Fuzzy pairwise \((r, s)\)-irresolute mappings

Seung On Lee\(^1\) and Eun Pyo Lee\(^2\)

\(^1\) Department of Mathematics, Chungbuk National University, Cheongju 361-763, Korea  
\(^2\) Department of Mathematics, Seonam University, Namwon 590-711, Korea

Abstract

In this paper, we introduce the concepts of fuzzy pairwise \((r, s)\)-irresolute, fuzzy pairwise \((r, s)\)-presemiopen and fuzzy pairwise \((r, s)\)-presemiclosed mappings in smooth bitopological spaces and then we investigate some of their characteristic properties.

**Key words**: \((T_i, T_j)\)-fuzzy \((r, s)\)-semiopen sets, fuzzy pairwise \((r, s)\)-irresolute mappings

1. Introduction

After the introduction of fuzzy sets by Zadeh [10], Chang [2] was the first to introduce the concept of a fuzzy topology on a set \(X\) by axiomatizing a collection \(T\) of fuzzy subsets of \(X\), where he referred to each member of \(T\) as an open set. In his definition of fuzzy topology, fuzziness in the concept of openness of a fuzzy subset was absent. These spaces and its generalizations are later studied by several authors, one of which, developed by Šostak [9], used the idea of degree of openness. This type of generalization of fuzzy topological spaces was later rephrased by Chattopadhyay, Hazra, and Samanta [3], and by Ramadan [7]. Kandil [4] introduced and studied the notion of fuzzy bitopological spaces as a natural generalization of fuzzy topological spaces. Lee [5] introduced the concept of smooth bitopological spaces as a generalization of smooth topological spaces and Kandil’s fuzzy bitopological spaces.

In this paper, we introduce the concepts of fuzzy pairwise \((r, s)\)-irresolute, fuzzy pairwise \((r, s)\)-presemiopen and fuzzy pairwise \((r, s)\)-presemiclosed mappings in smooth bitopological spaces and then we investigate some of their characteristic properties.

2. Preliminaries

Let \(I\) be the closed unit interval \([0, 1]\) of the real line and let \(I_0\) be the half open interval \((0, 1]\) of the real line. For a set \(X\), \(I^X\) denotes the collection of all mapping from \(X\) to \(I\). A member \(\mu\) of \(I^X\) is called a fuzzy set of \(X\). By \(0\) and \(\bar{1}\) we denote constant mappings on \(X\) with value 0 and 1, respectively. For any \(\mu \in I^X\), \(\mu^c\) denotes the complement \(1 - \mu\). All other notations are the standard notations of fuzzy set theory.

A Chang’s fuzzy topology on \(X\) [2] is a family \(T\) of fuzzy sets in \(X\) which satisfies the following properties:

1. \(\bar{0}, \bar{1} \in T\).
2. If \(\mu_1, \mu_2 \in T\) then \(\mu_1 \land \mu_2 \in T\).
3. If \(\mu_k \in T\) for all \(k\), then \(\bigvee \mu_k \in T\).

The pair \((X, T)\) be called a Chang’s fuzzy topological space. Members of \(T\) are called \(T\)-fuzzy open sets of \(X\) and their complements \(T\)-fuzzy closed sets of \(X\).

A system \((X, T_1, T_2)\) consisting of a set \(X\) with two Chang’s fuzzy topologies \(T_1\) and \(T_2\) on \(X\) is called a Kandil’s fuzzy bitopological space.

A smooth topology on \(X\) is a mapping \(T : I^X \to I\) which satisfies the following properties:

1. \(T(\bar{0}) = T(\bar{1}) = 1\).
2. \(T(\mu_1 \land \mu_2) \geq T(\mu_1) \land T(\mu_2)\).
3. \(T(\bigvee \mu_i) \geq \bigwedge T(\mu_i)\).

The pair \((X, T)\) is called a smooth topological space. For \(r \in I_0\), we call \(\mu\) a \(T\)-fuzzy \(r\)-open set of \(X\) if \(T(\mu) \geq r\) and \(\mu\) a \(T\)-fuzzy \(r\)-closed set of \(X\) if \(T(\mu^c) \geq r\).

