

# Overselling in A Competitive Environment: Boon or Bane?

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## Abstract

In this paper, we study the practice of overselling in a competitive environment where late arriving consumers value the good higher than early arriving ones but the former's arrival is uncertain. We show that overselling is a dominant strategy for the firms. However, it can lead to a prisoners' dilemma situation in which all firms are worse off overselling. We further show that only when demand from the late consumers far exceeds the supply and there is a sufficiently high profit margin from reselling does overselling result in a Pareto dominant outcome for the firms.

**Keywords:** overselling, overbooking, pricing, revenue management, competition, capacity constraints.

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# 1 Introduction

Overselling is a phenomenon that occurs as a result of firms overbooking their services, that is, selling access to a service which exceeds the capacity of the service. One typical example is the airline industry where airlines book more consumers onto a flight than can actually be accommodated by the aircraft. If every consumer shows up, or if more consumers than expected show up, overbooking will cause an oversale. In this instance, airlines often ask for volunteers to give away their seats and/or refuse boarding to certain consumers (a practice known as bumping) in exchange for a compensation that may include an additional free ticket, an upgrading in a later flight, or even cash. The main gist of the overbooking policy is that some consumers cancel their bookings nearer the date of departure, thus creating demand uncertainty. Overbooking therefore enables the airlines to take off with a higher load factor despite some no shows. It is often used as a tool to alleviate the demand uncertainty faced by the airlines and to improve their profits. Historical data suggests that in the airline industry, 10-15% of passengers do not claim the seat they reserved (Rothstein 1985, USA Today 1998). In 1999, Bialogorsky et al. showed that even if all consumers show up, overselling with opportunistic cancellations is profitable for a monopoly firm. Furthermore, it gives rise to a 'win-win' situation as the good is sold to consumers who value it most and the firm gains a higher profit.

For the past thirty years, there has been a constant stream of research on how firms respond towards demand uncertainty and they can be categorized into the areas of revenue management, overbooking and pricing (see Philips 2005, Talluri and van Ryzin 2004, Bitran and Caldentey 2003, Elmaghraby and Keskinocak 2003, Smith et al. 1992, Weatherford and Bodily 1992 for an overview.). Most of the related works have either studied revenue management without the option of overbooking (Shugan and Xie 2005, Dana 1999, Gallego and van Ryzin 1994, Staiger and Wolak 1992, Wilson 1988), or concentrated on the role of inventory decision to manage pricing

competition (Federgruen and Heching 1999, Van Mieghem and Dada 1999). More recently, the study of revenue management has been extended to include callable products (Gallego et al. 2008) or cancellation by customers (Xie and Gerstner 2007). Several other papers also proposed the use of reservation to reduce the risk of demand uncertainty (Lim 2008, Biyalogorsky and Gerstner 2004, Png 1989). Overbooking was first studied by Rothstein (1971). Subsequent studies on overbooking (Biyalogorsky et al. 1999, Desiraju and Shugan 1999) have focused on monopoly markets and showed that overbooking is profitable from the firm's perspective since tickets may remain un-consumed or hotel rooms left vacant otherwise. In addition, overbooking enhances market efficiency as tickets that would otherwise remain unused can be allocated to consumers who value them. In Desiraju and Shugan (1999), the authors found that yield management works best when price insensitive consumers prefer to buy later than price-sensitive consumers while Biyalogorsky et al. (1999) showed that even if all consumers show up, the use of overbooking (which leads to overselling with certainty in their model) can increase the expected profit for a monopoly and improve market efficiency. However, in the presence of competition, it is neither immediate nor straightforward that overselling remains more profitable for the competing firms. Overselling, by definition implies that units already sold can be resold to another consumer as long as some form of compensation is provided to and accepted by the first consumer. This inevitably puts more available supply into the market and may potentially drive prices lower than without overselling, thus compromising the profitability of the firms. In this paper, we are interested in whether overselling is indeed profitable for the firms in a duopoly and the parameters that determine the level of profitability. We seek to answer the question: How much does the overselling strategy improve the firms' profits, if any. This article is the first to extend existing research on overselling to a duopoly context.

Our work is also closely related to the classical newsboy problem where a firm

(newsboy) decides on the inventory level for a perishable good (newspaper) for resale. Although research in this area has been extended to the competitive model (Dai et al. 2006, Lippman and McCardle 1997, Kalai et al. 1992, Parlar 1988, Kirman and Sobel 1974), pricing decisions are omitted in these models. Recently, Zhao and Atkins (2008) extended the theory of  $N$  competitive newsvendor to consider *both* price and inventory competition. However, they did not consider the possibility of overselling.

In this paper, we consider a market where two firms selling substitutable goods (or services) compete. Consumers arrive at different times and late arriving consumers have higher valuations than early arriving ones. Goods that follow such demand patterns are aplenty. For example, it is widely accepted that airlines, hotels, car rentals and goods or services in the travel industry usually have business travelers with expense accounts arriving late (Desiraju and Shugan 1999). Demand uncertainty is modeled as the stochastic arrival of late consumers. The firms thus face a tradeoff if they sell to the early consumers because there may be more late arriving ones who are willing to pay a higher price for the goods. On the other hand, if the firms reserve or block the goods for the late arriving consumers, the latter may not arrive. Thus, it seems that one viable option for the firms in the midst of such demand uncertainty is to adopt an overselling strategy where if more late consumers arrive, firms resell the goods already sold to the early consumers to these late consumers but at the same time, provide some compensation to the former. Yet, adopting the overselling strategy also means that firms are creating more intense pricing competition than if overselling is not used since the supply to the late consumers is higher. In this paper, we develop a model that allows for explicit consideration and examination of the tradeoffs between adopting the conventional selling strategy and the overselling strategy. Our model captures three important elements in the study of the overselling strategy, namely competition, capacity constraint and demand uncertainty. To the best of our knowledge, this is the first attempt to include all three elements in an

overselling model. These elements enable us to gain the following policy insights:

- Overselling is a weakly dominant strategy. As a result, both firms adopting the overselling strategy is always an equilibrium.
- Much as being an equilibrium, both firms adopting the overselling strategy can lead to a Pareto dominated outcome with both firms being worse off than if both of them were to adopt the conventional selling strategy.
- Overselling presents a ‘everybody wins’ outcome if and only if the firms’ capacities are relatively low with respect to the demand from high valuation consumers *and* the profit margin for reselling is sufficiently high.
- Firms using overselling strategies enjoy higher profits when demand is high relative to capacity (as in the peak season) but find overselling less profitable when demand is low relative to capacity (as in the low season).

The paper is organized as follows. In Section 2, we present our model of a market structure with two firms which face demand uncertainty and capacity constraint. The question for each firm is one of whether to adopt the overselling strategy, and the number of units to block or reserve for the late arriving consumers in order to maximize its expected profit. We present the main results on the equilibrium strategies in Sections 3 and 4. In Section 5, we present some discussion on the results and conclude.

## 2 The Model

While not a general model of yield management, this article presents a stylized model that examines the role of overselling in a competitive setting.

Consider a market whereby there are two firms A and B selling competing brands of a substitutable good. We shall use the term good to refer to both products and

services in this paper. In our model, we focus on goods with the following characteristics. Firstly, it is perishable in nature and thus must be sold by a certain deadline or it will command no salvage value at all. Secondly, the good can be sold in advance but the actual consumption only takes place at a later time. Thirdly, the marginal cost of production of the good is negligible. Furthermore, the good is non-transferrable. Finally, the arrival of the consumers is uncertain. Examples of goods that exhibit these properties are abound and include air tickets, hotel rooms, allocation of medical appointments etc.. For example, once a flight is scheduled to take off at a certain date, the ticket will be worthless after that date, rendering the air ticket as a perishable good. Further, having an additional passenger on the aircraft does not make a significant difference to the overall cost of a flight as long as it is within the capacity of the aircraft. Air tickets go on sale as early as months before the actual departure and as is widely established, the arrival of consumers is uncertain. Note that air tickets are not transferrable. Concert tickets on the other hand, are transferrable and thus would not fall into our consideration here since such goods can certainly support the existence of secondary markets once the demand of late arriving consumers is sufficiently high. By the same token, the allocation of medical appointments either in a clinic or a hospital falls into the category of goods studied here where subsidized patients arrive earlier while private patients paying the full consultation fee arrive later and the medical appointment is not transferable. Our model is also relevant to business-to-business markets such as the selling of media advertising slots.

We model the arrival of consumers over two periods where the end of Period 2 represents the time of consumption. Thus, the salvage value of the good is zero if it is not sold by Period 2. We classify the consumers in the following way: In Period 1, the early consumers arrive while the late consumers arrive only in Period 2. The time of arrival of consumers is exogenous and each class of consumers makes their purchase decision upon their arrival. Consumers who arrive in Period  $t$  ( $t = 1, 2$ )

have valuation  $v_{i,t}$  ( $i = A, B$ ) for the good from Firm  $i$  and  $v_{i,1} < v_{i,2}$ <sup>1</sup>. For simplicity, we assume that the early consumers do not delay their purchase decisions till Period 2. Or rather, any unfulfilled demand in Period 1 does not spill over to Period 2. This assumption is not unreasonable since a holiday planner who is unable to buy a ticket for a particular destination would have bought tickets for another destination. Yet another alternative interpretation is that consumers are myopic (for example, business travellers who are not able to make travel plans way in advance), have high search cost or high waiting cost and thus will make their purchase decisions immediately upon arrival and do not or are not able to compare prices across different periods. As we shall see later, since the late arriving consumers have a higher valuation, the price of the good in Period 2 is no lower than that in Period 1, thus eliminating any possible gain for the early consumers to delay their purchase.

### **Table of Notations**

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<sup>1</sup>It was highlighted in Desiraju and Shugan (1999) that late consumers are often business travellers who have higher valuations. This assumption has been a constant feature in most research in revenue management. Furthermore, we assume that consumers within each period are homogeneous. Future research might consider the case where consumers are heterogeneous within each period.

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$A, B$	firms in a duopoly
$N_i$ ( $i = A, B$ )	capacity of Firm $i$
$v_{i,t}$ ( $= 1, 2$ )	valuation of good from Firm $i$ for consumers who arrive in Period $t$
$L$	random variable to denote the no. of consumers arriving in Period 2
$P(L = s)$	probability that the no. of consumers arriving in Period 2 is $s$
$\lambda_i$ ( $\lambda_i = CS, OS$ )	selling strategy of Firm $i$
$k_i(\lambda_i, \lambda_j)$	no. of units reserved for sale in Period 2 by Firm $i$ when the selling strategies are $\lambda_i, \lambda_j$
$p_{i,t}(\lambda_i, \lambda_j)$	price set by Firm $i$ for Period $t$ when the selling strategies are $\lambda_i, \lambda_j$
$v_{i,1} + R_i$	refund to early consumer if good is resold in Period 2
$\Pi_i^{\lambda_i, \lambda_j}$	expected profit for Firm $i$ if selling strategies are $\lambda_i, \lambda_j$
$\kappa_i$	largest integer no more than $N_i$ such that $P(L \geq \kappa_i) > \frac{v_{i,1}}{v_{i,2}}$
$\kappa_i^R$	largest integer no more than $N_i$ such that $P(L \geq \kappa_i^R) > \frac{v_{i,1}}{v_{i,1} + R_i}$

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In our model, we consider two selling strategies, namely conventional selling ( $CS$ ) and overselling ( $OS$ ). In conventional selling, firms do not resell units that were already sold in Period 1 to consumers who arrive in Period 2. If a firm adopts the overselling strategy, it can however, resell units of goods that were sold to the early consumers again upon the arrival of late consumers by offering a compensation amount that is acceptable to the early consumer. In both strategies, firms allocate their capacity into those to be sold in Period 1 to early consumers and those reserved for sale to the late consumers only in Period 2. These reserved units are thus not available for sale to the early consumers. We also assume that consumers who have bought the good would consume the good so that no show is not a possibility here and neither is it transferable. Demand uncertainty is instead modeled as the stochastic arrival pattern of the late consumers. As the focus of our paper is to examine and compare between the conventional selling strategy and the overselling strategy, we further assume that the demand in Period 1 far exceeds the total capacity of the two firms,  $N_A$  and  $N_B$



respectively. This assumption allows us to focus on the overselling strategy and the number of units to be reserved for the late consumers. This assumption has also been made in Biyalogorsky et al. (1999). We further assume that the late consumers value the good more than the early consumers, that is  $v_{i,1} < v_{i,2}$ <sup>2</sup>. We model the arrival of late consumers to be uncertain and the number of late consumers who arrive in Period 2, denoted by  $L$ , is a discrete random variable that follows a distribution function  $P(\cdot)$  where  $P(L = s)$  denotes the probability that the number of late consumers who arrive in Period 2 is  $s$ ,  $s \geq 0$ <sup>3</sup>.

