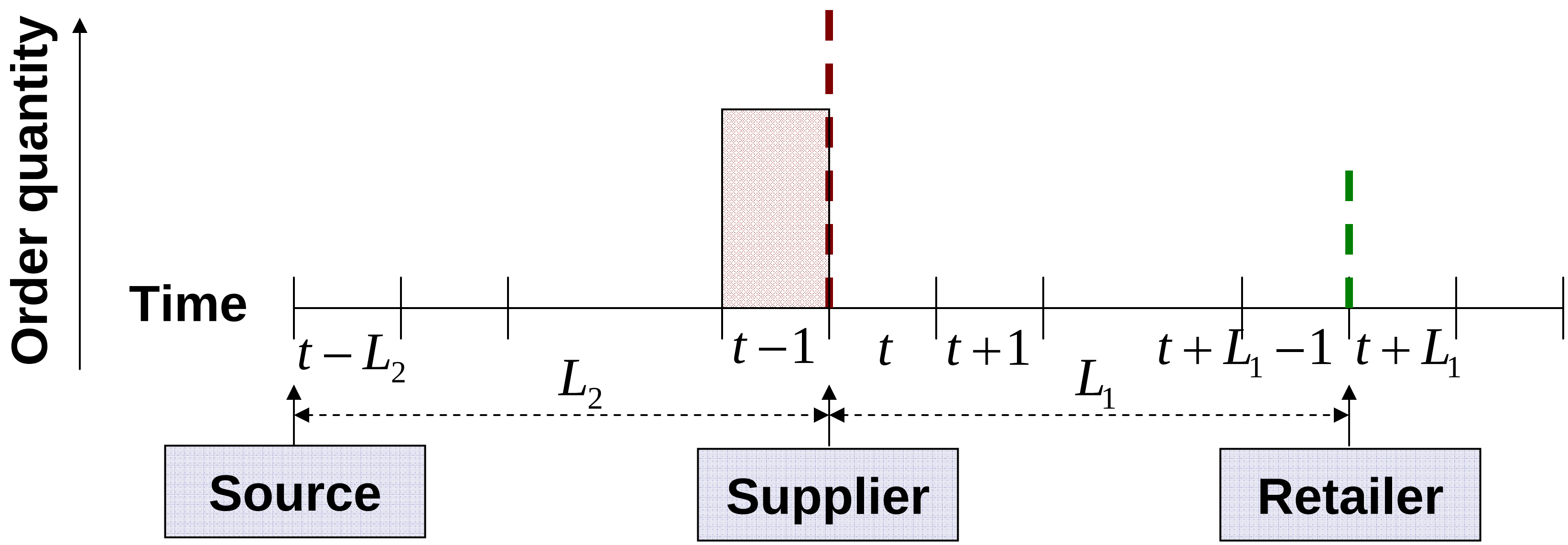
The Model



Inventory levels of interest:

