Optimal Uplink Pilot Time Interval Design for TDD MISO Beamforming Systems with Channel Estimation Error and Delay

Baolong Zhou1,2, Lingge Jiang1, Lei Zhang2, Chen He1, Shengjie Zhao2 and Kun Zhao2
Department of Electronic Engineering, Shanghai Jiao Tong University1, Shanghai, P. R. China
China Product R&D, Alcatel-Lucent Shanghai Bell2, Shanghai, P.R.China
Email: baolong.zhou@alcatel-sbell.com.cn  lgjiang@sjtu.edu.cn

Abstract—This paper proposes an optimization scheme on uplink pilots time interval (UPTI) in terms of maximum average post-processing SNR (signal to noise ratio) for a time division duplex (TDD) multiple input single output (MISO) beamforming system with channel estimation error and delay. In TDD system, the base station estimates the channel state information (CSI) at transmitter based on uplink pilots and then uses it to generate the beamforming vector in the downlink transmission. Because of the constraints of the TDD frame structure and the uplink pilot overhead, there inevitably exists delay and channel estimation error between CSI estimation and its use. In this paper, we first derive average post-processing SNR for TDD MISO beamforming system with channel estimation error and delay. We then obtain the optimal UPTI, which maximizes average post-processing SNR, given the normalized pilot overhead (the number of pilot symbols per data symbol). The simulation results validate that the optimal UPTI not only maximizes the average post-processing SNR but also maximizes the system ergodic rate. Especially our research is valuable for the uplink sounding reference signal design in LTE-Advanced system.

Index Terms—channel estimation error, delay, multiple input single output (MISO), time division duplex (TDD)

I. INTRODUCTION

The availability of accurate channel state information (CSI) at transmitter is very important for multiple input single output (MISO) beamforming system. However, in practice CSI is always imperfect due to the existence of CSI delay, channel estimation error and quantization error, which would degrade the system performance. Hence, it is significant to characterize and optimize the performance of MISO beamforming system in the case of imperfect CSI.

Most prior researches about the impact of imperfect CSI on beamforming pay attention to frequency division duplex (FDD) systems. In [1], symbol error rate was analyzed in the presence of feedback delay and quantization error. Bit error rate (BER) and outage probability were derived in [2] with the feedback delay and channel estimation error. In [3], the impact of channel estimation error, feedback delay and quantization error on MISO systems was analyzed. A Kalman-filter-based prediction method was proposed to overcome the feedback delay effect in [4]. In [5], ergodic capacity bounds were investigated with channel estimation error and quantization error, and then were optimized over the number of both finite training symbols and limited feedback bits under the condition of fixed frame length for a large number of transmitter antennas. The optimal bandwidth allocation between data channel and feedback channel were studied in [6] to maximize the average throughput in the data channel using MISO beamforming scheme. In [7] [8], the authors investigated the optimal precoding algorithm and corresponding power allocation in the presence of imperfect CSI feedback, such as an imperfect channel coefficient feedback, channel mean feedback and channel covariance feedback. The optimal spatial and temporal power allocation was studied in [9] to minimize outage transmission for a MISO system with delayed feedback.

To the authors’ knowledge, the impact of imperfect CSI on MISO beamforming in time division duplex (TDD) systems is almost not yet investigated. In [3], the authors tried to extend the analysis in FDD mode to TDD mode, but the assumption for TDD mode therein is not in line with the practice.

In this paper, we consider the optimization on uplink pilot time interval (UPTI) of uncoded TDD MISO beamforming system in terms of maximum average post-processing SNR (signal to noise ratio) in the presence of CSI delay and channel estimation error. For TDD system, the base station (BS) estimates CSI at transmitter based on uplink pilots periodically sent by the mobile station (MS) and then uses it via channel reciprocity to generate beamforming vector for the downlink data transmission. Because of the constraints of the TDD frame structure and the uplink pilot overhead, there inevitably exists both CSI delay and channel estimation error between channel estimate and its use, which causes system performance loss. In the practical communications system, such as TDD Long Term Evolution (LTE) system, UPTI which dominates CSI delay is configured by Layer 2 scheduler for each MS as described in 3GPP specifications Release 8 [10] and Release 9 [11]. To simplify the implementation of Layer 2 scheduler, the value of UPTI for a specific MS is generally fixed during its active time and can’t be adjusted dynamically. However, for high speed MSs, the channel environment could change dramatically during their active time. Hence this fixed UPTI mechanism could largely degrade the system throughput. In order to minimize the performance loss caused by CSI delay and channel estimation error, we propose that dynamic UPTI be used so that BS can get the matched CSI with downlink transmission channel as much as possible, that’s to say, when the channel environment of an active user changes, UPTI of this active user should be changed accordingly. In order to get the optimal UPTI, we first derive the average post-processing SNR, which is the function of CSI delay and channel estimation error. We then optimize the average post-processing SNR over CSI delay and channel estimation error under the constraint of the normalized uplink pilot overhead (the number of pilot symbols per data symbol). Our main contribution is: given the normalized uplink pilot overhead, a novel optimization scheme on UPTI is proposed to maximize the average post-processing SNR of a TDD MISO beamforming system with channel estimation error and CSI delay. Our
simulation results verify that the obtained optimal UPTI not only maximizes the average post-processing SNR but also maximizes the ergodic rate. Especially our research is valuable for the uplink sounding reference signal design in LTE-Advanced system.

