Abstract: A number of channel estimation algorithms for iterative receivers are compared for the case of an up-link orthogonal frequency division multiplexing (OFDM) system. Both pilot based algorithms, used to obtain an initial estimate, as well as semiblind decision-directed algorithms working as a component of the iterative receiver are considered. Algorithms performing either joint minimum mean square error (MMSE) channel estimation, or iterative estimation using space–alternating expectation maximization (SAGE), are evaluated. The considered algorithms differ in terms of complexity, as well as performance. The complexity versus performance tradeoff is at the focal point. There is no single channel estimator providing the best trade-off and the analysis shows how the system load (number of users) and the SNR influence the estimator choice.

Keywords—Channel estimation, OFDM-IDMA, algorithm complexity, expectation maximization (EM), SAGE, discrete prolate spheroidal (DPS) sequences.

I. INTRODUCTION

In recent years a new multiple access technique, where the users are separated through their unique interleaving patterns, has generated a large interest in the research community. The technique, referred to as interleave-division multiple access (IDMA) [1], has been shown to mitigate multiple access interference while simultaneously achieving a high spectral efficiency. IDMA shares many properties with code division multiple access (CDMA), where user separation is obtained through user-specific spreading codes, and has shown similar performance but with a reduced receiver complexity [1]–[4].

When the system bandwidth grows in single carrier systems, the equalization process becomes increasingly challenging due to the increase in the number of resolvable paths. Introducing orthogonal frequency division multiplexing (OFDM) simplifies this task by transforming the wideband channel into a set of orthogonal narrow band subchannels. A simple scalar equalization can then be performed separately for every subchannel. By combining IDMA and OFDM, an efficient mult-user system is formed which efficiently combats ISI and also reaches a high spectral efficiency [5], [6].

For interference cancellation and equalization in such systems, reliable channel estimates are needed. Channel estimates are usually obtained solely based on pilot symbols, which are known to the receiver. With breakthrough of turbo-like receivers, iterative decision-directed approaches to channel estimation have received increasing attention [7]. By using decoded symbols as pilots, more reliable estimates are obtained, at the same time as the pilot overhead is reduced. For a multi-user system that performs iterative MUD, such as OFDM-IDMA, iterative channel estimation can be incorporated in a straightforward way into the receiver structure.

There has only been limited research conducted on the performance of OFDM-IDMA systems employing channel estimation. For example, in an estimator based on the least mean square (LMS) algorithm is used, and in a least square (LS) estimate is performed in every iteration. Both methods iteratively perform per-user channel estimation using symbol estimates from the channel decoder. The iterative decision-directed channel estimation algorithms, and an evaluation of different algorithms is performed.

The expectation maximization (EM) like algorithm is performing MMSE based estimates on soft interference canceled single-user streams. Furthermore, due to shared properties with OFDM-CDMA, and multiple transmit antenna systems, algorithms available for these technology may be adopted, as will be discussed below. In a discrete Fourier transform (DFT) based estimator for an OFDM system with transmit diversity is developed. The estimator jointly estimates the channels to all antennas. To reduce complexity, a related estimator based on the EM algorithm is proposed, where the channels are estimated per user through indirect interference cancellation. Related to this work, a similar algorithm is proposed for multi-carrier CDMA systems in, and for multiple antenna systems. In a channel estimator that jointly estimates all user channels is developed for OFDM-CDMA, based on a low-rank discrete prolate spheroidal (DPS) sequence approximation of the channel. The algorithm is extended to multiple-input multiple output (MIMO) OFDM systems in and further developed to utilize time and frequency correlation.

The DPS sequences are used to efficiently exploit the frequency correlation. The algorithms all
make use of estimates on the transmitted symbols, and the receiver therefore needs to acquire an initial channel estimate. Three pilot based algorithms, based on well known principles, are evaluated for this purpose. The aim is not only to investigate algorithm performance, but also how their implementation complexity relates to performance. Due to limitations in power consumption and chip area, complexity considerations are of high importance when algorithms are implemented in real systems. The main contributions of this paper may be summarized as follows:

- Three different decision-directed channel estimation algorithms are evaluated in an OFDM-IDMA system for the first time. One of the algorithms jointly estimates the channels for all users, while the others perform peruser estimates based on the space-alternating EM (SAGE) algorithm. Different algorithms for obtaining an initial pilot based channel estimate are also evaluated. The algorithms effect on the overall system performance and convergence is studied, along with their complexity.

