Analytical Design Procedure for Resonant Inductively Coupled Wireless Power Transfer System With Class-DE Inverter and Class-E Rectifier

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Abstract—This paper presents a resonant inductive coupling wireless power transfer (RIC-WPT) system with a class-DE and class-E rectifier along with its analytical design procedure. By using the class-DE inverter as a transmitter and the class-E rectifier as a receiver, the designed WPT system can achieve a high power-conversion efficiency because of the class-E ZVS/ZDS conditions satisfied in both the inverter and the rectifier. In the simulation results, the system achieved 79.0 % overall efficiency at 5 W (50 Ω) output power, coil distance 10 cm, and 1 MHz operating frequency. Additionally, the simulation results showed good agreement with the design specifications, which indicates the validity of the design procedure.

Keywords—Class-DE inverter, class-E rectifier, wireless power transfer, resonant inductive coupling, class-E ZVS/ZDS conditions, high overall efficiency.

I. INTRODUCTION

Recently, there are many researches and development about the wireless power transfer (WPT) systems [1]-[5]. Resonant inductive coupling (RIC) [1] is one of the coupling methods for WPT systems and it has been widely used for a variety of applications benefiting from wireless power transfer, such as wireless battery charging for electric vehicles [2], bio-medical implants [3], and so on. For achieving a high power-delivery efficiency, it is important to design not only low power-loss RIC, but also high power-conversion efficiency inverter and rectifier.

The class-DE-E dc-dc converter [6], which consists of the class-DE inverter [7] and the class-E rectifier [8], is one of the high power-conversion efficiency dc-dc converter. By satisfying the class-E ZVS/ZDS conditions in the class-DE inverter at switch turn-on instant and those in the class-E rectifier at switch turn-off instant, high power-conversion efficiency can be achieved at high frequencies. In previous research [5], the WPT system applied class-DE-E dc-dc converter and its numerical design procedure have been presented. However, it is also important for designers to obtain the analytical design equations of the system. The analytical design approach may cultivate designer’s fundamental understanding and intuition for the WPT system design compared with the numerical one.

This paper presents an RIC-WPT system with class-DE inverter and class-E rectifier along with its analytical design procedure. By using the class-DE inverter as a transmitter and the class-E rectifier as a receiver, the designed WPT system can achieve a high power-conversion efficiency because of the class-E ZVS/ZDS conditions in both the inverter and the rectifier. In the simulation results, the system achieved 79.0 % overall efficiency at 5 W (50 Ω) output power, coil distance 10 cm, and 1 MHz operating frequency. Additionally, the simulation results showed good agreement with the design specifications, which indicates the validity of the design procedure.

II. WPT SYSTEM WITH CLASS-DE-E DC-DC CONVERTER

Figure 1(a) shows a topology of the designing WPT system in this paper. This system consists of class-DE inverter, resonant inductive coils, and class-E rectifier. In this system, it is regarded as the extended version of the class-DE-E dc-dc converter that the class-DE inverter is connected with the class-E rectifier by the resonant inductive coupling part. Figure 2 shows example waveforms of the WPT system, where \(\omega = \omega t = 2\pi ft\) is the angular time and \(f\) is the operating frequency. By applying the class-E switching technique to both the inverter and the rectifier, the WPT system can achieve a high power-delivery efficiency.

A. Class-DE Inverter

The class-DE inverter consists of dc-supply voltage \(V_{DD}\), two MOSFETs \(S_1\) and \(S_2\), which work as switching devices, two shunt capacitances \(C_{S1}\) and \(C_{S2}\), series resonant capacitance \(C_0\), and impedance transformation component \(X_p\) as shown in Fig. 1(a). The impedance transformation component has the function that is for transforming the output network into the optimal load. Figure 2(a) shows example waveforms of the class-DE inverter at 25 % switch on duty ratio. The switches of the inverter are driven by the driving signal \(D_1\) and \(D_2\). The driving pattern generates a dead time during the period when one switch has turned off before the other switch has turned on. Because of \(v_{S1} + v_{S2} = V_{DD}\), the electric charge moves from one shunt capacitance to the other one during the dead time. The midpoint voltage between two switches, namely \(v_{S}\), becomes \(V_{DD}\) or zero and the derivation of the voltage is also zero at the end of the dead time, allowing the class-E ZVS/ZDS conditions, namely,

\[ v_s(2\pi) = 0 \quad \text{and} \quad \frac{dv_s}{d\theta} \bigg|_{\theta=2\pi} = 0, \]  

