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Entry Trajectory Tracking Law via Feedback Linearization

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As a step toward extending the two-dimensional (longitudinal) entry predictive/tracking guidance scheme used by the U.S. Space Shuttle Orbiter to three dimensions, a control law for tracking a three-dimensional entry trajectory is designed. The tracking law commands the angles of attack and of bank that are required to follow a ground track specified as a function of energy. Feedback linearization is used to design the tracking law. Some extensions to the existing theory are required to accommodate features of the entry tracking problem. Downrange and crossrange angles serve as output variables for the feedback linearization and lead to state and control transformations that convert the entry dynamics to an equivalent linear system in an approximate sense, which is defined. A feedback tracking law is then designed, taking advantage of the linear structure of the system dynamics in the transformed variables. This tracking law is shown to achieve bounded tracking of the output variables. Simulation results indicate the effectiveness of the tracking law in compensating for initial offsets from a reference trajectory.

Introduction

CURRENT efforts to develop reusable launch vehicles (RLVs), a space station crew return vehicle, and a military spaceplane provide motivation for investigating potential improvements to the entry guidance capability for a lifting unpowered flight vehicle. The state of the art is represented by the U.S. Space Shuttle Orbiters' entry guidance, as described by Harpold and Graves.[1] The Shuttle's longitudinal guidance combines predictive guidance, i.e., model-based reference trajectory planning with in-flight updating, with reference trajectory tracking. Mease and Kremer[2] revisited the Shuttle tracking law derivation in the framework of feedback linearization and showed that the Shuttle law is a linearized version of the proportional integral derivative (PID) type law that the feedback linearization approach can produce. By not linearizing the tracking law, the nonlinearities in the dynamics can be compensated for, provided they can be adequately modeled. Roenneke and Well[3] simulated a variety of low-lift re-entry flights and showed that the longitudinal PID tracking law with feedback linearization yields uniformly good tracking performance and effectively compensates for 20% errors in air density and large initial position errors. A related nonlinear tracking law has been developed and applied to RLV guidance by Lu.[4]

The entry guidance requirements for future unpowered entry vehicles will likely be, as they are for the Shuttle, to steer the vehicle on a feasible trajectory, a trajectory within the entry corridor, defined by heating, acceleration, dynamic pressure, and controllability limits, that achieves the specified target condition within the specified error margin. Additional requirements that drive our study are flying over specified waypoints and increased crossrange capability. The former may be required to avoid flying over populated areas; the latter may be required for abort scenarios and to reduce the waiting time for return from orbit. The Shuttle entry guidance handles crossrange targeting by bank reversal logic. Crossrange targeting takes second priority to downrange targeting, and some crossrange capability is sacrificed as a result. The additional requirements suggest that it would be desirable to extend the Shuttle longitudinal predictive/tracking guidance to longitudinal and lateral predictive/tracking guidance.

The entry requirements can be met by a guidance scheme comprising a rapid trajectory planner that generates a feasible ground track and a control law that commands the appropriate angles of attack and of bank to follow the ground track. Feasible ground track means that a vehicle following this ground track will remain in the entry corridor. To ensure this, it is important to plan the ground track as a function of energy rather than time and to track the associated drag profile, as well as the ground track itself. It is the drag profile that can be directly related to the heating, acceleration, dynamic pressure, and controllability constraints that define the entry corridor. To realize more of the downrange/crossrange capability of an entry vehicle, both the angles of attack and of bank have to be considered as primary control variables for guidance. In the Shuttle entry guidance, the angle-of-attack profile is fixed for trajectory planning, and only modest modifications in angle of attack are used in tracking to smooth out transient behavior during bank reversals and winds. While this simplifies the guidance logic, it limits the entry capability. It is true that heating constraints dictate high angle of attack early in the entry and that it is desired to be on the front side of the L/D curve at the initiation of the terminal area energy management phase; nonetheless, these requirements still leave considerable freedom in the angle-of-attack profile. This freedom can be used to enlarge the landing footprint, increase guidance accuracy, and minimize bank reversals. Achieving these benefits requires extensions to both the trajectory planning and trajectory tracking algorithms relative to those used for the Shuttle. This paper concerns the required extensions to the tracking law.

We show that the translational entry dynamics can be approximately linearized through feedback linearization. Feedback linearization requires new state variables, and for these we use downrange and crossrange angles and their derivatives with respect to energy. The new state variables lead to approximate feedback linearization. A theory for approximate feedback linearization has been developed by Hauser et al.[5] We develop a small extension to their theory to handle the nonaffine nature of the controls. A linear control design method is applied to the approximately linearized dynamics to achieve bounded-input/bounded-output tracking globally in the absence of control saturation. The simulated entry of a reusable launch vehicle is conducted to validate the approximations made in the derivation of the tracking law and to determine the tracking performance under the ideal conditions of perfect state knowledge and no modeling error. A detailed error analysis and consideration of tracking law modifications for improving robustness are beyond the scope of this paper.

Entry Guidance Problem

To formulate the entry guidance problem for tracking law design, we assume that the vehicle's center of mass evolves according to the equations for unpowered atmospheric flight over a nonrotating, windless, spherical Earth given by[6]

$$\dot{r} = V \sin\gamma \tag{1}$$

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$$\dot{\theta} = \frac{V \cos\gamma \cos\chi}{r \cos\phi} \tag{2}$$

$$\dot{\phi} = \frac{V \cos\gamma \sin\chi}{r} \tag{3}$$

$$\dot{V} = -D - g \sin\gamma \tag{4}$$

$$\dot{\gamma} = (1/V)[L \cos\sigma - (g - (V^2/r)) \cos\gamma] \tag{5}$$

$$\dot{\chi} = \frac{1}{V}\left[\frac{L \sin\sigma}{\cos\gamma} - \frac{V^2}{r} \cos\gamma \cos\chi \tan\phi\right] \tag{6}$$

where the position coordinates are the distance from the center of the Earth r, the longitude θ, and the latitude ϕ and the velocity coordinates are the velocity magnitude V, the flight-path angle γ, and the heading angle χ. The heading angle is defined such that $\chi = 0$ is a due east heading. Also, σ is the bank angle defined such that the lift vector is in the local vertical plane at zero bank, and $g = \mu/r^2$ with μ as the gravitational parameter for the Earth. L and D represent the lift and drag accelerations and are given by

$$L = \frac{1}{2}\frac{\rho V^2 C_L S}{m}, \qquad D = \frac{1}{2}\frac{\rho V^2 C_D S}{m} \tag{7}$$

where S is the vehicle reference area, m is the mass, $\rho(r)$ is the r-dependent density, and $C_L(\alpha, M)$ and $C_D(\alpha, M)$ are the lift and the drag coefficients as functions of the angle of attack α and the Mach number M. We assume an exponential atmospheric density model

$$\rho = \rho_0 \exp[-(r - r_0)/H] \tag{8}$$

where ρ_0 is the density at sea level and r_0 is the radius of the Earth.