- A complexity versus performance analysis is performed, where the total number of complex multiplications needed to reach a bit error rate (BER) target is evaluated. For the evaluation, the complete receiver complexity incorporating channel estimation, MUD and data decoding is considered.

II. SYSTEM DESCRIPTION

In order to perform a comparison between the channel estimation algorithms, the studied system needs to be defined. First the OFDM-IDMA model is presented, followed by the channel model and its low-rank approximation.

A. OFDM-IDMA system model

The system considered in this paper is an uplink OFDM-IDMA system, as shown in Fig.1. It consists of K users transmitting blocks of S OFDM symbols, each with M sub-carriers. The first Sp OFDM symbols are reserved for pilots, known to the receiver, while the following S – Sp OFDM symbols contain data. The total number of signal constellation points per block becomes \( L = (S - Sp)M \). Low rate code words are formed by concatenating a forward error correcting code (FEC) with a repetition/spreading code. The code words are interleaved using, randomly generated, user specific interleavers, \( \pi_k \), of length \( 2L \), before mapped to QPSK symbols. Although QPSK is chosen here, the extension to other constellation sizes is conceptually straightforward.

After OFDM modulation and pilot insertion, the users transmit their signals over a frequency selective block fading channel. Furthermore, to allow for correct OFDM demodulation at the receiver, the users are assumed to be synchronized so that the difference in arrival times is less than the duration of the cyclic prefix minus the channel delay spread. The received signal, after appropriately demodulated into the complex baseband, is given by:

\[
r[m, s] = \sum_{k=1}^{K} h_k[m, s] x_k[m, s] + w[m, s],
\]

where \( h_j[m, s] \), \( x_j[m, s] \) and \( w[m, s] \) represent the complex valued channel coefficient, transmitted symbol and noise, respectively, at sub-carrier \( m \) in OFDM symbol \( s \). The complex valued noise ised according to \( \text{CN}(0, \sigma^2_w) \). Collecting the signal for all subcarriers, an equivalent description of the received signal is:

\[
r[s] = \sum_k X_k[s] h_k[s] + w[s],
\]

where \( X_k[s] \) is an \( M \times M \) diagonal matrix containing the transmitted symbols, on all subcarriers, from user \( k \), \( h_k \) the channel frequency response for user \( k \), and \( w[s] \) a vector containing noise.

Using the received baseband signal, the receiver implements an iterative MUD with channel estimation. Going through the receiver structure, as shown in Fig. 1, an initial pilot based estimate is first obtained. Using this, the receiver implements a per-symbol parallel interference canceling elementary signal estimator (PIC-ESE)[4]. The ESE, as detailed in Appendix A, models the interference plus noise as a complex Gaussian process and produces extrinsic log-likelihood ratio (LLR) outputs of the transmitted code bits. After the ESE, the peruser LLR streams are deinterleaved and despread before being fed to the soft-input soft-output (SISO) decoders. The extrinsic information output of the decoders are then respread and reinterleaved before fed back to the ESE and to
the second stage of the channel estimation process. The soft symbols are then used to update the channel estimates and LLRs. The channel estimation process is divided into two parts, one pilot based, and one decision-directed.

III. PILOT BASED CHANNEL ESTIMATORS

The Sp pilot OFDM symbols are transmitted at the beginning of each data block. The pilot symbols are used to obtain an initial channel estimate, which is used as a starting point for the iterative process in the receiver.

A. Per-user LS estimation

The conceptually simplest and most straightforward estimation approach is to perform a per-user LS estimate, based on the received signal and the transmitted pilot symbols, while ignoring the interference from the other users.

\[ r_k[s] = X_k[s]U \psi_k + w_k[s] \]

to represent the received signal for user k, where \( w_k[s] \) collects the interference plus noise. The LS estimates are then given by

\[ \hat{\psi}_k[s] = U^H X_k^H [s] r_k[s], \]

and equivalently for the frequency response

\[ \hat{h}_k[s] = U U^H X_k^H [s] r_k[s], \]

Note that the operation of U followed by UH may be seen as a subspace filtering, removing all received energy outside the channel space. If consecutive pilot OFDM symbols are transmitted, time-averaging is used to further improve performance.