(1)

and

\[ v_s(0) = 0 \quad \text{and} \quad \frac{dv_s}{d\theta} \bigg|_{\theta=0} = 0. \]  

(2)
As shown in Fig. 2(a), maximum voltages across switches are the same as the input voltage $V_{in}$. This is an advantage of the class-D type of inverter compared with the class-E inverter. Therefore, the class-DE inverter has the both merits of the class-D and class-E inverters, which are low switch voltage stress and high power-conversion efficiency at high frequencies, respectively.

### B. Class-E Rectifier

The class-E rectifier consists of diode $D$ as a switching device, shunt capacitance $C_D$, low-pass filter network $L_f - C_f$, and load resistor $R_L$. The waveforms of the class-E rectifier are a reversed version of those of the class-E inverter. The diode works as a half-wave voltage rectifier and the rectified voltage is converted into a dc voltage through the low-pass filter $L_f - C_f$. At the turn-off transition of the diode, both the diode voltage $v_D$ and the slope of $i_Dv_D/d\theta$ are zero as shown in Fig. 2(b), which are also the class-E ZVS/ZDS conditions. Therefore, the class-E rectifier can also achieve the high power conversion efficiency at high frequencies.

### III. ANALYTICAL DESIGN PROCEDURE

In this section, the analytical design procedure is given. First, the rectifier part is designed. After that, the rectifier part is expressed as the reflected impedance in the inverter side. Finally, the inverter part is designed to satisfy the class-E switching conditions and the optimal load.

#### A. Secondary Part

The rectifier part is designed using the model shown in Figs. 1(b) and (c). From [9], the parallel capacitor $C_p$ as a function of the load resistance $R_L$ and the diode on-duty ratio $D_d$ can be calculated from

$$C_D = \frac{1}{2\pi R_L} \left\{ \frac{(1 - \cos(2\pi D_d) - 2\pi^2(1 - D_d)^2)}{1 - \cos(2\pi D_d)} \right\}. \quad (3)$$

The class-E rectifier can be expressed as the input impedance at the operating frequency $f$ as shown in Fig. 1(c). The input capacitance $C_i$ and the input resistance $R_i$ are expressed as

$$C_i = \pi C_D \left[ \frac{\pi(1 - D_d) + \sin(2\pi D_d)}{4} - \frac{\sin(2\phi_d) \sin^2(2\pi D_d) - 2\pi(1 - D_d) \cdot \sin \phi_d \sin(2\pi D_d - \phi_d)}{1 - \cos(2\pi D_d)} \right], \quad (4)$$

and

$$R_i = 2 \cdot R_L \cdot \sin^2 \phi_d, \quad (5)$$

Fig. 1. Class-DE-E WPT system. (a) System overview. (b) Equivalent circuit of rectifier 1. (c) Equivalent circuit of rectifier 2. (d) Equivalent circuit of inverter. (e) Equivalent circuit boiled down to typical class-DE inverter.
where $\phi_d$ is the phase shift between the input current and the switch voltage. $\phi_d$ can be calculated from
\[
\tan \phi_d = \frac{1 - \cos(2\pi D_d)}{2\pi(1 - D_d) + \sin(2\pi D_d)}.
\] (6)

$C_2$ is the resonant capacitance, which resonates with $L_2$ and $C_i$. Therefore, $C_2$ can be obtained from
\[
C_2 = \frac{C_i}{\omega^2 L_2 C_i - 1}.
\] (7)

