Deconvolution Algorithms in Image Reconstruction for Aperture Synthesis Radiometers

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Abstract—In remote-sensing applications the inclusion of subgrid-scale variability in coarse resolution data still remains an elusive challenge. This paper is devoted to the development of an appropriate downscaling technique for future Aperture Synthesis Radiometer’s images. A comparative study of different deconvolution algorithms has been performed and particular emphasis is made on the use of least-squares Lagrangian methods and Fourier Wiener filtering. Results show that with this technique it is feasible to improve the spatial resolution of brightness temperature images from the Spatial Sensor Microwave Imager (SSMI) radiometer and from an upgraded version of the Soil Moisture and Ocean Salinity (SMOS) End-to-end Performance Simulator (SEPS).

Keywords—Aperture synthesis, deconvolution, least squares, Soil Moisture and Ocean Salinity (SMOS) mission, Spatial Sensor Microwave Imager (SSMI).

I. INTRODUCTION

The European Space Agency (ESA) with the Earth Explorer mission Soil Moisture and Ocean Salinity (SMOS) poses an important challenge for the entire scientific community. Its payload will be a totally new type of instrument: an L-band two-dimensional synthetic aperture radiometer with multiangular and dual-polarization/full-polarimetric capabilities. It will provide a new type of Earth observations, characterized for having different pixel size and orientation and a different noise level and spatial resolution for each pixel. Additionally, the development of new L-band and multiangular ocean and soil emission models and new geophysical parameter retrieval algorithms is a very relevant and involved issue. The work presented in this paper focuses on the study of the observations and on the potentiality of applying deconvolution algorithms to brightness temperature images in an attempt to improve its spatial resolution.

SMOS products are expected to have a relatively coarse resolution, of 30-50 km at best. This is inadequate to serve the full range of science needs for land surface hydrology modeling, weather forecasting, climate prediction, flood and drought monitoring and other water and energy cycle applications. With this work we aim at improving SMOS images’ spatial resolution. The downscaling limit will be determined by the noise amplification that the obtained smaller pixels will exhibit, but the goal is around 10 km. The grid chosen for the delivery of SMOS data is the Icosahedral Snyder Equal Area (ISEA) 4-9 [1], which has an inter-cell distance of ~15 km. In order for the deconvolution algorithms to be consistent and tailored to future SMOS data, the ISEA 4-11 grid, which has an intercell distance of ~4 km, will be used to process and represent the enhanced images.

The prospective deconvolution algorithms expected to be applied to the SMOS’ radiometric measurements should be newly created or have specific adaptations, due to the unique characteristics of the mission instrument and to its peculiar way of observing the Earth’s surface. Since this is such a novel technology, essential preparatory work is necessary when facing the study of SMOS’ Earth observations. With this motivation, a previous study and test of the algorithms have been performed on SSMI imagery. SSMI images have a homogeneous spatial resolution and a constant radiometric sensitivity for every pixel, hence constituting a simple model to begin with. In a second stage, these methods are applied to synthetic L-band brightness temperature images obtained with the SMOS End-to-end Performance Simulator (SEPS) [2].

A linear algebra framework is given to the deconvolution process, showing that frequency domain methods are more computational efficient than spatial domain ones. Section II shows that after testing the different deconvolution techniques, the method of Fourier Wiener filtering have resulted to be by far the one that better satisfies our requirements. The possibility of modifying the Fourier Wiener Filter by adding other constraints using Lagrange multipliers is also presented in Section II and results of its application will be shown at the conference. Section III illustrates the results of Wiener filtering SSMI images and synthetic L-band brightness temperature images
obtained with the SMOS End-to-end Performance Simulator (SEPS).

II. DECONVOLUTION ALGORITHMS

A. Discrete formulation

Satellite microwave radiometric observations may be expressed as the convolution of the sensor antenna beam projected onto the Earth’s surface with the Earth’s brightness temperature (BT) integrated over the sensor footprint. In the work presented on this paper, a cut-off value for the antenna beam has been conveniently set to -15dB of the maximum, in order to preserve high beam efficiency.

Adjacent observations cover mostly the same target features on the ground, but with different contributions to the overall signal. That overlap can be effectively utilized to estimate more accurately the BT of those grid cells. Typically, in a region scale study, the observations far outnumber the grid cells for which we want to estimate the unknown TB. Mathematically, it derives in an ill-conditioned or ill-posed linear problem that must be carefully inverted and proper regularization techniques must be considered to have under control noise amplification when inversion is accomplished [3]. Consequently, the existence, uniqueness and stability of the solution are not guaranteed for the general problem even when noise is not present. In addition, the presence of noise makes an exact solution unfeasible. Within this context, and following the lexicographic notation [4], the formation of a brightness temperature image can be described as:

\[ g = h \otimes f + n \]  

where \( g \) is a column vector containing the real observations, \( f \) is a column vector containing the unknown TB at the desired resolution, \( n \) is a column vector that includes all that might be considered noise, \( h \) is the matrix representation of the antenna response function, and \( \otimes \) indicates convolution.

