Localization Algorithm for Mobile Nodes in Wireless Sensor Networks Based on Discrete-Time H_\infty Filtering and Dynamic Node Model

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Abstract — Precise localization of mobile nodes in uncertain conditions is a fundamental and crucial topic in wireless sensor networks. In this paper a discrete-time H_\infty filtering and dynamic node model based algorithm is proposed. Accurate and complicate network models are not required and no assumptions are needed for the external noise characteristics but only have to be seen as energy-limited. State and measurement equation of unknown node are built with basic kinematic property and sensor node measurement method, including the impact of environment random uncertainties and node connection failure. The position of the mobile node is estimated by the filter using an integration of position information from other assisting nodes. Complying to Linear Matrix Inequality (LMI) criterion, a theorem of H_\infty filter designed for stochastic uncertain network is given. From the dynamics of the node, the solution existence of the proposed filter is obtained, and a low computational complexity method to get the optimum solution from the filter is provided in a simple motion model. The simulation results show that this method not only can achieve highly reliability but also the better localization accuracy under stochastic uncertainty.

Index Terms—WSN, mobile node localization, H_\infty filtering, simple kinematic model

I. INTRODUCTION

In recent years, wireless sensor networks (WSNs), have been widely used in a range of applications, such as military action, medical treatments, and the monitoring of animal activity and environment changes in the forest. In particular, location-based applications are among the first and most popular applications of WSNs [1], [2].

Many localization schemes have been proposed in the past few years, most of which are designed for static sensor networks [3]-[7]. They need a large density of anchor nodes around each unknown node, such as Centroid, APIT. Also network flooding and well-regulated nodes distribution, such as DV-HOP, Amorphous, are essential in others which is unrealistic. These kinds of algorithms need to update node's location information frequently, which consume majority of hardware and energy resources and further reduce the responding speed of the network, as well as the localization accuracy.

However, sensors are supposed to be highly dynamic and location aware in most applications. For example, in target tracking, sensor nodes detect surrounding areas by tracking positions of moving objects. In addition, once sensors are set to be mobile the sensing region will be enlarged. Thus, a specially designed mobile localization algorithm is highly required in dynamic WSN.

In this paper a discrete-time H_\infty filtering and dynamic node model based mobile algorithm is proposed. Accurate and complicate network models are not required and no assumptions are needed for the external noise characteristics but only have to be seen as energy-limited. The position of the mobile node is estimated by the filter using an integration of position information from other assisting nodes. Complying to Linear Matrix Inequality (LMI) criterion, a full theorem of H_\infty filter designed for stochastic uncertain network is given. From the dynamics of the node, the solution existence of the proposed filter is obtained and a low computational complexity localization algorithm to get an optimum coordinate solution from the filter is provided.

The remaining paper is organized as follows: Section II describes related work about mobile localization algorithms in WSN. System modeling and algorithm formation are given in Section III. Simulation validations and conclusions are described in Section IV and Section V respectively.

II. RELATED WORK

The existing localization schemes can be mainly divided into two parts: range-based and range-free [8]. The former depends on calculating absolute distances or angles between two nodes. Each node can estimate distance by Angle of Arrival (AoA), Time of Arrival (ToA), Time Difference of Arrival (TDoA), or Received Signal Strength Indicator (RSSI). In ToA and TDoA methods, distance between any two sensors can be calculated using the velocity of signal and the signal...
propagation time interpolation between them. In AoA, each sensor node is equipped with directional antenna and estimates the relative angle with its neighbors. Sensor nodes use the RSSI [9] value to reckon real physical distance accurately. Once distance information between nodes can be obtained, coordinates calculation will be finished by triangulation or rigidity rule [10]. On the contrary, range-free localization algorithms estimate node position only by network connectivity instead of absolute distance or angle information. Each sensor node confirms the connectivity with neighbors by transmitting a packet and estimates its location by gathering information from them. Although its accuracy is not as well as range-based it is easy to implement and needs less energy consumption than range-based. So it is more suitable for WSN environment and become main research directions of localization algorithms recently. Until now many range-free schemes have been proposed but only for static sensors, in which unknowns estimate their positions by collecting the coordinates of neighbor anchors. Unknown nodes can receive information packages directly from near anchors which are defined as neighbors. Centroid, APIT, DV-HOP, Amorphous, and MDS-MAP all belong to this kind [11].