We consider a multistage game with observed actions where the firms have complete information of the game<sup>4</sup>. The sequence of the game is as follows. In Period 0, Firms  $A$  and  $B$  simultaneously determine their selling strategies  $\lambda_A, \lambda_B$  ( $\lambda_A, \lambda_B \in \{CS, OS\}$ ). Upon observation of the selling strategy  $\lambda_j$  adopted by Firm  $j$ , Firm  $i$  then determines the number of units  $k_i(\lambda_i, \lambda_j)$  to reserve for sale only in Period 2 ( $i, j = A, B, j \neq i$ ). In Period 1, Firms  $A$  and  $B$  simultaneously set prices  $p_{A,1}(\lambda_i, \lambda_j), p_{B,1}(\lambda_i, \lambda_j)$  upon observation of  $k_A(\lambda_A, \lambda_B), k_B(\lambda_A, \lambda_B)$ <sup>5</sup>. Early consumers arrive in this period and make their purchase decisions after they observe  $p_{A,1}(\lambda_i, \lambda_j), p_{B,1}(\lambda_i, \lambda_j)$ . Firm  $i$  sells at most  $(N_i - k_i(\lambda_i, \lambda_j))$  units in this period. In Period 2, Firms  $A$  and  $B$  again simultaneously set prices  $p_{A,2}(\lambda_i, \lambda_j), p_{B,2}(\lambda_i, \lambda_j)$ . Late con-

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<sup>2</sup>If  $v_{i,1} > v_{i,2}$ , then the optimum pricing strategy for Firm  $i$  will be straightforward: sell as many units as possible in Period 1 at price  $v_{i,1}$ .

<sup>3</sup>We assume that  $P(L = s) > 0$  for all  $s \in [\underline{s}, \bar{s}]$ ,  $\underline{s} < \bar{s}$  and  $\underline{s} < \min(N_i, N_j)$ . The last inequality is to ensure that the capacity of each firm is high enough such that the probability of filling all the units is less than 1. Otherwise, the problem becomes trivial.

<sup>4</sup>For multistage games with observed actions, all strategies or actions taken in earlier periods are observable before strategies in the current period are chosen.

<sup>5</sup>I thank an anonymous reviewer for drawing my attention to expertflyer.com where subscribers can see on a real-time basis the remaining inventory on each flight for each fair code. Seatcounter.com positioned as the booking class availability machine also provides similar information. As pointed out by an anonymous reviewer, if firms cannot observe the quantities, the implications of the model may change.

sumers arrive in this period and make their purchase decisions. If Firm  $i$  has opted for the conventional selling strategy in Period 0 ( $\lambda_i = CS$ ), it sells at most  $k_i(\lambda_i, \lambda_j)$  units in this period. On the other hand, if Firm  $i$  has opted for the overselling strategy in Period 0 ( $\lambda_i = OS$ ), it can re-sell another  $(N_i - k_i(\lambda_i, \lambda_j))$  units in addition to the blocked  $k_i(\lambda_i, \lambda_j)$  units to the late consumers by providing a refund of  $(v_{i,1} + R_i)$  ( $R_i > 0$ ) to each of the early consumers. In order that we need not consider the strategic role of  $R_i$  (i.e., firms use  $R_i$  to adjust their expected prices), and be distracted from the main focus of the paper, we assume that  $R_i$  is exogenous<sup>6</sup>. This is not an unreasonable assumption as there is some general expectation in each industry as to what is an acceptable amount of compensation. In Period 3, the game ends and the remaining goods have no salvage value.

### 3 Reservation Rule and Pricing Strategy Given Selling Strategies

We assume that the selling strategies are observable to the firms when they are deciding on their pricing and reservation rules and we use subgame perfection as our solution concept<sup>7</sup>. Thus, we apply backward induction and solve for the equilibrium strategy of each firm by solving first for the Nash equilibrium of the pricing competition in Period 2. There are three subgames in Period 1, namely, when both firms choose the conventional selling strategy ( $(\lambda_A, \lambda_B) = (CS, CS)$ ), when both firms choose the overselling strategy ( $(\lambda_A, \lambda_B) = (OS, OS)$ ) and when one firm chooses

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<sup>6</sup>Furthermore, as we shall see in the analysis, the choice of  $R_i$  does not affect the decision of Firm  $j$ . And since the choice of  $R_i$  is not pre-announced to the consumers before they make their purchase decisions,  $R_i$  does not affect the decisions of the consumers as well. Thus, Firm  $i$  is likely to choose  $R_i$  to be as small as possible. Perhaps in future studies, the model can be extended to consider the case where  $R_i$  can take on a more strategic role.

<sup>7</sup>We will see that this assumption is not critical to our main result.

the overselling strategy while the other chooses the conventional selling strategy  $((\lambda_A, \lambda_B) = (OS, CS) \text{ or } (CS, OS))$ . Unless there is a possibility of confusion, we will exclude the term  $(\lambda_A, \lambda_B)$  in our notations here.

*Subgame 1:*  $((\lambda_A, \lambda_B) = (CS, CS))$ .

By virtue of the demand in Period 1,  $p_{A,1}^* = v_{A,1}$ ,  $p_{B,1}^* = v_{B,1}$ . However, the pricing competition in Period 2 is less straightforward.

The pricing competition in Period 2 differs from that in the Bertrand model as the firms here face capacity constraint. Yet, our formulation of the market demand also differs from that in the Edgeworth competition in that in addition to being uncertain, the late consumers have the same valuation for the same good. Our formulation is thus more in accordance with that in Narasimhan (1988). In the following, we present an informal argument to illustrate that the equilibrium pricing strategy in Period 2 must necessarily be a mixed strategy. For the purpose here, we shall assume without loss of generality that  $v_{A,2} \leq v_{B,2}$ . If  $p_{A,2} < p_{B,2} + (v_{A,2} - v_{B,2})$ , late arriving consumers prefer to buy from Firm *A* than from Firm *B*. By symmetry, if  $p_{A,2} > p_{B,2} + (v_{A,2} - v_{B,2})$ , late arriving consumers prefer to buy from Firm *B* instead. Intuitively, if Firm *A* adopts a pure pricing strategy and sets  $p_{A,2}$  such that  $p_{A,2} > p_{B,2} + (v_{A,2} - v_{B,2})$ , then  $p_{A,2} = v_{A,2}$  strictly dominates any other price levels  $p$  where  $p \in [p_{B,2} + (v_{A,2} - v_{B,2}), v_{A,2})$  since the expected sales remain the same while the profit margin increases. However, Firm *B*'s best response to  $p_{A,2} = v_{A,2}$  must then be  $p_{B,2} = v_{B,2} - \epsilon$ , where  $\epsilon$  is a small positive number. In turn, given  $p_{B,2} = v_{B,2} - \epsilon$ , the best response for Firm *A* is actually to undercut Firm *B*'s price by choosing  $p_{A,2} = v_{A,2} - 2\epsilon$ . Undercutting is a best response to a competitor's pure pricing strategy because it increases the market share while sacrificing only a minuscule profit margin. As a result of a series of undercutting of prices, the firm with a less preferred good, (Firm *A* in this case) will eventually price at zero while the competing firm (Firm *B*) sets  $p_{B,2} = v_{B,2} - v_{A,2} - \epsilon$ . At this point, it is actually better off for Firm *A* to increase its price. As we have seen earlier in

this paragraph, Firm  $A$  will again increase its price to  $p_{A,2} = v_{A,2}$ , thus triggering another round of price cutting. Undercutting is clearly a preferred strategy if the competing firm adopts a pure pricing strategy when there is demand uncertainty because undercutting increases the expected sales while sacrificing only a small profit margin. Thus, we must necessarily expect a mixed strategy at the equilibrium. Taking this into consideration, we derive the equilibrium pricing strategies<sup>8</sup>. Applying the results of the equilibrium pricing strategies, we deduce the firms' decisions on the number of units to reserve for sale only in Period 2 which is given in the lemma below<sup>9</sup>. We present the expected payoffs in Table 1.

**Lemma 3.1**  $((\lambda_A, \lambda_B) = (CS, CS))$ . *Suppose both firms adopt the conventional selling strategy. Then Firm  $i$  ( $i = A, B$ ) reserves  $k_i^*(CS, CS)$  units for sale exclusively in Period 2, where  $k_i^*(CS, CS)$  is the largest integer no more than  $N_i$  such that*

$$P(L \geq k_i^*(CS, CS) + k_j^*(CS, CS)) > \frac{v_{i,1}}{v_{i,2}}, j = A, B, j \neq i.$$

*More specifically, if  $\frac{v_{i,1}}{v_{i,2}} = \frac{v_{j,1}}{v_{j,2}}$ ,  $k_i^*(CS, CS) = \min(\max(0, \kappa_i - k_j^*(CS, CS)), N_i)$ ,  $k_j^*(CS, CS) = \min(\max(0, \kappa_j - k_i^*(CS, CS)), N_j)$ . If  $\frac{v_{i,1}}{v_{i,2}} < \frac{v_{j,1}}{v_{j,2}}$ ,  $k_i^*(CS, CS) = \min(\kappa_i, N_i)$ ,  $k_j^*(CS, CS) = \min(\max(0, \kappa_j - k_i^*(CS, CS)), N_j)$ <sup>10</sup>.*

**Table 1: Expected Payoffs At Equilibrium**

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<sup>8</sup>Proofs of lemmas are given in the Online Appendix while all other technical proofs are in the Appendix.

<sup>9</sup>This result is analogous to Littlewood (1972)'s seat allocation problem for a monopoly where the total number of reserved units is  $k_i^*(CS, CS) + k_j^*(CS, CS)$  in a duopoly.

<sup>10</sup>Much as Firm  $i$  may not be able to verify directly that  $k_j^*(CS, CS)$  is indeed the quantity Firm  $j$  reserves for Period 2, by virtue of being an equilibrium would mean that Firm  $j$  has no incentive to unilaterally deviate from  $k_j^*(CS, CS)$  to gain a higher expected profit. I thank an anonymous Area Editor for highlighting this point.