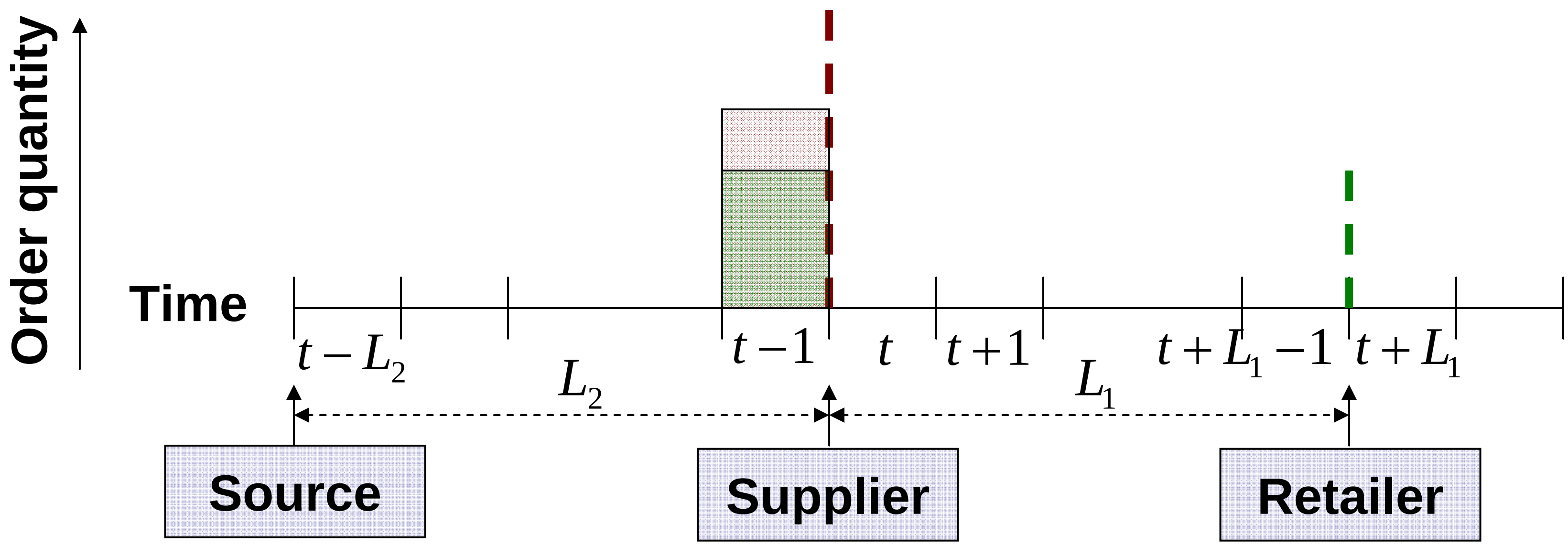
- Echelon inventory position: $IP_{it} = IL_{it} + IT_{it}$
- Local inventory position: $\overline{IP}_{it} = \overline{IL}_{it} + IT_{it}$

The Model

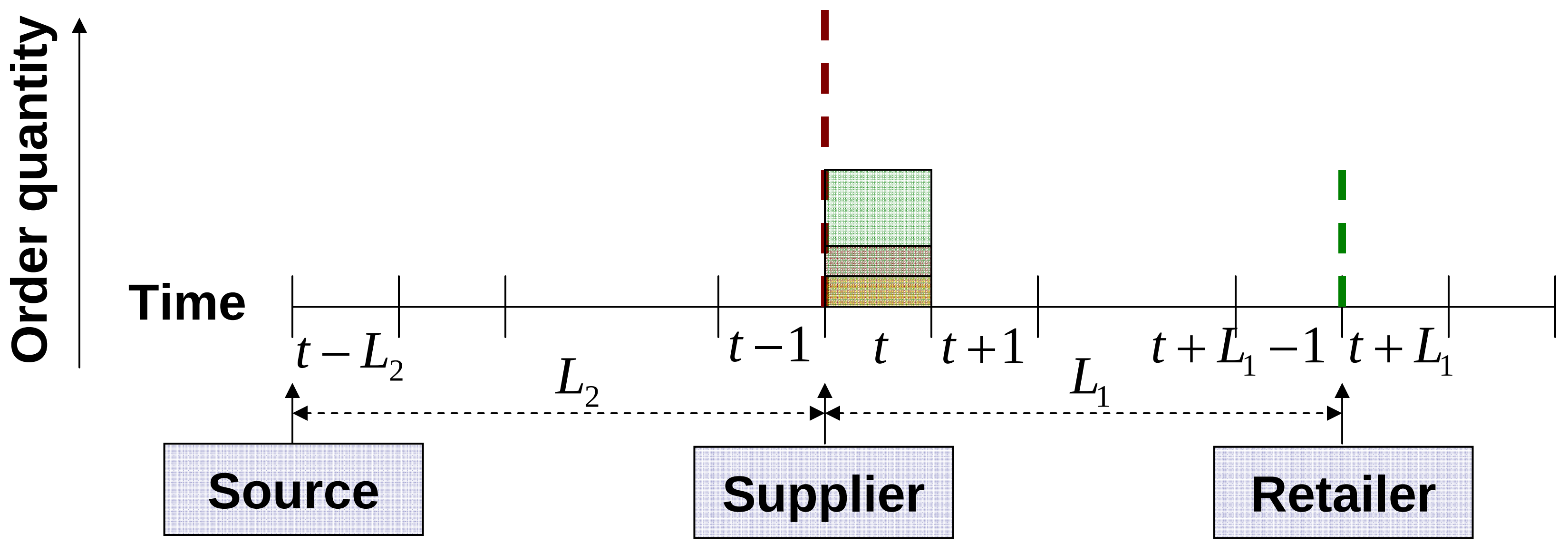


Order quantity
Time
$t-L_2$
$t-1$
t
$t+1$
$t+L_1-1$
$t+L_1$
L_2
L_1
Source
Supplier
Retailer