Mobile node localization algorithm is highly needed but no perfect schemes have appeared. A classic Monte Carlo Localization (MCL) scheme specifically designed for mobile sensor networks is proposed in [12]. In MCL, all sensors are supposed to be mobile. There are two basic assumptions to make mobile sensors simple. One is that the time is divided into several time slots. The other is each sensor’s maximum moving distance in each time slot is no more than a specified value. Each anchor node periodically forwards its physical location to two-hop neighbors. Unknown nodes collect the location coordinates of their one-hop and two-hop anchor nodes by message exchange, and a new possible location set is constructed in each time slot. The possible location set consists of various coordinates where unknown nodes may locate. They are also constrained by the communication range of anchor nodes and the moving region of location set in the previous time slot. But the defect of MCL is that the localization error is high when the anchor density in the network is very low.

A Monte Carlo Box (MCB) algorithm based on MCL is designed to solve the problem in [13]. In MCB algorithm anchor constraints are bounded by a square which is called an anchor box. By constructing anchor boxes and sample box in two hops of anchors it can reduce sampling area and increase sampling rate effectively. But it has fixed sample times which results in low efficiency, long cycle and large amount of computation that can not to be widely used in energy limited wireless sensor nodes. Two more main drawbacks can be listed as following. Firstly in WSN with low anchor density, each unknown gets tighter anchor constraint, which makes the estimated location error become larger. Secondly in WSN with high anchor density conversely, unknowns get more location constraints from anchors, which results that the most possible located region of the unknowns will be smaller. And also MCL and MCB both cannot reduce the localization error if the two have the same limit number of valid samples.

Multi-hop based Monte Carlo Localization (MMCL) [14] is another range-free algorithm which uses Monte Carlo method combining principles of MCL and DV-Hop and possessing advantages of the two. In MMCL average distance of each two nodes can be obtained by DV-Hop and the area where unknowns may locate will be sure by using each average hop distance and hops between anchors and unknowns. In the end, possible coordinate information of unknown nodes will be filtered by basic MCL. The drawback of MMCL is that the communication range of each node should be known in advance which is hard to be realized and also introduces error accumulation. Mobile and Static Sensor Networks Localization (MSL) [15] realizes localization based on the weight of particles in the progress of construction and sampling. The performance is improved by one and two orders of neighbor nodes. MSL* is also derived from MCL which uses two kinds of neighbor nodes from unknown nodes, which means it uses the information collected from all one hop and two-hop neighbors of unknown nodes and anchors [16]. Its two main drawbacks are also as following. Firstly MSL* will have lower location accuracy in high dynamic environment. Secondly MSL* needs a lot of communication cost in forwarding location information packages. Additionally both of MSL and MSL* need high energy consumption and complicated calculation.

III. SYSTEM MODELING AND ALGORITHM FORMATION

In this section, the statement of the problem and H-filtering and dynamic node model based localization algorithm are introduced and discussed. In order for clarity, required motion and kinematics relationships of the mobile node in three-dimensional space are given.

A. Parameter Definitions

Fig. 1 depicts a node moving statement with velocity $V_i$ in three dimensional spaces. $\{I\}$ denotes a global coordinate system. The following symbols are used.

\[ P = [x, y, z] \] : Measured inertial position of the moving node in three dimensional space. $[\{\bullet\}]$ denotes the transpose of a matrix.

\[ P = [x, y, z] \] : Measured inertial position of the moving node in three-dimensional space. $[\{\bullet\}]$ denotes the transpose of a matrix.
\( V_s \): Measured inertial instantaneous velocity of the moving node in WSN.
\( \alpha \): Unknown node acceleration
\( w \): External disturbance with unknown statistical character but only bounded energy.
\( v_{nx}, v_{ny}, v_{nz} \): Inertial instantaneous node velocity along axis line \( OX, OY, OZ \) respectively.
\( \alpha_{nx}, \alpha_{ny}, \alpha_{nz} \): Unknown node’s acceleration along axis line \( OX, OY, OZ \) direction respectively.
\( \phi \): Angle between node moving direction and axis line \( OZ \).
\( \theta \): Angle between vehicle velocity projection on plane \( XOY \) and \( OX \).