$(\lambda_i, \lambda_j)$	$\Pi_i^{\lambda_i, \lambda_j}, i, j = A, B.$
$(CS, CS)$	$\Pi_i^{CS, CS} = v_{i,1}(N_i - k_i) + v_{i,2}(\sum_{s=k_j+1}^{k_j+k_i} (s - k_j)P(L = s) + k_i P(L > k_i + k_j)),$ $k_i = k_i^*(CS, CS), k_j = k_j^*(CS, CS).$
$(OS, OS)$	$\Pi_i^{OS, OS} = v_{i,1}(N_i - k_i) + v_{i,2}(\sum_{s=N_j+1}^{N_j+k_i} (s - N_j)P(L = s) + k_i P(L > N_j + k_i)) +$ $(v_{i,2} - v_{i,1} - R_i)(\sum_{s=N_j+k_i+1}^{N_j+N_i} (s - N_j - k_i)P(L = s) +$ $(N_i - k_i)P(L > N_j + N_i)), k_i = k_i^*(OS, OS).$
$(CS, OS)$	$\Pi_i^{CS, OS} = v_{i,1}(N_i - k_i) + v_{i,2}(\sum_{s=N_j+1}^{N_j+k_i} (s - N_j)P(L = s) + k_i P(L > N_j + k_i)),$ $\Pi_j^{CS, OS} = v_{j,1}(N_j - k_j) + v_{j,2}(\sum_{s=k_i+1}^{k_i+k_j} (s - k_i)P(L = s) + k_j P(L > k_i + k_j)) +$ $(v_{j,2} - v_{j,1} - R_j)(\sum_{s=k_i+k_j+1}^{k_i+N_j} (s - k_i - k_j)P(L = s) +$ $(N_j - k_j)P(L > N_j + k_i)), k_i = k_i^*(CS, OS), k_j = k_j^*(CS, OS).$

The critical ratio for the reservation rule is the ratio between the valuations in Periods 1 and 2 and the reservation rule reflects the tradeoffs between selling immediately in Period 1 at  $v_{i,1}$  and waiting for the possible arrival of the late consumer. In essence, the reservation rule states that the  $k_i^*(CS, CS)^{th}$  unit is reserved for Period 2 if and only if its expected return (denoted by  $v_{i,2}P(L \geq k_i^*(CS, CS) + k_j^*(CS, CS))$ ) is greater than the profit it can obtain by selling in Period 1 at  $v_{i,1}$ . More units are reserved if and only if the ratio  $\frac{v_{i,1}}{v_{i,2}}$  is small since a smaller ratio implies a higher benefit for reserving the unit. Not unexpectedly, the number of units reserved by a firm for Period 2 decreases as the number of units reserved by the competing firm increases. If the critical ratios are equal, i.e.,  $\frac{v_{i,1}}{v_{i,2}} = \frac{v_{j,1}}{v_{j,2}}$ , then any  $(k_i^*(CS, CS), k_j^*(CS, CS))$  is an equilibrium as long as the total number of reserved units is equal to  $\kappa_i (= \kappa_j)$ . If  $\frac{v_{i,1}}{v_{i,2}} < \frac{v_{j,1}}{v_{j,2}}$ , then Firm  $i$  reserves  $\min(\kappa_i, N_i)$  units while Firm  $j$  reserves  $\min(\max(0, \kappa_j - k_i^*(CS, CS)), N_j)$  units.

We further note from Table 1 that the expected payoff of Firm  $i$  from the sales in Period 2 (as given by  $v_{i,2}(\sum_{s=k_j+1}^{k_j+k_i} (s - k_j)P(L = s) + k_i P(L > k_i + k_j))$ ) is the same as if Firm  $i$  sets a price of  $v_{i,2}$  in Period 2 and only gets to sell after Firm  $j$  has sold all the reserved  $k_j$  units.

*Subgame 2:  $((\lambda_A, \lambda_B) = (OS, OS))$ .*

When both firms adopt the overselling strategy, each firm can potentially sell its entire capacity in Period 2. Thus, we expect the firms' capacities to play a more direct role on the reservation rule. In addition, since all units can be sold in Period 2 regardless of whether it is reserved or already sold, the tradeoff between reserving a unit or not is no longer about selling in Periods 1 or 2. Rather, the tradeoff in this case is between the opportunity loss from not selling immediately in Period 1 at  $v_{i,1}$  and the cost of reselling the unit should demand in Period 2 warrant it, i.e.,  $\frac{v_{i,1}}{v_{i,1}+R_i}$ . Using the same argument as in Subgame 1, we can obtain the expected profits of the firms (Table 1) and thus the reservation rule at the equilibrium.

**Lemma 3.2**  $((\lambda_A, \lambda_B) = (OS, OS))$ . *Suppose both firms adopt the overselling strategy. Then Firm  $i$  ( $i = A, B$ ) reserves  $k_i^*(OS, OS)$  units for sale exclusively in Period 2, where  $k_i^*(OS, OS)$  is the largest integer no more than  $N_i$  such that*

$$P(L \geq N_j + k_i^*(OS, OS)) > \frac{v_{i,1}}{v_{i,1} + R_i}, j = A, B, j \neq i.$$

*More specifically,  $k_i^*(OS, OS) = \min(\max(0, \kappa_i^R - N_j), N_i)$ .*

When both firms adopt the overselling strategy, the optimum number of units to reserve does not depend on the number of units reserved by the competing firm, but rather by the capacity of the competing firm. This is because with overselling, each firm can sell up to its capacity in Period 2. In other words, the optimum choice of  $k_i^*(OS, OS)$  is independent of  $k_j^*(OS, OS)$  and the expected profit of Firm  $i$  is also independent of  $k_j^*(OS, OS)$ . We further note that if  $N_i = N_j$ ,  $k_i^*(OS, OS)$  is less than  $k_j^*(OS, OS)$  if and only if  $\frac{v_{i,1}}{v_{i,1}+R_i}$  is larger than  $\frac{v_{j,1}}{v_{j,1}+R_j}$ .

When a firm adopts the overselling strategy, its upside benefit of reserving a unit decreases since it can always resell a unit previously sold in Period 1. This is true regardless of whether a firm is a monopoly or is part of a duopoly in the market. In the latter case, price competition between the firms in Period 2 further reduces the

incentives for reserving a unit for sale exclusively in Period 2. Hence, we have the following proposition.

**Proposition 3.1** *The number of units reserved by each firm when both firms adopt the overselling strategy is less than that reserved when the firms adopt the conventional selling strategy, i.e.,  $k_i^*(OS, OS) \leq k_i^*(CS, CS)$ ,  $i = A, B$ .*

*Subgame 3:  $((\lambda_i, \lambda_j) = (CS, OS)), i, j = A, B, j \neq i$ .*

When Firm  $j$  adopts the overselling strategy, the number of units that it reserves for Period 2 does not affect the number of units that Firm  $i$  reserves but rather, it is the capacity of Firm  $j$  that will have a bearing on the reservation rule for Firm  $i$ . However, since the latter adopts the conventional selling strategy, its reservation rule does affect the number of units that Firm  $j$  reserves. The following lemma describes the reservation rules for both Firms  $i$  and  $j$ .

**Lemma 3.3**  $((\lambda_i, \lambda_j) = (CS, OS))$ . *Suppose Firm  $i$  adopts the conventional selling strategy while Firm  $j$  adopts the overselling strategy,  $i, j = A, B, j \neq i$ . Then Firm  $i$  reserves  $k_i^*(CS, OS)$  units for sale in Period 2, where  $k_i^*(CS, OS)$  is the largest integer no more than  $N_i$  satisfying*

$$P(L \geq N_j + k_i^*(CS, OS)) > \frac{v_{i,1}}{v_{i,2}},$$

*while  $k_j^*(CS, OS)$  is the largest integer no more than  $N_j$  satisfying*

$$P(L \geq k_i^*(CS, OS) + k_j^*(CS, OS)) > \frac{v_{j,1}}{v_{j,1} + R_j}.$$

*More specifically,  $k_i^*(CS, OS) = \min(\max(0, \kappa_i - N_j), N_i)$ ,  $k_j^*(CS, OS) = \min(\max(\kappa_j^R - k_i^*(CS, OS), 0), N_j)$ .*

Reviewing the reservation rules in Subgame 3 with those in Subgames 1 and 2, we obtain the following proposition.

**Proposition 3.2** For  $i, j = A, B, j \neq i$ , (i)  $k_j^*(CS, OS) \geq k_j^*(OS, OS)$ , (ii)  $k_i^*(CS, OS) \leq k_i^*(CS, CS)$ .

The above proposition states that regardless of the selling strategy of a firm, it always reserves more units if the competing firm adopts the conventional selling policy. The rationale is as follows. When Firm  $i$  chooses the conventional selling strategy while Firm  $j$  chooses the overselling strategy, the pricing competition in Period 2 is less intense than if both firms were to adopt the overselling strategy (as in Subgame 2). This is because Firm  $i$  can sell at most  $k_i^*(CS, OS)$  units. Thus, the upside potential for Firm  $j$  to reserve additional units is higher, resulting in  $k_j^*(CS, OS) \geq k_j^*(OS, OS)$ . From Firm  $i$ 's perspective however, competing with a firm which adopts the overselling strategy is more intense than when competing with one who adopts the conventional selling strategy (as in Subgame 1) since the competing firm can sell up to its capacity in Period 2. The upside potential for reserving units for sale exclusively in Period 2 is thus reduced. Hence,  $k_i^*(CS, OS) \leq k_i^*(CS, CS)$ .

## 4 Equilibrium Selling Strategies

In this section, we apply our results on the three subgames obtained in §3 to determine the selling strategies of the firms at equilibrium. We present the respective expected payoffs of the firms contingent on their choice of selling strategy in Period 0 in the table below. Firm  $A$  ( $B$ )'s strategy choice is indicated as the rows (columns) and the payoffs are obtained from Table 1.



Table 2

$A / B$	Conventional Selling	Overselling
Conventional Selling	$\Pi_A^{CS,CS}, \Pi_B^{CS,CS}$	$\Pi_A^{CS,OS}, \Pi_B^{CS,OS}$
Overselling	$\Pi_A^{OS,CS}, \Pi_B^{OS,CS}$	$\Pi_A^{OS,OS}, \Pi_B^{OS,OS}$

When Firm  $i$  adopts the overselling strategy, the number of units that Firm  $i$  reserves for Period 2 does not matter to Firm  $j$  at all. Rather, it is the capacity of Firm  $i$  that affects the expected profit of Firm  $j$ . If the latter adopts a conventional selling strategy, it is competing with  $N_i$  units from Firm  $i$  in Period 2, with itself selling no more than  $k_j$  units. On the other hand, by adopting an overselling strategy, Firm  $j$  has the opportunity to sell more units albeit at the expense of a further erosion in prices as a result of the additional capacity. Can the increase in sales compensate for the decrease in prices? As we have observed in §3, the expected payoff for Firm  $j$  in Period 2 at the equilibrium is the same as if Firm  $j$  sets the price at  $v_{j,2}$ . Hence, any increase in expected sales increases the expected profit of Firm  $j$ , thus ensuring that Firm  $j$  is indeed better off adopting the overselling strategy. By the same token, when Firm  $i$  adopts the conventional selling strategy, can the competing Firm  $j$  sell more units if it adopts the overselling strategy rather than follow the same conventional selling strategy? Again, the downside is the potentially lower prices from the increase supply in Period 2 if the overselling strategy is adopted. Using the same argument as before, an increase in expected sales in Period 2 increases the expected payoff of Firm  $j$ . Thus, regardless of whether Firm  $i$  adopts an overselling strategy or a conventional selling strategy, Firm  $j$  is better off adopting an overselling strategy.

**Theorem 4.1** (i) *Overselling is a weakly dominant strategy.* (ii) *Both firms adopting the overselling strategy is a subgame perfect equilibrium.*

The fact that the overselling strategy is a dominant strategy (albeit a weak one) further implies that it really does not matter if firms choose their selling strategies in

Period 0 simultaneously (as we have modeled here) or sequentially (in a Stackelberg manner) or whether each firm's selling strategy is observable by the other firm. This is because the first mover firm choosing either a conventional selling strategy or an overselling strategy does not change the decision of the second mover firm to adopt an overselling strategy because of its dominance. As a result, any first mover firm will also find it optimum to adopt an overselling strategy. Furthermore, regardless of the selling strategy of one firm, the other firm is always no worse off by adopting the overselling strategy because of the dominance of the overselling strategy. Any observation of the competing firm's selling strategy does not change the decision of the firm. Thus, our assumption in the model (on Page 9) that each firm observes the selling strategies of both firms before making their respective reservation choices and pricing decision is not crucial for our main results.

Clearly, from the consumers' perspective, overselling results in a more efficient allocation where more goods are sold to late consumers (who have a higher valuation for the goods) than when firms follow the conventional selling strategy. More specifically, when both firms adopt the conventional selling strategy, only the consumers who purchase in Period 2 may potentially gain a positive consumer surplus. However, when both firms adopt the overselling strategy, more consumers arriving in Period 2 may be able to make a purchase and at a potentially lower price because of the more intense pricing competition as a result of the overselling strategy thus increasing the consumer surplus. In addition, the consumers who have bought earlier in Period 1 are better off too if the units they have bought are resold and they are compensated. Hence, the overall consumer surplus is higher when both firms adopt the overselling strategy. This is given in the proposition below.