In the entry guidance of an unpowered vehicle, energy is a more appropriate independent variable than time. The specific energy E is given by

$$E = \tfrac{1}{2}V^2 - [(\mu/r) - (\mu/r_0)] \tag{9}$$

defined such that the potential energy is zero at the Earth's surface. Using $\dot{E} = -VD$ and denoting $\mathrm{d}(\cdot)/\mathrm{d}E$ by $(\cdot)'$, Eqs. (1–6) with energy as the independent variable are given by

$$r' = -(\sin\gamma/D) \tag{10}$$

$$\theta' = -\frac{\cos\gamma \cos\chi}{Dr \cos\phi} \tag{11}$$

$$\phi' = -\frac{\cos\gamma \sin\chi}{Dr} \tag{12}$$

$$V' = \frac{D + g \sin\gamma}{DV} \tag{13}$$

$$\gamma' = -(1/DV^2)[L \cos\sigma - (g - (V^2/r)) \cos\gamma] \tag{14}$$

$$\chi' = \frac{-1}{DV^2}\left[\frac{L \sin\sigma}{\cos\gamma} - \frac{V^2 \cos\gamma \cos\chi \tan\phi}{r}\right] \tag{15}$$

An advantage gained by the choice of energy as the independent variable is that the system order is reduced from six to five. This is because the r' equation and the V' equation are not independent. This can be seen by differentiating both sides of Eq. (9) with respect to energy, which gives

$$E' = \frac{\mathrm{d}E}{\mathrm{d}E} = 1 = VV' + gr'$$

Therefore, a minimal realization of the system need only retain either the r' equation or the V' equation. We will retain the r' equation.

The entry guidance problem is: given the model, Eqs. (10–15), of the vehicle's longitudinal (vertical-plane) and lateral (horizontal-plane) dynamics, determine the controls, namely, the angles of attack α and of bank σ, as functions of the state $\boldsymbol{x} = [r\ \theta\ \phi\ \gamma\ \chi]^T$ and energy E, that steer the vehicle on a feasible trajectory, a trajectory within the entry corridor, defined by heating, acceleration, dynamic pressure, and controllability limits, that achieves the specified target condition and flies over selected waypoints within the specified error margin. We limit our considerations primarily to the hypersonic entry phase, and hence the target, i.e., final, state of our reference trajectory is a state, e.g., a longitude, latitude point at an energy corresponding to a speed of about Mach 2, from which further guidance phases [such as the terminal area energy management (TAEM) phase in the Shuttle entry guidance] would proceed.

Predictive/Tracking Guidance Scheme

We propose a three-dimensional entry guidance scheme that is a natural extension of the two-dimensional predictive/tracking guidance scheme for the longitudinal guidance of the Shuttle during entry. The predictive part of the guidance plans a feasible reference trajectory based on a model of the translational entry dynamics. The tracking law commands angles of attack and of bank to the autopilot for following the reference trajectory. With no updating of the reference trajectory, the guidance scheme reduces to reference trajectory tracking. Tracking guidance is less sensitive to modeling errors. As the vehicle approaches the target, the effects of modeling errors lessen, and a better reference can be computed. The new reference trajectory can also proceed from the current state, which may be offset from the previous reference trajectory. Thus, the reference trajectory should be updated several times during the entry. The capability for rapid, onboard trajectory planning is also desirable for abort scenarios.

We develop a tracking law. We assume that the tracking law will operate together with a trajectory planning algorithm. The planning algorithm generates a reference trajectory and control consistent with the dynamic model. The planning algorithm ensures that the reference trajectory and controls are feasible and leave sufficient margin in satisfying the constraints that the tracking problem can be solved with little concern for the trajectory constraints. In the case of the control α, there may be segments of the entry during which it should not deviate much from α_{ref}, for example, when α_{ref} is a high value for the purpose of temperature control.

To synthesize a feedback tracking law, we shall use feedback linearization, specifically, input–output linearization via static state feedback. The theory of feedback linearization for the control affine case is presented in Refs. 7 and 8. The entry dynamics are not affine in the control variables α and σ. We, therefore, need to extend the existing theory of feedback linearization to apply to systems in which the control does not appear affinely. We will see that the reference drag acceleration D is an important variable to track. Both α and σ influence D. One means of avoiding large deviations in α is to build a priority for changing drag via σ into the tracking law. This leads us to the use of approximate feedback linearization as described in the next section.

Feedback Linearization and Bounded Tracking

The theory to support the tracking law design is presented. To present the theory, we refer to a dynamical system of the general form

$$\dot{\boldsymbol{x}} = f(\boldsymbol{x}, \boldsymbol{u}), \qquad \boldsymbol{y} = h(\boldsymbol{x}, \boldsymbol{u}) \tag{16}$$

where $\boldsymbol{x} \in \mathbb{R}^n$ is the state, $\boldsymbol{u} \in \mathbb{R}^m$ is the control, and $\boldsymbol{y} \in \mathbb{R}^m$ is the output. In exact feedback linearization,[7,8] each element of the output vector is differentiated with respect to the independent variable until the first explicit dependence of one or more control variables. The number of required derivatives ρ for a given output variable is called the relative degree of the system with respect to that output. Sometimes, however, the control effect on the ρth derivative is weak (and perhaps nonminimum phase). In applying feedback linearization for aircraft autopilot design, Lane and Stengel[9] neglected the weak effect of the elevator on lift and differentiated further to arrive at the more significant effect of elevator on the pitching moment. Hauser et al.[5] used a similar approach for autopilot design and developed a supporting theory called approximate feedback linearization. Their theory applies to control-affine systems, i.e., systems of the form

$$\dot{\boldsymbol{x}} = f(\boldsymbol{x}) + G(\boldsymbol{x})\boldsymbol{u} \tag{17}$$

where $G(\boldsymbol{x}) \in \mathbb{R}^{n \times m}$. In the following, we extend their theory to noncontrol-affine systems of the general form $\dot{\boldsymbol{x}} = f(\boldsymbol{x}, \boldsymbol{u})$.

We consider the problem of tracking a reference trajectory $\boldsymbol{y}_{\text{ref}}(t)$, which is smooth and bounded and has components that have bounded derivatives of requisite order over a time interval $[0, \infty)$. We assume knowledge of $\boldsymbol{u}_{\text{ref}}(t)$ for all $t \in [0, \infty)$ and of $\boldsymbol{x}_{\text{ref}}(0)$ that produce the reference output $\boldsymbol{y}_{\text{ref}}$ on $[0, \infty)$ consistent with the system dynamics in Eq. (16).

The output function $\boldsymbol{y} = h(\boldsymbol{x}, \boldsymbol{u})$ can be expanded in a Taylor's series about the reference control to get

$$\boldsymbol{y} = h(\boldsymbol{x}, \boldsymbol{u}) = h(\boldsymbol{x}, \boldsymbol{u}_{\text{ref}}(t)) + \frac{\partial h}{\partial u}(\boldsymbol{x}, \boldsymbol{u}_{\text{ref}}(t)) \cdot (\boldsymbol{u}(t) - \boldsymbol{u}_{\text{ref}}(t)) + \cdots \tag{18}$$

If for the output $\boldsymbol{y}_i$ all of the components of $(\partial h_i/\partial \boldsymbol{u})$ are small, we express $\boldsymbol{y}_i$ as $\boldsymbol{y}_i = \phi_{(i,0)}(\boldsymbol{x}, \boldsymbol{u}_{\text{ref}}(t)) + \epsilon\psi_{(i,0)}(\boldsymbol{x}, \boldsymbol{u}, \boldsymbol{u}_{\text{ref}}(t))$, where $\phi_{(i,0)}(\boldsymbol{x}, \boldsymbol{u}_{\text{ref}}(t)) = h(\boldsymbol{x}, \boldsymbol{u}_{\text{ref}}(t))$ and $\epsilon\psi_{(i,0)}(\boldsymbol{x}, \boldsymbol{u}, \boldsymbol{u}_{\text{ref}}(t)) = h_i(\boldsymbol{x}, \boldsymbol{u}) - \phi_{(i,0)}(\boldsymbol{x}, \boldsymbol{u}_{\text{ref}})$. This means that, for reasonable values of the controls, $\epsilon\psi_{(i,0)}$ is negligible compared to $\phi_{(i,0)}$. Although our application does not involve the use of outputs that have a direct dependence on the control, we present the following theory for the more general case.