**Notation:** \( E(\cdot), (\cdot)^H, (\cdot)^T, (\cdot)^* \) denote expectation, conjugate transpose, transpose, complex conjugation, and Frobenius norm respectively. \( \mathcal{CN}(\mu, \Sigma) \) denotes the complex Gaussian distribution with mean vector \( \mu \) and variance matrix \( \Sigma \).

**II. System Model**

Consider a MISO system with \( M \) antennas at the BS and 1 antenna at the MS. The channel is modeled as a time-varying, spatially uncorrelated, frequency flat and independently and identically distributed (i.i.d.) Rayleigh fading channel. The channel vector from BS to MS is denoted by \( \mathbf{h} = [h_1, h_2, \ldots, h_M]^T \), where \( h_i \sim \mathcal{CN}(0,1) \). Let \( s \) denote the transmitted modulated symbol and \( \mathbf{w} \) be the unit norm beamforming vector. Then, the received signal at the \( m \)-th symbol interval is

\[
y[m] = \mathbf{h}^H[m] \mathbf{w} [n[m]] + n[m],
\]

where \( n[m] \sim \mathcal{CN}(0, \sigma_n^2) \) is an additive white Gaussian noise (AWGN), \( s[m] \) satisfies \( E(\|s[m]\|^2) = E_s \), \( E_s \) is the average symbol energy.

There are seven kinds of TDD frame configurations as defined in 3GPP specifications [10] [11]. Without loss of generality, we choose the TDD frame configuration 2 for analysis. Fig. 1 describes the structure of TDD frame configuration 2. One radio frame includes ten subframes and one subframe includes fourteen symbols per subcarrier. The uplink pilots can be sent via the last one or several symbols in one subframe. The structure of TDD frame configuration 2 is shown in Fig. 1.

![TDD frame configuration 2](image)

**Step 1:** BS obtains the delay estimated version \( \hat{\mathbf{h}}[m-M_d] \) of CSI based on received uplink pilots at the \( (m-M_d) \)-th symbol interval. Here \( M_d \) denotes the delay in symbol between the channel estimation and its actual use, and its value is ranged from 1 to \( n_d \) for the different downlink data symbol as in Fig. 2.

\[
\hat{\mathbf{h}}[m] = \mathbf{h}[m-M_d]
\]

**Step 2:** BS generates the normalized beamforming vector as follows and sends out the beamformed stream.

\[
\mathbf{w}[m] = \frac{\hat{\mathbf{h}}[m-M_d]}{\|\hat{\mathbf{h}}[m-M_d]\|_2}
\]

**Step 3:** MS estimates the downlink equivalent channel as below via UE specific reference signal [10] [11] and then detects the received signal.

\[
\hat{h}_n[m] = \mathbf{h}^H[m] \mathbf{w}[m].
\]

Assuming that \( \mathbf{h}[m-M_d] \) and \( \mathbf{h}[m] \) are jointly Gaussian distributed, the relationship between them is given by [3]

\[
\mathbf{h}[m] = \rho \hat{\mathbf{h}}[m-M_d] + \eta[m],
\]

where the channel error vector \( \eta[m] = \begin{bmatrix} \eta_1, \eta_2, \ldots, \eta_M \end{bmatrix} \sim \mathcal{CN}(0,1-\rho^2) \mathbf{I} \) is uncorrelated with \( \hat{\mathbf{h}}[m-M_d] \), \( \rho = E[h[m]\hat{h}^H[m-M_d]] \). The correlation coefficient between channel \( h[m] \) and its delay estimation \( \hat{h}[m-M_d] \). As in [3], \( \rho \) can be further expressed as

\[
\rho = \rho_s \rho_e,
\]

where \( \rho_s \) and \( \rho_e \) are delay-only complex correlation coefficient and estimation-error-only complex correlation coefficient, respectively.