A low-complexity version of the algorithm is also considered. By using an identity matrix instead of the DPS sequences in the algorithm, i.e., setting \( U = IM \) in (6)-(8), the frequency filtering and its associated cost is removed. Though this modification leads to a reduced complexity, the loss in performance is also significant.

B. Per-user SIC based LS estimation

In the second pilot based algorithm, SIC is used in an attempt to decrease the effect of inter-user interference. By estimating one user channel at the time, and removing that user’s signal component, the interference will decrease while going through the users. The procedure may be repeated in additional iterations \( i \), using the output of the last step to reinitialize the first, for improved performance.

C. Joint MMSE estimate

For the two previous algorithms, the channel is estimated per user, either by ignoring interference, or by trying to cancel it. In the last pilot based algorithm, the channels for all users are estimated jointly, taking inter-user correlation into account. Based on the model of the received signal given in (5), the MMSE estimate of \( \psi \) is produced.

IV. DECISION-DIRECTED CHANNEL ESTIMATION

The different algorithms for performing decision-directed channel estimation represented. The first algorithm, referred to as Full MMSE, performs a joint MMSE estimate of the channel coefficients for all users based on both pilots and soft estimates of the data symbols. The second algorithm iteratively obtains the maximum-likelihood (ML) solution, using SAGE based on pilots and hard decisions of the decoded data symbols, and is referred to as SAGE ML. The third algorithm, SAGE MMSE, borrows ideas from the first two and computes an estimate in a similar way as SAGE ML but using soft estimates of the data symbols.

A. Joint MMSE estimator using soft decisions (Full MMSE)

When estimating the channel, the optimal and at the same time most costly approach is to estimate the channel for all users jointly.

\[ \hat{\psi} = (\hat{\Sigma}^H \Delta^{-1} \hat{\Sigma} + C_{\psi}^{-1})^{-1} \hat{\Sigma}^H \Delta^{-1} \tau, \]

B. SAGE based estimator (SAGE ML)

The algorithm presented in the previous section requires the creation and multiplication of large matrices, and a matrix inversion, leading to a large algorithm complexity.

\[ r_k[s] = X_k[s]U \psi_k + w_k[s], \]  for \( k = 1, \ldots, K, \]

C. Modified SAGE estimator (SAGE MMSE)

The drawback of SAGE ML is that it does not support direct use of soft symbol estimates. In this section, an algorithm is proposed where SAGE ML is altered to support the use of soft symbols by deriving the MMSE solution instead of the ML solution in (12). The resulting algorithm share some properties with the single-carrier algorithm presented in [13].

\[ r_k = \Sigma_k \hat{\psi} + \hat{\Sigma}_k, \]

V. SIMULATION RESULTS

The evaluations of the proposed algorithms are performed using system simulations. In the simulations, each user transmits code words covering \( S = 19 \) OFDM symbols, with \( M = 256 \) subcarriers. One OFDM symbol is dedicated for training information, i.e., \( Sp = 1 \), which is generated randomly for each user.

A rate 1/2 convolutional code with generator polynomial \( (7,5)8 \) is used, followed by a rate 1/8 repetition code (spreading). Since QPSK is used as modulation, each code word consists of 608 information bits. With the chosen coding and spreading rates, the maximum number of users is
observed to be K=17, given perfect channel state information (PCSI) at the receiver. Similar numbers are observed in [6] and [5].

For the simulations, a fading multi-path channel model, mimicking a rich scattering environment, is used. The channel impulse response for user k is given by

\[ g_k(\tau) = \sum_{p=0}^{P-1} \alpha_{p,k} \delta(\tau - \tau_{p,k}), \]

Where \( \alpha_{p,k} \) are zero-mean complex Gaussian random variables with an exponential power delay profile, \( \theta(\tau_{p,k}) = Ce^{-\tau_{p,k}/\tau_{rms}} \), where C is a constant, and the delays \( \tau_{p,k} \) are uniformly distributed within the cyclic prefix (CP). In this paper, the length of the channel, normalized to the symbol duration, is \( \tau_{max} = 0.1 \), the root mean square delay spread set to \( \tau_{rms} = 0.03 \), and the number of multi-path components \( P = 100 \). The channel delay is assumed to be no longer than the cyclic prefix, and the block fading channel is generated independently for each user. The number of DPS sequences used in the channel estimation process is given by \( \tau_{max}M+1 = 27 \), which is a sufficient number as discussed in Section II-B. It is interesting to note that for the given choice of parameters, the channel is initially under-sampled if \( K > 9 \). But as will be seen, with the assistance of estimated data symbols, accurate channel estimates can be obtained.

