Performance Analysis of a Distributed Resource Allocation Scheme for D2D Communications

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Abstract—Device-to-device (D2D) communications underlaying a cellular infrastructure has recently been proposed as a means of increasing the cellular capacity, improving the user throughput and extending the battery lifetime of user equipments by facilitating the reuse of spectrum resources between D2D and cellular links. In network assisted D2D communications, when two devices are in the proximity of each other, the network can not only help the devices to set the appropriate transmit power and schedule time and frequency resources but also to determine whether communication should take place via the direct D2D link (D2D mode) or via the cellular base station (cellular mode). In this paper we formulate the joint mode selection, scheduling and power control task as an optimization problem that we first solve assuming the availability of a central entity. We also propose a distributed suboptimal joint mode selection and resource allocation scheme that we benchmark with respect to the centralized optimal solution. We find that the distributed scheme performs close to the optimal scheme both in terms of resource efficiency and user fairness.

I. INTRODUCTION

Device-to-device (D2D) communications supported by a cellular infrastructure holds the promise of four types of gains. The proximity of user equipments (UE) may allow for very high bit rates, low delays and low power consumption [1]. Secondly, the hop gain refers to using a single link in the D2D mode rather than using an uplink and a downlink resource when communicating via the base station (BS) in the cellular mode. Thirdly, the reuse gain implies that radio resources may be simultaneously used by cellular as well as D2D links thereby tightening the reuse factor even of a reuse-1 system [1]. Finally, the pairing gain refers to the degree of freedom of selecting the UEs communicating with the BS and the UE pairs using a direct D2D link that should use the same time and frequency resources. Additionally, D2D communications may also facilitate new types of wireless peer-to-peer services [1], [2].

However, D2D communications utilizing cellular spectrum poses new challenges, because relative to cellular communication scenarios, the system needs to cope with new interference situations. For example, in an orthogonal frequency division (OFDM) system in which D2D communication links may reuse some of the OFDM physical resource blocks (PRB), intra-cell interference is no longer negligible. Solution approaches to deal with this problem include power control [3], [4], various interference avoiding multiple-input-multiple-output (MIMO) techniques [5] that can be combined with proper mode selection [6] and advanced (network) coding schemes [7]. However, to the best of our best knowledge, prior works have not proposed a global optimization model that takes into account mode selection, OFDM resource assignment and power allocation in an unified framework with the aim of minimizing the overall power consumption in a multicell OFDM system.

Therefore, the purpose of the current paper is to develop a unified optimization problem that minimizes the used sum power in an OFDM system that may reuse PRBs for D2D links. Although the proposed centralized mathematical model turns out to be strongly NP-hard and the corresponding resolution methods might be cumbersome and not-practically useful, it serves as a benchmark for the development of distributed heuristic and power control schemes in D2D multicell systems. By adopting the proposed framework, we gain insight in the potential gains of using the direct D2D link as compared to using cellular links between two communicating UEs (Tx UE - Rx UE) when transmitting in both (i.e. cellular and D2D) operational modes.

Although conceptually D2D links may use downlink (DL) as well as uplink (UL) cellular resources, UEs transmitting in DL spectrum may not be acceptable by regulatory bodies in certain geographical regions. Therefore, in this paper we focus on an OFDM system in which the same uplink PRBs may be used simultaneously for a cellular and one (or possibly more) D2D link(s) tightening the reuse factor below 1 (as in Figure 1). For a particular UE pair, the sum power minimization problem is combined with mode selection that determines whether the UE pair (Tx UE - Rx UE of Figure 1) should use the direct D2D link or they should communicate via the cellular BS. Therefore, we compare the performance of these two communications modes, i.e. cellular mode and D2D mode, and provide a simulation analysis of the impact of resource sharing, when the positions of both the D2D pair and the cellular UE vary within the cells.