If the BS obtains the CSI through the minimum mean-square error (MMSE) estimation from \( n_p \) pilot symbols, \( |\rho|_p \) can be deduced as [12]

\[
|\rho|_p = \sqrt{\frac{1}{1+\frac{1}{\gamma_p^*n_p}}},
\]

where \( \gamma_p \) is the SNR of the pilot symbol, \( n_p \) is the number of pilot symbols in one equivalent block as shown in Fig. 2.

On the other hand, the power spectrum of time-varying Rayleigh fading channel follows the Jakes model, then [13]

\[
|\rho|_d = J_0(2\pi M \gamma_p T_s \cdot F_d),
\]

where \( J_0 \) is a zero-th order Bessel function of the first kind,
$F_d$ is the maximal Doppler frequency shift of MS, $T_s$ is the symbol duration and $M_dT_s$ is the time delay between channel estimation and its use. So

$$\rho = J_0 (2\pi M_d T_s F_d) \sqrt{\frac{1 + \frac{1}{\gamma_p \kappa_n}}{1}} \gamma_p \kappa_n,$$

where $M_d \in [1, n_d]$ as shown in Fig. 2.

III. UPLINK PILOTS TIME INTERVAL OPTIMIZATION

In this section we first derive the average post-processing SNR, and then propose an optimization scheme on UPTI in terms of maximum average post-processing SNR.

Substituting (4) into (1), the received signal (1) can be rewritten as

$$y[m] = (\rho \hat{h}[m - M_d] + \eta[m])^H w[m]s[m] + n[m],$$

$$= h_{eq}[m]s[m] + n[m]. \tag{9}$$

As mentioned before, in TDD system MS can get the estimated version $\hat{h}_{eq}[m]$ of $h_{eq}[m]$ via UE specific reference signal, so $\eta'[m]w[m]s[m]$ should be considered as signal.

Based on (9), the covariance of signal can be computed as follows. For the simplification of expression, we omit discrete time index $m$ in the following computing process.

$$E \left[ h_{eq}[m]s[m] \right] = E \left[ (\rho \hat{h} + \eta)^H w s \right] = E \left[ h_{eq}[m]s[m] \right] = E \left[ (\rho \hat{h} + \eta)^H w s \right].$$

$$E \left[ h_{eq}[m]s[m] \right] = E \left[ (\rho \hat{h} + \eta)^H w s \right] = E \left[ (\rho \hat{h} + \eta)^H w s \right]. \tag{10}$$

$$= E_i \left[ |\rho|^2 E \left[ |\hat{h}|^2 \right] + E \left[ \rho^* \hat{h}^* \eta \right] + E \left[ \rho \eta^* \hat{h}^* \right] + E \left[ |\rho^* w|^2 \right] \right)$$

$$= E_i \left[ M |\rho|^2 + \rho \sum_{i=1}^M \left( \hat{h}_i \eta_i + \rho \sum_{i=1}^M \left( \eta_i \hat{h}_i + \sum_{j=1}^M \eta_j w_j \right) \right) \right].$$

$\approx E_i \left[ M |\rho|^2 + \rho \sum_{i=1}^M \left( \hat{h}_i \eta_i + \rho \sum_{i=1}^M \left( \eta_i \hat{h}_i \right) \right) \right].$}

where $\approx$ follows from the fact that $E(\hat{h}_i \eta_i) = 0$ and $E(\eta_i \hat{h}_i) = 0$ because Gaussian random variables $\eta_i$ and $\hat{h}_i$ are uncorrelated and $E \eta_i = 0$ as described before.

So average post-processing SNR, denoted by $\overline{\gamma} \gamma$, can be obtained as

$$\overline{\gamma} = \frac{E \left[ h_{eq}[m]s[m] \right]}{E \left[ |n[m]|^2 \right]} = \gamma \left[ (M - 1)|\rho|^2 + 1 \right], \tag{11}$$

where $\gamma = E_i / \sigma^2$.