A discrete convolution formulation can be derived from Eq. (1). Assuming that \( f \) and \( h \) are two dimensional periodic functions of periods \( M \) and \( N \) adequately padded with zeros to avoid overlap between different periods, and using the lexicographic notation as above:

\[ g = H \cdot f + n \]  

where \( f, g, \) and \( n \) are of dimension \((MN) \times 1\) and \( H \) is of dimension \( MN \times MN \). This matrix consists on \( M^2 \) partitions, each partition being size \( N \times N \) and ordered according to:

\[
H = \begin{bmatrix}
H_0 & H_{u,1} & H_{u,2} & \cdots & H_{u,2}
H_1 & H_0 & H_{u,1} & \cdots & H_2
H_2 & H_1 & H_0 & \cdots & H_3
\vdots & \vdots & \vdots & \ddots & \vdots
H_{u-1} & H_{u-2} & H_{u-3} & \cdots & H_0
\end{bmatrix}
\]  

Each partition \( H_j \) is constructed from the \( j \)th row of the extended function \( h \) by a circular shifting it to the right (see [5] for more details).

A direct solution of (2) is not computationally feasible as, just for images of a practical size, it will require an inversion of a too high number of simultaneous linear equations. Fortunately, as the matrix \( H \) is block-circulant, it can be diagonalized and therefore the problem can be considerably reduced by working in frequency domain [4].

The Fourier space equivalent of (2) can be written as:

\[ G = H \cdot F + N \]  

where \( G, H, F \) and \( N \) are the Fourier transforms of \( g, H, f \) and \( n \) respectively. Using frequency-domain based deconvolution methods the computation time is no longer a limitation since nowadays there are very powerful tools to perform Fast Fourier Transforms.

B. Wiener filter

Linear deconvolution can be posed as the task of finding a linear operator \( K \) such that:

\[ F = K \cdot G \]

The most elementary deconvolution will be performed by the simple inverse filter, given by \( K = H^{-1} \). However, such filtering tends to be very error sensitive and instable.

To solve deconvolution usefully, a constrained least-squares filter can be developed in which the constraint allows the designer additional control over the process. This approach consists on minimizing functions of the form \( ||Q||^2 \), where \( Q \) is a linear operator on \( f \), subject to the constraint \( ||g - Hf||^2 = ||n||^2 \). Then, using the method of Lagrange multipliers [5],

\[
J(f) = ||Qf - J||^2 + \alpha \left( ||g - Hf||^2 - ||n||^2 \right)
\]

Differentiating (6) with respect to \( f \) and setting the result equal to zero yields:

\[
\frac{\partial J(f)}{\partial f} = 0 = 2Q^T Q f - 2 \alpha H^T (g - Hf)
\]

The solution is obtained by solving (7) for \( f \):

\[
f = \left( H^T H + \gamma \cdot Q^T Q \right)^{-1} H^T g
\]

where \( \gamma \equiv 1/\alpha \) for simplification. This quantity must be adjusted such that the initial constraint is satisfied.

The choice of \( Q \) generates different deconvolution techniques. However, in order to work on the frequency domain, as it has been demonstrated to be highly desirable, \( Q \) should be a block-circulant matrix. With this premise, it has been confirmed that a well-working solution is the Wiener Filter, which establishes \( Q = R_f^{-1} R_n \), being \( R_f \) and \( R_n \) the correlations matrices of \( f \) and \( n \) respectively.

Performing the convenient operations, the Wiener filter can be expressed in the frequency domain as:
\[ K(u,v) = \left[ \frac{H'(u,v)}{|H(u,v)|^2 + \frac{S_r(u,v)}{S_s(u,v)}} \right] G(u,v) \] (9)

for \( u,v = 0,1,2,...,N-1 \), where \( S_r(u,v) \) and \( S_s(u,v) \) are the Fourier transforms of \( R_s \) and \( R_r \) and it is assumed that \( M = N \). Note that in the absence of noise, \( S_r(u,v) = 0 \), the Wiener filter reduces to the simple inverse filter.

When \( S_r(u,v) \) and \( S_s(u,v) \) are unknown, the whole factor \( \gamma \frac{S_r(u,v)}{S_s(u,v)} \) is usually reduced to a constant \( \phi_0 \) [6]. On the results presented on Section III, the value of \( \phi_0 \) has been empirically adjusted to 0.05 in the SSM/I case, and to 0.01 in the SEPS case.

### C. Modified Wiener filter

Following the least-squares procedure, a modified Wiener filter has been developed with the possibility of having an extra constraint and so improving its performance.