**Proposition 4.1** *When both firms adopt the overselling strategy, the total expected consumer surplus is no less than if both firms adopt the conventional selling strategy.*

However, is the outcome of both firms adopting the overselling strategy also a win-

win one for the firms, or does the strategic interaction between the firms here lead to an outcome that is Pareto-dominated from the firms' perspectives? We proceed to illustrate this with two numerical examples below. These examples will also be used to motivate our results on the Pareto efficiency of the equilibrium outcome.

## 4.1 Expected Revenue and Pareto Dominance

Consider two symmetric firms. Let  $L$  follow the uniform distribution where  $P(L = s) = \frac{1}{4}$  for  $s = 0, 1, 2, 3$ , and  $P(L = s) = 0$  otherwise.

**Numerical Example 1:**  $v_{i,1} = 1, v_{i,2} = 3, R_i = 1, N_i = 2, i = A, B$ .

Using Lemmas 3.1, 3.2, 3.3, we deduce that  $k_i^*(CS, CS) = k_j^*(CS, CS) = 1$ ,  $k_i^*(OS, OS) = k_j^*(OS, OS) = 0$  and  $k_i^*(CS, OS) = 0, k_j^*(CS, OS) = 1$ . The Period 0 game is illustrated in Table 3. In this case, the overselling strategy strictly dominates the conventional selling strategy and there is a unique subgame perfect equilibrium here, namely, both firms adopt the overselling strategy. More importantly, we note that although both firms adopting the overselling strategy is indeed an equilibrium, it gives the firms the worst outcomes and is Pareto-dominated. In this example, the capacity of each firm is relatively large such that when a firm adopts the overselling strategy, the competing firm does not reserve any goods for sale in Period 2. In the case where the competing firm adopts the conventional selling strategy, the former acts as a monopoly in Period 2, thus resulting in a high expected payoff of  $\frac{15}{4}$ . If the competing firm also adopts the overselling strategy, then the overcapacity in Period 2 results in intensive pricing competition with each firm having an expected payoff of  $\frac{9}{4}$  only. In essence, this example demonstrates that both firms adopting the overselling strategy, albeit being a subgame perfect equilibrium, leads to a prisoners' dilemma situation where each firm turns out to be worse off.

Table 3

$A / B$	Conventional Selling	Overselling
Conventional Selling	$\frac{5}{2}, \frac{5}{2}$	$2, \frac{15}{4}$
Overselling	$\frac{15}{4}, 2$	$\frac{9}{4}, \frac{9}{4}$

Based on the example above, shall we then conclude that we should object to the practice of overselling? We show in the next example that the answer is no.

**Numerical Example 2:**  $v_{i,1} = 1, v_{i,2} = 3, R_i = 0.1, N_i = 1, i = A, B$ .

In this example, both  $R_i$  and  $N_i$  are smaller than in Numerical Example 1.  $k_i^*(CS, CS) = k_j^*(CS, CS) = 1, k_i^*(OS, OS) = k_j^*(OS, OS) = 0$  and  $k_i^*(CS, OS) = 1, k_j^*(CS, OS) = 0$ . Table 4 below presents the game in Period 0.

Table 4

$A / B$	Conventional Selling	Overselling
Conventional Selling	$\frac{3}{2}, \frac{3}{2}$	$\frac{3}{2}, 1.95$
Overselling	$1.95, \frac{3}{2}$	$1.95, 1.95$

Here, overselling is not only a dominant strategy but its outcome is Pareto dominant as well. From the two examples above, we observe that much as the overselling strategy is a dominant strategy, it does not always ensure that the expected payoffs are also Pareto dominant. In fact, as seen in Numerical Example 1 above, it can give rise to a Pareto dominated outcome where consumers win but firms, collectively lose.

A natural question at this point is, under what condition is the outcome where both firms oversell Pareto dominant? Intuitively, overselling seems optimum as a firm cannot be worse off by being able to resell the already sold units at a higher price. In fact, any price higher than the sum of the earlier price and the compensation should give the firm an additional profit. This is indeed true in the case of a monopoly (Biyalogorsky et al. 1999). However, when there is a competing firm, strategic

interaction between the firms changes the dynamics of the situation. Firstly, when both firms oversell, each firm reserves less units for Period 2 and sell more in Period 1 than when they do not (Proposition 3.1). The rationale being that the goods can always be resold should the demand from the late arriving consumers warrant it. Firms therefore make a higher profit in Period 1 when both oversell than when they do not. Secondly, overselling increases the price competition between the firms in Period 2 and each firm has less reserved units available for sale in that period, resulting in a lower profit from the sale of these reserved units. In Proposition 4.2 below, we show that the increase in profit from selling more units in Period 1 cannot compensate for the decrease in profit in Period 2 as a result of selling less reserved units. The intuition is that the expected per unit loss in profit margin in Period 2 is more than the per unit gain in profit margin from selling in Period 1. Thus, for the overselling outcome to be Pareto dominant, it is crucial that the expected gain from reselling the units already sold in Period 1 is sufficiently high to make up for this overall loss. And reselling can give rise to a higher expected profit if and only if the profit margin from overselling *and* the demand in Period 2 are sufficiently high.

**Proposition 4.2** *The expected profit from selling in Period 1 and selling the reserved units in Period 2 is lower if both firms adopt the overselling strategy than if they follow the conventional selling strategy.*

**Theorem 4.2** *The outcome where both firms oversell is Pareto dominant if and only if both  $(v_{i,2} - v_{i,1} - R_i)$  and  $P(L > N_j + k_i^*(OS, OS))$  are sufficiently large,  $i = A, B$ <sup>11</sup>. More specifically, the expected profit from reselling must be large enough to compensate for the loss from selling less in Period 2 and selling more in Period 1 at a lower price.*

Theorem 4.2 spells out the conditions under which overselling gives rise to a ‘win-win’ situation for the firms. More specifically, the capacities of the firms relative to

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<sup>11</sup>The exact quantification is given in the proof in the Appendix.

the demand in Period 2 is an important determinant. This poses a dilemma for firms that face seasonal demand shifts. That is, in some seasons when demand is high, firms may find the overselling outcome to be ‘win-win’. On the other hand, during the lull period when demand is low, firms may find themselves stuck in a prisoners’ dilemma situation. In the following table, we illustrate and compare the extent of the positive effect of overselling (when demand is high) with the negative effect (when demand is low)<sup>12</sup>.

Table 5 shows that when  $\frac{v_{i,1}}{v_{i,2}} = 0.25$ ,  $\frac{v_{i,1}}{v_{i,1}+R_i} = 0.5$  and the capacity of the firm ( $N_i$ ) is 10, there is a downside ratio of  $-0.067$  from overselling as a result of an over capacity of 10% but the upside ratio of overselling with an under capacity by the same amount is 0.090. In this instance, the upside ratio is higher than the magnitude of the downside ratio as demand changes from an over to an under capacity while the capacities of the firms are fixed. In general, firms trapped into overselling strategies, enjoy higher profits when demand is high relative to capacity but find overselling less profitable when demand is low relative to capacity. This is driven by the fact that excessive demand leads to less intense pricing competition which insulates firms from the competitive effects of overselling in Period 2. In the extreme case when demand is sufficiently high, firms act like monopolies in Period 2. However, from the table, we also observe that there are both instances of upside ratio being lower than the magnitude of the downside ratio (the asterix cases) as well as instances of upside ratio being higher than the magnitude of the downside ratio. Our numerical results suggest that as the ratio  $\frac{v_{i,1}}{v_{i,1}+R_i}$  increases, the upside ratio is higher than the magnitude of the downside ratio.<sup>13</sup> This is not surprising as a lower  $R_i$  (and thus a higher  $\frac{v_{i,1}}{v_{i,1}+R_i}$ ) enhances the attractiveness of overselling as the cost of doing so is

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<sup>12</sup>I thank an anonymous reviewer for highlighting this point.

<sup>13</sup>The same trend is observed for other values of  $\frac{v_{i,1}}{v_{i,2}}$ . These results are presented in the Online Appendix.

lower. In particular, when  $\frac{v_{i,1}}{v_{i,1}+R_i} = 0.833$ , an over capacity of 10% can give rise to a positive downside ratio.

As overselling is a weakly dominant strategy, firms may sometimes be indifferent between adopting the overselling strategy and the conventional selling strategy. In this instance, there may be other subgame perfect equilibria (in addition to the one where both firms oversell) such as when both firms adopt the conventional selling strategy, or when one firm oversells but the other does not. The following proposition outlines the two scenarios under which one firm oversells while the other does not constitutes an equilibrium. In the first scenario, the demand in Period 2 is so high that the likelihood of the total capacity of the firms not being able to meet the demand is at least  $\frac{v_{j,1}}{v_{j,1}+R_j}$ . As such, Firm  $j$  does not sell in Period 1 at all but reserves all units for Period 2. As a result, there is no possibility of reselling and Firm  $j$  is indifferent between the conventional and overselling strategies. The second scenario presents the other extreme case where the demand is so low that the capacity of the competing Firm  $i$  is more than sufficient to meet the demand in Period 2, i.e.,  $P(L > N_i) = 0$ . Thus, Firm  $j$  sells only in Period 1 and as the demand is low, there is no opportunity to resell at all in Period 2, which renders Firm  $j$  indifferent between the two selling strategies.

**Table 5: Numerical Results<sup>14</sup>**

$\frac{v_{i,1}}{v_{i,2}}$	$\frac{v_{i,1}}{v_{i,1}+R_i}$	$N_i$	$ 1 - \frac{N_i+N_j}{\bar{s}} $	$(\rho^D, \rho^U)$
0.25	0.5	10	0.5	$(-0.200, 0.000)^*$
			0.25	$(-0.145, 0.107)^*$
			0.1	$(-0.067, 0.09)$
		20	0.5	$(-0.202, 0.000)^*$
			0.25	$(-0.150, 0.100)^*$
			0.1	$(-0.070, 0.040)^*$
0.25	0.667	10	0.5	$(-0.184, 0.030)^*$
			0.25	$(-0.103, 0.212)$
			0.1	$(-0.004, 0.190)$
		20	0.5	$(-0.188, 0.033)^*$
			0.25	$(-0.109, 0.199)$
			0.1	$(-0.010, 0.138)$
0.25	0.833	10	0.5	$(-0.175, 0.102)^*$
			0.25	$(-0.077, 0.287)$
			0.1	$(0.035, 0.257)$
		20	0.5	$(-0.180, 0.101)^*$
			0.25	$(-0.085, 0.274)$
			0.1	$(0.027, 0.200)$

**Proposition 4.3** *The outcome where Firm  $i$  oversells and Firm  $j$  adopts the conventional pricing strategy is a subgame perfect equilibrium if and only if (i) the demand in Period 2 is sufficiently high, i.e.,  $P(L \geq N_i + N_j) \geq \frac{v_{j,1}}{v_{j,1}+R_j}$  or (ii) the demand in*

<sup>14</sup>If  $1 - \frac{N_i+N_j}{\bar{s}} > 0$ , there is under capacity while if  $1 - \frac{N_i+N_j}{\bar{s}} < 0$ , there is over capacity.  $\rho^D$  ( $\rho^U$ ) denotes the downside (upside) ratio  $\frac{\Pi_i^{OS,OS} - \Pi_i^{CS,CS}}{\Pi_i^{CS,CS}}$  when  $1 - \frac{N_i+N_j}{\bar{s}} < 0$  ( $> 0$ ). In these numerical examples, we assume that firms are symmetric and  $L$  follows a uniform distribution on  $[0, \bar{s}]$ .



Period 2 is low with  $P(L > N_i) = 0$ . In (i), Firm  $j$  never sells in Period 1 while in (ii), Firm  $j$  sells every unit in Period 1.

Finally, the outcome where both firms adopt the conventional selling strategy is a subgame perfect equilibrium too when the demand in Period 2 is so high that both firms always reserve all the units for Period 2 and nothing is sold at all in Period 1 regardless of their selling strategy as long as the competing firm adopts the conventional selling strategy. This occurs when the total capacity of the firms is low compared to the demand. Since nothing is sold in Period 1, there is no reselling in Period 2. Each firm is thus indifferent between the two selling strategies.