We now successively differentiate the output until the control appears and is such that its effect on the appropriate derivative of y_i is significant. This involves neglecting additional terms in the derivatives of the output. Each output y_i is differentiated ρ_i times

$$\begin{aligned}
\dot{y}_i &= \phi_{(i,1)}(\boldsymbol{x}, \boldsymbol{u}_{\text{ref}}(t)) + \epsilon\psi_{(i,1)}(\boldsymbol{x}, \boldsymbol{u}, \boldsymbol{u}_{\text{ref}}(t), \dot{\boldsymbol{u}}_{\text{ref}}(t)) \\
\vdots &= \quad \vdots \quad + \quad \vdots \\
y_i^{(\rho_i - 1)} &= \phi_{(i,\rho_i - 1)}(\boldsymbol{x}, \boldsymbol{u}_{\text{ref}}(t)) + \epsilon\psi_{(i,\rho_i - 1)}(\boldsymbol{x}, \boldsymbol{u}, \boldsymbol{u}_{\text{ref}}(t), \dot{\boldsymbol{u}}_{\text{ref}}(t), \ldots) \\
y_i^{(\rho_i)} &= \phi_{(i,\rho_i)}(\boldsymbol{x}, \boldsymbol{u}, \boldsymbol{u}_{\text{ref}}(t)) + \epsilon\psi_{(i,\rho_i)}(\boldsymbol{x}, \boldsymbol{u}, \boldsymbol{u}_{\text{ref}}(t), \dot{\boldsymbol{u}}_{\text{ref}}(t), \ldots)
\end{aligned} \tag{19}$$

where ρ_i is the smallest integer such that the control $\boldsymbol{u}$ has a significant effect on $y_i^{(\rho_i)}$ and $\epsilon\psi_{(i,j)}(\boldsymbol{x}, \boldsymbol{u})$ contains the time derivative of $\epsilon\psi_{(i,j-1)}(\boldsymbol{x}, \boldsymbol{u})$ and possibly some other terms arising from the time derivative of $\phi_{(i,j-1)}$ that are small. For example, if the term $(\partial\phi_{(i,j-1)}/\partial\boldsymbol{u})\dot{\boldsymbol{u}}$ in the time derivative of $\phi_{(i,j-1)}$ has a weak sensitivity to control, i.e., $(\partial\phi_{(i,j-1)}/\partial\boldsymbol{u})$ is small, then it is included in the $\epsilon\psi_{(i,j)}$ term. At each stage, the significant terms are included in the ϕ term, and the terms that are small are included in the $\epsilon\psi$ term. Notice that the earlier derivatives of the output in which the control appears prematurely are approximated by substituting $\boldsymbol{u}_{\text{ref}}(t)$ for $\boldsymbol{u}(t)$ in the corresponding ϕ term. Hence, for each i, only $\phi_{(i,\rho_i)}$ has a dependence on $\boldsymbol{u}$, whereas all of the previous $\phi_{(i,j)}$ do not depend on $\boldsymbol{u}$. We decide that the effect of the control on the corresponding derivative of the output is significant when $(\partial\phi/\partial\boldsymbol{u}) = \mathcal{O}(\phi)$ as $(\boldsymbol{x}, \boldsymbol{u}) \to (\boldsymbol{x}_{\text{ref}}, \boldsymbol{u}_{\text{ref}})$. For all earlier derivatives, the effect of the control is weak because the corresponding $(\partial\phi/\partial\boldsymbol{u}) = \mathcal{O}(\epsilon\psi)$ as $(\boldsymbol{x}, \boldsymbol{u}) \to (\boldsymbol{x}_{\text{ref}}, \boldsymbol{u}_{\text{ref}})$. Here, the order symbol $\mathcal{O}(\cdot)$, has the usual definition[10] that when $p(\boldsymbol{x}, \boldsymbol{u}) = \mathcal{O}(q(\boldsymbol{x}, \boldsymbol{u}))$, there exists a neighborhood of $(\boldsymbol{x}_{\text{ref}}, \boldsymbol{u}_{\text{ref}})$ in which the ratio $p(\boldsymbol{x}, \boldsymbol{u})/q(\boldsymbol{x}, \boldsymbol{u})$ is finite as $(\boldsymbol{x}, \boldsymbol{u}) \to (\boldsymbol{x}_{\text{ref}}, \boldsymbol{u}_{\text{ref}})$.

Definition 1 (global approximate vector relative degree): A system of the form

$$\begin{aligned}
\dot{\boldsymbol{x}} &= f(\boldsymbol{x}, \boldsymbol{u}), \qquad \boldsymbol{x} \in \mathbb{R}^n, \qquad \boldsymbol{u} \in \mathbb{R}^m \\
\boldsymbol{y} &= h(\boldsymbol{x}, \boldsymbol{u}_{\text{ref}}(t)) + \epsilon\psi_0(\boldsymbol{x}, \boldsymbol{u}), \qquad \boldsymbol{y} \in \mathbb{R}^m
\end{aligned} \tag{20}$$

is said to have a well-defined approximate vector relative degree $[\rho_1, \rho_2, \ldots, \rho_m]^T$ where ρ_i is the least-order time derivative of y_i on which the effect of the control is significant, if the equation

$$\begin{bmatrix} \dfrac{\mathrm{d}^{(\rho_1)} y_1}{\mathrm{d}t^{(\rho_1)}} \\ \dfrac{\mathrm{d}^{(\rho_2)} y_2}{\mathrm{d}t^{(\rho_2)}} \\ \vdots \\ \dfrac{\mathrm{d}^{(\rho_m)} y_m}{\mathrm{d}t^{(\rho_m)}} \end{bmatrix} = \Phi(\boldsymbol{x}, \boldsymbol{u}, \boldsymbol{u}_{\text{ref}}(t)) + \epsilon\Psi(\boldsymbol{x}, \boldsymbol{u}, \boldsymbol{u}_{\text{ref}}(t), \dot{\boldsymbol{u}}_{\text{ref}}(t), \ldots) \tag{21}$$

where

$$\Phi(\boldsymbol{x}, \boldsymbol{u}, \boldsymbol{u}_{\text{ref}}(t)) = [\phi_{(1,\rho_1)}(\boldsymbol{x}, \boldsymbol{u}, \boldsymbol{u}_{\text{ref}}(t)), \ldots, \phi_{(m,\rho_m)}(\boldsymbol{x}, \boldsymbol{u}, \boldsymbol{u}_{\text{ref}}(t))]^T$$

and

$$\Psi(\boldsymbol{x}, \boldsymbol{u}) = [\psi_{(1,\rho_1)}(\boldsymbol{x}, \boldsymbol{u}, \boldsymbol{u}_{\text{ref}}(t)), \ldots, \psi_{(m,\rho_m)}(\boldsymbol{x}, \boldsymbol{u}, \boldsymbol{u}_{\text{ref}}(t))]^T$$

is such that the Jacobian $\partial\Phi/\partial\boldsymbol{u}$ is nonsingular $\forall\boldsymbol{x} \in \mathbb{R}^n$ and $\forall\boldsymbol{u} \in \mathbb{R}^m$ and for all values of $\boldsymbol{u}_{\text{ref}}(t)$ for $t \in [0, \infty)$.