The Model

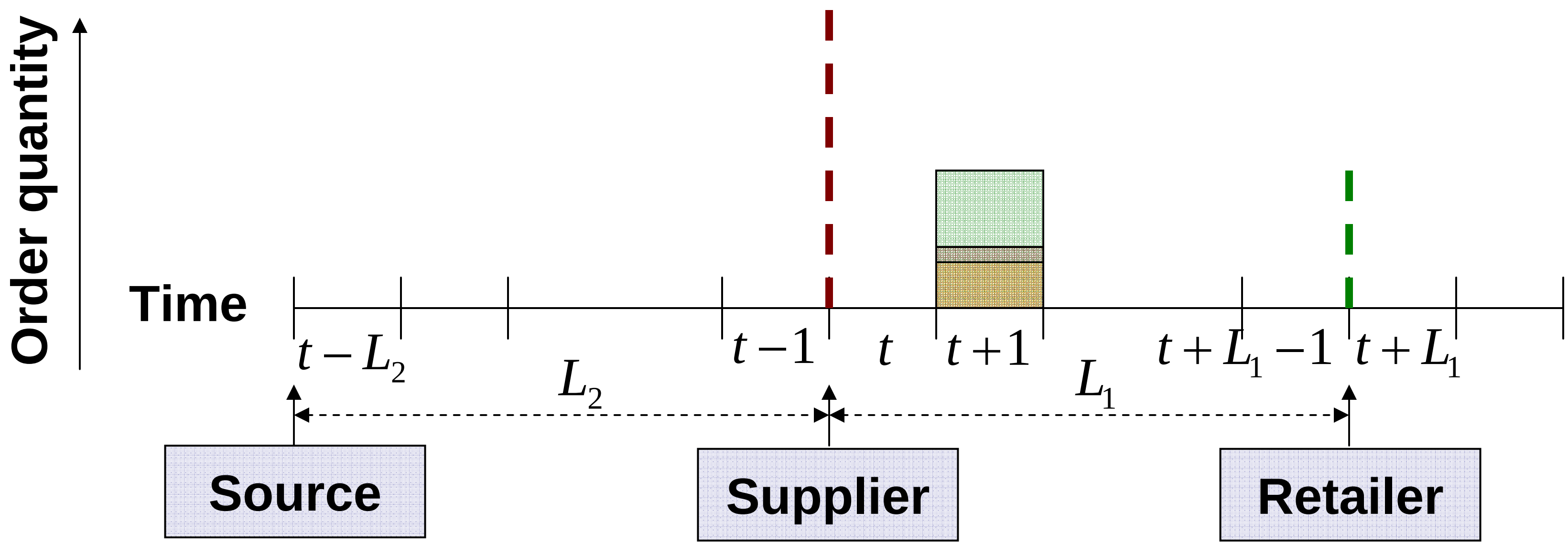

Order quantity
Time
$t - L_2$
$t-1$
t
$t+1$
$t + L_1 - 1$
$t + L_1$
L_2
L_1
Source
Supplier
Retailer

The Model



Order quantity
Time
$t-L_2$
L_2
$t-1$
t
$t+1$
L_1
$t+L_1-1$
$t+L_1$
Source
Supplier
Retailer

