Estimating subcatchment runoff coefficients using weather radar and a downstream runoff sensor

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ABSTRACT

This paper presents a method for estimating runoff coefficients of urban drainage subcatchments based on a combination of high resolution weather radar data and flow measurements from a downstream runoff sensor. By utilising the spatial variability of the precipitation it is possible to estimate the runoff coefficients of the separate subcatchments. The method is demonstrated through a case study of an urban drainage catchment (678 ha) located in the city of Aarhus, Denmark. The study has proven that it is possible to use corresponding measurements of the relative rainfall distribution over the catchment and downstream runoff measurements to identify the runoff coefficients at subcatchment level.

Key words | remote sensing, runoff coefficient, urban drainage modelling, weather radar

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INTRODUCTION

The runoff coefficient and the initial loss are essential surface parameters in urban drainage modelling of stormwater runoff. The runoff coefficient (φ) is implemented in urban drainage modelling to account for the hydrological losses except for the initial loss during rainfall. The runoff coefficient is therefore crucial for estimating the overall mass balance of the stormwater runoff.

In the context of modelling stormwater runoff from urban drainage systems, the runoff coefficient is the product of the hydrological reduction factor and the impervious fraction. When modelling the overall runoff from a catchment it is reasonable to use the runoff coefficient and not distinguish between the impervious fraction and the hydrological reduction factor.

The runoff coefficient and the initial loss are usually considered as constant in stormwater runoff modelling. However, the literature indicates that this is often not the case and consequently the runoff coefficient can be difficult to estimate directly and unequivocally (Jensen 1990; Arnbjerg-Nielsen & Harremoës 1996; Becciu & Paoletti 1997, 2000; Chaubey *et al.* 1999; Thorndahl *et al.* 2006; Chen & Adams 2007; Thorndahl 2008).

The variation of the runoff coefficient found in the literature is not only caused by poor measurements and varying hydrological surface responses. It is also due to a lack of spatial information of the rainfall (Chaubey *et al.* 1999; Thorndahl 2008). This observation is supported by Faurès *et al.* (1995) and Goodrich *et al.* (1995) who performed a joint study of the effects of spatial rainfall variability on small-scale catchments using rain gauges.

The traditional engineering approach to estimate runoff coefficients is based on a regression analysis of corresponding measurements of rainfall depth and runoff depth from multiple rainfall events obtained from a rain gauge and an *in-situ* flow sensor. However this approach is expensive and impractical when the objective is to estimate the runoff coefficients at subcatchment level. To estimate the runoff coefficients at subcatchment level it is necessary to install rain gauges and flow meters in each subcatchment.

Other approaches have consequently been presented, e.g. Rasmussen *et al.* (2008) who showed, that it is possible to estimate the runoff coefficient on the basis of the duration of multiple combined sewer overflow (CSO) events. Liu *et al.* (2011) used Geographic Information System (GIS) to optimise the runoff coefficients of GIS subareas, and Schultz (1988), Carlson & Traci Arthur (2000), Ravagnani *et al.* (2009), and Weng (2012) presented methods to estimate the impervious area using high resolution satellite imagery.

The temporal and spatial resolution of rainfall measurements required for rural and urban runoff applications to account for the rainfall variability have been studied by

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Ogden & Julien (1994) and Berne *et al.* (2004), respectively. Berne *et al.* (2004) concluded that a temporal resolution of 5 min and a spatial resolution of 3 km are required at urban catchments scales of 1,000 ha. At catchment scales of 100 ha the required resolutions are about 3 min and 2 km. These conclusions are supported by Schilling (1991) who stated that the necessary rainfall data resolutions required for urban drainage design and analysis are 1–5 min and 1 km.

The purpose of this study is to develop a method for estimating the runoff coefficients of urban drainage subcatchments for urban drainage modelling by the use of the spatial rainfall variability and *in-situ* flow measurements from a single downstream flow meter.