**Proposition 4.4** *The outcome where both firms adopt the conventional selling strategy is a subgame perfect equilibrium if and only if  $P(L \geq N_j + N_i) > \max(\frac{v_{j,1}}{v_{j,1} + R_j}, \frac{v_{i,1}}{v_{i,1} + R_j})$ .*

Propositions 4.3 and 4.4 above show that only under conditions of extremely high or extremely low demand do we expect other subgame perfect equilibria in addition to the dominant strategy equilibrium where both firms oversell. This is because when the demand in Period 2 is extremely high, the firms reserve all the units for Period 2, thus rendering reselling irrelevant. Thus, the firms are indifferent between the conventional selling and overselling strategies and both firms adopting the conventional selling strategy or one firm oversells but the other does not, is a subgame perfect equilibrium. On the other hand, when the demand in Period 2 is extremely low such that one firm's capacity is sufficient to meet the demand, a firm sells all units in Period 1 and the likelihood of reselling any of these units in Period 2 is zero even if an overselling strategy has been adopted. Thus, the outcome when one firm oversells while the other adopts the conventional selling strategy is a subgame perfect equilibrium. We summarize the condition under which both firms oversell is the unique subgame perfect equilibrium in the following proposition.

**Proposition 4.5** *Both firms overselling is the unique subgame perfect equilibrium if and only if  $P(L \geq N_j + N_i) < \max(\frac{v_{j,1}}{v_{j,2}+R_j}, \frac{v_{i,1}}{v_{i,2}+R_j})$  and  $P(L > N_i) > 0$ ,  $i, j \in \{A, B\}, i \neq j$ .*

## 5 Discussion and Conclusion

Firms face a non trivial tradeoff in their pricing strategies when demand is uncertain. It is well known in the literature that overselling is one way to mitigate such demand uncertainty and to improve the profitability of the firms. However, the practice of overselling has never been explicitly examined in a duopoly context. The parsimonious model we have developed here allows us to investigate this phenomenon. We show that overselling is a weakly dominant strategy. However, it also tends to intensify competition since more supply enters the market in the later period than if firms do not oversell. As a result, overselling can lead to a prisoners' dilemma in which all firms are worse off. We further show that market characteristics and firm characteristics can help soften the effect of price competition as a result of overselling. More specifically, market characteristics such as the difference in valuations between the early and the late consumers as well as the amount of compensation needed to be given to the early buyer are important parameters in determining the profitability of the overselling strategy. Essentially, a big spread between the valuations and a lower compensation such that the net profit margin from overselling is sufficiently high provide justification to the overselling strategy. This is in contrast to the case when the firm is a monopoly. As a monopoly, as long as the net profit margin for overselling is positive, no matter how small, overselling is optimum. Firm characteristics such as capacity is also an important determinant. The lower the firms' capacities relative to the demand from the high valuation consumers, the less intense the price competition and the more likely it is for overselling to be profitable. Again, this result is in stark contrast to

the monopoly context where overselling is profitable regardless of the firm's capacity. In other words, the condition for overselling to be Pareto dominant is more stringent in a duopoly than in a monopoly. In the latter, overselling is always profitable.

To a large extent, overselling is in essence similar to the concept of short selling in stocks. In the latter, short sellers sell stocks that they do not already own, in anticipation of a fall in prices. When prices fall, they buy back these stocks and return them to the account from which they were borrowed and make a profit from short selling earlier at a higher price. In the case of overselling, firms resell units that they have already sold at a higher price. Both overselling and short selling make perfect sense in the context of a monopoly. However, when everyone short sells and needs to buy back the stocks at the same time, the price may rise instead of fall because of the unexpectedly high demand of the short sellers. In this case, short sellers do not make a profit. Likewise, when all the firms in the market practice the overselling strategy, price competition can lead to a loss rather than a profit for these firms if the supply in the market is relatively more than the demand. In other words, overselling is not always a 'everybody wins' strategy. It depends on both market and firm characteristics as we have highlighted earlier. Companies should therefore consider these factors carefully before adopting the overselling strategy 'as a tactic to improve the bottom line and benefit consumers' (Biyalogorsky et al. (2000)), thus creating a 'win-win-win' situation. Our conclusion also provides some explanation as to why anecdotal evidence can be found in some industries but not others. For example, it is commonly known that some airlines sell tickets at a full fare to late arriving consumers when in actual fact, these tickets had already been sold earlier and these airlines give a compensation to the early consumers. On the other hand, not all airlines follow the same practice. A check on the ticket price in several routes in summer 2008 shows that the price difference between booking four weeks before departure and one week before can range from 9% to a staggering 190% (a sample of

prices over the weeks is shown in Table 6)! Clearly, the airline with a price difference of 190% is more likely to be profitable practicing the overselling strategy than one with a price difference of 9%.

**Table 6 Sample of Prices Over A 4-Week Period**

Airlines	Week 4	Week 3	Week 2	Week 1	Maximum % Change
<i>A</i> Business Class	\$14,285	\$16,664	\$16,664	\$15,854	17 %
<i>B</i> Business Class	\$9,432	\$9,485	\$9,466	\$8,654	9%
<i>C</i> Economy Class	\$1,547	\$1,917	\$1,855	\$4,495	190%
<i>D</i> Economy Class	\$2,449	\$2,449	\$3,506	\$3,491	43%

One can also draw the analogy between overselling and arbitraging opportunities in secondary markets. If goods are transferable, secondary markets are likely to develop if valuations between consumers are sufficiently different. And the existence of these secondary markets improve the allocation efficiency of the markets. It is not unheard of that buyers of properties after signing the option to purchase, transfer the option (before exercising it) to another buyer who is willing to pay more for the properties. In this way, the first buyer makes a profit while the second buyer gets to buy the property that he wants. Overselling serves the same purpose except that instead of a separate secondary market, the seller plays the role of the secondary market as well to facilitate the efficient allocation of goods.

Our research thus suggests that marketers and managers alike examine the market structure and firm characteristics when considering their pricing and selling strategies. More importantly, policy makers need to convey the value proposition to the consumers that overselling is an ‘always win’ strategy for the consumer even if the good he or she has purchased has been resold. This is because in place of the good, this consumer has been offered a compensation (usually in cash) and a replacement (such as a comparable air ticket at an alternative time). The format in which this can be communicated and managed clearly depends on the industry and setting. While there are obvious challenges, the benefit for all parties can be substantial as we have shown. Much as firms should always oversell, they may not always be more prof-

itable doing so. In order to ensure the profitability of overselling, marketers could do well by focusing their efforts on enhancing *both* the valuation *and* the demand of late consumers, or alternatively, by managing the firms' capacities especially as demand fluctuates either as a seasonality factor or as the broad economic condition changes. It has been reported that Singapore Airlines, one of Asia's major carriers 'will cut 17% of its operating fleet ... amid a global economic slump.' (The China Post, February 17 (2009)).

In this paper, we have studied the use of overselling as a strategy to maximize a firm's expected profit. As per our intuition, we find that the overselling strategy is a dominant strategy. However, unlike in the case where a firm is a monopoly, overselling may not result in a Pareto dominant outcome once they are in a competitive environment. Hence, whether overselling is indeed a boon or a bane depends on a number of parameters. Indeed firms can work together to limit their respective capacities *and* to increase their profit margin for reselling the already sold units to ensure that overselling is a real boon.

## 6 References

- Bitran, G., R. Caldentey. 2003. An overview of pricing models for revenue management. *Manufacturing & Service Operations Management*, 5(3), 203-229.
- Biyalogorsky, E., Z. Carmon, G. E. Fruchter, E. Gerstner. 1999. Overselling with opportunistic cancellations. *Marketing Science*, 18, 4, 605-610.
- Biyalogorsky, E., Z. Carmon, G. E. Fruchter, E. Gerstner. 2000. Should airlines and hotels use short selling? *OR/MS Today*, 27, 5, October.
- Biyalogorsky, E., E. Gerstner. 2004. Note: Contingent pricing to reduce price risks. *Marketing Science*, 23(1), 146-155.
- Dai, Y., X. Chao, S. Fang, H. Nuttle. 2006. Capacity allocation with traditional and

- Internet channels. *Navel Research Logistics*, 53, 772-787.
- Dana, J. D. 1999. Using yield management to shift demand when the peak time is unknown. *RAND Journal of Economics*, 30,3, 456-474.
- Desiraju, R., S. Shugan. 1999. Strategic service planning and yield management. *Journal of Marketing*, 63,1, 44-56.
- Elmaghraby, W., P. Keskinocak. 2003. Dynamic pricing in the presence of inventory considerations: Research overview, current practices, and future directions. *Management Science*, 49,10, 1287-1309.
- Federgruen, A., A. Heching. 1999. Combined pricing and inventory control under uncertainty. *Operations Research*, 47,3, 454-475.
- Gallego, G., G. van Ryzin. 1994. Optimal dynamic pricing of inventories with stochastic demand over finite horizons. *Management Science*, 40,8, 999-1020.
- Gallego, G., S.G. Kou, R. Philips. 2008. Revenue management of callable products. *Management Science*, 54(3), 550-564.
- Kalai, E., M. Kamien, M. Rubinovitch. 1992. Optimal service speeds in a competitive environment. *Management Science*, 38, 1154-1163.
- Kirman, A., M. Sobel. 1974. Dynamic oligopoly with inventories. *Econometrica*, 42, 279-287.
- Lim, W. 2008. Can sellers do better than simply let buyers compete? The strategic role of reservation. working paper.
- Lippman, S., K. McCardle. 1997. The competitive newsboy. *Operations Research*, 45(1), 54-65.
- Littlewood, K. 1972. Forecasting and control of passengers bookings. *AGIFORS 12th Annual Symposium Proceedings*, 95-117.
- Narasimhan, C. 1988. Competitive promotional strategies. *Journal of Business*, 61,4, 427-449.
- Parlar, M. 1988. Game theoretic analysis of the substitutable product inventory

- problem with random demand. *Naval Research Logistics*, 35, 397-409.
- Philips, R. L. 2005. *Pricing and Revenue Optimization*. Stanford University Press, Stanford, CA.
- Png, I. 1989. Reservations: Customer insurance in the marketing of capacity. *Marketing Science*, 8(3), 248-264.
- Rothstein, M. 1971. An airline overbooking model. *Transportation Science*, 5, 2, 180-92.
- Rothstein, M. 1985. O.R. and the airline over-booking problem. *Operations Research*, 33, 237-248.
- Shugan, S., J. Xie. 2005. Advance selling as a competitive marketing tool. *International Journal of Research in Marketing*, 22(3), 351-373.
- Smith, B.C., J.F. Leimkuhler, R.M. Darrow. 1992. Yield management at American Airlines. *Interfaces*, 22, 8-31.
- Staiger, R., F. Wolak. 1992. Collusive pricing with capacity constraints in the presence of demand uncertainty. *Rand Journal of economics*, 23,2, 203-220.
- Talluri, K. G. van Ryzin. 2004. *The Art and Science of Revenue Management*. Kluwer, New York.
- The China Post*. 2009. Singapore Airlines to cut 17 percent of fleet. February 17.
- USA Today*. 1998. Giving up jet seat can be ticket to free ride. April 28, 05B.
- Van Mieghem, J., M. Dada. 1999. Price versus production postponement: Capacity and competition. *Management Science*, 45,12, 1631-1649.
- Weatherford, L., S. Bodily. 1992. A taxonomy and research overview of perishable-asset revenue management: Yield management, overbooking and pricing. *Operations Research*, 40,5, 831-844.
- Wilson, C. 1988. On the optimal pricing policy of a monopolist. *Journal of Political Economy*, 96,1, 164-176.
- Xie, J., E. Gerstner. 2007. Service escape: Profiting from customer cancellations.



*Marketing Science*, 26(1), 18-30.

Zhao, A., D. Atkins. 2008. Newsvendors under simultaneous price and inventory competition. *Manufacturing & Service Operations Management*, 10(3), 539-546.