Theorem 1 (global approximate input–output linearizability): For every system of the form (20) that has a well-defined global approximate vector relative degree $[\rho_1, \rho_2, \ldots, \rho_m]^T$, there exists a state-dependent control transformation $\boldsymbol{u} = \boldsymbol{k}(\boldsymbol{x}, \boldsymbol{w}, \boldsymbol{u}_{\text{ref}}(t))$ to the new control $\boldsymbol{w} = [w_1, w_2, \ldots, w_m]^T = \Phi(\boldsymbol{x}, \boldsymbol{u}, \boldsymbol{u}_{\text{ref}}(t))$ that globally transforms the system into m decoupled approximately linear systems of the form

$$\begin{aligned}
\frac{\mathrm{d}^{(\rho_1)} y_1}{\mathrm{d}t^{(\rho_1)}} &= \boldsymbol{w}_1 + \epsilon\psi_{(1,\rho_1)}(\boldsymbol{x}, \boldsymbol{u}, \boldsymbol{u}_{\text{ref}}(t), \dot{\boldsymbol{u}}_{\text{ref}}(t), \ldots) \\
\frac{\mathrm{d}^{(\rho_2)} y_2}{\mathrm{d}t^{(\rho_2)}} &= \boldsymbol{w}_2 + \epsilon\psi_{(2,\rho_2)}(\boldsymbol{x}, \boldsymbol{u}, \boldsymbol{u}_{\text{ref}}(t), \dot{\boldsymbol{u}}_{\text{ref}}(t), \ldots) \\
\vdots &= \quad \vdots \\
\frac{\mathrm{d}^{(\rho_m)} y_m}{\mathrm{d}t^{(\rho_m)}} &= \boldsymbol{w}_m + \epsilon\psi_{(m,\rho_m)}(\boldsymbol{x}, \boldsymbol{u}, \boldsymbol{u}_{\text{ref}}(t), \dot{\boldsymbol{u}}_{\text{ref}}(t), \ldots)
\end{aligned} \tag{22}$$

Proof: Because the system has a well-defined global approximate vector relative degree, Eq. (21) holds such that $\partial\Phi/\partial\boldsymbol{u}$ is nonsingular $\forall\boldsymbol{x} \in \mathbb{R}^n$, $\forall\boldsymbol{u} \in \mathbb{R}^m$, and all values of $\boldsymbol{u}_{\text{ref}}(t)$ for $t \in [0, \infty)$. For a global static feedback transformation $\boldsymbol{u} = \boldsymbol{k}(\boldsymbol{x}, \boldsymbol{w}, \boldsymbol{u}_{\text{ref}}(t))$ that makes the system approximately input–output linear to exist, we should be able to invert the implicit equation

$$\boldsymbol{w} - \Phi(\boldsymbol{x}, \boldsymbol{u}, \boldsymbol{u}_{\text{ref}}(t)) = 0 \tag{23}$$

Invoking the implicit function theorem (stated in Appendix A of Isidori[7]), we find that there exists a function $\boldsymbol{u} = \boldsymbol{k}(\boldsymbol{x}, \boldsymbol{w}, \boldsymbol{u}_{\text{ref}}(t))$ that satisfies $\boldsymbol{w} - \Phi(\boldsymbol{x}, \boldsymbol{k}(\boldsymbol{x}, \boldsymbol{w}, \boldsymbol{u}_{\text{ref}}(t))) = 0$ if and only if the Jacobian $\partial\Phi/\partial\boldsymbol{u}$ is nonsingular for all $\boldsymbol{u} \in \mathbb{R}^m$ and all values of $\boldsymbol{u}_{\text{ref}}(t)$ for $t \in [0, \infty)$. But this is true because the system has a well-defined global approximate relative degree. Hence, we have shown that for all systems of the form (20) that have a well-defined global approximate relative degree, there exists a static state feedback transformation that renders the system globally approximately input–output linear. □

Consider a system of the form (20) that is globally approximately input–output linearizable. If the sum of the approximate relative degrees

$$\bar{\rho} = \sum_{i=1}^{m} \rho_i$$

is equal to n, then the closed-loop system is approximately linearized both from input-to-state and input-to-output by the state transformation to the new state variables $\boldsymbol{\xi}$, where

$$\boldsymbol{\xi} = \left[\phi_{(1,0)}, \ldots, \phi_{(1,\rho_1-1)}, \ldots, \phi_{(m,0)}, \ldots, \phi_{(m,\rho_m-1)}\right]$$

has dimension n. If $\bar{\rho} < n$, we have leftover dynamics that are unobservable in the input–output system. To avoid the potential complications when $\bar{\rho} < n$, we shall seek output functions that achieve input-to-state linearization.

Bounded Tracking of Approximate Outputs

Consider systems of the form (20) that are globally approximately input-output linearizable via static state feedback.

Definition 2 (bounded output tracking): Given a smooth bounded reference trajectory $\boldsymbol{y}_{\text{ref}}$, which has the derivatives of $y_{i_{\text{ref}}}$ up to order ρ_i also bounded, $1 \le i \le m$, the bounded output tracking problem is said to be solvable for a globally approximately input–output linearizable system of the form (20), if there exists a control $\boldsymbol{u} = \boldsymbol{k}(\boldsymbol{x}, \boldsymbol{Y}_1, \boldsymbol{Y}_2, \ldots, \boldsymbol{Y}_m, \boldsymbol{Y}_{1_{\text{ref}}}, \boldsymbol{Y}_{2_{\text{ref}}}, \ldots, \boldsymbol{Y}_{m_{\text{ref}}})$ where

$\boldsymbol{Y}_i = [y_i, y_i', \ldots, y_i^{(\rho_i)}]^T$ and $\boldsymbol{Y}_{i_{\mathrm{ref}}} = [y_{i_{\mathrm{ref}}}, y_{i_{\mathrm{ref}}}', \ldots, y_{i_{\mathrm{ref}}}^{(\rho_i)}]^T$, such that for any initial condition $\boldsymbol{x}(0) \in \mathbb{R}^n$, the closed-loop system

$$\dot{\boldsymbol{x}} = f\Big(\boldsymbol{x}, \boldsymbol{k}\big(\boldsymbol{x}, \boldsymbol{Y}_1, \boldsymbol{Y}_2, \ldots, \boldsymbol{Y}_m, \boldsymbol{Y}_{1_{\mathrm{ref}}}, \boldsymbol{Y}_{2_{\mathrm{ref}}}, \ldots, \boldsymbol{Y}_{m_{\mathrm{ref}}}\big)\Big)$$
$$\boldsymbol{y} = \boldsymbol{h}(\boldsymbol{x}, \boldsymbol{u}_{\mathrm{ref}}) + \epsilon\psi_0(\boldsymbol{x}, \boldsymbol{u}) \tag{24}$$

is such that 1) $\|\boldsymbol{x}(t)\|$ is bounded $\forall t \geq 0$; 2) for each i, $|y_i(t) - y_{i_{\mathrm{ref}}}(t)| \leq \epsilon\beta M$, where ϵM is an upper bound on the error due to the neglected terms and β is a finite positive constant; and 3) $\lim_{t\to\infty}(\boldsymbol{y}(t) - \boldsymbol{y}_{\mathrm{ref}}(t)) = 0$ when $\epsilon = 0$.

Theorem 2 (bounded output tracking): Bounded tracking of the approximately flat outputs for a system of the form (20) is achieved by setting

$$w_i = y_{i_{\mathrm{ref}}}^{(\rho_i)} - k_{(i,1)}\Big(y_i^{(\rho_i-1)} - y_{i_{\mathrm{ref}}}^{(\rho_i-1)}\Big) + \cdots - k_{(i,\rho_i)}\big(y_i - y_{i_{\mathrm{ref}}}\big) \tag{25}$$

where the gains are chosen such that the polynomial $s^{\rho_i} + k_{i,1}s^{\rho_i-1} + \cdots + k_{i,\rho_i}$ is Hurwitz.