The method presented here will be based on already existing sensor equipment in the study catchment area; weather radar, rain gauge, and flow meter. It will also be possible to use other indirect measurements of runoff than flow meters, e.g. in-sewer water level measurements. These other indirect flow measurement methods will not be handled further in this study.

METHODOLOGY

The fundamental principle of this method is that different rainfall patterns over an urban drainage catchment will give different response hydrographs in a downstream point even if the average rain depth is the same. It is therefore possible to set up a system of linear equations to estimate the runoff coefficients of individual subcatchments on the basis of the spatial rainfall variability. The relation between rainfall and runoff is shown in Equation (1):

$$ra\,\vec{a}\,\vec{\varphi} = \vec{ro} \tag{1}$$

where,

$$ra = \begin{bmatrix} ra_{1,1} & ra_{1,2} & \cdots & ra_{1,m} \\ ra_{2,1} & ra_{2,2} & \cdots & ra_{2,m} \\ \vdots & \vdots & \ddots & \vdots \\ ra_{n,1} & ra_{n,2} & \cdots & ra_{n,m} \end{bmatrix}, \quad \vec{a} = \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_m \end{bmatrix},$$

$$\vec{\varphi} = \begin{bmatrix} \varphi_1 \\ \varphi_2 \\ \vdots \\ \varphi_m \end{bmatrix}, \quad \vec{ro} = \begin{bmatrix} ro_1 \\ ro_2 \\ \vdots \\ ro_n \end{bmatrix}$$

The input coefficients (ra) are the rainfall depth of each individual subcatchment for each rainfall event. The variables ($\vec{\varphi}$) are the individual runoff coefficients of each subcatchment, and the output coefficients ($\vec{ro}$) are the stormwater runoff depth from the whole catchments for each rainfall event. Variable $\vec{a}$ is the areas of the individual subcatchments (constants). The indexes n and m denote rainfall event (n) and subcatchment (m), respectively.

In practice it is very difficult to solve this equation system due to uncertainties between precipitation and runoff in a downstream point. However, this method requires that there is unambiguity between precipitation and runoff in a point downstream. In order to fulfil this assumption, the rainfall data are normalised to the runoff data using the mean runoff coefficient for the whole catchment and data period. The weather radar data are therefore only used to determine the spatial variation of the rainfall and thereby only used to determine the relative distribution of the mean runoff coefficient at subcatchment level.

The method consists of four steps. The first three steps are preliminary steps to prepare the data before the actual estimation of the runoff coefficients can be performed (Step 4). First, a weather radar rain gauge bias correction is performed. Secondly, the mean runoff coefficient for the whole catchment and the whole data period is calculated. Thirdly, the mean runoff coefficient is used to normalise the weather radar data to the runoff measurements.

Step 1: The weather radar Quantitative Precipitation Estimate (QPE) product is bias corrected according to rain gauges, based on rainfall event accumulations using the method proposed by Krajewski & Smith (2002):

$$\text{rg\_bias}_n = \frac{\frac{1}{E}\left(\sum e_{\text{rg}}\right)}{\frac{1}{E}\left(\sum e_{\text{ra}}\right)} \tag{2}$$

where e_{rg} is the rain gauges-based event accumulated rainfall depth, e_{ra} is the event accumulated rainfall depth of the QPE product in the corresponding pixels of the rain gauges locations, and E is the number of rain gauges used in the bias correction.

It is important to point out, that the aim of the bias correction is to adjust the QPE product locally to the catchment and thereby be able to better estimate the mean runoff coefficient (Step 2). Consequently only rain gauges located within the catchment are used for the bias correction.

Step 2: The mean runoff coefficient (φ_{mean}) is the mean relation between the rainfall depth and the stormwater runoff depth and is estimated on the basis of the whole data period. The estimation is performed by linear Least Squares Error (LSE) regression (Equation (3)). The initial

loss is neglected in this study to minimise the degree of freedom in the optimisation:

$$\mathrm{LSE} = \min\left(\sum_{n=1}^{N}\left(ro_{\mathrm{meas},n} - \varphi_{\mathrm{mean}}\, ra_{\mathrm{mean},n}\right)^2\right) \tag{3}$$

where ($ro_{\mathrm{meas},n}$) is the event runoff depth, ($ra_{\mathrm{mean},n}$) is the event mean radar rainfall depth of all radar pixels covering the catchment, and N is the total number of rainfall events.