A system \((X, T_1, T_2)\) consisting of a set \(X\) with two smooth topologies \(T_1\) and \(T_2\) on \(X\) is called a smooth bitopological space. Throughout this paper the indices \(i, j\) take values in \(\{1, 2\}\) and \(i \neq j\).

Let \((X, T)\) be a smooth topological space. Then it is easy to see that for each \(r \in I_0\), an \(r\)-cut

\[ T_r = \{\mu \in I^X \mid T(\mu) \geq r\} \]

is a Chang’s fuzzy topology on \(X\).
Let \((X, T)\) be a Chang’s fuzzy topological space and \(r \in I_0\). Then the mapping \(T^r : I^X \rightarrow I\) is defined by
\[
T^r(\mu) = \begin{cases} 
1 & \text{if } \mu = [0, 1], \\
r & \text{if } \mu = T - \{0, 1\}, \\
0 & \text{otherwise}
\end{cases}
\]
becomes a smooth topology.

Hence, we obtain that if \((X, T_1, T_2)\) is a smooth bitopological space and \(r, s \in I_0\), then \((X, (T_1)_r, (T_2)_s)\) is a Kandil’s fuzzy bitopological space. Also, if \((X, T_1, T_2)\) is a Kandil’s fuzzy bitopological space and \(r, s \in I_0\), then \((X, (T_1)^r, (T_2)_s)\) is a smooth bitopological space.

**Definition 2.1.** [5] Let \((X, T)\) be a smooth topological space. For each \(r \in I_0\) and for each \(\mu \in I^X\), the \(T\)-fuzzy \(r\)-closure is defined by
\[
T\text{-Cl}(\mu, r) = \{\rho \in I^X | \rho \leq \mu, T(\rho) \geq r\}
\]
and the \(T\)-fuzzy \(r\)-interior is defined by
\[
T\text{-Int}(\mu, r) = \{\rho \in I^X | \rho \geq \mu, T(\rho) \geq r\}.
\]

**Lemma 2.2.** [5] Let \(\mu\) be a fuzzy set of a smooth topological space \((X, T)\) and let \(r \in I_0\). Then we have:

1. \(T\text{-Cl}(\mu, r)^c = T\text{-Int}(\mu^c, r)\).
2. \(T\text{-Int}(\mu, r)^c = T\text{-Cl}(\mu^c, r)\).

**Definition 2.3.** [5] Let \(\mu\) be a fuzzy set of a smooth bitopological space \((X, T_1, T_2)\) and \(r, s \in I_0\). Then \(\mu\) is said to be

1. \((T_1, T_2)\)-fuzzy \((r, s)\)-semiopen set if there is a \(T_1\)-fuzzy \(r\)-open set \(\rho \in X\) such that \(\rho \leq \mu \leq T_1\text{-Cl}(\rho, s)\),
2. \((T_1, T_2)\)-fuzzy \((r, s)\)-semi closed set if there is a \(T_1\)-fuzzy \(r\)-closed set \(\rho \in X\) such that \(T_1\text{-Int}(\rho, s) \leq \rho \leq \mu\).

**Definition 2.4.** [5] Let \((X, T_1, T_2)\) be a smooth bitopological space. For each \(r, s \in I_0\) and for each \(\mu \in I^X\), the \((T_1, T_2)\)-fuzzy \((r, s)\)-semi closed is defined by
\[
(T_1, T_2)\text{-sCl}(\mu, r, s) = \{\rho \in I^X | \rho \leq \mu, \rho \text{ is } (T_1, T_2)\text{-fuzzy } (r, s)\text{-semi closed}\}
\]
and the \((T_1, T_2)\)-fuzzy \((r, s)\)-semi interior is defined by
\[
(T_1, T_2)\text{-sInt}(\mu, r, s) = \{\rho \in I^X | \rho \geq \mu, \rho \text{ is } (T_1, T_2)\text{-fuzzy } (r, s)\text{-semi open}\}.
\]

**Lemma 2.5.** [5] Let \(\mu\) be a fuzzy set of a smooth bitopological space \((X, T_1, T_2)\) and let \(r, s \in I_0\). Then we have:

1. \((T_1, T_2)\)-sCl(\(\mu, r, s\))^c = \((T_1, T_2)\)-sInt(\(\mu^c, r, s\)).
2. \((T_1, T_2)\)-sInt(\(\mu, r, s\))^c = \((T_1, T_2)\)-sCl(\(\mu^c, r, s\)).