Proof: With the control (25), the closed-loop system for $y_i^{\rho_i}$ becomes

$$\Big(y_i^{(\rho_i)} - y_{i_{\mathrm{ref}}}^{(\rho_i)}\Big) + k_{(i,1)}\Big(y_i^{(\rho_i-1)} - y_{i_{\mathrm{ref}}}^{(\rho_i-1)}\Big) + \cdots + k_{(i,\rho_i)}\big(y_i - y_{i_{\mathrm{ref}}}\big) = \epsilon\psi_{(i,\rho_i)}$$

because $y_i^{(\rho_i)} = \phi_{(i,\rho_i)} + \epsilon\psi_{(i,\rho_i)}$ and $w_i = \phi_{(i,\rho_i)}$. The tracking-error dynamics are

$$e^{(\rho_i)} + k_{(i,1)}e^{(\rho_i-1)} + \cdots + k_{(i,\rho_i)}e = \epsilon\psi_{(i,\rho_i)} \tag{26}$$

where $e = (y_i - y_{i_{\mathrm{ref}}})$. The coefficients $k_{(i,j)}$ are now chosen so that the unforced error dynamics

$$e^{(\rho_i)} + k_{(i,1)}e^{(\rho_i-1)} + \cdots + k_{(i,\rho_i)}e = 0 \tag{27}$$

are exponentially stable. The true error dynamics (26) can now be viewed as the single-input/single-output (SISO) linear system (27) with the right-hand-side of Eq. (26) as (disturbance) input and e as the output. From a well-known result in robust control theory[11] we know that this SISO linear system is bounded-input/bounded-output stable, i.e., the ratio of 2 norm of the output to the 2 norm of the input is bounded by the ∞ norm of the transfer function of the linear system. Therefore, $\|e(t)\|_2 \leq \epsilon\beta M, \forall t$ where

$$M = \|\psi_{(i,\rho_i)}\|_2 = \left(\int_0^\infty \psi_{(i,\rho_i)}^2\,\mathrm{d}t\right)^{\frac{1}{2}}$$

and where

$$\beta = \|E(s)\|_\infty = \sup_\omega |E(j\omega)|$$

is the ∞ norm of the transfer function E of the SISO linear system (27). Hence, the second condition of definition (2) is satisfied by the control (25). Also, the choice of gains in Eq. (27) ensures that the tracking error in each of the outputs tends to zero asymptotically in the absence of approximations, and thus the third condition in definition (2) is satisfied by the control (25). Moreover, because we have assumed that the choice of output functions is such that the sum of the relative degrees $\bar{\rho} = n$, the bounded behavior of the tracking error ensures bounded behavior of the state. Therefore, the first condition of definition (2) is also satisfied.

Because all of the requirements of definition (2) are satisfied by the control law in Eq. (25), it achieves bounded-output tracking. □

Remarks: 1) The theory only considers stability in the choice of the gains. There are many sets of gains that satisfy this condition. Performance objectives, such as minimizing β, can be used to uniquely specify the gains. 2) Adding an integral feedback term of the form

$$k_{(i,\rho_i+1)}\int_0^t \big(\phi_{(i,0)} - \phi_{(i,0)_{\mathrm{ref}}}\big)\,\mathrm{d}t$$

has the well-known effect of zeroing the steady-state output error for a constant disturbance without any detrimental effect on the other properties of the control law that we have described. We will include integral feedback in applying the theory in the next section.

Tracking Law Derivation

Downrange and Crossrange Angles

The vectors from the center of the Earth to the nominal reference entry point and the nominal target point define our reference entry plane. The longitude, latitude, and heading angles can be redefined, by an orthogonal rotation of the Earth frame, such that the zero longitude line corresponds to the great circle in the reference entry plane and such that, for flight in the reference entry plane, the target is at zero heading. For a nonrotating, windless Earth, no changes in the equations of motion are necessary. With these redefinitions, we will refer to θ as the downrange angle and ϕ as the crossrange angle.

Approximate Feedback Linearization

To illustrate the tracking law derivation we shall take the system dynamics to be represented by

$$\theta' = -(1/rD) \tag{28}$$

$$r' = -(\gamma/D) \tag{29}$$

$$\gamma' = (1/DV^2)[(g - (V^2/r)) - L\cos\sigma] \tag{30}$$

$$\phi' = -(\chi/rD) \tag{31}$$

$$\chi' = -\frac{1}{D}\left[-\frac{\phi}{r} + \frac{L\sin\sigma}{V^2}\right] \tag{32}$$

where θ, ϕ, and χ assume their new definitions. Equations (28–30) represent the longitudinal dynamics. Equations (31) and (32) represent the lateral dynamics. The small angle approximations for ϕ, γ, and χ serve to simplify the tracking law derivation for the presentation. The approach, however, can be carried out without making these approximations.

We want the bank angle σ to be the primary means of controlling drag. The bank angle has a less direct effect on drag than the angle of attack: σ affects γ, γ in turn affects r, and r in turn affects drag via the air density. For example, at a given energy, if the drag is less than required to achieve the specified θ', we want the vehicle to bank, reducing the vertical component of lift, and to fly to a lower altitude, where the higher density will increase the drag. If the increased drag does not produce the specified ϕ', then α is adjusted to change the heading angle. The use of bank angle as the primary drag control is consistent with the operation of the Shuttle. Especially in the early part of the Shuttle entry, α is kept high (≈40 deg) for temperature control and cannot be modified too much. Note, however, that although we want σ to be the primary control of drag, changes in α will affect the drag. The desired roles we have described for the controls α and σ are not achievable, strictly speaking, due to the nature of their effects. The desired roles serve as guiding considerations rather than hard constraints.

We pick the natural candidates for output functions for feedback linearization, $y_1 = \theta$ and $y_2 = \phi$. These outputs are now differentiated with respect to energy until the effect of the control appears explicitly. The control α first appears through the drag in the first derivative of the outputs as

$$y_1' = \frac{-1}{D(\alpha)r} \tag{33}$$

$$y_2' = \frac{-\chi}{D(\alpha)r} \tag{34}$$

Because this dependence is insufficient to control the outputs independently and because we do not want the effect of α on drag as the means for trajectory control, we differentiate further after approximating the drag through the first term of its Taylor's series about the reference control $\alpha_{\text{ref}}(E)$, i.e.,

$$D = \left(\frac{qS}{m}\right) \cdot C_D(\alpha_{\text{ref}}(E), M) + \left(\frac{qS}{m}\right) \cdot \frac{\partial C_D}{\partial \alpha}(\alpha - \alpha_{\text{ref}}) + \cdots \tag{35}$$

where $q = \frac{1}{2}\rho(r)V^2$ is the dynamic pressure. Also, we can conceptually identify the parameter $(\partial C_D/\partial\alpha)$ with ϵ used in the theory section. Thus,

$$D \approx \tilde{D} = (qS/m) \cdot C_D(\alpha_{\text{ref}}(E), M) \tag{36}$$

Therefore,

$$y_1' = (-1/\tilde{D}r) + \epsilon\psi_{(1,1)} \tag{37}$$

$$y_2' = (-\chi/\tilde{D}r) + \epsilon\psi_{(2,1)} \tag{38}$$

where $\epsilon\psi_{(1,1)}$ and $\epsilon\psi_{(2,1)}$ are used as in the preceding section to denote the errors introduced by the approximation. Further differentiation yields

$$y_1'' = (\gamma/\tilde{D}^2 r)[(1/H) - (1/r) + (2g/V^2)] + (2/\tilde{D}V^2 r) + \epsilon\psi_{(1,2)} \tag{39}$$

and

$$y_2'' = l(x, \tilde{D}) + m(\boldsymbol{x}, \tilde{D}) \cdot L(\alpha)\sin\sigma + \epsilon\psi_{(2,2)} \tag{40}$$

where

$$l(\boldsymbol{x}, \tilde{D}) = \left(\frac{\gamma\chi}{\tilde{D}^2 r}\right)\left(\frac{1}{H} - \frac{1}{r} + \frac{2g}{V^2}\right) + \frac{2\chi}{\tilde{D}V^2 r} - \frac{\phi}{\tilde{D}^2 r^2}$$
$$m(\boldsymbol{x}, \tilde{D}) = \left(\frac{1}{\tilde{D}^2 V^2 r}\right) \tag{41}$$