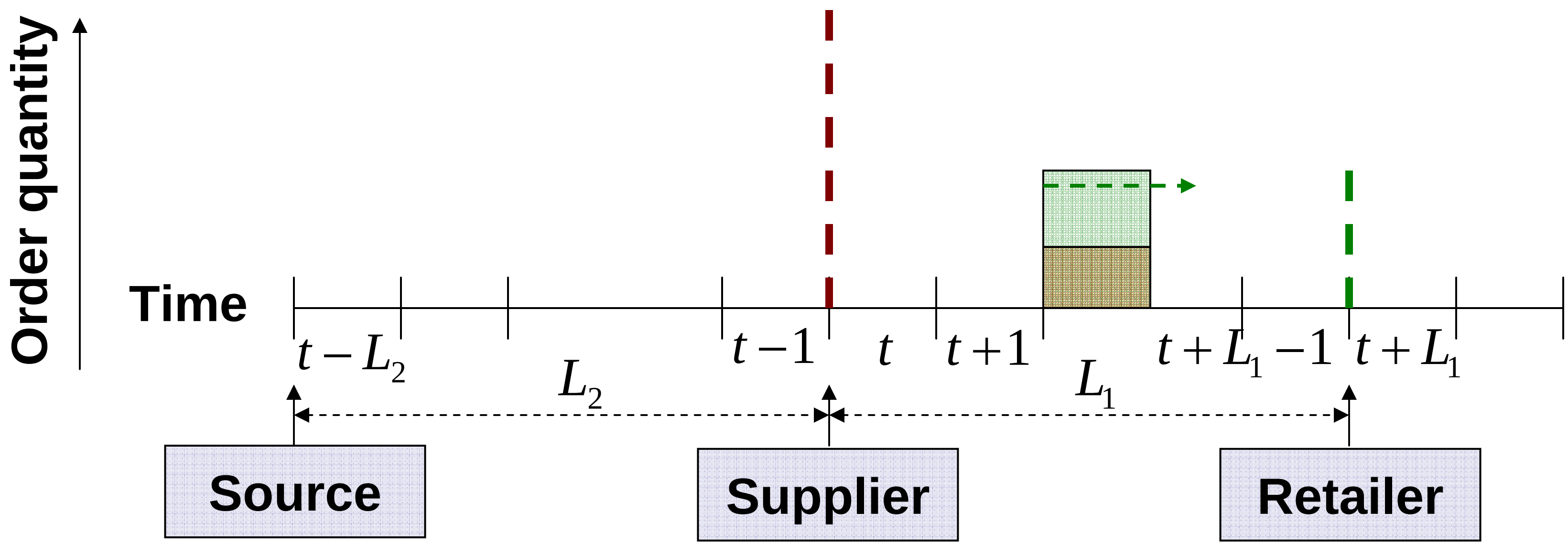
The Model



Holding costs:

- Supplier: h_2 per period for each unit in its stock or en route to the retailer
- Retailer: $h_1 + h_2$ per period for each unit in its stock
- Assumption: $h_2 > 0,\ h_1 \geq 0$

The Model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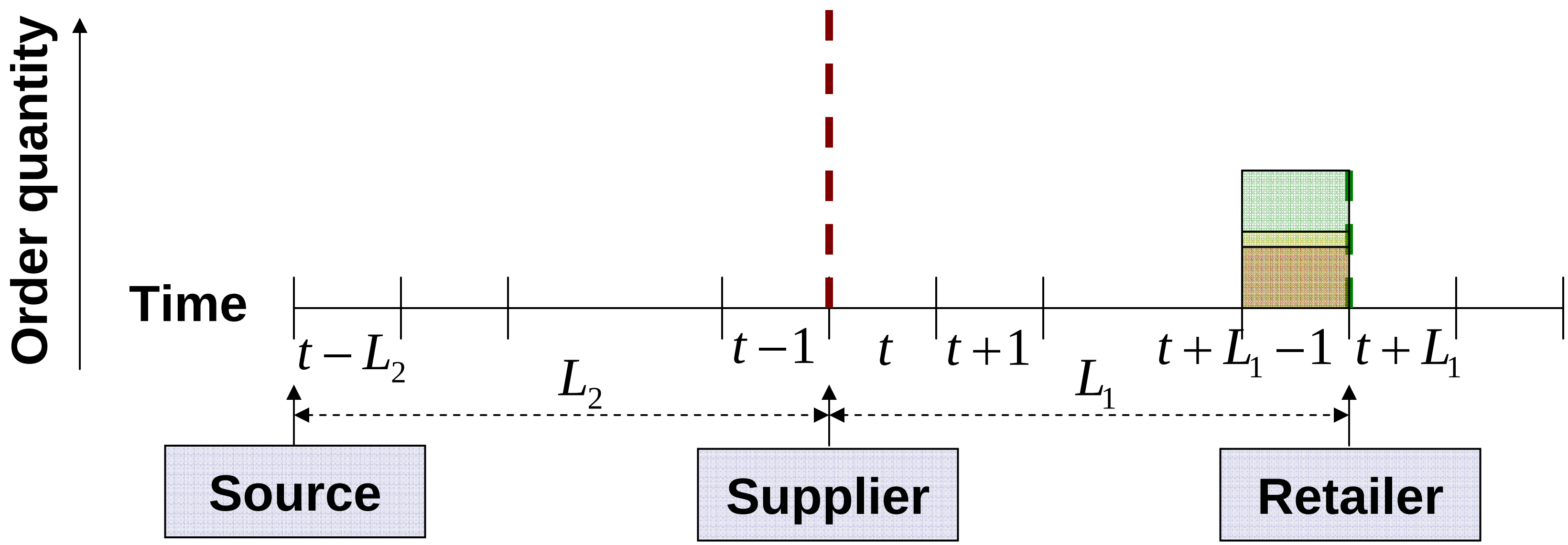