## Appendix

**Proof of Proposition 3.1.** We recall that overselling is profitable if and only if  $v_{i,2} > v_{i,1} + R_i$ , which implies that  $\frac{v_{i,1}}{v_{i,2}} < \frac{v_{i,1}}{v_{i,1}+R_i}$ . Since  $P(L > s)$  is decreasing in  $s$ , ( $s > 0$ ), it follows immediately from the reservation rules stated in Lemmas 3.1 and 3.2 that  $N_j + k_i^*(OS, OS) \leq k_i^*(CS, CS) + k_j^*(CS, CS)$ . As  $k_j^*(CS, CS)$  is no more than  $N_j$ , we deduce that  $k_i^*(OS, OS) \leq k_i^*(CS, CS)$ .  $\square$

**Proof of Proposition 3.2.** (i) From Lemmas 3.2 and 3.3, we know that  $k_j^*(OS, OS)$  is such that  $P(L \geq N_i + k_j^*(OS, OS)) > \frac{v_{j,1}}{v_{j,1}+R_j}$  and  $k_j^*(CS, OS)$  is such that  $P(L \geq k_i^*(CS, OS) + k_j^*(CS, OS)) > \frac{v_{j,1}}{v_{j,1}+R_j}$ . Since  $N_i \geq k_i^*(CS, OS)$ , it follows immediately that  $k_j^*(OS, OS) \leq k_j^*(CS, OS)$ . (ii) We use the results from Lemmas 3.1 and 3.2 in our proof. If  $\frac{v_{i,1}}{v_{i,2}} \geq \frac{v_{j,1}}{v_{j,2}}$ ,  $k_i^*(CS, OS) = \min(\max(0, \kappa_i - N_j), N_i) \leq \min(\max(0, \kappa_i - k_j^*(CS, CS)), N_i) = k_i^*(CS, CS)$ . If  $\frac{v_{i,1}}{v_{i,2}} < \frac{v_{j,1}}{v_{j,2}}$ ,  $k_i^*(CS, OS) = \min(\max(0, \kappa_i - N_j), N_i) \leq \min(\max(0, \kappa_i), N_i) = \min(\kappa_i, N_i) = k_i^*(CS, CS)$ .  $\square$

**Proof of Theorem 4.1.** We will prove a series of lemmas which will lead to our result that the overselling strategy is a weakly dominant strategy.

**Lemma A1.** (i)  $\Pi_j^{OS,OS}(k_i, k_j)$ ,  $\Pi_j^{OS,CS}(k_i, k_j)$  are independent of  $k_i$ . (ii)  $\Pi_j^{OS,OS}(k_i, k_j) \geq \Pi_j^{OS,CS}(k_i, k_j)$  for all  $k_j$ . In particular, when  $k_j = k_j^*(OS, CS)$ , equality holds if and only if  $k_j^*(OS, CS) = N_j$  or  $k_j^*(OS, CS) = 0$  (or  $P(L > N_j) = 0$ ).

(iii)  $\Pi_j^{OS,OS}(k_i^*(OS, OS), k_j^*(OS, OS)) \geq \Pi_j^{OS,OS}(k_i^*(OS, OS), k_j^*(OS, CS))$  and equality holds if and only if  $k_j^*(OS, OS) = k_j^*(OS, CS)$ .

Proof: (i) This result is obvious. (ii)  $\Pi_j^{OS,OS}(k_i, k_j) - \Pi_j^{OS,CS}(k_i, k_j) = (v_{j,2} - v_{j,1} - R_j)(\sum_{s=N_i+k_j+1}^{N_i+N_j} (s - N_i - k_j)P(L = s) + (N_j - k_j)P(L > N_i + N_j)) \geq 0$ . When  $k_j = k_j^*(OS, CS)$ , equality holds in the expression above if and only if the right-hand-side term is equal to zero, i.e.,  $k_j^*(OS, CS) = N_j$  or  $P(L > N_i + k_j^*(OS, CS)) = 0$ . In the

latter case,  $k_j^*(OS, CS)$  must necessarily be zero from the optimality condition given in Lemma 3.3. (iii)  $\Pi_j^{OS,OS}(k_i^*(OS, OS), k_j^*(OS, OS)) \geq \Pi_j^{OS,OS}(k_i^*(OS, OS), k_j^*(OS, CS))$  by the optimality of  $k_j^*(OS, OS)$ . Furthermore, equality holds if and only if  $k_j^*(OS, OS) = k_j^*(OS, CS)$ .

**Lemma A2.**  $\Pi_j^{OS,OS}(k_i^*(OS, OS), k_j^*(OS, OS)) \geq \Pi_j^{OS,CS}(k_i^*(OS, CS), k_j^*(OS, CS))$  and equality holds if and only if (i)  $k_j^*(OS, CS) = k_j^*(OS, OS) = N_j$  or (ii)  $k_j^*(OS, CS) = k_j^*(OS, OS) = 0$  and  $P(L > N_i) = 0$ .

Proof:  $\Pi_j^{OS,OS}(k_i^*(OS, OS), k_j^*(OS, OS)) \geq \Pi_j^{OS,OS}(k_i^*(OS, OS), k_j^*(OS, CS)) \geq \Pi_j^{OS,CS}(k_i^*(OS, OS), k_j^*(OS, CS)) = \Pi_j^{OS,CS}(k_i^*(OS, CS), k_j^*(OS, CS))$ , where the first and second inequalities follow from Lemma A1(iii) and Lemma A1(ii) while the last equality follows from Lemma A1(i). From Lemma A1(ii) and (iii), we deduce that strict equality holds if and only if (i)  $k_j^*(OS, CS) = k_j^*(OS, OS) = N_j$  or (ii)  $k_j^*(OS, CS) = k_j^*(OS, OS) = 0$  and  $P(L > N_i) = 0$ .

**Lemma A3.** (i)  $\Pi_j^{CS,OS}(k_i^*(CS, OS), k_j^*(CS, OS)) \geq \Pi_j^{CS,OS}(k_i^*(CS, OS), k_j^*(CS, CS))$  and equality holds if and only if  $k_j^*(CS, OS) = k_j^*(CS, CS)$ . (ii)  $\Pi_j^{CS,OS}(k_i^*(CS, OS), k) \geq \Pi_j^{CS,CS}(k_i^*(CS, CS), k)$  for all  $k$ . In particular, when  $k = k_j^*(CS, CS)$ , equality holds if and only if  $k_i^*(CS, OS) = k_i^*(CS, CS)$  and  $k_j^*(CS, CS) = N_j$ .

Proof: (i) The proof is clear from the optimality of  $k_j^*(CS, OS)$ . Equality holds if and only if  $k_j^*(CS, OS) = k_j^*(CS, CS)$ . (ii) From Proposition 3.2,  $k_i^*(CS, OS) \leq k_i^*(CS, CS)$ . Thus,  $\Pi_j^{CS,OS}(k_i^*(CS, OS), k) - \Pi_j^{CS,CS}(k_i^*(CS, CS), k) = v_{j,2}(\sum_{s=k_i^*(CS,OS)+1}^{k_i^*(CS,CS)} (s - k_i^*(CS, OS))P(L = s) + \sum_{s=k_i^*(CS,OS)+1}^{k_i^*(CS,OS)+k} (k_i^*(CS, CS) - k_i^*(CS, OS))P(L = s) + \sum_{s=k_i^*(CS,OS)+k+1}^{k_i^*(CS,OS)+k} (-s + k_i^*(CS, CS) + k)P(L = s)) + (v_{j,2} - v_{j,1} - R_j)(\sum_{s=k_i^*(CS,OS)+k+1}^{k_i^*(CS,OS)+N_j} (s - k_i^*(CS, OS) - k)P(L = s) + (N_j - k)P(L > N_j + k_i^*(CS, OS))) \geq 0$ . In particular, when  $k = k_j^*(CS, CS)$ , equality holds if and only if  $k_i^*(CS, OS) = k_i^*(CS, CS)$  and  $P(L \geq k_i^*(CS, CS) + k_j^*(CS, CS)) = 0$  or  $k_i^*(CS, OS) = k_i^*(CS, CS)$  and  $k_j^*(CS, CS) = N_j$ . However,  $P(L \geq k_i^*(CS, CS) + k_j^*(CS, CS)) > 0$  by the definition of  $k_i^*(CS, CS), k_j^*(CS, CS)$ .  $\Pi_j^{CS,OS}(k_i^*(CS, OS), k_j^*(CS, CS)) = \Pi_j^{CS,CS}(k_i^*(CS, CS), k_j^*(CS, CS))$

if and only if  $k_i^*(CS, OS) = k_i^*(CS, CS)$ ,  $k_j^*(CS, CS) = N_j$ .

**Lemma A4.**  $\Pi_j^{CS,OS}(k_i(CS, OS), k_j(CS, OS)) \geq \Pi_j^{CS,CS}(k_i(CS, CS), k_j(CS, CS))$  and equality holds if and only if  $k_j^*(CS, OS) = k_j^*(CS, CS) = N_j$  and  $k_i^*(CS, OS) = k_i^*(CS, CS)$ .

Proof: From Lemma A3(i) and (ii), we have  $\Pi_j^{CS,OS}(k_i^*(CS, OS), k_j^*(CS, OS)) \geq \Pi_j^{CS,OS}(k_i^*(CS, OS), k_j^*(CS, CS)) \geq \Pi_j^{CS,CS}(k_i^*(CS, CS), k_j^*(CS, CS))$  where equality holds if and only if  $k_j^*(CS, OS) = k_j^*(CS, CS) = N_j$ ,  $k_i^*(CS, OS) = k_i^*(CS, CS)$ .

The result that overselling is a weakly dominant strategy follows directly from Lemmas A2 and A4. As an immediate consequence, the outcome where both firms adopt the overselling strategy is a subgame perfect equilibrium.  $\square$

**Proof of Proposition 4.1.** For ease of exposition, we write  $k_i^*(CS, CS)$  as  $k_i^C$  and  $k_i^*(OS, OS)$  as  $k_i^O$ . When both firms adopt the conventional selling strategies, the  $(N_i - k_i^C)$  consumers who buy in Period 1 at  $v_{i,1}$  does not enjoy any consumer surplus ( $i = A, B$ ). The remaining  $k_i^C$  potential consumers (they are termed potential consumers because of their uncertain arrival in Period 2) may enjoy some surplus, depending on the expected price paid. Essentially, the total consumer surplus when both firm adopt the conventional selling strategy is the expected surplus of the  $k_i^C$  consumers. On the other hand, when both firms adopt the overselling strategy, all consumers, regardless of whether they buy in Period 1 or in Period 2 may gain some surplus - the latter depending on the expected price in Period 2 and the former depending on whether the good they have purchased in Period 1 has been resold and they have been paid a positive compensation  $R_i$ . Thus, we will proceed to compare the consumer surplus for the  $k_i^C$  consumers (who potentially make a purchase in Period 2) in the case when both firms adopt the conventional selling strategy with the consumer surplus of  $(k_i^O + (k_i^C - k_i^O))$  consumers (the  $k_i^O$  consumers potentially buy in Period 2 while the remaining  $(k_i^C - k_i^O)$  may potentially receive a refund )when both firms adopt the overselling strategy. Let  $\Delta_{CS,CS}$  ( $\Delta_{OS,OS}$ ) denote the expected

revenue of Firm  $i$  from the  $k_i^C$  units described above when both firms adopt the conventional selling (overselling) strategy. Then  $\Delta_{CS,CS} = v_{i,2}(\sum_{s=k_j^C+1}^{k_j^C+k_i^C} (s-k_j^C)P(L=s) + k_i^C P(L > k_i^C + k_j^C))$ ,  $\Delta_{OS,OS} = v_{i,2}(\sum_{s=N_j+1}^{N_j+k_i^O} (s-N_j)P(L=s) + k_i^O P(L > N_j + k_i^O) + \sum_{s=N_j+k_i^O+1}^{N_j+k_i^C} (s-N_j-k_i^O)P(L=s))$ . We shall first show that  $\Delta_{CS,CS} \geq \Delta_{OS,OS}$ . Since  $k_j^C + 1 \leq k_j^C + k_i^C \leq N_j + k_i^C$ , we need only consider the following three cases in our analysis as  $k_i^O \leq k_i^C$  (Proposition 3.1).