Step 3: The compensation for the ambiguities between rainfall and runoff are performed by normalising the mean radar rainfall depth to the runoff depth by applying a bias correction factor on rainfall event basis to the radar rainfall depth. The normalising bias correction factor (Equation (4)) is calculated on the basis of ra_{mean}, ro_{meas}, and φ_{mean}:

$$\mathrm{ro\_bias}_n = \frac{\left(\dfrac{ro_{\mathrm{meas},n}}{\varphi_{\mathrm{mean}}}\right)}{ra_{\mathrm{mean},n}} \tag{4}$$

Step 4: The runoff coefficients at subcatchment level are estimated by a LSE minimisation of the difference between measured and calculated runoff (Equation (5)):

$$\mathrm{LSE} = \min\left(\sum_{n=1}^{N}\left(ro_{\mathrm{cal},n}(\vec{\varphi}) - ro_{\mathrm{meas},n}\right)^2\right) \tag{5}$$

where $ro_{\mathrm{cal},n}(\vec{\varphi})$ is the event calculated runoff volume for a specific set of subcatchment runoff coefficients. Variable $ro_{\mathrm{cal},n}(\vec{\varphi})$ is determined by Equation (6):

$$ro_{\mathrm{cal},n}(\vec{\varphi}) = \sum_{m=1}^{M} ra_{\mathrm{norm},n,m} \cdot a_m \cdot \varphi_m \tag{6}$$

where $ra_{\mathrm{norm},n,m}$ is the normalised event accumulated rainfall depth for the specific subcatchments and M is the total number of subcatchments. If multiple radar pixels are covering the subcatchment, the area weighted mean rainfall of these radar pixels is used.

The LSE minimisation is constrained by the mean runoff coefficient for the catchment. The area weighted mean of the subcatchment runoff coefficients have to be equal to the mean runoff coefficient for the whole catchment (Equation (7)):

$$\varphi_{\mathrm{mean}} = \varphi_{\mathrm{AWM}} = \sum_{m=1}^{M} a'_m \varphi_m \tag{7}$$

The boundaries of the runoff coefficients for the minimisation are 0 (lower) and 1 (upper).

CASE STUDY AREA AND DATA

The case study area is located in the city of Aarhus, Denmark. The catchment is 1,426 ha and consists of both combined (678 ha) and separate (748 ha) sewer networks, Figure 1. The runoff is only measured for the combined sewer catchments, and the demonstration of the presented method is therefore limited to this area. The combined sewer catchments are divided into seven subcatchments in this study, see Figure 1.

The area weighted average of the impervious fraction of the combined sewer catchments is 39%, estimated on the available GIS data. For the impervious fraction of each subcatchment, see Table 2.

In 2011, changes were performed in the catchment to minimise the number of CSO events. A 15,700 m^3 underground basin was constructed at Viby Wastewater Treatment Plant (WWTP) in order to reduce the number of CSO events for the catchment to one a year.

The Viby WWTP plant is equipped with a flow meter in the inlet which is used to measure the total runoff. The dry weather and infiltration flow must be removed from the measurements to isolate the stormwater runoff.

A weekly variation of the dry weather flow has been computed on the basis of dry weather flow measurements from 1 May 2011 to 1 February 2012, according to the methodology used in Jensen (1990). The dry weather flow has been separated in variation and base flow. The variation has then been subtracted from the inlet measurements. Subsequently, the base flows before and after a rainfall event are identified. The base flow during the rainfall event is estimated by linear interpolation and subtracted from the dry weather adjusted flow measurements.

The new basin at Viby WWTP was inaugurated on 2 September 2011. In February 2012, Denmark experienced several snowfalls. The data period available for this study is therefore limited to the period from 2 September 2011 to 1 February 2012. No CSO events have occurred during this period.