In Section III three different pilot based algorithms were presented. Here, the effect of the initial estimates on the overall system performance is investigated, and Fig. 2 shows the BER when different pilot based estimators are used. For the comparison we are only presenting the performance obtained when using SAGE ML in the decision-directed mode. Similar results are observed for the other two algorithms. To illustrate the impact of user load, results are shown for \( K = 6 \), 10 and 14 users, at an \( Eb/N_0 = 6dB \). The SIC based estimator is set to perform 3 internal iterations, giving a tradeoff between reasonable interference cancellation at high SNR and poor convergence at low SNR. As can be observed in Fig. 2, the difference between the algorithms is relatively small for \( K = 6 \) users, with a possible save of 1–2 iterations. When increasing the user load, the LS estimator not making use of the frequency correlation, i.e., \( U = IM \), is observed to degrade. Due to the poor initial estimate, the convergence speed is decreased and the error floor is elevated by more frequent convergence failures. This is especially evident at \( K = 14 \) users.

Overall, the joint MMSE algorithm shows the best performance, especially at high user loads, followed by the LS-SIC. At a user load of \( K = 14 \), the interference cancellation process has an insignificant gain due to unreliable cancellation. Thus, the LS and LS-SIC algorithms show a similar performance.

B. Performance of the decision-directed algorithms

Having covered the pilot based estimators, the performance of the decision-directed algorithms is investigated next. In this investigation, the initial estimate is obtained using the joint MMSE approach. Again, \( S = 20 \) and \( Sp = 10 \) OFDM symbols, \( M = 256 \) subcarriers. In Fig. 3, the BER is shown for \( K = 14 \) users, at \( Eb/N_0 = 6dB \). For comparison, single user performance with PCSI at the receiver is also shown. As seen, with the chosen pilot density, using decoded data in the estimation process greatly improves performance. The exception being for SAGE ML with \( U = IM \), where convergence is not observed. The other algorithms reach close to single user performance after 20–30 iterations.

Amongst these, Full MMSE has the fastest convergence. SAGE MMSE, using soft decisions, shows the best performance among the SAGE based estimators.

To begin with, the impact of the initial pilot based estimate is investigated by looking at the system performance. Then, the performance of the decision-directed channel estimation algorithms is evaluated. The results show both the BER, as well as mean square estimation error (MSE), after each receiver iteration, averaged over all users in the system. Different number of users \( K \) and SNR per bit, \( Eb/N_0 \), are used to investigate the performance of the algorithms. Here, \( Eb \) is the average received energy per bit.

A. Influence of the initial channel estimate

![Fig. 2. The BER is shown for the decision-directed SAGE-ML estimator for 6, 10 and 14 users. Different algorithms are used to obtain the initial pilot based estimate, and the results are shown for \( S = 20 \) OFDM symbols, \( Sp = 10 \) OFDM pilot symbol, \( M = 256 \) subcarriers and at an \( Eb/N_0 = 6dB \).](image)

![Fig. 3. Convergence in terms of BER, when using the different decision-directed algorithms with \( S = 20 \) OFDM symbols, \( Sp = 10 \) OFDM pilot symbol, \( M = 256 \) subcarriers. The initial estimate is obtained using the joint MMSE estimator, and the case of PCSI and when pilots only are used in the CI, are shown for comparison.](image)
CONCLUSION

A number of channel estimation algorithms for OFDM-IDMA have been evaluated in terms of algorithm complexity and system performance. The channel estimation procedure is divided into two parts, one initial part where the estimate is obtained using pilot symbols, and one decision-directed part where both pilots and estimates of the transmitted symbols are used. For the initial pilot based algorithms, the best performance is obtained by the joint MMSE estimator, which jointly estimates the channel for all users. Overall, the algorithm also gives the best trade-off between complexity and performance, given that the estimator matrices are pre-calculated and stored in memory. For the decision-directed algorithms, the best performance, in terms of convergence speed, is obtained using the Full MMSE estimator. Taking complexity into account, SAGE ML is the best choice in most situations. If operating on the limit of the maximal system load, the SAGE MMSE estimator provides the best trade-off. SAGE ML is most attractive due to its low complexity, and by allowing the frequency filtering to be switched on/off, the complexity can be further reduced at low user loads.

REFERENCES


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