II. SYSTEM MODEL

We consider the uplink of an OFDM cellular system with $N$ cells and a complete reuse of the available frequency resources among different cells and possibly also within the same cell due to D2D communications. In each cell a base station (BS) controls a fixed number of UEs consisting of cellular UEs (c-UEs) and D2D transmitters. While all c-UEs
communicate directly with the BS, D2D pairs can either exploit the direct link or communicate via the BS (see Figure 1). We represent the cellular network topology by a graph in which BSs belonging to the set $B$ are labeled with $n = 1, \ldots, N$; UEs belonging to the set $M$ with $l = 1, \ldots, L$. We denote with $\mathcal{U} \subset M$ and $\mathcal{D} \subset M$ the set of c-UEs and D2D transmitters respectively. Hence, we define with $h(l)$ the receiver of any UE $l$, i.e., the serving BS $n(l)$ for any c-UE $l \in \mathcal{U}$ and either the serving BS $n(l)$ or the D2D receiver $d(l)$ for any D2D transmitter $l \in \mathcal{D}$.

We consider the total frequency bandwidth $W$ divided into $F$ PRBs each of bandwidth $B = W/F$. Each UE $l$ can be assigned to any of the $f = 1, \ldots, F$ PRBs belonging to the set $\mathcal{F}$. We model the effective channel gain between the transmitter $l$ and its receiver when using PRB $f$ as $G_{h(l),f}$. Let $\sigma$ be the thermal noise at the PRB operation bandwidth $B = W/F$ and $P_{l,f}$ be the actual transmission power of user $l$ on PRB $f$.

The perceived signal to noise and interference ratio (SINR) on PRB $f$ for user $l$ is:

$$\gamma_{l,f}(p_f) = \frac{G_{h(l),f} P_{l,f}}{I_{l,f}(p_f) + \sigma} \tag{1}$$

where $p_f = (P_{1,f}, \ldots, P_{L,f})^T$ is the vector of powers allocated to UEs overall the cellular network and transmitting on PRB $f$, $I_{l,f}(p_f) = \sum_{j \neq l} G_{h(l),f}^j P_{j,f}$ is the interference experienced at the receiver $h(l)$ of user $l$, and $G_{h,l,f}^j$ accounts for the effect of distance-based attenuation, beamforming, fading etc., between $h(l)$ and the interferer $j$ transmitting on PRB $f$. Neglecting the details of practical coding schemes, we employ the Shannon capacity as a measure of the achievable bit rate. Specifically, given a certain SINR, the spectral efficiency in bit/s/Hz for user $l$ employing PRB $f$ is $\eta_{l,f} = \log_2(1 + \gamma_{l,f}(p_f))$ and the maximum achievable theoretical rate in bit/s is $R_{l,f} = B \eta_{l,f}$. For the sake of tractability, in this paper we disregard link adaptation (LA). Thus, we assume that PRB $f$ allows user $l$ to transmit with a spectral efficiency $\eta_{l,f} = \log_2(1 + \gamma_{l,f}(p_f))$ if the experienced $\gamma_{l,f}(p_f)$ exceeds a fixed target $\gamma_{l,f}^{\text{tgt}}$. Otherwise, user $l$ cannot transmit on PRB $f$.

We impose equal $\gamma_{l,f}^{\text{tgt}}$ for each user-PRB pair.

### III. A CENTRALIZED JOINT MODE SELECTION, RESOURCE ALLOCATION, AND POWER ASSIGNMENT FRAMEWORK

According to the underlying D2D infrastructure, we select two possible transmission modes, i.e. $q = 0$ in which a given UE communicates with the serving BS (cellular mode) and $q = 1$ in which a D2D pair communicates directly either sharing PRBs or with an exclusive assignment of PRBs (D2D mode).

Obviously, for c-UEs, $q$ is by definition always 0, while for D2D pairs mode selection can select $q = 0$ or $q = 1$. Let $x_{l,f}(q)$ be a binary variable equal to 1 if user $l$ is assigned PRB $f$ with mode $q$ and 0 otherwise, and let $P_{l,f}(q) \in \mathbb{R}$ be a positive real variable denoting the transmission power allocated to user $l$ on PRB $f$ when adopting mode $q$.