As a first approach, the additional constraint is set to be \( \|Tb - Hf\| = \|e\|^2 \), where \( Tb \) is a brightness temperature model extracted adequately from a land/sea mask and \( e \) is a tolerance error. Hence, two Lagrange multipliers, \( \lambda_1 \) and \( \lambda_2 \), will be used to include the two constraints so that:

\[ J(f) = \|Qf\|_2^2 + \lambda_1 \left( \|g - Hf\|_2^2 + \|Hg\|_2^2 \right) + \lambda_2 \left( \|Tb - Hf\|_2^2 + \|Hf\|_2^2 \right) \] (10)

Differentiating with respect to \( f \), setting the result to zero and solving it for \( f \) gives us the following expression in the spatial domain:

\[ f = \left( Q^T Q + \lambda_1 H^T H + \lambda_2 (H^T H)^{-1} \right) \left( \lambda_1 g + \lambda_2 Tb \right) \] (11)

At this point, the Wiener concept is used to set the value of \( Q \). Making use of the diagonalization procedure, we get the following expression in the frequency domain:

\[ K(u,v) = \left[ \frac{H'(u,v) (G(u,v) + \beta Tb)}{(1 + \beta |H(u,v)|^2) + \gamma \frac{S_r(u,v)}{S_s(u,v)}} \right] \] (12)

where \( \gamma \equiv 1/\lambda_1 \) and \( \beta \equiv \lambda_2/\lambda_1 \) for ease of notation and \( Tb \) is the Fourier transform of \( Tb \). Note that if we don’t add the second constraint, \( \lambda_2 = 0 \), this new filter reduces to the Wiener filter.

### III. APPLICATION OF THE WIENER FILTER DECONVOLUTION TECHNIQUE TO SSM/I AND SEPS IMAGES

To test the Wiener filter technique, an area surrounding Catalonia, in the North-East coast of Spain, has been selected. It is an area that comprises a variety of land cover types, orographic features, an abrupt coast line and the Balearic Islands which, because of their sizes, are outstanding features upon which the resolution improvement can be easily tested.

#### A. SSM/I results

The effective antenna pattern function (EAPF) of the SSM/I sensor is commonly approximated by a two-dimensional Gaussian function with frequency-dependent half-width values as published in Hollinger et al [7]. We have used this Gaussian response function for this study, as it has been proved sufficient for data fusion applications [8].

The SSM/I imagery under study, with a mean resolution of 25 km, is interpolated to the ISEA 4-9 grid, which imposes a Nyquist frequency of 0.03 km\(^{-1}\). It is important to notice that in order to deconvolve a specific SSM/I image, it is required that the EAPF conserves any significant spatial frequency below 0.03 km\(^{-1}\).

The regridding of the data introduces some error in the measurements. The noise-to-signal ratio in the Fourier space at the frequency 0.03 km\(^{-1}\) must be considerably smaller than unity, to allow the retrieval of components up to this limit.

The image in Fig. 1a shows the brightness temperature values, represented as colours, and interpolated to the ISEA4h9 grid. These values have been Wiener-filtered according to the algorithm described in previous sections and the result is plotted in Fig. 3b. A contour line delineating the coastlines and countries’ boundaries has been overlaid to the images for clarity.

![Figure 1.](image-url)

Comparing Fig. 1a and Fig. 1b it is possible to see some progress in the definition of the coast line. The nominal resolution has been improved from a mean resolution of 25 km to a nominal resolution of 15 km. To further improve the results, a possibility could be to better adjust the value of \( \phi_0 \) according to the noise-to-signal statistics. Still, the inclusion of additional information could also improve the method performance.

#### B. SEPS results

With the aim of obtaining a spatial resolution of ~4 km, SEPS images and the EAPF of the sensor are interpolated to the ISEA4-11 grid. It establishes a Nyquist frequency of 0.125 km\(^{-1}\). As in the SSM/I case, it has been confirmed that the EAPF conserves any spatial frequency up to 0.125 km\(^{-1}\) and that the noise-to-signal ratio in the Fourier space at the Nyquist frequency is considerably smaller than unity.
The original brightness temperature values of the study area, represented as colours, are depicted on Fig. 2a. These are the values used as SEPS input. The image in Fig. 2b shows the brightness temperature values obtained directly from a SEPS simulation and interpolated to the ISEA4-11 grid. These values have been Wiener-filtered according to the algorithm described in previous sections and the result is plotted in Fig. 2c.

Comparing Fig. 2c and 2b with respect to Fig. 2a, a clear improvement on the resolution of the overall image can be seen. In particular, there is a visible improvement in the coast line and in the island of Mallorca.

In order to better assess the algorithm performance, the BT values of Fig 2a, 2b and 2c at constant latitude are plotted on Fig.3. It shows that, in mean, the error between the deconvolved values and the original ones is smaller than between the retrieved and the original. However, the deconvolved values are still too close to the recovered ones. This is due to the fact that the deconvolution has been performed only using the EAPF of the sensor and the noise-to-signal statistics, but no additional information from the original image has been applied. It becomes then necessary the use of auxiliary data to obtain deconvolved BT values at a higher resolution.

![Image of Fig. 2a, 2b, and 2c showing brightness temperature values.]

IV. CONCLUSIONS

The well-known technique of Wiener filtering has been implemented here for the specific case of SSM/I and SEPS data. It has been shown that with this method the resolution enhancement is possible but limited. Further improvements are subject to the inclusion of additional information from the original image, and for this purpose, a modified Wiener filter has been proposed to conveniently combine external input data to obtain a solution closer to the original image. Results of its application will be shown at the conference.

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