From the output current $I_o$, the input current $I_2$, which is the effective value of $i_2$, is described as
\[
I_2 = \frac{I_o}{\sqrt{2} \sin \phi_d}.
\] (8)

The impedance $Z_{sec}$ seen by the induced voltage source $V_{ind}$ is expressed as
\[
Z_{sec} = R_{L2} + R_i + j\left(\omega L_2 - \frac{1}{\omega C_2} - \frac{1}{\omega C_1}\right).
\] (9)

At resonance, the imaginary part of $Z_{sec}$ equals zero. Therefore, $V_{ind}$ can be obtained as
\[
V_{ind} = I_2 \cdot (R_{L2} + R_i).
\] (10)

B. Inverter Part

From the equivalent transformer model as shown in Fig. 1(d), the equivalent impedance $Z_{eq}$, which includes the rectifier part and the transmitting coil, can be obtained as
\[
Z_{eq} = R_{eq} + jX_{eq},
\] (11)
where
\[
R_{eq} = R_{L1} + \frac{k^2 \omega L_1 L_2 (R_{L2} + R_i)}{(R_{L2} + R_i)^2 + \left(\omega L_1 L_2 - \frac{C_2 + C_i}{\omega C_2 C_i}\right)^2},
\] (12)
and
\[
X_{eq} = \frac{k^2 \omega L_1 \left[(R_{L2} + R_i)^2 - \frac{L_2 (C_2 + C_i)}{C_2 C_i} + \left(\frac{C_2 + C_i}{\omega C_2 C_i}\right)^2\right]}{1 - k^2},
\] (13)
where $k = M \sqrt{L_1 L_2}$ is the coupling coefficient between $L_1$ and $L_2$. The current through the transmitting coil $I_1$, which is the effective value of $i_1$, can be calculated from
\[
I_1 = \frac{V_{ind}}{\omega k \sqrt{L_1 L_2}}.
\] (14)

For satisfying the optimal load, $Z_{eq}$ is transformed into $Z_{out} = R_{out} + jX_{out}$ using the impedance transformation component $X_p$. $R_{out}$ and $X_{out}$ are obtained as
\[
R_{out} = \frac{R_{eq} X_p^2}{R_{eq}^2 + (X_{eq} + X_p)^2},
\] (15)
and
\[
X_{out} = \omega L_{out} = \frac{R_{eq}^2 X_p + X_p X_{eq} (X_{eq} + X_p)}{R_{eq}^2 + (X_{eq} + X_p)^2}.
\] (16)

From this transformation, the WPT system becomes an typical class-DE inverter as show in Fig. 1(e). In the typical class-DE inverter at switch-on duty ratio $D_s = 0.25$, $R_{out}$ for the optimal load is obtained from [10] as
\[
R_{out} = \frac{V_{DD}^2}{2\pi^2 R_{eq}^2 k^2}.
\] (17)

From (15) and (17), $X_p$ can be given as
\[
X_p = \frac{R_{out} X_{eq} - R_{eq} R_{out}}{R_{eq} - \frac{R_{out} R_{eq} (R_{eq} - R_{out}) (R_{eq}^2 + X_{eq}^2)}}.
\] (18)

The loaded-quality factor $Q$ in the inverter is expressed as
\[
Q = \frac{X_{out}}{R_{out}},
\] (19)
From (17) and (19), $C_0$, $C_{S1}$, and $C_{S2}$ satisfying the class-E ZVS/ZDS conditions are expressed as
\[
C_0 = \frac{1}{\omega R_{out} (Q - \pi/2)},
\] (20)
and
\[
C_{S1} = C_{S2} = \frac{1}{2\pi ko R_{out}},
\] (21)
respectively.