**Definition 2.6.** [5] Let \(f : (X, T_1, T_2) \rightarrow (Y, U_1, U_2)\) be a mapping from a smooth bitopological space \(X\) to a smooth bitopological space \(Y\) and \(r, s \in I_0\). Then \(f\) is said to be

1. a fuzzy pairwise \((r, s)\)-continuous mapping if the induced mapping \(f : (X, T_1) \rightarrow (Y, U_1)\) is a fuzzy \(r\)-continuous mapping and the induced mapping \(f : (X, T_2) \rightarrow (Y, U_2)\) is a fuzzy \(s\)-continuous mapping,
2. a fuzzy pairwise \((r, s)\)-semicontinuous mapping if \(f^{-1}(\mu)\) is a \((T_1, T_2)\)-fuzzy \((r, s)\)-semi open set of \(X\) for each \(U_1\)-fuzzy \(r\)-open set \(\mu\) of \(Y\) and \(f^{-1}(\nu)\) is a \((T_2, T_1)\)-fuzzy \((s, r)\)-semi open set of \(X\) for each \(U_2\)-fuzzy \(s\)-open set \(\nu\) of \(Y\).
3. a fuzzy pairwise \((r, s)\)-precontinuous mapping if \(f^{-1}(\mu)\) is a \((T_1, T_2)\)-fuzzy \((r, s)\)-pre open set of \(X\) for each \(U_1\)-fuzzy \(r\)-open set \(\mu\) of \(Y\) and \(f^{-1}(\nu)\) is a \((T_2, T_1)\)-fuzzy \((s, r)\)-pre open set of \(X\) for each \(U_2\)-fuzzy \(s\)-open set \(\nu\) of \(Y\).

**3. Fuzzy pairwise \((r, s)\)-irresolute, fuzzy pairwise \((r, s)\)-pre semi open and fuzzy pairwise \((r, s)\)-presemi closed mappings**

**Definition 3.1.** Let \(f : (X, T_1, T_2) \rightarrow (Y, U_1, U_2)\) be a mapping from a smooth bitopological space \(X\) to a smooth bitopological space \(Y\) and \(r, s \in I_0\). Then \(f\) is called

1. fuzzy pairwise \((r, s)\)-irresolute if \(f^{-1}(\mu)\) is a \((T_1, T_2)\)-fuzzy \((r, s)\)-semi open set of \(X\) for each \((U_1, U_2)\)-fuzzy \((r, s)\)-semi open set \(\mu\) of \(Y\),
2. fuzzy pairwise \((r, s)\)-pre semi open if \(f(\rho)\) is a \((U_1, U_2)\)-fuzzy \((r, s)\)-semi open set of \(Y\) for each \((T_1, T_2)\)-fuzzy \((r, s)\)-semi open set \(\rho\) of \(X\),
3. fuzzy pairwise \((r, s)\)-presemi closed if \(f(\rho)\) is a \((U_1, U_2)\)-fuzzy \((r, s)\)-semi closed set of \(Y\) for each \((T_1, T_2)\)-fuzzy \((r, s)\)-semi closed set \(\rho\) of \(X\).

**Theorem 3.2.** Let \(f : (X, T_1, T_2) \rightarrow (Y, U_1, U_2)\) be a mapping and \(r, s \in I_0\). Then the following statements are equivalent:

1. \(f\) is a fuzzy pairwise \((r, s)\)-irresolute mapping.
2. \(f^{-1}(\mu)\) is a \((T_1, T_2)\)-fuzzy \((r, s)\)-semi open set of \(X\) for each \((U_1, U_2)\)-fuzzy \((r, s)\)-semi open set \(\mu\) of \(Y\).
3. For each fuzzy set \(\rho\) of \(X\),
\[
f((T_1, T_2)\text{-sCl}(\rho, r, s)) \leq (U_1, U_2)\text{-sCl}(f(\rho), r, s).
\]
For each fuzzy set $\mu$ of $Y$,

$$\mu 
\leq f^{-1}(\mu)
\mu
\leq f^{-1}(\mu).$$

(5) Let $\mu$ be any $(\mathcal{U}, \mathcal{U}_1)$-fuzzy $(r, s)$-semiopen set of $X$. Then $(\mathcal{U}, \mathcal{U}_1)$-sInt(\mu, r, s) = \mu$. By (5),