Order quantity
Time
$t-L_2$
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L_2
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Supplier
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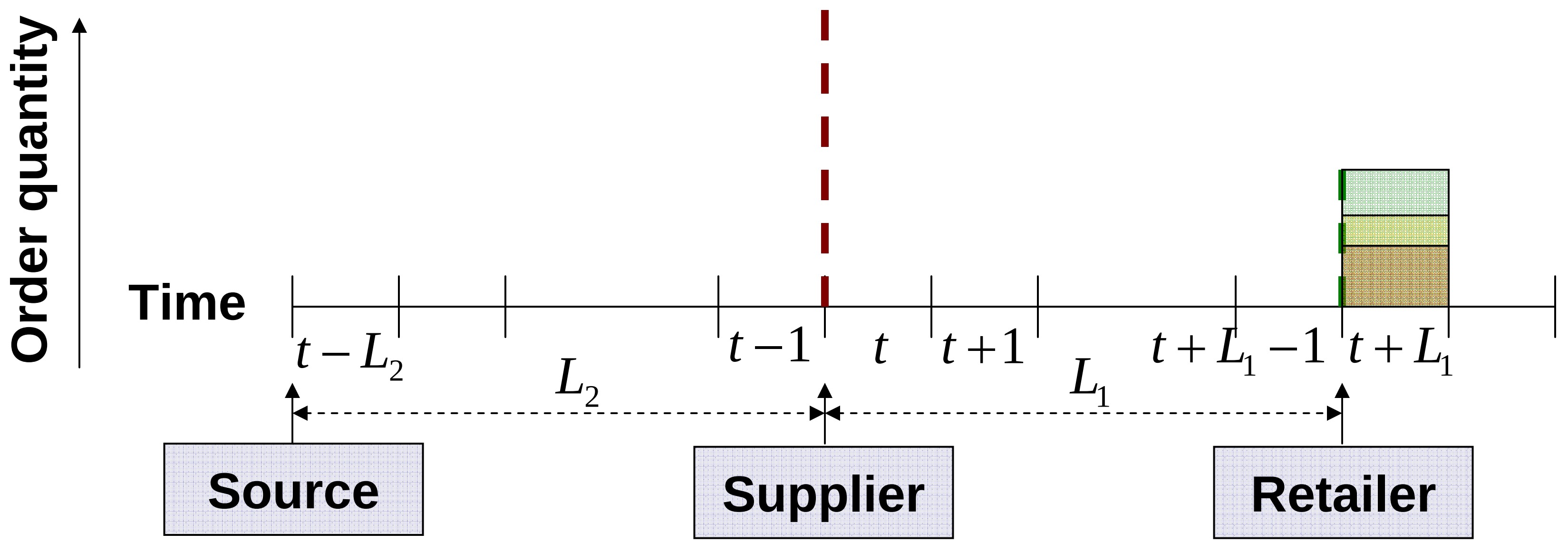