The radar data used in this study are derived from one 250 kW dual-polarisation C-band radar located in Virring approx. 14 km southwest of Viby WWTP. The radar is part

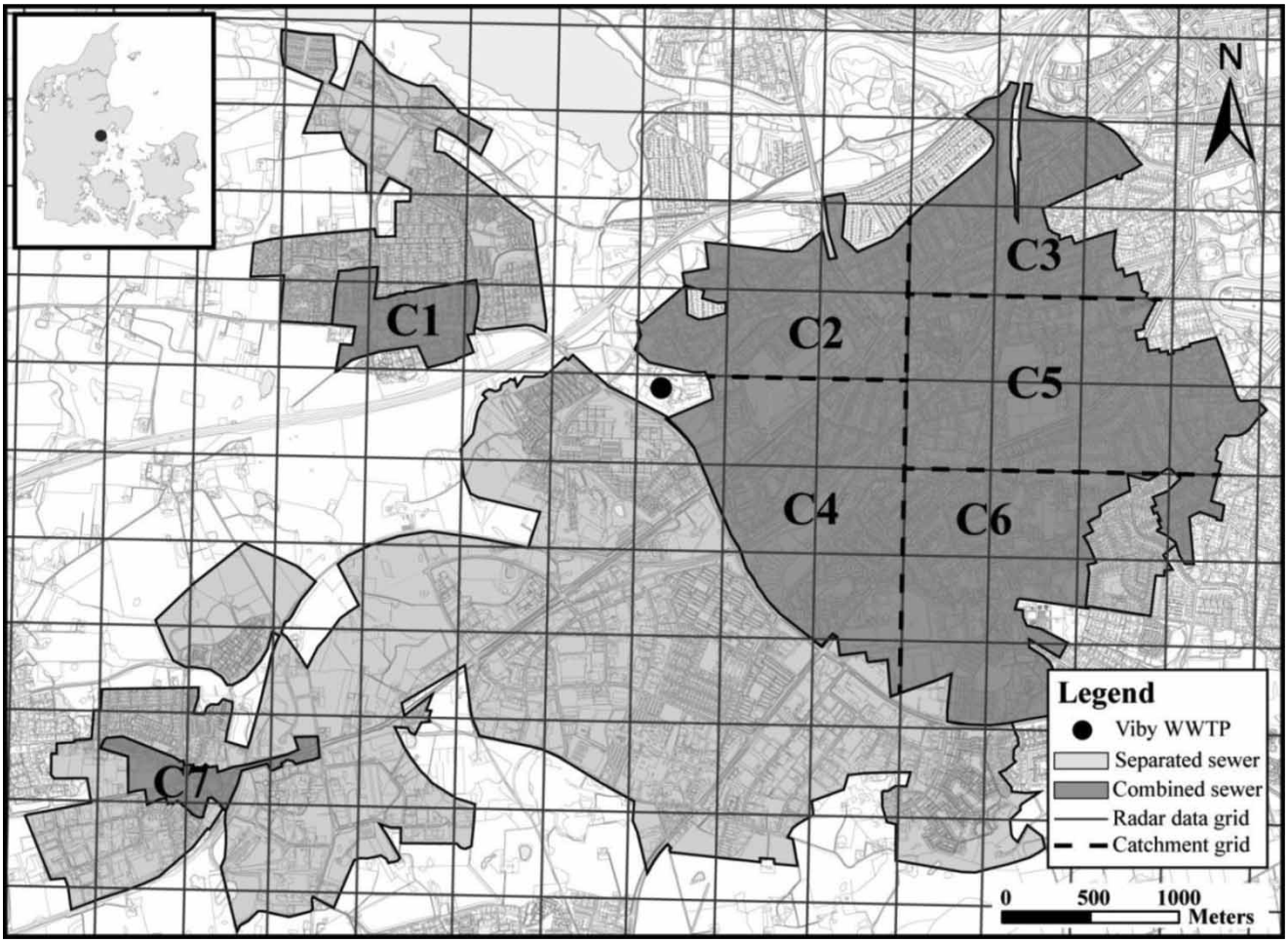

Figure 1 | Case study area, Viby Catchment, located in the city of Aarhus, Denmark.

of the Danish weather radar network owned and operated by the Danish Meteorological Institute (DMI). The radar has a total range of 240 km and a DMI QPE range of 100 km (Gill *et al.* 2006).