Observing (10), we can know that $\overline{\gamma}$ increases monotonously as $|\rho|$ increases. Naturally we have such an idea: optimize system performance through maximizing $|\rho|$ (equivalent to maximizing average post-processing SNR) under the constraint of given normalized pilot overhead $\kappa = n_p / n_d$. Further, we find that $|\rho|$ is a monotonic increasing function of $n_p$ and a monotonic decreasing function of $M_d$, so it is possible to find a tradeoff between $n_p$ and $M_d$ to maximize $|\rho|$. For simplicity, we only optimize maximal delay $n_p$ in one equivalent block, as shown in Fig. 2. Because $UPTI = (1 + \kappa) n_d$ in symbol, the optimization on $n_d$ implies the optimization on $UPTI$.

The above optimization idea can be formulated as

$$n_p^* = \arg \max_{n_p} |\rho|$$

$$= \arg \max_{n_p} \gamma \left[ (M - 1)|\rho|^2 + 1 \right]$$

$$= \arg \max_{n_p} \left[ (1 - (2\pi T_0 M_d F_d)^2) \right] \left[ (1 + \frac{1}{\gamma_p \kappa_n}) \right], \tag{11}$$

where $\gamma$ depends on $J_0(z) = (1 - z^2 / 4)^{1/2}$. The derivative of $|\rho|$ with respect to $n_d$ is as below:

$$\frac{d |\rho|}{dn_d} = \frac{n_p^*}{8c_1} \left( 1 + \frac{1}{c_2 n_d} \right)^{-1/2} \left( -4c_1 c_2 c_3 n_d^3 + 5c_2 c_3 n_d^2 + 4 \right), \tag{12}$$

where $c_1 = 2\pi F_d T_0$ and $c_2 = \gamma_p \kappa$. Letting $d |\rho| / dn_d = 0$, we get the optimal $n_p^*$ in (13) at the top of next page. Furthermore, $n_p^*$ can be obtained by $n_p^* = \kappa n_d^*. \gamma$.

If $n_d^* T_s$ is less than coherent time, Wiener filter can be used to combine several neighborly channel estimations to improve CSI quality further.

IV. SIMULATION RESULTS

Consider a TDD MISO system, where the BS with 4 antennas transmits the beamformed data to an MS with 1 antenna. The channels are assumed to be time-varying, spatially uncorrelated, frequency flat and Rayleigh fading. Jakes model is used to simulate the time-varying channels. MMSE channel estimation and ideal channel estimation are used by BS and MS respectively. No channel code is considered for simplicity.

Figs. 3 and 4 are drawn according to (8) and show the variation of $|\rho|$ with $n_d$ for $F_d = 5$Hz and 70Hz respectively. Simulation parameters are seen in Table 1. One can see from Fig. 3 that $n_{d,\text{sim}}^*$ is 98, 78.5 and 68.5 respectively for $\kappa = 0.02, 0.04$ and 0.06 in case $F_d = 5$Hz. $n_{d,\text{theory}}^*$ which is calculated with (13) is 97.7, 78.2 and 68.5 respectively for $\kappa = 0.02, 0.04$ and 0.06 in case $F_d = 5$Hz. $n_{d,\text{theory}}^*$ is almost equal to $n_{d,\text{sim}}^*$, which proves $n_{d,\text{sim}}^*$ in (13)
\[ n_{d}^{\text{opt}} = A + 25 \sqrt{(B \cdot A) - C} \]

\[ A = \frac{1}{\sqrt{26(\pi F_d T_s)^{1} (\gamma_p k)^{4}}} \cdot \frac{125}{6192(\pi F_d T_s)^{1} (\gamma_p k)^{4}} \cdot \frac{1}{1728(\gamma_p k)^{4}} + \frac{1}{2^3(\pi F_d T_s)^{1} (\gamma_p k)^{4}}, \quad B = 144 (\gamma_p k)^{2}, \quad C = \frac{5}{12 \gamma^3 p} \]  

(13)

---

**Fig. 3.** Relationship between \( \rho \) and \( n_d \), \( F_d = 5 \text{Hz} \)

**Fig. 4.** Relationship between \( \rho \) and \( n_d \), \( F_d = 70 \text{Hz} \)

**Fig. 5.** Relationship between average post-SNR and \( n_d \), \( F_d = 5 \text{Hz} \)

**Fig. 6.** Relationship between average post-SNR and \( n_d \), \( F_d = 70 \text{Hz} \)

**Fig. 7.** Relationship between ergodic rate and \( n_d \), \( F_d = 5 \text{Hz} \)

**Fig. 8.** Relationship between ergodic rate and \( n_d \), \( F_d = 70 \text{Hz} \)
gotten by the approximation of $J_0(z)$ is reasonable. Furthermore, one can find that $n_{\text{opt}}^d$ decreases as $\kappa$ increases, the reason is that the bigger $\kappa$ is, the smaller $n_{\text{opt}}^d$ is under the same estimation accuracy. The same conclusion can be also obtained in Fig. 4.