Order quantity
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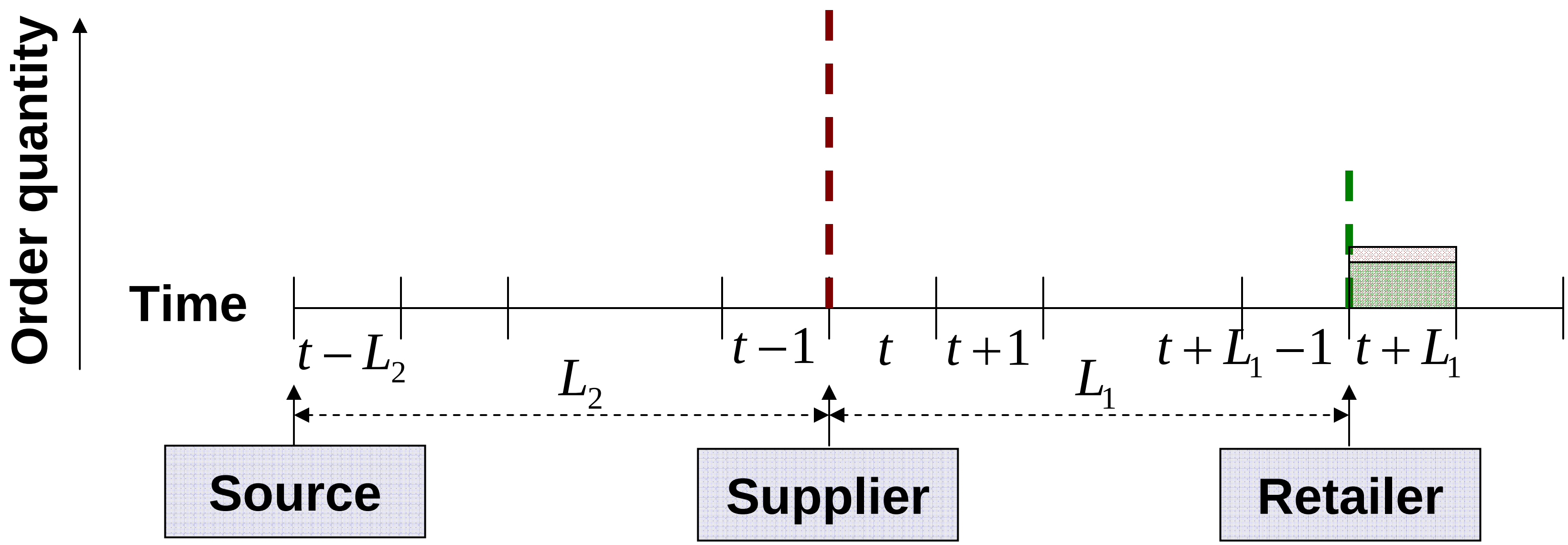
Supplier
Retailer

The Model



Backorder costs:

- System backorder cost: p per unit backorder
- Supplier: αp
- Retailer: $(1-\alpha)p$
- Assumption: $0 \le \alpha \le 1$

Cost Functions

Retailer:

- Cost in period *t:* $\hat{G}_1(IL_{1t} - D^1)$

$$\hat{G}_1(y) = (h_1 + h_2)[y]^+ + \alpha p[y]^-$$

- Expected cost in period $t + L_1$: $G_1(IP_{1t})$

$$G_1(y) = E[\hat{G}_1(y - D^{L_1+1})]$$

- IP_{1t} depends on supplier’s order and demand up to time *t*
  - If $s_2 - D^{L_2} \geq s_1$, $IP_{1t} = s_1$
  - If $s_2 - D^{L_2} < s_1$, $IP_{1t} = s_2 - D^{L_2}$.
- Total expected cost

$$H_1(s_1, s_2) = E[G_1(\min\{s_2 - D^{L_2}, s_1\})]$$

Cost Functions

Supplier:

- Backorder cost in period *t:* $\hat{G}_2(IL_{1t} - D^1)$

$$\hat{G}_2(y) = (1-\alpha)p[y]^-$$

- Expected backorder cost in period $t + L_1$: $G_2(IP_{1t})$

$$G_2(y) = E[\hat{G}_2(y - D^{L_1+1})]$$

- Let

$$\hat{H}_2(s_1, x) = h_2\mu^{L_1} + h_2[x]^+ + G_2(s_1 + \min\{x, 0\})$$

- Total expected cost

$$H_2(s_1, s_2) = E[\hat{H}_2(s_1, s_2 - s_1 - D^{L_2})]$$

System optimal solution

- A system optimal solution minimizes the total average cost per period.

$$(s_1^o, s_2^o) = \arg\min_{(s_1, s_2)} H_1(s_1, s_2) + H_2(s_1, s_2)$$

Echelon Inventory (EI) Game

- Players: $i = 1,2$
- Strategies: $s_i \in \sigma = [0, S]$, $i = 1,2$
- Payoffs: $-H_i(s_1, s_2)$, $i = 1,2$

- Pure strategy Nash equilibrium (s_1^e, s_2^e)
  such that,

$$s_2^e \in r_2(s_1^e) \quad s_1^e \in r_1(s_2^e)$$

$$r_1(s_2) = \{s_1 \in \sigma \mid H_1(s_1, s_2) = \min_{x \in \sigma} H_1(x, s_2)\}$$

$$r_2(s_1) = \{s_2 \in \sigma \mid H_2(s_1, s_2) = \min_{x \in \sigma} H_2(s_1, x)\}$$

- Game is common knowledge

Local Inventory (LI) Game

- Players: $i = 1,2$
- Strategies: $s_i \in \sigma = [0, S]$, $i = 1,2$
- Payoffs: $-H_i(\bar{s}_1, \bar{s}_2 + \bar{s}_1)$, $i = 1,2$

- Pure strategy Nash equilibrium $(\bar{s}_1^l, \bar{s}_2^l)$
  such that,

$$\bar{s}_2^l \in \bar{r}_2(\bar{s}_1^l) \quad \bar{s}_1^l \in \bar{r}_1(\bar{s}_2^l)$$

$$\bar{r}_1(\bar{s}_2) = \{\bar{s}_1 \in \sigma \mid H_1(\bar{s}_1, \bar{s}_2 + \bar{s}_1) = \min_{x \in \sigma} H_1(x, \bar{s}_2 + x)\}$$

$$\bar{r}_2(\bar{s}_1) = \{\bar{s}_2 \in \sigma \mid H_2(\bar{s}_1, \bar{s}_2 + \bar{s}_1) = \min_{x \in \sigma} H_2(\bar{s}_1, x + \bar{s}_1)\}$$

- Game is common knowledge

Nash equilibria

- **Theorem 4:** For $0 < \alpha < 1$, EI game has a unique Nash equilibrium.

$$(s_1^e = s_1^a, s_2^e = r_2(s_1^a))$$

$$\Phi^{L_1+1}(s_1^a) = \frac{\alpha p}{h_1 + h_2 + \alpha p}$$

- **Theorem 8:** For $0 < \alpha < 1$, LI game has a unique Nash equilibrium.

Nash equilibria (cont')

- **Theorem 9:** For $\alpha = 1$, EI game has the following Nash equilibria.

$$(s_1^e \in [s_2^e, S], s_2^e \in [0, s_1^a])$$

- **Theorem 10:** For $\alpha = 1$, LI game has a unique Nash equilibrium.

$$(\bar{s}_1^l = \bar{r}_1(0), \bar{s}_2^l = 0)$$

- **Theorem 11:** For $\alpha = 0$, both EI and LI games have unique Nash equilibrium and they are identical.

$$(s_1^e = 0, s_2^e = r_2(0))$$

$$(\bar{s}_1^l = s_1^e, \bar{s}_2^l = s_2^e - \bar{s}_1^l)$$

Comparing Nash equilibria

- **Theorem 12:** For $0 < \alpha < 1$, the base stock levels for both firms are higher in the LI game equilibrium than in the EI game equilibrium, i.e. $s_2^l > s_2^e \; and \; s_1^l > s_1^e$

- **Theorem 13:** For $0 < \alpha < 1$, the supplier's cost in the LI game equilibrium is lower than its cost in the EI game equilibrium.