$$f^{-1}(\mu) = f^{-1}(\mu) \leq (T_i, T_j)$$

$$\leq f^{-1}(\mu).$$

Proof. (1) $\Rightarrow$ (2) Let $\mu$ be any $(\mathcal{U}, \mathcal{U}_1)$-fuzzy $(r, s)$-semiopen set of $Y$. Then $\mu^c$ is a $(\mathcal{U}, \mathcal{U}_1)$-fuzzy $(r, s)$-semiopen set of $X$. Since $f$ is a fuzzy pairwise $(r, s)$-irresolute mapping, $f^{-1}(\mu^c)$ is a $(T_i, T_j)$-fuzzy $(r, s)$-semiopen set of $X$. Thus $f^{-1}(\mu)$ is a $(T_i, T_j)$-fuzzy $(r, s)$-semiopen set of $X$.

(2) $\Rightarrow$ (3) Let $\rho$ be any fuzzy set of $X$. Then $(\mathcal{U}, \mathcal{U}_1)$-sCl($f(\rho), r, s$) is a $(\mathcal{U}, \mathcal{U}_1)$-fuzzy $(r, s)$-semiopen set of $Y$. By (2), $f^{-1}(\mathcal{U}, \mathcal{U}_1)$-sCl($f(\rho), r, s$) is a $(T_i, T_j)$-fuzzy $(r, s)$-semiopen set of $X$. Since $f(\rho) = (T_i, T_j)$-sCl($f(\rho), r, s$), we have

$$f((T_i, T_j) \setminus C_l(\rho, r, s))$$

$$\leq (T_i, T_j) \setminus C_l(f(\rho), r, s)$$

$$\leq (T_i, T_j) \setminus C_l(f^{-1}(\mu), r, s)$$

$$= f^{-1}((U_i, U_j) \setminus C_l(f(\rho), r, s)).$$

Hence

$$f((T_i, T_j) \setminus C_l(\rho, r, s))$$

$$\leq f^{-1}(f(U_i, U_j) \setminus C_l(f(\rho), r, s))$$

$$\leq f^{-1}(f(U_i, U_j) \setminus C_l(\mu, r, s)).$$

(3) $\Rightarrow$ (4) Let $\mu$ be any fuzzy set of $Y$. By (3),

$$f((T_i, T_j) \setminus C_l(\mu), r, s)$$

$$\leq (U_i, U_j) \setminus C_l(f^{-1}(\mu), r, s)$$

$$\leq (U_i, U_j) \setminus C_l(f^{-1}(\mu), r, s).$$

Thus

$$f((T_i, T_j) \setminus C_l(\mu), r, s)$$

$$\leq f^{-1}(f(U_i, U_j) \setminus C_l(\mu), r, s)$$

$$\leq f^{-1}(f(U_i, U_j) \setminus C_l(\mu), r, s)).$$

(4) $\Rightarrow$ (5) Let $\mu$ be any fuzzy set of $Y$. Then $\mu^c$ is a fuzzy set of $Y$. By (4),

$$f((T_i, T_j) \setminus C_l(\mu^c), r, s)$$

$$= (T_i, T_j) \setminus C_l(f^{-1}(\mu^c), r, s)$$

$$\leq f^{-1}(f(U_i, U_j) \setminus C_l(\mu^c), r, s).$$

By Lemma 2.5,

$$f^{-1}(f(U_i, U_j) \setminus C_l(\mu, r, s))$$

$$= f^{-1}(f(U_i, U_j) \setminus C_l(\mu^c, r, s))^c$$

$$\leq (T_i, T_j) \setminus C_l(f^{-1}(\mu^c), r, s)^c$$

$$= (T_i, T_j) \setminus C_l(f^{-1}(\mu), r, s).$$

(5) $\Rightarrow$ (1) Let $\mu$ be any $(\mathcal{U}, \mathcal{U}_1)$-fuzzy $(r, s)$-semiopen set of $X$. Then $(\mathcal{U}, \mathcal{U}_1)$-sInt(\mu, r, s) = \mu$. By (5),

$$f^{-1}(\mu) = f^{-1}(\mu) \leq (T_i, T_j)$$

$$\leq f^{-1}(\mu).$$

So $f^{-1}(\mu) = (T_i, T_j) \setminus C_l(f^{-1}(\mu), r, s)$ and hence $f^{-1}(\mu)$ is a $(T_i, T_j)$-fuzzy $(r, s)$-semiopen set of $X$. Thus $f$ is a fuzzy pairwise $(r, s)$-irresolute mapping.