Accordingly,

$$G_{h(l),f}(q) = \begin{cases} G_{n(l),f} & \text{if user } l \text{ adopts mode } q = 0 \text{ on PRB } f \\ G_{d(l),f} & \text{if user } l \text{ adopts mode } q = 1 \text{ on PRB } f \end{cases} \tag{2}$$

A Mixed Integer Linear Programming (MILP) formulation for jointly optimizing mode selection, resource allocation and power assignment (JOMSRAP) can be derived as follows

\text{minimize} \quad \sum_{l,f,q} x_{l,f}(q) P_{l,f}(q) \tag{C1}

\text{subject to} \quad P_{l,f}(q) \leq K x_{l,f}(q) \quad \forall l, f, q \tag{C2}

$$G_{h(l),f}(q) P_{l,f}(q) - \gamma_{l,f}^{\text{tgt}} \sum_{j \neq q} G_{h(l),f}(q) P_{j,f}(q) \geq \gamma_{l,f}^{\text{tgt}} (1 - K(1 - x_{l,f}(q))) \quad \forall l, f, q \tag{C3}$$

$$\sum_{l \in n(l)} x_{l,f}(0) \leq 1 \quad \forall n, f \tag{C4}$$

$$\sum_{f,q} x_{l,f}(q) \geq R_{l,f}^{\text{min}} \quad \forall l \tag{C5}$$

$$P_{l,f}(q) \geq \frac{\gamma_{l,f}^{\text{tgt}} \sigma}{G_{h(l),f}(q)} x_{l,f}(q) \quad \forall l, f, q \tag{C6}$$

$$\sum_{f,q} x_{l,f}(q) \leq P_{l,f}^{\text{max}} \quad \forall l, q \tag{C7}$$

$$x_{l,f}(1) = 0 \quad \forall l \in \mathcal{U}, f \tag{C8}$$

$$x_{l,f}(q) \in \{0, 1\} \quad \forall l, f, q \tag{C9}$$

The objective function of JOMSRAP minimizes the overall transmission power. In Constraints (C1) and (C2), $K$ is a large positive number. Specifically, constraints (C1) force the transmission power of user $l$ on a given PRB $f$ to be 0 in case $x_{l,f}(q) = 0$. Constraints (C2) can be derived by direct inspection of Equation 1 and they state that if user $l$ is assigned to PRB $f$ with mode $q = 1$, i.e. $x_{l,f}(q) = 1$, its transmission power must guarantee that $\gamma_{l,f}(q) \geq \gamma_{l,f}^{\text{tgt}}$ (see Equation (1)). Otherwise, if $x_{l,f}(q) = 0$, the large constant $K$ ensures that constraints (C2) are always satisfied. Constraints (C3) guarantee the exclusivity of PRBs assignment for UEs selecting mode $q = 0$ and belonging to the same cell. Constraints (C4) impose that at least $R_{l,f}^{\text{min}}$ PRBs must be allocated to each user $l \in \mathcal{M}$. Constraints (C5) are redundant but enforce Constraints (C2). Constraints (C6) ensure that each user $l$ can exploit PRB $f$ by adopting one mode at most, and each UE $l \in \mathcal{U}$ is forced to adopt cellular mode $q = 0$. 

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**Figure 1.** Illustration of D2D communications, when a user equipment (UE1) and a D2D pair (Tx UE - Rx UE) may use the same OFDM PRB. Due to the D2D link, intracell interference as well as intercell interference between D2D and cellular links (UE3 to Rx UE) can be very high. (In this example assuming that the D2D link uses cellular UL resources.)
From JOMSRAP, we note that each PRBs can be assigned to at most one c-UE within a cell in order to avoid intra-cell interference among c-UEs, while we let D2D users decide whether share PRBs or not. Hence, while in classical OFDMA cellular networks the multiple access interference (MAI) can only affect users belonging to different cells, in this scenario MAI can jeopardize the system performances even within a cell. To this end, we note that in classical uplink cellular systems, interfering users are located at a distance that amounts at least to the radius of the cell, while in D2D communications MAI can operate at any distance thus leading to a potentially much heavier impairment of performances.