**B. Network Mobile Model**

According to the geometry of the mobile node localization problem in Fig. 1, the following equations can be easily obtained by resolution of velocity \( V_s \).

\[
\begin{align*}
v_{nx} &= V_s \sin \phi \cos \theta \\
v_{ny} &= V_s \sin \phi \sin \theta \\
v_{nz} &= V_s \cos \phi
\end{align*}
\]  

(1)

By using kinematic laws and discretization method the continuous mobile node localization problem will be made discrete by sampling with time slot \( T \), the following relations can be fixed:

\[
\begin{align*}
x_{s}(k+1) &= x_{s}(k) + T \cdot v_{nx}(k) + \frac{1}{2} \alpha_{nx}(k) \cdot T^2 \\
y_{s}(k+1) &= y_{s}(k) + T \cdot v_{ny}(k) + \frac{1}{2} \alpha_{ny}(k) \cdot T^2 \\
z_{s}(k+1) &= z_{s}(k) + T \cdot v_{nz}(k) + \frac{1}{2} \alpha_{nz}(k) \cdot T^2
\end{align*}
\]  

(2)

As in (2) \( P_s(k+1) = [x_{s}(k+1), y_{s}(k+1), z_{s}(k+1)] \) denotes node real position at time slot \( (k+1) \). In the same way \( P_n(k) = [x_n(k), y_n(k), z_n(k)] \) denotes node position at time slot \( k \). \( \{v_{nx}(k), v_{ny}(k), v_{nz}(k)\} \) meanwhile denotes instantaneous node velocity along coordinate axis line \( OX, OY, OZ \) respectively at time slot \( k \).

\( \{\alpha_{nx}(k), \alpha_{ny}(k), \alpha_{nz}(k)\} \) is denoted as unknown real mobile node acceleration along coordinate axis line \( OX, OY, OZ \) respectively at time \( k \).

**C. Problem Description**

The purpose in this part is as follows.

Given the observation position \( P_s(k) \) and \( V_s(k) \) at time slot \( k \), predict the position \( P_s(k+1) \) at time slot \( (k+1) \). That is to say how to design a filter to recognize the trajectory of unknown node moving in complicated WSN environment only with system uncertainties and unknown external disturbance in three-dimensional space.

The equations listed above in system (2) all have the same form and all the equations are independent with each other, so we can simplify system (2) into unified one, which means three-dimensional node localization problem will be changed into one-dimensional target observation problem and rewrite system (2) into system (3).

\[
L(k+1) = L(k) + T \cdot V(k) + \frac{1}{2} A(k) \cdot T^2
\]  

(3)

where all parameters in equation (3) are the following.

\[
L(k+1) \in \{x_s(k+1), y_s(k+1), z_s(k+1)\}
\]

\[
L(k) \in \{x_s(k), y_s(k), z_s(k)\}
\]

\[
V_k \in \{v_{nx}(k), v_{ny}(k), v_{nz}(k)\}
\]

\[
A(k) \in \{\alpha_{nx}(k), \alpha_{ny}(k), \alpha_{nz}(k)\}
\]

We define the following for convenience.

\[
X(k) = \left[ L(k) \ V(k) \right] \ W(k) = \left[ A(k) \ w \right]^T
\]  

(4)

In this way the state space model of system (2) and the observation equation can be written as

\[
X(k+1) = (A + \Delta A) X(k) + B W(k)
\]

\[
Y(k) = (C + \Delta C) X(k) + D w
\]

\[
Z(k) = LX(k)
\]  

(5)

And \( X(k) \) is the state, \( Y(k) \) is observation and \( Z(k) \) is the state combination to be estimated. \( \Delta A, \Delta C \) represent the system uncertainty and measurement uncertainty respectively. Combining (2), (3), (4) and (5), (6) will be obtained:

\[
A = [1 \ T \ 0 \ 1], B = [T^2/2 \ 1], C = [1 \ 1], D = [1 \ 1], L = [1 \ 1 \ 0](6)
\]

In this way the key problem in this algorithm is to be thoroughly investigated in this paper will become as: Given the observation \( F(k) \), design a filter for system (5) to realize localization of \( X(k) \) with system uncertainties and unknown external disturbance in three-dimensional space. \( X(k) \) is the position and velocity of unknown node in WSN.

**D. Proposed Algorithm**

We will give an accurate robust discrete-time H\(_\infty\) filtering algorithm with stochastic uncertainties. Consider the uncertain stochastic discrete-time system:

\[
\begin{align*}
x_{s,k+1} &= (A + E[\beta_1]) x_s + B w_k \\
y_s &= (C + E[\alpha_2]) x_s + D v_s \\
z_s &= L x_s, \quad k = 0, 1, 2, \ldots
\end{align*}
\]  

(7)

where \( y_s \in \mathbb{R} \) is measurement, \( x_s \in \mathbb{R} \) is the state, \( v_s \in \mathbb{R} \) is measurement noise, \( w_t \in \mathbb{R} \) is the input noise, and \( z_s \in \mathbb{R} \) is state combination to be estimated. Also \( x_0, w_t \) and \( v_s \) are uncorrelated noises not only with