**Case 1:**  $k_j^C + 1 \leq k_j^C + k_i^C \leq N_j \leq N_j + k_i^O \leq N_j + k_i^C$ . Upon simplification,  $\Delta_{CS,CS} - \Delta_{OS,OS} \geq v_{i,2}(k_i^C P(L > k_j^C + k_i^C) - \sum_{s=N_j+1}^{N_j+k_i^O} (s-N_j)P(L=s) - k_i^O P(L > N_j + k_i^O) - \sum_{s=N_j+k_i^O+1}^{N_j+k_i^C} (s-N_j-k_i^O)P(L=s)) \geq v_{i,2}(\sum_{s=N_j+1}^{N_j+k_i^O} (k_i^C + N_j - s)P(L=s) + (k_i^C - k_i^O)P(L > N_j + k_i^C) + \sum_{s=N_j+k_i^O+1}^{N_j+k_i^C} (k_i^C - k_i^O - s + N_j + k_i^O)P(L=s)) \geq 0$ .

**Case 2:**  $k_j^C + 1 \leq N_j \leq N_j + k_i^O \leq k_j^C + k_i^C \leq N_j + k_i^C$ .  $\Delta_{CS,CS} - \Delta_{OS,OS} = v_{i,2}(\sum_{s=k_j^C+1}^{N_j} (s-k_j^C)P(L=s) + \sum_{s=N_j+1}^{N_j+k_i^O} (s-k_j^C-s+N_j)P(L=s) + \sum_{s=N_j+k_i^O+1}^{k_j^C+k_i^C} (s-k_j^C-s+N_j+k_i^O-k_i^O)P(L=s) + \sum_{s=k_j^C+k_i^C}^{N_j+k_i^C} (k_i^C-k_i^O-s+N_j+k_i^O)P(L=s) + (k_i^C-k_i^O)P(L > N_j + k_i^O)) \geq 0$ .

**Case 3:**  $k_j^C + 1 \leq N_j \leq k_j^C + k_i^C \leq N_j + k_i^O \leq N_j + k_i^C$ .  $\Delta_{CS,CS} - \Delta_{OS,OS} = v_{i,2}(\sum_{s=k_j^C+1}^{N_j} (s-k_j^C)P(L=s) + \sum_{s=N_j+1}^{k_j^C+k_i^C} (s-k_j^C-s+N_j)P(L=s) + \sum_{s=k_j^C+k_i^C}^{N_j+k_i^O} (k_j^C-s+N_j)P(L=s) + \sum_{s=N_j+k_i^O+1}^{N_j+k_i^C} (k_i^C-k_i^O-s+N_j+k_i^O)P(L=s) + (k_i^C-k_i^O)P(L > N_j + k_i^O)) \geq 0$ .

Since the total value created from these  $k_i^C$  units is the same at  $v_{i,2} \sum_{s=1}^{k_i^C} sP(L=s)$  regardless of the selling strategies of the firms and we have shown above that the expected revenue of Firm  $i$  for these  $k_i^C$  units is higher when both firms adopt the conventional selling strategy than when both firms adopt the overselling strategy, we can conclude that the consumer surplus must be higher in the latter case for all  $N_i$  units when both firms adopt the overselling strategy since in addition to the  $k_i^C$  consumers having a higher consumer surplus, even the remaining  $(N_i - k_i^C)$  consumers may have a positive consumer surplus.  $\square$

**Proof of Proposition 4.2.** Essentially, we compare the expected payoffs when

both firms adopt the conventional selling strategy and when both firms adopt the overselling strategy. For simplicity, we write  $k_i^*(CS, CS)$  as  $k_i^C$  and  $k_i^*(OS, OS)$  as  $k_i^O$ . We need to consider three cases, namely, when  $k_j^C + k_i^C \leq N_j (\leq N_j + k_i^O)$ , when  $N_j < k_j^C + k_i^C \leq N_j + k_i^O$  and when  $N_j \leq N_j + k_i^O \leq k_j^C + k_i^C$ .

**Case 1:**  $k_j^C + k_i^C \leq N_j (\leq N_j + k_i^O)$ . We define  $\hat{\Pi}_i^{OS, OS}$  as the expected payoff obtained by Firm  $i$  from selling in Period 1 and selling the reserved units in Period 2, i.e.,  $\hat{\Pi}_i^{OS, OS} = v_{i,1}(N_i - k_i^O) + v_{i,2}(\sum_{s=N_j+1}^{N_j+k_i^O-1} (s - N_j)P(L = s) + k_i^O P(L \geq N_j + k_i^O))$ . Then,  $\hat{\Pi}_i^{OS, OS} - \Pi_i^{CS, CS}$  simplifies to  $\hat{\Pi}_i^{OS, OS} - \Pi_i^{CS, CS} = (k_i^C - k_i^O)(v_{i,1} - v_{i,2}P(L \geq N_j + k_i^O)) + v_{i,2}(\sum_{s=k_j^C+1}^{k_j^C+k_i^C} (-s + k_j^C)P(L = s) - k_i^C P(k_i^C + k_j^C < L \leq N_j) + \sum_{s=N_j+1}^{N_j+k_i^O-1} (k_j^C - N_j)P(L = s)) \leq (k_i^C - k_i^O)(v_{i,1} - v_{i,2}\frac{v_{i,1}}{v_{i,1}+R_i}) + v_{i,2}(\sum_{s=k_j^C+1}^{k_j^C+k_i^C} (-s + k_j^C)P(L = s) - k_i^C P(k_i^C + k_j^C < L \leq N_j) + \sum_{s=N_j+1}^{N_j+k_i^O-1} (k_j^C - N_j)P(L = s)) \leq 0$ . Thus, we have Proposition 4.2. The proofs for the other two cases are similar so we omit them here.  $\square$

**Proof of Theorem 4.2.** We shall first compare the payoffs  $\Pi_i^{CS, CS}$  and  $\Pi_i^{OS, OS}$ . Suppose  $N_j + k_i(OS, OS) \geq k_j(CS, CS) + k_i(CS, CS)$ . Upon simplification,  $\Pi_i^{OS, OS} - \Pi_i^{CS, CS} = v_{i,1}(k_i(CS, CS) - k_i(OS, OS)) + v_{i,2}(\sum_{s=k_j(CS, CS)+1}^{N_j} (-s + k_j(CS, CS))P(L = s) + \sum_{s=N_j+1}^{k_j(CS, CS)+k_i(CS, CS)} (-N_j + k_j(CS, CS))P(L = s) + \sum_{s=k_j(CS, CS)+k_i(CS, CS)+1}^{N_j+k_i(OS, OS)-1} (s - N_j - k_i(CS, CS))P(L = s) + (k_i(OS, OS) - k_i(CS, CS))P(L \geq N_j + k_i(OS, OS))) + (v_{i,2} - v_{i,1} - R_i)(\sum_{s=N_j+k_i(OS, OS)+1}^{N_j+N_i} (s - N_j - k_i(OS, OS))P(L = s) + (N_i - k_i(OS, OS))P(L > N_j + N_i))$ . Note that the sum of the first two terms are negative since  $P(L \geq k_i(OS, OS)) > \frac{v_{i,1}}{v_{i,2}}$ . Hence,  $\Pi_i^{OS, OS} > \Pi_i^{CS, CS}$  if and only if  $(v_{i,2} - v_{i,1} - R_i)(\sum_{s=N_j+k_i(OS, OS)+1}^{N_j+N_i} (s - N_j - k_i(OS, OS))P(L = s) + (N_i - k_i(OS, OS))P(L > N_j + N_i)) > -v_{i,1}(k_i(CS, CS) - k_i(OS, OS)) - v_{i,2}(\sum_{s=k_j(CS, CS)+1}^{N_j} (-s + k_j(CS, CS))P(L = s) + \sum_{s=N_j+1}^{k_j(CS, CS)+k_i(CS, CS)} (-N_j + k_j(CS, CS))P(L = s) + \sum_{s=k_j(CS, CS)+k_i(CS, CS)+1}^{N_j+k_i(OS, OS)-1} (s - N_j - k_i(CS, CS))P(L = s) + (k_i(OS, OS) - k_i(CS, CS))P(L \geq N_j + k_i(OS, OS)))$ . Also, we note that since  $\Pi_i^{CS, OS} \leq \Pi_i^{CS, CS}$ , the outcome where both firms oversell is Pareto dominant if and only if the condition earlier is satisfied. The proof for the case where  $N_j + k_i(OS, OS) < k_j(CS, CS) + k_i(CS, CS)$  is similar so we omit it here.

□

**Proof of Proposition 4.3.** The outcome where Firm  $i$  adopts the overselling strategy while Firm  $j$  adopts the conventional selling strategy is a subgame perfect equilibrium if and only if given the overselling strategy of Firm  $i$ , Firm  $j$  is indifferent between adopting the overselling and the conventional selling strategy. From Lemma A2, this means that either (i)  $k_j^*(OS, CS) = k_j^*(OS, OS) = N_j$ , which implies that  $P(L \geq N_i + N_j) \geq \frac{v_{j,1}}{v_{j,1} + R_j}$ , or (ii)  $k_j^*(OS, CS) = k_j^*(OS, OS) = 0$ ,  $P(L > N_i) = 0$ . □

**Proof of Proposition 4.4.** For the outcome where both firms adopt the conventional selling strategy to be a subgame perfect equilibrium, we require that given the conventional selling strategy of one firm, the other firm is indifferent between overselling or conventional selling. More specifically, Lemma A4 implies that  $k_j^*(CS, CS) = k_j^*(CS, OS) = N_j$  for  $j = A, B$ . From the definitions of  $k_j^*(CS, CS)$  and  $k_j^*(CS, OS)$ ,  $k_j^*(CS, CS) = k_j^*(CS, OS) = N_j$  is equivalent to the condition where  $P(L \geq N_i + N_j) \geq \max(\frac{v_{j,1}}{v_{j,1} + R_j}, \frac{v_{i,1}}{v_{i,1} + R_i})$ . □

**Proof of Proposition 4.5.** The proof follows immediately from Propositions 4.3 and 4.4. □

# Online Appendix

## Proof of Lemma 3.1.

Suppose  $(\lambda_A, \lambda_B) = (CS, CS)$ . If Firm  $i$  has a lower relative price, i.e.,  $p_{i,2} < p_{j,2} + (v_{i,2} - v_{j,2})$ ,  $i, j = A, B, i \neq j$ , its expected payoff is  $\Pi_i^{CS,CS} = \Pi'_i(p_{i,2}, p_{j,2}) = v_{i,1}(N_i - k_i) + p_{i,2}(\sum_{s=1}^{k_i} sP(L = s) + k_iP(L > k_i))$  while that of Firm  $j$  (with a higher relative price) is  $\Pi_j^{CS,CS} = \Pi''_j(p_{i,2}, p_{j,2}) = v_{j,1}(N_j - k_j) + p_{j,2}(\sum_{s=k_i+1}^{k_i+k_j} (s - k_i)P(L = s) + k_jP(L > k_i + k_j))$ . We shall first establish that the pricing strategy of the firms must necessarily be a mixed strategy at the equilibrium. If Firm  $j$  adopts a pure strategy and sets its price at  $p_{j,2}$  such that  $p_{j,2} > p_{i,2} + (v_{j,2} - v_{i,2})$ , then  $p_{j,2} = v_{j,2}$  strictly dominates all other prices  $p_{j,2} \in (p_{i,2} + (v_{j,2} - v_{i,2}), v_{j,2}]$  since the market share remains unchanged but the profit margin is higher. However, Firm  $i$ 's best response to  $p_{j,2} = v_{j,2}$  must thus be  $p_{i,2} = v_{i,2} - \epsilon$ , where  $\epsilon$  is a small positive number, since again the market share remains unchanged but the profit margin is at its highest. Given that  $p_{i,2} = v_{i,2} - \epsilon$ , should Firm  $j$  price at  $p_{j,2} = v_{j,2} - \epsilon$ , maintain the price at  $p_{j,2} = v_{j,2}$ , or undercut Firm  $i$  a little by setting  $p_{j,2} = v_{j,2} - 2\epsilon$ ? It is quite clear that Firm  $j$  will always be better off by undercutting its price by  $\epsilon$ ; the  $\epsilon$  loss in profit margin is more than compensated for by the increase in market share. Thus, the best response for Firm  $j$  is to set  $p_{j,2} = v_{j,2} - 2\epsilon$ . Hence, Firm  $i$  will also respond by undercutting its price until one firm gets zero profit, at which point, the firm sets the price at its valuation, as we have argued at the beginning of the proof. As a result, any subgame perfect equilibrium must necessarily be a mixed strategy.