\textbf{Proposition 1:} JOMSRAP is NP-hard in the strong sense. 

\textit{Proof:} The proof follows by reducing the strongly NP-hard 3-partition problem [8] to JOMSRAP. \hfill\Box

IV. A DISTRIBUTED SUBOPTIMAL MODE SELECTION AND RESOURCE ALLOCATION SCHEME

As showed in Proposition 1, the resolution of JOMSRAP by means of a brute-force approach can be demanding in terms of computational complexity that renders it infeasible in most scenarios in practice. This calls for the development of more practical distributed algorithms that can be applied in a real-world system. To this end, we reduce JOMSRAP to a single-cell mode selection and resource allocation problem (S-MSRAP) in which each BS is in charge of selecting the proper mode and PRB to each mobile.

\begin{equation}
\begin{aligned}
\text{minimize} & \sum_{l,f,q} P_{l,f}(q) x_{l,f}(q) \\
\text{subject to} & \quad \sum_{n(l)} x_{l,f}(0) \leq 1 \quad \forall f \quad (C10) \\
& \quad \sum_{f,q} x_{l,f}(q) \geq R_{l}^{\min} \quad \forall l \quad (C11) \\
& \quad \sum_{q} x_{l,f}(q) \leq 1 \quad \forall l, f, q \quad (C12) \\
& \quad x_{l,f}(q) \in \{0, 1\} \quad \forall l, f, q
\end{aligned}
\end{equation}

Similarly to related constraints in Problem (2), constraints (C10) impose the exclusivity of PRB assignment for UEs, constraints (C11) impose that at least $R_{l}^{\min}$ PRBs must be allocated to each user in the cell, constraints C12 state that each user $l$ can exploit PRB $f$ by adopting at most one mode. The computation burden of JOMSRAP is distributed among the BSs that have now to solve a simple polynomial algorithm.

\textbf{Proposition 2:} The linear programming (LP) problem obtained by relaxing constraints (C13) has an integer optimal solution

\textit{Proof:} Constraints (C10) - (C12) can be grouped as

\begin{equation}
A b \leq c
\end{equation}

where $b = [x_{1,1}(0), \ldots, x_{L,1}(0), \ldots, x_{L,1}(1), \ldots, x_{L,F}(1)]$ and $c = [1, \ldots, 1, -R_{l}^{\min}, \ldots, -R_{l}^{\min}, 1, \ldots, 1]$. The proof follows from the total unimodularity of the constraint matrix $A$, i.e. every square submatrix of $A$ has determinant equal to 0,1, or -1 [9]. \hfill\Box

V. NUMERICAL RESULTS

We study an OFDM system featuring a total uplink bandwidth of $W = 5$ MHz with $N = 7$ cells. Both JOMSRAP (2) and S-MSRAP with LC have been coded in the MATLAB-based RUDIMENTARY NETWORK EMULATOR (RUNE) that has been extended to incorporate a D2D-OFDMA system. For a more accurate analysis, we consider the realistic 3GPP macro-cell propagation environment for urban and suburban scenarios presented in [14]. In Table I, we provide the input parameters of our simulation study.

Figures 2 - 3 illustrate the potential benefit of a D2D network
D2D mode. The advantage becomes negligible and not effective for $R_{D2D} \geq 120m$.

![Figure 2](image)

**Figure 2.** Comparison between cellular and D2D mode: the percentage gain of D2D against cellular mode is about 100% while progressively reduces as $R_{D2D}$ and $R_{UE}$ increase. The advantage becomes negligible and not effective for $R_{D2D} \geq 120m$.