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bounded energy. \( A, E, B, C, F, D, L \) are fixed known matrices of appropriate dimensions. Also \( E(\beta_i) \) and \( E(\alpha_i) \) represent system uncertainty and measurement uncertainty respectively. \( E(\beta_i) \) and \( E(\alpha_i) \) are both uncorrelated standard random sequences with mean zero which satisfies following equation.

\[
E(\beta_i, \beta_j) = \delta_{ij}, E(\alpha_i, \alpha_j) = \delta_{ij}, E(\alpha_i, \beta_j) = \lambda_i \delta_{ij} \quad (8)
\]

In (8), \( E(\cdot) \) denotes the mathematical expectation. The core idea of \( H_\infty \) filtering is to design an estimator to estimate the unknown state combination via output measurement, which guarantees the gain and includes external disturbance and estimation error, to be less than the prescribed level. The performance index is defined as following.

\[
J = \inf_{\hat{X}_k} \sup_{x_0 \in \Omega_0} \int_0^T \int|e_k|^2 \, dt < \gamma^2 \quad (9)
\]

In (9) above, \( T_c/F_r \) is the mapping function from unknown prior information \( \{(x_0 - \hat{x}_0), \xi_k \} \) to filtering error \( e_k = \hat{x}_k - L_x x_k \). The initial estimation error matrix is

\[
\begin{bmatrix}
-P & 0 & A'P - C'Q' & 0 \\
0 & -\gamma^2 I & B'P - D'Q' & 0 \\
PA - QC & PB_i - QD_i & -P & 0 \\
0 & 0 & 0 & -P \\
PA - QC & 0 & 0 & 0 \\
L & 0 & 0 & 0 \\
PE - \lambda QF & 0 & 0 & 0 \\
-\lambda QF & 0 & 0 & 0 
\end{bmatrix} < 0 
\quad (15)
\]

Let \( \gamma^2 = \varphi \), the LMI (15) can be written down as:

\[
\psi(P, Q, \varphi) < 0 
\quad (16)
\]

Then the optimal filter design could be obtained by solving the following convex optimization problem:

\[
\begin{aligned}
\min_{\delta_{ij} \rightarrow 0} \varphi \\
\text{subject to LMI (15)}
\end{aligned}
\quad (17)
\]

With the mathematical software the robust \( H_\infty \) filter design can be obtained with the least \( H_\infty \) bound for system (10):

\[
B_j = P^{-1}Q \quad A_j = A - P^{-1}QC + \Xi 
\quad (18)
\]

Once the optimum solution is calculated the estimated position of each unknown node will be obtained accordingly.

IV. SIMULATION VALIDATION

In order to demonstrate the effectiveness of the proposed algorithm, simulations under dynamic 2D and 3D environment with random noise respectively have been made with the MATLAB software.

Parameter settings are listed in the following.

- \( V_x, V_y, V_z \): Node moving velocity along \( X, Y, Z \) coordinate axis directions and will be given the same value for simplicity and unity.
- \( T \): Sampling period of this algorithm and is set at 5 intervals.
- \( N \): Sampling points of unknown node at the moving trajectory and is set as 25 points per interval.
- \( \lambda \): System setup parameters and is set as 0.05.
- \( L \): Side length of localization area and is set at 100 units.

A. Localization Performance in 2D Space

Main parameters are set as the values previous. Mobile node is moving in 2D plain and its movement routes are random. But it must comply with the following moving limits.

The node starts from inertial coordinate origin \((0,0)\). Moving velocity along \( X \) and \( Y \) axis directions will be
changed randomly from \((0, V_x)\) and \((-V_y, V_y)\) in each step period respectively. In this way the dynamic route of mobile node becomes uncertain.

Fig. 2 gives the tracking trajectory comparisons under different moving velocities, which is changed from 2 to 8 m/s with interval as 2 m/s respectively. From the whole no matter how the velocity of mobile nodes changes the proposed algorithm realizes node localization in dynamic environment with low location errors. By using this mobile method the mobile trajectory of each node can be well followed and the coordinates in each time slot can be computed quickly and accurately, which is better applicable in different mobile conditions. We can also find directly from the figure localization errors present an increasing trend in different conditions of node velocities. In order to better explore the location errors, Fig. 3 gives the error comparisons in the same environment settings.

![Fig. 2. Tracking trajectory in 2D plane](image)

In order to give the accurate localization error comparison values in each velocity condition Fig. 3 is given below which gives different errors under different moving velocity settings.