The applied weather radar data in this study are 1 km CAPPI layer data with a spatial resolution of 500×500 m and a temporal resolution of 5 min.

The QPE product is generated using a power law Z–R relationship ($Z = aR^b$) with standard parameters ($a = 20$ and $b = 1.6$) (Marshall & Palmer 1948). The QPE product has been bias corrected on event basis according to one rain gauge located within the catchment at Viby WWTP. The rain gauge is operated and quality controlled by DMI.

Before a rainfall event can be used, in the presented method for estimation of the runoff coefficients at subcatchment level, it has to pass some criteria.

- A complete dataset of rain gauge data, radar data, and flow measurement data in the rainfall event period have to be available. The rainfall period is defined by the stormwater runoff.
- The accumulated rainfall depth of rain gauge measurement has to be larger than 1 mm.
- The mean accumulated rainfall depth of the radar pixels covering the catchment has to be larger than 1 mm.
- The dry weather period after a rainfall event has to be longer than 3 hours to ensure complete runoff. This criterion depends on the size and runoff lag time of the specific catchment. If the dry weather period is shorter than 3 hours, the events will be treated as one combined rainfall event.

A summary of the available rainfall events in the data period are listed in Table 1.

Table 1 | Available rainfall events data, normalisation bias and normalised rainfall data

Event no.	Runoff volume [m³]	ro_{meas} Runoff depth [mm]	ra_{mean} Mean radar rain depth [mm]	Normalisation bias	$ra_{norm,\ mean}$ Normalisation mean radar rain depth [mm]
1	13,035	1.92	11.57	0.639	7.39
2	32,980	4.86	18.59	1.006	18.71
3	11,247	1.66	5.42	1.177	6.38
4	6,033	0.89	4.03	0.849	3.42
5	12,941	1.91	9.26	0.793	7.34
6	15,451	2.28	5.98	1.466	8.76
7	3,099	0.46	1.66	1.060	1.76
8	2,663	0.39	3.09	0.488	1.51
9	2,346	0.35	1.94	0.684	1.33
10	21,800	3.21	12.33	1.003	12.36
11	10,582	1.56	5.35	1.123	6.00
12	3,442	0.51	2.01	0.969	1.95
13	11,557	1.70	6.16	1.065	6.55
14	6,100	0.90	3.04	1.139	3.46
15	10,194	1.50	4.41	1.310	5.78
16	18,417	2.72	6.85	1.525	10.45
17	7,246	1.07	3.74	1.098	4.11
18	9,035	1.33	4.34	1.181	5.12
19	2,860	0.42	1.39	1.165	1.62
20	7,408	1.09	3.35	1.253	4.20
21	7,970	1.18	3.09	1.462	4.52

RESULTS

The method has been tested on the seven subcatchments shown in Figure 1 with the catchment data shown in Table 2, rows 1 and 2. Figure 2 shows the estimated mean runoff coefficient (slope).

The result of the optimisation of the runoff coefficients for each subcatchment is shown in Table 2 along with the