Theorem 3.3. Let $f : (X, T_1, T_2) \rightarrow (Y, U_1, U_2)$ be a bijection and $r, s \in I_0$. Then $f$ is a fuzzy pairwise $(r, s)$-irresolute mapping if and only if $(\mathcal{U}, \mathcal{U}_1)$-sInt($f(\rho), r, s$) $\leq$ $(T_i, T_j) \setminus C_l(f^{-1}(\mu), r, s)$ for each fuzzy set $\rho$ of $X$.

Proof. Let $f$ be a fuzzy pairwise $(r, s)$-irresolute mapping and $\rho$ any fuzzy set of $X$. Since $(\mathcal{U}, \mathcal{U}_1)$-sInt($f(\rho), r, s$) is a $(\mathcal{U}, \mathcal{U}_1)$-fuzzy $(r, s)$-semiopen set of $Y$, we have

$$f^{-1}(\mathcal{U}, \mathcal{U}_1) \setminus C_l(f(\rho), r, s)$$

is a fuzzy pairwise $(r, s)$-irresolute and one-to-one, we have

$$f^{-1}(\mathcal{U}, \mathcal{U}_1) \setminus C_l(f(\rho), r, s)$$

$$\leq (T_i, T_j) \setminus C_l(f^{-1}(\mu), r, s)$$

$$= (T_i, T_j) \setminus C_l(f^{-1}(\mu), r, s).$$

Since $f$ is onto,

$$(\mathcal{U}, \mathcal{U}_1) \setminus C_l(f(\rho), r, s)$$

$$f^{-1}(\mathcal{U}, \mathcal{U}_1) \setminus C_l(f^{-1}(\mu), r, s)$$

$$\leq (T_i, T_j) \setminus C_l(f^{-1}(\mu), r, s).$$

Conversely, let $\mu$ be any $(\mathcal{U}, \mathcal{U}_1)$-fuzzy $(r, s)$-semiopen set of $Y$. Then $(\mathcal{U}, \mathcal{U}_1) \setminus C_l(\mu, r, s) = \mu$. Since $f$ is onto,

$$f((T_i, T_j) \setminus C_l(\mu), r, s)$$

$$\leq (T_i, T_j) \setminus C_l(f^{-1}(\mu), r, s)$$

$$= (T_i, T_j) \setminus C_l(f^{-1}(\mu), r, s).$$

Since $f$ is one-to-one, we have

$$f^{-1}(\mu) \leq f^{-1}(f((T_i, T_j) \setminus C_l(\mu), r, s))$$

$$= (T_i, T_j) \setminus C_l(f^{-1}(\mu), r, s)$$

$$\leq f^{-1}(\mu).$$

Thus $f^{-1}(\mu) = (T_i, T_j) \setminus C_l(f^{-1}(\mu), r, s)$ and hence $f^{-1}(\mu)$ is a $(T_i, T_j)$-fuzzy $(r, s)$-semiopen set of $X$. Therefore $f$ is a fuzzy pairwise $(r, s)$-irresolute mapping.

Theorem 3.4. Let $f : (X, T_1, T_2) \rightarrow (Y, U_1, U_2)$ be a mapping and $r, s \in I_0$. Then the following statements are equivalent:

$$f^{-1}(\mu) \leq f^{-1}(f((T_i, T_j) \setminus C_l(\mu), r, s))$$

$$= (T_i, T_j) \setminus C_l(f^{-1}(\mu), r, s)$$

$$\leq f^{-1}(\mu).$$
(1) $f$ is a fuzzy pairwise $(r, s)$-presemiopen mapping.