Differentiating Eq. (39) again yields

$$y_1''' = a(\boldsymbol{x}, \tilde{D}) + b(\boldsymbol{x}, \tilde{D})L(\alpha)\cos\sigma + \epsilon\psi_{(1,3)} \tag{42}$$

where

$$a(\boldsymbol{x}, \tilde{D}) = \left(\frac{1}{\tilde{D}^3 V^2 r}\right)\left(\frac{1}{H} - \frac{1}{r} + \frac{2g}{V^2}\right)\left(g - \frac{V^2}{r}\right) - \frac{\gamma^2}{\tilde{D}^3 r^3} - \frac{4(\tilde{D} + g\gamma)^2}{\tilde{D}^3 V^4 r} - \frac{2\tilde{D}'}{\tilde{D}^2 V^2 r} + \left(\frac{1}{H} - \frac{1}{r} + \frac{2g}{V^2}\right)\left(\frac{\gamma^2}{\tilde{D}^3 r^2} - \frac{2\tilde{D}'\gamma}{\tilde{D}^3 r}\right)$$
$$b(\boldsymbol{x}, \tilde{D}) = \left(\frac{-1}{\tilde{D}^3 V^2 r}\right)\left(\frac{1}{H} - \frac{1}{r} + \frac{2g}{V^2}\right) \tag{43}$$

and where Eqs. (7) and (8) and their energy derivatives can be used to show that

$$\tilde{D}' = (\gamma/H) + (2\tilde{D}/V^2) + (2g\gamma/V^2) \tag{44}$$

assuming $C_D' \approx 0$. We can now make a direct association of these terms in the derivatives of the outputs with the notation used in the theory to get expressions for $\phi_{(1,0)}, \phi_{(1,1)}, \phi_{(1,2)}, \phi_{(1,3)}, \phi_{(2,0)}, \phi_{(2,1)}$, and $\phi_{(2,2)}$. This gives

$$\begin{bmatrix} y_1'''(E) \\ y_2''(E) \end{bmatrix} = \Phi + \epsilon\Psi \tag{45}$$

where $\Phi = [\phi_{(1,3)}, \phi_{(2,2)}]^T$ and $\Psi = [\psi_{(1,3)}, \psi_{(2,2)}]^T$.

To have a well-defined approximate global relative degree, $\partial\Phi/\partial\boldsymbol{u}$ should be globally nonsingular:

$$\frac{\partial\Phi}{\partial u} = \begin{bmatrix} \dfrac{\partial\phi_{(1,3)}}{\partial\alpha} & \dfrac{\partial\phi_{(1,3)}}{\partial\sigma} \\ \dfrac{\partial\phi_{(2,2)}}{\partial\alpha} & \dfrac{\partial\phi_{(2,2)}}{\partial\sigma} \end{bmatrix}$$

Because

$$\frac{\partial\phi_{(1,3)}}{\partial\alpha} = \frac{\partial L}{\partial\alpha} b(\boldsymbol{x}, \tilde{D})\cos\sigma, \qquad \frac{\partial\phi_{(1,3)}}{\partial\sigma} = -b(\boldsymbol{x}, \tilde{D})L(\alpha)\sin\sigma$$
$$\frac{\partial\phi_{(2,2)}}{\partial\alpha} = \frac{\partial L}{\partial\alpha} m(\boldsymbol{x}, \tilde{D})\sin\sigma, \qquad \frac{\partial\phi_{(2,2)}}{\partial\sigma} = m(\boldsymbol{x}, \tilde{D})L(\alpha)\cos\sigma \tag{46}$$

we have

$$\det\left(\frac{\partial\Phi}{\partial\boldsymbol{u}}\right) = b(\boldsymbol{x}, \tilde{D})m(\boldsymbol{x}, \tilde{D})L\frac{\partial L}{\partial\alpha} \tag{47}$$

For the class of entry trajectories considered here, this determinant is always nonzero. Assuming that α will be restricted to values below stall, $(\partial L/\partial\alpha)$ will be nonzero, and thus the Jacobian is nonsingular if $b(\boldsymbol{x}, \tilde{D}) \neq 0$ and $m(\boldsymbol{x}, \tilde{D}) \neq 0$. This is true everywhere. Therefore, the Jacobian is globally nonsingular. Hence, the global approximate vector relative degree is $[3, 2]^T$.

Because the sum of the relative degrees $\bar{\rho} = 5$, which equals the state dimension for the dynamics given by Eqs. (28–32), there are no leftover dynamics that would be unobservable with respect to the outputs. Hence, this choice of outputs result in complete input to state approximate linearization.

Bounded Tracking

Invoking theorem 1, we find that the system (29–32) can be globally approximately linearized to get

$$y_1'''(E) = w_1 + \epsilon\psi_{(1,3)}, \qquad y_2''(E) = w_2 + \epsilon\psi_{(2,2)} \tag{48}$$

Based on theorem 2, a control of the form

$$w_1 = y_{1_{\text{ref}}}''' - k_{(1,1)}\left(y_1'' - y_{1_{\text{ref}}}''\right) - k_{(1,2)}\left(y_1' - y_{1_{\text{ref}}}'\right) - k_{(1,3)}\left(y_1 - y_{1_{\text{ref}}}\right) - k_{(1,4)}\int_{E_0}^{E}\left(y_1 - y_{1_{\text{ref}}}\right)\mathrm{d}E$$
$$w_2 = y_{2_{\text{ref}}}'' - k_{(2,1)}\left(y_2' - y_{2_{\text{ref}}}'\right) - k_{(2,2)}\left(y_2 - y_{2_{\text{ref}}}\right) - k_{(2,3)}\int_{E_0}^{E}\left(y_2 - y_{2_{\text{ref}}}\right)\mathrm{d}E \tag{49}$$

with proper gain selection achieves bounded tracking of the outputs. The unforced closed-loop error dynamics are given by

$$e_1''' + k_{(1,1)}e_1'' + k_{(1,2)}e_1' + k_{(1,3)}e_1 + k_{(1,4)}\int_{E_0}^{E} e_1\,\mathrm{d}E = 0 \tag{50}$$

$$e_2'' + k_{(2,1)}e_2' + k_{(2,2)}e_2 + k_{(2,3)}\int_{E_0}^{E} e_2\,\mathrm{d}E = 0 \tag{51}$$

where $e_1 = (y_1 - y_{1_{\text{ref}}})$ and $e_2 = (y_2 - y_{2_{\text{ref}}})$. The integral terms have been added to improve the performance of the controller. Because the independent variable energy is decreasing, the gains must be chosen such that the eigenvalues of the closed-loop error dynamics lie in the open right-half complex plane for asymptotic stability.