Nash equilibria and Optimal Solution

- **Theorem 14:** In an EI game equilibrium, the retailer's base stock level is lower than in the optimal solution.
- **Theorem 15 & 16:** For $\alpha \leq 1$, the supplier's base stock level in both the LI and the EI equilibria is lower than in the system optimal solution.
- **Theorem 17:** For $\alpha < 1$, the system optimal solution is not a Nash equilibrium in either game.
- **Theorem 18:** For $\alpha = 1$, the system optimal solution is a Nash equilibrium in the LI game only when

$$\Phi^{L_2+L_1+1}(s_1^o) = \frac{p}{h_1 + h_2 + p}$$

Linear Contracts

- Period t transfer payment from supplier to retailer

$$\iota_1 I_{1t} + \beta_2 B_{2t} + \beta_1 B_{1t}$$

- Expected transfer payment in period $t + L_1$ due to retailer inventory and backorders

$$T_1(y) = E[\iota_1[y - D^{L_1+1}]^+ + \beta_1[y - D^{L_1+1}]^-]$$

- Expected per period transfer payment from supplier to retailer

$$T(\bar{s}_1, \bar{s}_2) = E[\beta_2[\bar{s}_2 - D^{L_2}]^-$$

System with modified costs

- Costs accounting for transfer payments

$$H_1^c(\bar{s}_1, \bar{s}_1 + \bar{s}_2) = H_1(\bar{s}_1, \bar{s}_1 + \bar{s}_2) - T(\bar{s}_1, \bar{s}_2)$$

$$H_2^c(\bar{s}_1, \bar{s}_1 + \bar{s}_2) = H_2(\bar{s}_1, \bar{s}_1 + \bar{s}_2) + T(\bar{s}_1, \bar{s}_2)$$

- **Objective:** To determine the set of contracts, $(\iota_1, \beta_2, \beta_1)$, such that $(\bar{s}_1^c, \bar{s}_2^c)$ is a Nash equilibrium for the cost functions $H_i^c(\bar{s}_1, \bar{s}_1 + \bar{s}_2)$, where $\bar{s}_1^c = s_1^o$, and $\bar{s}_1^c + \bar{s}_2^c = s_2^o$.

Finding optimal linear contract

- Assuming H_i^c to be strictly concave in $\bar{s}_i$, find the contracts satisfying

$$\frac{\partial H_1^c}{\partial \bar{s}_1} = 0 \quad \text{and} \quad \frac{\partial H_2^c}{\partial \bar{s}_2} = 0$$

  at systems optimal

- Out of this set of contracts, select the subset of contracts that make the cost functions strictly concave.

Set of optimal linear contracts

- **Theorem 19:** When the firms choose a contract $(\iota_1, \beta_2, \beta_1)$ that satisfies the following properties,

$$(1-\alpha)p = \left(\frac{p}{h_1 + h_2}\right)\iota_1 - \beta_1,$$

$$h_2 = \left(\frac{h_2}{h_1 + h_2}\right)\iota_1 + \left(\frac{1-\gamma_2}{\gamma_2}\right)\beta_2.$$

$$h_1 + h_2 > \iota_1 \geq 0$$

$$\beta_2 > 0$$

$$\alpha p > \beta_1 \geq -(1-\alpha)p,$$

then the optimal policy is a Nash equilibrium.

Conclusion

- When both players care about consumer backorders, there is a unique Nash equilibrium in EI game as well as LI game, and these equilibria differ.
- The Nash equilibrium of such EI and LI game does not provide optimal solution of supply chain.
- Competition lowers the supply chain inventory relative to the optimal solution.
- Appropriate linear contracts can help achieving optimal supply chain solution at some Nash equilibrium.

Future Scope

- Multi product supply chains where demands of different products are correlated and are stationary, can be studied similarly by considering joint distribution of demands of these products.

- Other contracts can be investigated which ensure that all Nash equilibria provide optimal supply chain solution.

- The work can be extended to incorporate processing times of orders.

Thank you!