(2) For each fuzzy set $\rho$ of $X$,
\[
 f((T_i, T_j)_{\text{-sInt}}(\rho, r, s)) \\
\leq (U_i, U_j)_{\text{-sInt}}(f(\rho), r, s).
\]

(3) For each fuzzy set $\mu$ of $Y$,
\[
 (T_i, T_j)_{\text{-sInt}}(f^{-1}(\mu), r, s) \\
\leq f^{-1}((U_i, U_j)_{\text{-sInt}}(\mu, r, s))
\]

Proof. (1) $\Rightarrow$ (2) Let $\rho$ be any fuzzy set of $X$. Clearly $(T_i, T_j)_{\text{-sInt}}(\rho, r, s)$ is a $(T_i, T_j)$-fuzzy $(r, s)$-semiopen set of $X$. Since $f$ is a fuzzy pairwise $(r, s)$-presemiopen mapping, $f((T_i, T_j)_{\text{-sInt}}(\rho, r, s))$ is a $(U_i, U_j)$-fuzzy $(r, s)$-semiopen set of $Y$. Thus
\[
f((T_i, T_j)_{\text{-sInt}}(\rho, r, s)) \\
= (U_i, U_j)_{\text{-sInt}}(f((T_i, T_j)_{\text{-sInt}}(\rho, r, s)), r, s) \\
\leq (U_i, U_j)_{\text{-sInt}}(f(\rho), r, s).
\]

(2) $\Rightarrow$ (3) Let $\mu$ be any fuzzy set of $Y$. Then $f^{-1}(\mu)$ is a fuzzy set of $X$. By (2),
\[
f((T_i, T_j)_{\text{-sInt}}(f^{-1}(\mu), r, s)) \\
\leq (U_i, U_j)_{\text{-sInt}}(f f^{-1}(\mu), r, s) \\
\leq (U_i, U_j)_{\text{-sInt}}(f \mu, r, s).
\]

Thus we have
\[
(T_i, T_j)_{\text{-sInt}}(f^{-1}(\mu), r, s) \\
\leq f^{-1} f((T_i, T_j)_{\text{-sInt}}(f^{-1}(\mu), r, s)) \\
\leq f^{-1}((U_i, U_j)_{\text{-sInt}}(f \mu, r, s)).
\]

(3) $\Rightarrow$ (1) Let $\rho$ be any $(T_i, T_j)$-fuzzy $(r, s)$-semiopen set of $X$. Then $(T_i, T_j)_{\text{-sInt}}(\rho, r, s) = \rho$ and $f(\rho)$ is a fuzzy set of $Y$. By (3),
\[
\rho = (T_i, T_j)_{\text{-sInt}}(\rho, r, s) \\
\leq (T_i, T_j)_{\text{-sInt}}(f^{-1} f(\rho), r, s) \\
\leq f^{-1}((U_i, U_j)_{\text{-sInt}}(f(\rho), r, s)).
\]

Hence we have
\[
f(\rho) \leq f f^{-1}((U_i, U_j)_{\text{-sInt}}(f(\rho), r, s)) \\
\leq (U_i, U_j)_{\text{-sInt}}(f(\rho), r, s) \\
\leq f(\rho).
\]

Thus $f(\rho) = (U_i, U_j)_{\text{-sInt}}(f(\rho), r, s)$ and hence $f(\rho)$ is a $(U_i, U_j)$-fuzzy $(r, s)$-semiopen set of $Y$. Therefore $f$ is a fuzzy pairwise $(r, s)$-presemiopen mapping.

Theorem 3.5. Let $f : (X, T_1, T_2) \rightarrow (Y, U_1, U_2)$ be a mapping and $r, s \in I_0$. Then the following statements are equivalent:

(1) $f$ is a fuzzy pairwise $(r, s)$-presemiclosed mapping.

(2) For each fuzzy set $\rho$ of $X$,
\[
(U_i, U_j)_{\text{-sCl}}(f(\rho), r, s) \\
\leq f(((T_i, T_j)_{\text{-sCl}}(\rho, r, s)).
\]

Proof. (1) $\Rightarrow$ (2) Let $\rho$ be any fuzzy set of $X$. Clearly $(T_i, T_j)_{\text{-sCl}}(\rho, r, s)$ is a $(T_i, T_j)$-fuzzy $(r, s)$-semiclosed set of $X$. Since $f$ is a fuzzy pairwise $(r, s)$-presemiclosed mapping, $f((T_i, T_j)_{\text{-sCl}}(\rho, r, s))$ is a $(U_i, U_j)$-fuzzy $(r, s)$-semiclosed set of $Y$. Thus we have
\[
(U_i, U_j)_{\text{-sCl}}(f(\rho), r, s) \\
\leq (U_i, U_j)_{\text{-sCl}}(f((T_i, T_j)_{\text{-sCl}}(\rho, r, s)), r, s) \\
= f(((T_i, T_j)_{\text{-sCl}}(\rho, r, s)).
\]