In Fig. 3 explicit average localization errors during a whole moving process are computed and given (sampling points \(N\) is also set as 25). With \(V_x\) and \(V_y\) increasing average localization error is increasing too. \(V_x\) and \(V_y\) are set at 2, 4, 6 and 8 m/s respectively in each time slot. The error is increased by 95.25%, 77.40%, 27.48% and 31.96% respectively. That is because with velocity increased the random uncertainties of nodes movement direction are raised. In this way it’s hard to forecast the future status and directions and also localization time of each stop is reduced which adds the location errors. But in the whole localization errors are in an accepted scope. Meanwhile margin of error presents a downward trend, which proves the stability of proposed method. The maximum value is 7.77145 m which is much lower than the side length of localization area which is only by 7.77%. The effects of velocity on this algorithm can be neglected.

![Fig. 3. Average localization errors with different 3D velocity](image)

As said in part II many classic mobile node localization algorithms have been proposed, in which MCL and MCB are the most famous. Here comparisons are made between them under the same condition settings shown in Fig. 4.
We assume a number of nodes and anchors deployed in an obstacle-free area of $500 \times 500$ units but with random noise. We thus allow all algorithms to use possible negative information. Both the nodes are mobile randomly. The anchors know their location in priority, for example by using GPS or other ways. The radio range $r$ set to 40 units for all nodes and kept unchanged.

**Fig. 4. Localization error of three algorithms**

Fig. 4 gives the simulation results of MCL, MCB and proposed method in this paper. It is obvious MCL is the worst of the three no matter how node speed changes. Its error is the largest and with least stability. Especially when maximum speed is larger than 0.4 the error increase sharply and the summit is nearly 35m which is 35% of side length of localization area. Also MCL is the most sensitive to node moving speed. However MCB is the most stable and stays at low level all the time because it introduces a box to limit the estimated error and accumulated uncertainties. The box is playing an important role in MCB. It is obvious our proposed algorithm has the best performance. The smallest error is only 2.3m which is improved 44% and 69.3% compared with MCB and MCL respectively. When maximum speed arrives at 1, MCB and proposed algorithm will have the same accuracy that is 80% better than MCL. So it proves that our proposed algorithm has both best accuracy and stability. And it is very suitable for the mobile WSN environment.

**B. Localization Performance in 3D Space**

Simulation results in 3D space are given in this part. The parameters are set as before except the environment is changed into three dimensional spaces. The side length is still set as 100 units. Fig. 5 gives the comparisons of estimated node moving route and the true locations in 3D conditions. In Fig. 5, $V_x$ and $V_y$ are set as 2m/s. From the figure we can see the node is moving randomly in direction and velocity in 3D space. Also by using the proposed method tracking can be well realized. Tracking can be well realized for each node in each time slot. In order to find out the accurate errors Fig. 6 is given.

Fig. 6 average localization errors both in 2D and 3D are given. The two both present an upward trend which can be sees as almost linear. Smallest values are 1.33372 and 1.78276 respectively in two different conditions. The differences in each velocity between 2D and 3D spaces are very small and the biggest is 1.1 m which is only 1.1% compared with localization side length as shown in Fig. 6. That is equally to say the performance in 3D space is more or less the same as MCL and MCB in 2D plain which is much better compared with the classic two.

We can see in the two figures above our proposed algorithm proposed algorithm can be well used in 3D space and the accuracy is as high as in 2D plain. The effectiveness of this method for mobile node localization in WSN environment is proved.

**Fig. 6. Accuracy comparisons in 2D and 3D space**

From the simulation curves above we can find the proposed method can be both used in 2D and 3D environments. In each condition it can realize node localization well with low position errors. The accuracy differences in 2D and 3D conditions are very small, which proves to have universality. It receives better accuracy compared with MCL and MCB by the comparisons above.

**V. CONCLUSIONS**

Many applications in WSNs must know accurate locations of sensor nodes. In order to get location information, many localization schemes have been proposed to estimate sensors’ positions automatically. In mobile sensor networks, the localization scheme becomes
difficult to implement because of node mobility and network uncertainties. In this way developing a simple localization scheme with low estimated error is a big challenge for mobile sensor networks. In this paper a H∞ filtering and dynamic node model based mobile algorithm is proposed. State and measurement equation of each node are built with basic kinematic property and sensor node measurement method, including the impact of environment random uncertainties and node failure. The position of the mobile node is estimated by the filter using an integration of position information from other nodes, based on Linear Matrix Inequality (LMI) criterion. Simulation results show that this method can achieve highly reliable and accurate localization performance under stochastic uncertainty conditions compared with the classic mobile MCL and MCB mobile schemes. With the simulation results, our scheme has lower localization error than the previous work in most scenarios.

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