The actual controls $[\alpha, \sigma]^T$ are obtained by inverting the equation $\boldsymbol{w} = \Phi(\boldsymbol{x}, \boldsymbol{u}, \boldsymbol{u}_{\text{ref}}(t))$. A feature of our formulation is that this inversion can be performed semianalytically. From the equations

$$w_1 = a(\boldsymbol{x}, \tilde{D}) + (L(\alpha)\cos\sigma)b(\boldsymbol{x}, \tilde{D})$$
$$w_2 = l(\boldsymbol{x}, \tilde{D}) + (L(\alpha)\sin\sigma)m(\boldsymbol{x}, \tilde{D})$$

we can compute $A = L(\alpha)\sin\sigma$ and $B = L(\alpha)\cos\sigma$ as

$$A = \left(\frac{w_1 - a(\boldsymbol{x}, \tilde{D})}{b(\boldsymbol{x}, \tilde{D})}\right), \qquad B = \left(\frac{w_2 - l(\boldsymbol{x}, \tilde{D})}{m(\boldsymbol{x}, \tilde{D})}\right)$$

The values of A and B computed from the preceding expression determine values of $L\cos\sigma$ and $L\sin\sigma$ needed to meet the downrange and crossrange tracking objectives. This can be viewed as a direct generalization of the Shuttle entry guidance logic that computes the value of $L\cos\sigma$ needed to meet the downrange objective. In the Shuttle, this desired vertical-plane component of the lift is achieved through a combination of α and σ commands. For the most part, α is held fixed at $\alpha_{\text{ref}}(E)$ and the desired $L\cos\sigma$ is achieved using bank-angle modulation. But during bank reversals, the bank angle is undergoing a fixed transition, so the angle of attack is varied to achieve the desired $L\cos\sigma$. (Note that angle-of-attack modulation has a direct effect on drag, which is being ignored in the Shuttle logic, a feature in common with our approach.) In our extended formulation, both the angle of attack and the bank commands are uniquely determined and can be found using

$$L(\alpha) = \sqrt{A^2 + B^2} \tag{52}$$

$$\sigma = \arctan(B/A) \tag{53}$$

The value of α has then to be backed out from the lift equation using the aerodynamic model for $C_L(\alpha, M)$. This then gives the feedback law that achieves bounded tracking of the downrange and crossrange distances as functions of energy.

This procedure does not, however, limit the angle of attack to stay close to the reference value. During certain segments of entry, it may be important to restrict α to an allowable band around the reference, $\alpha_{\text{low}}(E) < \alpha < \alpha_{\text{high}}(E)$, for example, during the high-heating segment. If the solution to the preceding equations results in an α command that is outside the allowed band, it implies that both tracking objectives cannot be simultaneously met at the current energy with acceptable controls. Therefore, we can trade off between the two objectives of tracking downrange and crossrange by solving for the α and σ commands that minimize the cost function

$$J = \eta(L\cos\sigma - A)^2 + (1-\eta)(L\sin\sigma - B)^2 \tag{54}$$

subject to the constraint $\alpha_{\text{low}}(E) \le \alpha \le \alpha_{\text{high}}(E)$ and possibly $\sigma_{\text{low}}(E) \le \sigma \le \sigma_{\text{high}}(E)$ and where $0 < \eta < 1$ is a weighting that is chosen to emphasize either the downrange or crossrange tracking objective. A similar approach where the control commands are calculated by solving a pointwise optimization problem is described in Ref. 12. The bounded tracking result still holds, provided the tracking errors remain bounded down to an energy E_1, beyond which $J \equiv 0$ is achieved.

Simulation Results

Simulations were conducted to validate the approximations made in the derivation of the tracking law and to determine the tracking performance under the ideal conditions of perfect state knowledge and no modeling error. Note that the tracking law used for simulation was derived without the small heading angle assumption. The derivation of this tracking law is similar to the one shown, but involves many more terms in the computation of the derivatives of the output. In the interest of clarity and brevity in explaining the methodology, the simpler derivation was shown. The reader is referred to Ref. 13 for an earlier version of this research.

The aerodynamic model used to represent an RLV corresponds to the SX-2 vehicle of McDonnell Douglas Aerospace, which has a maximum lift-to-drag ratio of 1.6. The data for $C_L(\alpha)$ and $C_D(\alpha)$ include the effect of any flap deflections required to balance the pitching moment on the vehicle. A sufficiently accurate polynomial fit of the aerodynamic data for $C_L(\alpha)$ and $C_D(\alpha)$ over the range $5 < \alpha < 25$ deg is given by $C_L(\alpha) = 2.5457\alpha - 0.0448$ and $C_D(\alpha) = 3.7677\alpha^2 - 0.1427\alpha + 0.1971$, where α is in radians. The vehicle weight during descent is 82,310 lb, and the reference area is 709.2 ft^2. The exponential density model parameters are $H = 2.3443 \times 10^4$ ft and $\rho_0 = 2.3769 \times 10^{-3}$ lb/ft^3. The reference state and output trajectories for the simulations are obtained by open-loop integration of some chosen reference controls $\alpha_{\text{ref}}(E)$ and $\sigma_{\text{ref}}(E)$ so that the resulting trajectory satisfies all of the constraints with enough margin to accommodate dispersions. The initial reference state is

$$[r, \theta, \phi, V, \gamma, \chi]^T = [r_0 + 200{,}000\,\text{ft}, 0, 0, 17{,}000\,\text{ft/s}^2, 0, 0]^T$$

where r_0 is Earth's equatorial radius, in feet.

Simulations were performed for a range of representative initial perturbations from the reference trajectory. In each of the simulations, the initial state variables were perturbed one at a time from their initial reference values by an amount within the ranges $\delta r = \pm 1000$ ft, $\delta\theta = \pm 0.1$ deg, $\delta\phi = \pm 0.1$ deg, $\delta V = \pm 100$ ft/s, $\delta\gamma = \pm 0.1$ deg, and $\delta\chi = \pm 0.1$ deg. The final errors in downrange and crossrange were found to be less than 1 n mile in all cases.

Figures 1–6 show the results for an initial perturbation vector

$$[\delta r, \delta\theta, \delta\phi, \delta V, \delta\gamma, \delta\chi] = [1000\,\text{ft}, 0.1\,\text{deg}, 0.1\,\text{deg}, 0, 0, 0]$$

The initial deviations in longitude and latitude correspond to deviations of about 6 n mile in downrange and crossrange from their reference values. Figure 1 shows the downrange vs crossrange plot of the reference and actual ground tracks. The vehicle successfully follows the desired ground track, as shown more clearly by the tracking errors plotted in Fig. 2. It can be seen that the downrange and crossrange are nearly asymptotically tracked and the final errors are reduced to within 1 n mile.

Figures 3 and 4 show the reference and commanded control profiles. The α and σ commands were computed by solving the constrained optimization problem described earlier with equal weighting on both downrange and crossrange, i.e., $\eta = 0.5$. The constraints α_{low}, α_{high}, σ_{low}, and σ_{high} are shown as functions of Mach number in the respective plots. The commanded angle of attack rides the upper and lower constraint lines in the initial portions and remains close to the reference subsequently. Note that there is a reversal in

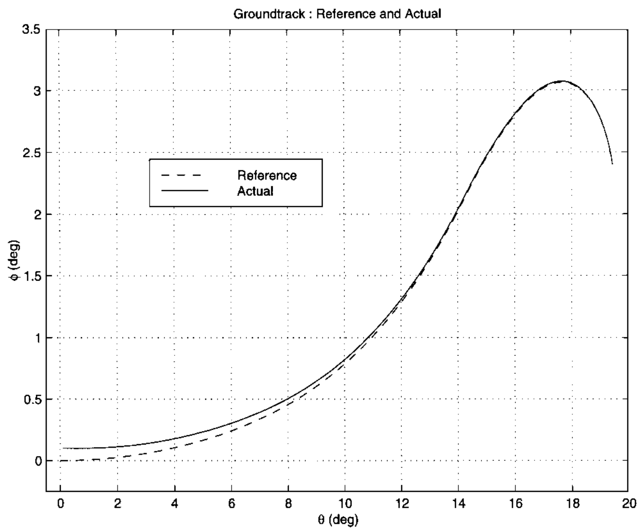

Fig. 1 Reference and actual downrange vs crossrange.