(2) $\Rightarrow$ (1) Let $\rho$ be any $(T_i, T_j)$-fuzzy $(r, s)$-semiclosed set of $X$. Then $(T_i, T_j)_{\text{-sCl}}(\rho, r, s) = \rho$. By (2),
\[
(U_i, U_j)_{\text{-sCl}}(f(\rho), r, s) \leq f((T_i, T_j)_{\text{-sCl}}(f(\rho), r, s)) \\
= f(\rho) \\
\leq (U_i, U_j)_{\text{-sCl}}(f(\rho), r, s).
\]

Thus $f(\rho) = (U_i, U_j)_{\text{-sCl}}(f(\rho), r, s)$ and hence $f(\rho)$ is a $(U_i, U_j)$-fuzzy $(r, s)$-semiclosed set of $Y$. Therefore $f$ is a fuzzy pairwise $(r, s)$-presemiclosed mapping.

Theorem 3.6. Let $f : (X, T_1, T_2) \rightarrow (Y, U_1, U_2)$ be a bijection and $r, s \in I_0$. Then $f$ is a fuzzy pairwise $(r, s)$-presemiclosed mapping and only if $f^{-1}((U_i, U_j)_{\text{-sCl}}(\mu, r, s)) \leq (T_i, T_j)_{\text{-sCl}}(f^{-1}(\mu), r, s)$ for each fuzzy set $\mu$ of $Y$.

Proof. Let $f$ be a fuzzy pairwise $(r, s)$-presemiclosed mapping and let $\mu$ be any fuzzy set of $Y$. Then $f^{-1}(\mu)$ is a fuzzy set of $X$. Since $f$ is fuzzy pairwise $(r, s)$-presemiclosed and onto,
\[
(U_i, U_j)_{\text{-sCl}}(\mu, r, s) \\
= (U_i, U_j)_{\text{-sCl}}(f f^{-1}(\mu), r, s) \\
\leq f(((T_i, T_j)_{\text{-sCl}}(f^{-1}(\mu), r, s)).
\]

Since $f$ is one-to-one, we have
\[
f^{-1}((U_i, U_j)_{\text{-sCl}}(\mu, r, s)) \\
\leq f^{-1} f(((T_i, T_j)_{\text{-sCl}}(f^{-1}(\mu), r, s)) \\
= (T_i, T_j)_{\text{-sCl}}(f^{-1}(\mu), r, s).
\]

Conversely, let $\rho$ be any $(T_i, T_j)$-fuzzy $(r, s)$-semiclosed set of $X$. Then $(T_i, T_j)_{\text{-sCl}}(\rho, r, s) = \rho$. Since $f$ is one-to-one,
\[
f^{-1}((U_i, U_j)_{\text{-sCl}}(f(\rho), r, s)) \\
\leq (T_i, T_j)_{\text{-sCl}}(f^{-1}(f(\rho), r, s)) \\
= (T_i, T_j)_{\text{-sCl}}(\rho, r, s) = \rho.
\]
Since $f$ is onto, we have
\[(U_i, U_j)\text{-sCl}(f(\rho), r, s) = f^{-1}((U_i, U_j)\text{-sCl}(f(\rho), r, s)) \leq f(\rho) \leq (U_i, U_j)\text{-sCl}(f(\rho), r, s).
\]
Thus $f(\rho) = (U_i, U_j)\text{-sCl}(f(\rho), r, s)$ and hence $f(\rho)$ is a $(U_i, U_j)$-fuzzy $(r, s)$-semiclosed set of $Y$. Therefore $f$ is a fuzzy pairwise $(r, s)$-presemiclosed mapping.

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**References**


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**Seung On Lee**
Professor of Chungbuk National University
E-mail : solee@chungbuk.ac.kr

**Eun Pyo Lee**
Professor of Seonam University
E-mail : eplee@seonam.ac.kr