The simulation results indicate that the obtained optimal UPTI can not only maximize the average post-processing SNR but also maximize the ergodic rate. Especially our research is valuable for the uplink sounding reference signal design in LTE-Advanced system. In the future work, we will extend the study to multiple-input multiple-output orthogonal frequency division multiplexing (MIMO-OFDM) system.

Table 1  Simulation parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>value</th>
<th>Comment</th>
</tr>
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<tbody>
<tr>
<td>$T_s$ (sec.)</td>
<td>$10^{-3}/14$</td>
<td>Symbol Duration</td>
</tr>
<tr>
<td>$F_d$ (Hz)</td>
<td>5, 70</td>
<td>Maximum Doppler Frequency Shift</td>
</tr>
<tr>
<td>$\gamma_p$ (dB)</td>
<td>10</td>
<td>Pre-processing Pilot SNR</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>0.02, 0.04, 0.06</td>
<td>Normalized Pilot Overhead</td>
</tr>
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</table>

Figs. 5 and 6 show the variation of average post-processing SNR with $n_d$ for $F_d=5$Hz and 70Hz respectively, where simulation average post-processing SNR curves are drawn by Monte Carlo simulation and theory average post-processing SNR curves are drawn by numerical calculation according to (10). Some more simulation parameters are listed in Table 2 in addition to those in Table 1. One can observe that theory curves exactly match simulation curves in these two figures, which validates that average post-processing SNR in (10) is correct. Furthermore, one can find in Fig. 5 that the optimal maximal delay $n_{d,\text{sim}}^{\text{opt}}$ of maximal average post-processing SNR does exist under the constraint of given normalized pilot overhead and is 98, 78 and 69 respectively for $\kappa=0.02$, 0.04 and 0.06 given $F_d=5$Hz, which is almost equal to $n_{d,\text{theory}}^{\text{opt}}$ given $F_d=5$Hz. It verifies that our optimization method can maximize average post-processing SNR indeed. The same conclusion can be also obtained in Fig. 6.

Table 2  Simulation parameters

<table>
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<th>value</th>
<th>Comment</th>
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<tbody>
<tr>
<td>$\gamma_p$ (dB)</td>
<td>10</td>
<td>Pre-processing Data SNR</td>
</tr>
<tr>
<td>$M$</td>
<td>4</td>
<td>Number of BS Antenna</td>
</tr>
</tbody>
</table>

Figs. 7 and 8 are drawn by Monte Carlo simulation and show the variation of ergodic rate with $n_d$ for $F_d=5$Hz and 70Hz respectively. Some more simulation parameters are seen in Table 2 in addition to those in Table 1. One can see from Fig. 7 that the optimal maximal delay $n_{d,\text{sim}}^{\text{opt}}$ of maximal ergodic rate does exist under the constraint of given normalized pilot overhead and is 98, 78 and 69 respectively for $\kappa=0.02$, 0.04 and 0.06 given $F_d=5$Hz, which is also almost equal to $n_{d,\text{theory}}^{\text{opt}}$ given $F_d=5$Hz. It validates that our optimization method can maximize ergodic rate, and is correct and useful. The same conclusion can be also obtained in Fig. 8. Furthermore, by comparing Fig. 7 with Fig. 8, one can find that the bigger the Doppler frequency shift is, the more sensitive to the deviation from $n_{\text{opt}}^d$ the ergodic rate loss is. The reason is that the bigger the Doppler frequency shift is, the faster the channel changes, which causes the much more mismatch between CSI estimate and its use, so ergodic rate loss is the bigger under the same deviation from $n_{\text{opt}}^d$ in the case of bigger Doppler frequency shift. It indicates that it is much more important for high speed mobile user to obtain the optimal $n_{\text{opt}}^d$.

V. CONCLUSION

In this paper, we have investigated the optimal UPTI design for a TDD MISO beamforming system with channel estimation error and delay. Given the normalized uplink pilots overhead, we have proposed a novel optimization scheme on UPTI in terms of maximum average post-processing SNR. The simulation results indicate that the obtained optimal UPTI can not only maximize the average post-processing SNR but also maximize the ergodic rate. Especially our research is valuable for the uplink sounding reference signal design in LTE-Advanced system. In the future work, we will extend the study to multiple-input multiple-output orthogonal frequency division multiplexing (MIMO-OFDM) system.

VI. ACKNOWLEDGEMENT

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