Figure 3. Power consumption when both cellular and D2D mode are available: compared with Figure 2, mode selection allows to drastically improve the system performances.

![Figure 3](image)

**Figure 3.** Power consumption when both cellular and D2D mode are available: compared with Figure 2, mode selection allows to drastically improve the system performances.

Figure 4. Performances of S-MSRAP with LC.

![Figure 4](image)

**Figure 4.** Performances of S-MSRAP with LC.

when the optimal centralized JOMSRAP (2) is used. To solve JOMSRAP, RUNE has been integrated with the optimization solver ILOG CPLEX 12.2. Hence, we perform extensive simulations varying the distance $R_{UE}$ between each c-UE and the serving BS, as well as the proximity $R_{D2D}$ between a D2D pair. For each $\{R_{UE}, R_{D2D}\}$, we average over 500 feasible Monte Carlo experiments and we consider the channel gains constant during the JOMSRAP procedure. In Figure 2, we solve JOMSRAP by forcing cellular mode, i.e. $q = 0 \forall l$ (see Figure 2(a)), and D2D mode, i.e. $q = 1 \forall l$ (see Figure 2(b)), and compare the gain of D2D over conventional cellular communications (see Figure 2(c)). Figure 3 depicts the advantage of the jointly mode selection, PRB assignment and power allocation scheme. Such an advantage is triggered by an accurate exploitation of the multi-user diversity and mode selection. Specifically, the gain of mode selection strategy over a conventional cellular communication can be significant for $R_{D2D} \leq 150m$ and become less effective only for higher distances. The impact of mode selection is analyzed in Figure 5 and Figure 6. In particular, Figure 5 illustrates the percentage of selecting cellular mode by D2D pairs, while Figure 6 indicates the occurrence of resource sharing in D2D communications.

The performances of the distributed S-MSRAP with LC heuristic are depicted in Figure 4. Compared with the optimal results provided by JOMSRAP (Figure 3(a)), the distributed scheme achieves suboptimal performances in terms of power consumption (Figure 4(a)), but rather than relying on a central entity
to solve JOMSRAP the computational burden is distributed among the BSs in the network. As shown in Figure 4(b), LC comes into play when the distances \( \{R_{\text{UE}}, R_{\text{D2D}}\} \) are relatively high involving a reduction on the total spectral efficiency. Nevertheless, the method is able to guarantee fair share of the overall network capacity (see Figure 4(c)).

VI. CONCLUSIONS

In this paper we developed a model for a joint optimization of mode selection, resource assignment and power allocation for D2D communications underlaying a cellular infrastructure. Resource management in such a scenario is particularly challenging, since intracell and intercell interference need to be managed between the cellular and D2D layers. We used this model to develop a distributed single-cell mode selection and resource allocation algorithm (S-MSRAP) that we benchmarked relative to the centralized optimal multi-cell algorithm. We implemented the S-MSRAP algorithm in a system simulator. The numerical results indicate that the heuristic approach shows consistently good performance with respect to the optimum solution. The results also indicate that when D2D communications can reuse the cellular spectrum resources under network control, it can improve the system capacity, especially when joint mode selection and power control are used along with proper resource block allocation. The S-MSRAP algorithm effectively extends the range for which D2D communications is useful, it helps to protect the cellular layer from interference from D2D links and reduces the overall power consumption in the network.

APPENDIX

Derivation of \( \Delta P \) in Algorithm 1. For a given PRB \( f \) and iteration \( t \), we compute

\[
P_{t,f}' = \Delta \frac{P_{t,f}}{G_{\text{ME}t,f}} (I_{t,f} + \sigma) / G_{\text{ME}t,f}
\]

The SINR reduction by \( \Delta \) involves a power reduction of \( \Delta P_t = |P_t' - P_t| \). Similarly, it is possible to derive the reduction \( \Delta \eta_l = |\eta_l' - \eta_l| \) in the spectral efficiency.

<table>
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REFERENCES