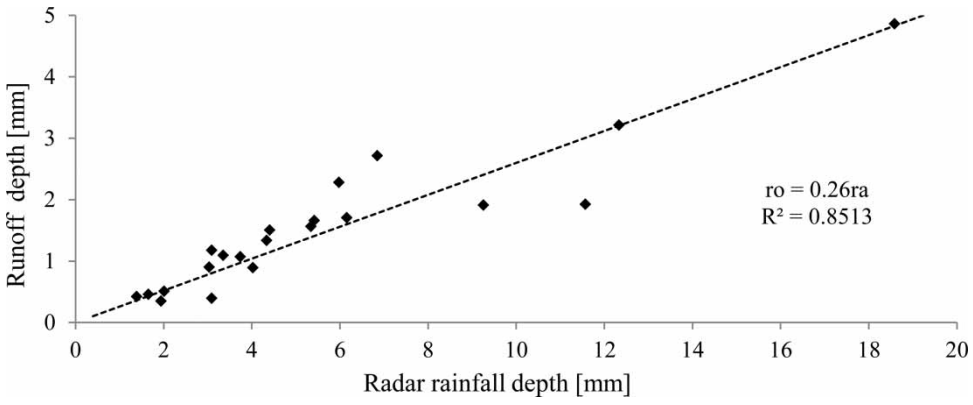

Figure 2 | Corresponding measurements of the mean radar rainfall depth and runoff depth.

Table 2 | Area, average impervious fractions based on GIS data, estimated runoff coefficients and area weighted mean (AWM) and standard deviation (STD) of the two last-mentioned parameters

Subcatchment	C1	C2	C3	C4	C5	C6	C7	AWM	STD
Area [ha]	31.2	84.0	89.2	145.5	166.7	144.7	16.9	–	–
Average impervious fraction (GIS) [–]	0.26	0.44	0.37	0.45	0.34	0.38	0.49	0.39	0.08
Estimated runoff coefficient [–]	0.25	0.31	0.32	0.24	0.23	0.22	0.57	0.26	0.12

The result of the optimisation is consistent due to the equality between the AWM runoff coefficient (0.26) and the mean runoff coefficient (slope) shown in Figure 2.

area and average impervious fraction of each subcatchment.

DISCUSSION

By comparing rows 2 and 3 in Table 2, some of the same features can be seen in the two different estimates. However a direct comparison cannot be performed because the impervious fraction does not tell anything about which areas are actually connected to the sewer system. This could cause the relative differences in the estimated runoff coefficients and the average impervious fractions. However it is also important to emphasise, that the uncertainties related to the rainfall estimates and the flow measurements also can influence the results.

The radar rainfall data are only bias adjusted to a single rain gauge located within the Viby catchment. To minimise the uncertainty in the rainfall input a larger number of rain gauges can be used with advantage. However, it is more important to use a few rain gauges within the catchment area rather than many far from the catchment. Rain gauges located far from the catchment are not representative for the catchment area and can therefore not be used to determine the precipitation over the catchment on an event basis.

The optimisation of the runoff coefficients for the seven subcatchments has been performed on 21 rainfall events. The equation system shown in Equation (1) is therefore overdetermined in theory. Consequently it should be possible to divide the catchment into more subcatchments than the seven chosen in this paper. However it is only rainfall events with different high spatial rainfall variability over the catchment which provided new information to the optimisation of the equation system.

In order to be able to estimate the number of possible subcatchments, the number of rainfall events with a different high spatial variability has to be identified. One proposal is to use the Coefficient of Variation (CV) to classify the spatial variability of a rainfall event. Additionally, the maximal number of subcatchments is also limited by the resolution of the weather radar data.

It has been possible to identify seven individual runoff coefficients with the available data period of four months. If a higher subcatchment resolution is wanted, a longer data period with more spatially distributed rainfall events is needed.

CONCLUSION

This work proposes a method for estimating the runoff coefficients at subcatchment level by the use of spatial information of rainfall obtained from weather radars. The method utilises the relative distribution of the rainfall over the catchment in combination with corresponding stormwater runoff measurements from a single downstream flow meter. Varying rainfall distributions will result in different runoff responses. Hence it is possible to optimise the runoff coefficients of the system under the assumption of unambiguity between rainfall and runoff.

The study has proven that it is possible to identify realistic runoff coefficients at subcatchment level in a 678 ha city catchment. The estimated runoff coefficients, found by the means of the presented method, are reasonable when compared to the impervious fractions of the subcatchments.

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