IV. DESIGN EXAMPLE

A design example is shown in this section. First, the design specifications were given as follows: operating frequency $f = 1$ MHz, dc-supply voltage $V_{DD} = 48$ V, output power $P_o = 5$ W, output resistance $R_o = 50$ Ω, switch and diode on-duty ratios $D_s = 0.25$ and $D_d = 0.5$, and distance between the primary coil and the secondary one $d_{cows} = 10$ cm. The low-pass filter $L_f - C_f$ was selected to be 300 μH and 47 μF, which generates direct output voltage with sufficient low ripple. The specifications of the coupling coils are: diameter $D = 15.5$ cm, number of turn $N = 10$, and diameter of wire $D_w = 1$ mm, particularly the height $h = 1$ cm. Both the specifications of the primary coil are the same as those of the secondary one. Self inductances, ESRs of the coils, and coupling coefficient were analytically calculated by Nagaoka coefficient [11], Dowell’s equation, and Neumann’s formula [12] as $L_1 = L_2 = 35.4$ μH, $R_{L1} = R_{L2} = 1.36$ Ω, and $k = 0.0725$, respectively. From the previous design procedure, the component values were obtained as given in Table I. $X_p$ was obtained as $X_p = -789$, particularly $C_P = 1/\omega X_p = 202$ pF. The IRFZ24N MOSFET and the STPS5H100B Schottky Barrier Diode were chosen as switching devices because the maximum switch voltage equals $V_{DD} = 48$ V and the diode voltage were obtained from [9] as
\[
V_{Dmax} = \frac{V_o}{\omega C_{DL}} \left(2\phi_d - \pi + \frac{2}{\tan \phi_d} \right) = 56.3$ V. (22)

Figure 3 shows the waveforms of the designed system obtained from PSpice simulation. All the switch voltages in this system, particularly $u_{S1}$, $u_{S2}$, and $u_{DP}$, satisfy the class-E ZVS/ZDS conditions and the output power obtained from the PSpice simulation is $P_o = 4.70$ W. From these results, the validity of the design procedure can be confirmed. Additionally, 79.0 % overall efficiency was achieved, where the overall
TABLE I. DESIGN VALUES OF DESIGN EXAMPLE

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
<th>Analytical</th>
<th>Simulated</th>
</tr>
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<tr>
<td>Inductance</td>
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<td>-</td>
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<tr>
<td>Inductance</td>
<td>$L_2$</td>
<td>35.4 µH</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Capacitance</td>
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<td>-</td>
<td>-</td>
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<tr>
<td>Capacitance</td>
<td>$C_2$</td>
<td>1 nF</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Capacitance</td>
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<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Capacitance</td>
<td>$C_4$</td>
<td>0.1 µF</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Resistance</td>
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<td>-</td>
<td>-</td>
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<td>-</td>
</tr>
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<td>-</td>
<td>-</td>
</tr>
<tr>
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<td>-</td>
</tr>
<tr>
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<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Distance</td>
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<td>10 cm</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Efficiency</td>
<td>$\eta$</td>
<td>79.0%</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

Fig. 3. PSpice waveforms for design example.

The overall efficiency is calculated from

$$\eta = \frac{P_o}{V_{DD} I_{DD}}.$$ (23)

Figure 4 shows the overall efficiencies as functions of the load resistance $R_D$ and the diode on-duty ratio $D_d$ obtained from PSpice simulation, where the design values are recalculated for each load resistance and duty ratio. It is seen from Fig. 4 that the overall efficiency highly depends on the load resistance and the diode on-duty ratio. Additionally, we can confirm from Fig. 4 that there is an optimal load resistance and duty ratio for maximizing the overall efficiency in this system.

V. CONCLUSION

In this paper, an RIC-WPT system with class-DE inverter and class-E rectifier along with its analytical design procedure has been presented. By using the class-DE inverter as a transmitter and the class-E rectifier as a receiver, the designed WPT system can achieve a high power-conversion efficiency because of the class-E ZVS/ZDS conditions in both the inverter and the rectifier. In the simulation results, the system achieved 79.0% overall efficiency at 5 W (50 Ω) output power, coil distance 10 cm, and 1 MHz operating frequency. Additionally, the simulation results showed good agreement with the design specifications, which indicates the validity of the design procedure.

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