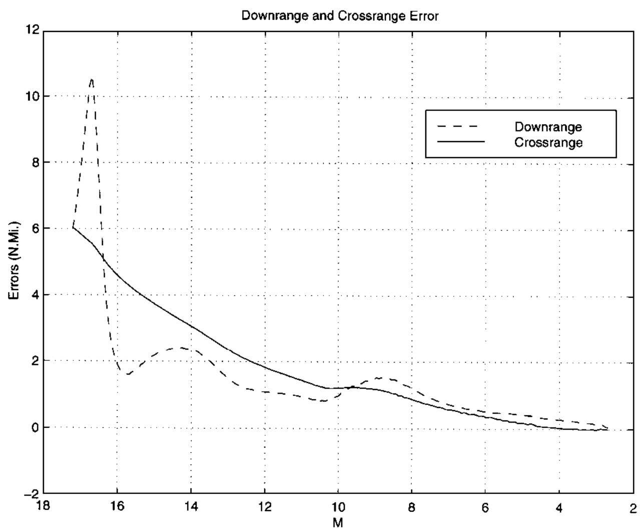

Fig. 2 Downrange error $r_0\Delta\theta$ and crossrange error $r_0\Delta\phi$ vs Mach number.

the reference bank angle at a velocity of about 10,000 ft/s. Figure 5 shows that the actual altitude tracks the reference altitude profile following a quick recovery from the initial deviation of 1000 ft. Figure 6 shows the reference drag and actual drag, which remain well within the entry corridor determined by the constraints on heating, normal load, dynamic pressure, and equilibrium glide. The final altitude is about 84,000 ft at a velocity close to Mach 2.7. It is assumed that the TAEM guidance would take over from there.

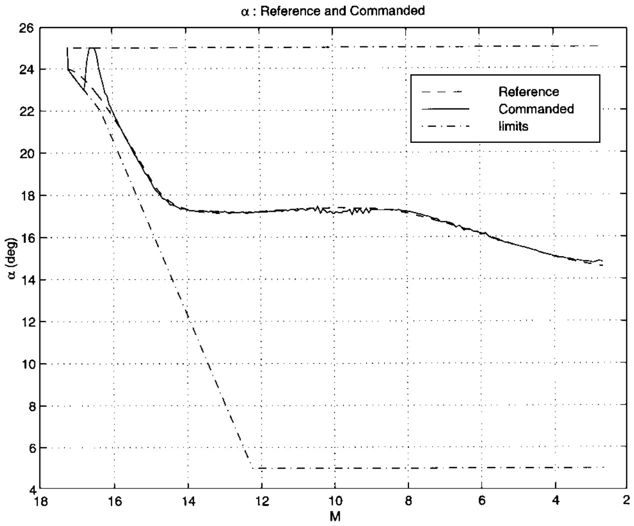

Fig. 3 Reference and commanded angle of attack vs Mach number.

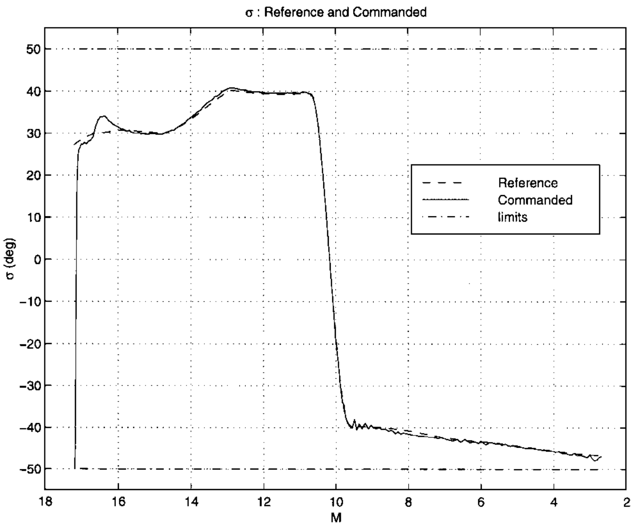

Fig. 4 Reference and commanded angle of bank vs Mach number.

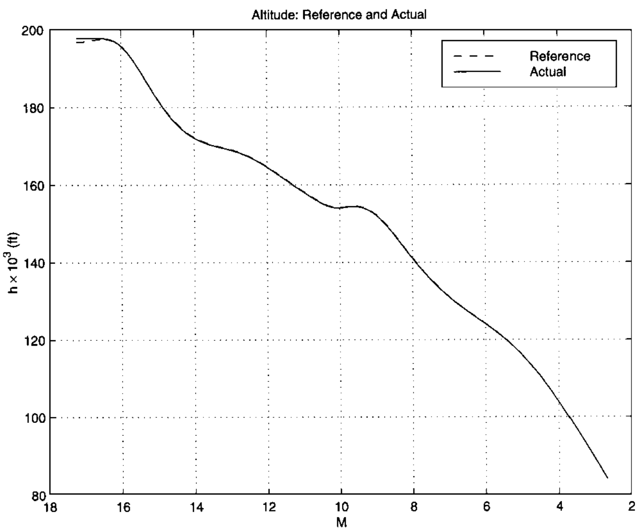

Fig. 5 Reference and actual altitude vs Mach number.

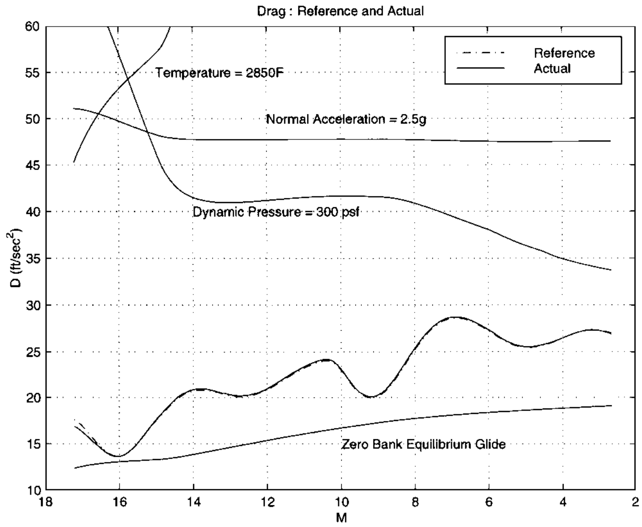

Fig. 6 Reference and actual drag vs Mach number.

Conclusions

A reference trajectory tracking law for a new entry guidance concept that generalizes the two-dimensional Shuttle entry guidance scheme to a full three-dimensional entry guidance scheme was developed. Both the angles of attack and of bank are commanded to make the vehicle follow a reference ground track. Some extensions to approximate feedback linearization theory were developed to enable tracking law design. By posing the inverse control transformation as a constrained optimization problem, tradeoffs between tracking and other concerns, such as keeping the angle of attack high during high-speed flight to reduce heating, are enabled. Simulation results indicate the effectiveness of the tracking law in compensating for initial offsets from the reference trajectory.

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