Next, we will proceed to characterize the mixed strategy Nash equilibrium,  $\sigma_A, \sigma_B$ , where  $\sigma_i(b)$  denotes the probability that Firm  $i$  chooses a price  $p_{i,2}$  less than or equal to  $b$ ,  $b \in [i_0, v_{i,2}]$ ,  $i = A, B$ <sup>1</sup>. Consider Firm  $i$ . The expected payoff for Firm  $i$  is  $\Pi_i^{CS,CS}(b, p_{j,2}) = Prob(p_{j,2} < b + (v_{j,2} - v_{i,2}))\Pi''_i(b, p_{j,2}) + Prob(p_{j,2} > b + (v_{j,2} - v_{i,2}))\Pi'_i(b, p_{j,2})$ , which simplifies to

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<sup>1</sup>For technical properties of such a mixed strategy equilibrium, please refer to Narasimhan (1988).

$$\Pi_i^{CS,CS}(b, p_{j,2}) = \Pi'_i(b, p_{j,2}) + Prob(p_{j,2} < a)(\Pi''_i(b, p_{j,2}) - \Pi'_i(b, p_{j,2})), \quad (1)$$

where  $a = b + (v_{j,2} - v_{i,2})$ . In particular, when  $p_{i,2} = b = v_{i,2}$  (and thus  $a = v_{j,2}$ ),

$$\Pi_i^{CS,CS}(v_{i,2}, v_{j,2}) = \Pi''_i(v_{i,2}, v_{j,2}). \quad (2)$$

Since Firm  $i$  must be indifferent between  $p_{i,2} = v_{i,2}$  and  $p_{i,2} = b$  for all  $b \in [i_0, v_{i,2}]$  at the equilibrium, by equating (1) and (2), we obtain

$$Prob(p_{j,2} < a) = \frac{\Pi'_i(b, p_{j,2}) - \Pi''_i(v_{i,2}, v_{j,2})}{\Pi''_i(b, p_{j,2}) - \Pi'_i(b, p_{j,2})}. \quad (3)$$

Put  $Prob(p_{j,2} < j_0) = 0$ , we deduce that  $i_0$  is such that

$$\Pi'_i(i_0, p_{j,2}) = \Pi''_i(v_{i,2}, v_{j,2}). \quad (4)$$

Thus,  $\sigma_i$ ,  $i = A, B$  is characterized by (3) and (4) and the expected payoff for Firm  $i$  at the equilibrium is given by  $\Pi_i^{CS,CS}(v_{i,2}, v_{j,2}) = \Pi''_i(v_{i,2}, v_{j,2})$ , which we re-write as (5) below:

$$\Pi_i^{CS,CS} = v_{i,1}(N_i - k_i) + v_{i,2}\left(\sum_{s=k_j+1}^{k_j+k_i} (s - k_j)P(L = s) + k_i P(L > k_j + k_i)\right). \quad (5)$$

Having obtained the expected profits after analyzing the price competition in Period 2, we can now determine  $k_i$ , the number of units to be reserved for sale in Period 2. From (5), let  $\Pi_i(k, k_j)$  denote the expected payoff of Firm  $i$  when the number of units reserved by Firm  $i$  is  $k$  and the number of units reserved by Firm  $j$  is  $k_j$  (we omit the notation (CS,CS) here). Upon simplification,  $\Pi_i(k, k_j) - \Pi_i(k-1, k_j) = -v_{i,1} + v_{i,2}P(L \geq k + k_j) > 0$  if and only if  $P(L \geq k + k_j) > \frac{v_{i,1}}{v_{i,2}}$ . Hence, we deduce that for Firm  $i$ , the optimum number of units to reserve is  $k_i^*(CS, CS)$ , where  $k_i^*(CS, CS)$  is the largest integer no more than  $N_i$  such that  $P(L \geq k_i^*(CS, CS) +$



$k_j^*(CS, CS) > \frac{v_{i,1}}{v_{i,2}}$ ,  $i, j = A, B, j \neq i$ <sup>2</sup>. In particular, if  $\frac{v_{i,1}}{v_{i,2}} = \frac{v_{j,1}}{v_{j,2}}$ ,  $k_i^*(CS, CS) = \min(\max(0, \kappa_i - k_j^*(CS, CS)), N_i)$ ,  $k_j^*(CS, CS) = \min(\max(0, \kappa_j - k_i^*(CS, CS)), N_j)$  such that  $k_i^*(CS, CS) + k_j^*(CS, CS) = \kappa_i (= \kappa_j)$ . If however,  $\frac{v_{i,1}}{v_{i,2}} < \frac{v_{j,1}}{v_{j,2}}$ , then  $\kappa_i > \kappa_j$ . Our earlier analysis implies that  $k_i^*(CS, CS) = \min(\max(0, \kappa_i - k_j^*(CS, CS)), N_i)$  given any  $k_j^*(CS, CS)$  while  $k_j^*(CS, CS) = \min(\kappa_j - k_i^*(CS, CS), N_j)$  given any  $k_i^*(CS, CS)$ . Since  $\kappa_i > \kappa_j$ ,  $k_i^*(CS, CS) = \min(\kappa_i, N_i)$ ,  $k_j^*(CS, CS) = \min(\max(0, \kappa_j - k_i^*(CS, CS)), N_j)$ .  $\square$

### Proof of Lemma 3.2.

Suppose  $(\lambda_A, \lambda_B) = (OS, OS)$ . For  $i, j = A, B, i \neq j$ , if  $p_{i,2} < p_{j,2} + (v_{i,2} - v_{j,2})$ , the expected payoffs for Firms  $i$  and  $j$  are given by  $\Pi_i^{OS,OS} = \tilde{\Pi}'_i(p_{i,2}, p_{j,2}) = v_{i,1}(N_i - k_i) + p_{i,2}(\sum_{s=1}^{k_i} sP(L = s) + k_iP(L > k_i)) + (p_{i,2} - v_{i,1} - R_i)(\sum_{s=k_i+1}^{N_i} (s - k_i)P(L = s) + (N_i - k_i)P(L > N_i))$ ,  $\Pi_j^{OS,OS} = \tilde{\Pi}'_j(p_{i,2}, p_{j,2}) = v_{j,1}(N_j - k_j) + p_{j,2}(\sum_{s=N_i+1}^{N_i+k_j} (s - N_i)P(L = s) + k_jP(L > N_i + k_j)) + (p_{j,2} - v_{j,1} - R_j)(\sum_{s=N_i+k_j+1}^{N_i+N_j} (s - N_i - k_j)P(L = s) + (N_j - k_j)P(L > N_i + N_j))$ . Using a similar argument as that in the proof of Lemma 3.1, we can deduce that the pricing competition in Period 2 must result in a mixed strategy at the equilibrium (we omit the details here but the detailed analysis is available from the authors upon request). Let  $\phi_A, \phi_B$  denote the equilibrium pricing strategy at the equilibrium where  $\phi_i(b)$  denote the probability that Firm  $i$  chooses a price  $p_{i,2}$  less than or equal to  $b$ ,  $b \in [\tilde{i}_0, v_{i,2}]$ . We proceed to specify  $\phi_i$  and  $\tilde{i}_0$ . Consider Firm  $i$ . For any  $b \in [\tilde{i}_0, v_{i,2}]$ , the expected payoff for Firm  $i$  can be written as  $\Pi_i^{OS,OS}(b, p_{j,2}) = Prob(p_{j,2} < b + (v_{j,2} - v_{i,2}))\tilde{\Pi}''_i(p_{i,2}, p_{j,2}) + Prob(p_{j,2} > b + (v_{j,2} - v_{i,2}))\tilde{\Pi}'_i(p_{i,2}, p_{j,2})$ , which simplifies to

$$\Pi_i^{OS,OS}(b, p_{j,2}) = \tilde{\Pi}'_i(b, p_{j,2}) + Prob(p_{j,2} < a)(\tilde{\Pi}''_i(b, p_{j,2}) - \tilde{\Pi}'_i(b, p_{j,2})), \quad (6)$$

where  $a = b + (v_{j,2} - v_{i,2})$ . In particular, when  $p_{i,2} = b = v_{i,2}$ , the expected payoff of

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<sup>2</sup>This result is analogous to the seat allocation problem for a monopoly considered in Littlewood (1972) where  $k_i^*(CS, CS) + k_j^*(CS, CS)$  is the total capacity reserved for Period 2.

Firm  $i$  is

$$\Pi_i^{OS,OS}(v_{i,2}, v_{j,2}) = \tilde{\Pi}_i''(v_{i,2}, v_{j,2}). \quad (7)$$

Since Firm  $i$  must be indifferent between  $p_{i,2} = v_{i,2}$  and  $p_{i,2} = b$  for all  $b \in [\tilde{i}_0, v_{i,2}]$  at the equilibrium, by equating (6) and (7) gives the specification for  $\phi_j(a)$  as

$$Prob(p_{j,2} < a) = \frac{\tilde{\Pi}_i'(b, p_{j,2}) - \tilde{\Pi}_i''(v_{i,2}, v_{j,2})}{\tilde{\Pi}_i''(b, p_{j,2}) - \tilde{\Pi}_i'(b, p_{j,2})}. \quad (8)$$

Putting  $Prob(p_j, 2 < \tilde{j}_0) = 0$ , we conclude that  $\tilde{i}_0$  solves

$$\tilde{\Pi}_i'(\tilde{i}_0, p_{j,2}) = \tilde{\Pi}_i''(v_{i,2}, v_{j,2}). \quad (9)$$

(8) and (9) together specify the pricing strategy at the equilibrium. The expected payoff for Firm  $i$  at the equilibrium is given by  $\Pi_i^{OS,OS}(v_{i,2}, v_{j,2}) = \tilde{\Pi}_i''(v_{i,2}, v_{j,2})$ , or specifically

$$\begin{aligned} \Pi_i^{OS,OS} &= v_{i,1}(N_i - k_i) + v_{i,2} \left( \sum_{s=N_j+1}^{N_j+k_i} (s - N_j)P(L = s) + k_i P(L > N_j + k_i) \right) + \\ &\quad (v_{i,2} - v_{i,1} - R_i) \left( \sum_{s=N_j+k_i+1}^{N_j+N_i} (s - N_j - k_i)P(L = s) + (N_i - k_i)P(L > N_i + N_j) \right). \end{aligned} \quad (10)$$

By backward induction, Firm  $i$  chooses  $k_i(OS, OS)$  in Period 1 to maximize its expected payoff  $\Pi_i(OS, OS)$  given in (10). Given  $k_j$ , let  $\Pi_i(k, k_j)$  denote the expected payoff of Firm  $i$  if it reserves  $k$  units for Period 2. Here we omit the notation  $(OS, OS)$ . After some algebraic manipulation,  $\Pi_i(k, k_j) - \Pi_i(k - 1, k_j)$  simplifies to  $-v_{i,1} + (v_{i,1} + R_i)P(L \geq N_j + k)$ , which is greater than zero if and only if  $P(L \geq N_j + k) > \frac{v_{i,1}}{v_{i,1} + R_i}$ .  $\square$

### Proof of Lemma 3.3.

Suppose  $(\lambda_A, \lambda_B) = (CS, OS)$ . As the firms use different selling strategies, we need to solve for the equilibrium pricing strategies separately. If  $p_{A,2} < p_{B,2} + (v_{A,2} - v_{B,2})$ ,

$\Pi_A^{CS,OS} = \bar{\Pi}'_A(p_{A,2}, p_{B,2}) = v_{A,1}(N_A - k_A) + p_{A,2}(\sum_{s=1}^{k_A} sP(L = s) + k_A P(L > k_A)),$   
 $\Pi_B(CS, OS) = \bar{\Pi}'_B(p_{A,2}, p_{B,2}) = v_{B,1}(N_B - k_B) + p_{B,2}(\sum_{s=k_A+1}^{k_A+k_B} (s - k_A)P(L = s) +$   
 $k_B P(L > k_A + k_B)) + (p_{B,2} - v_{B,1} - R_B)(\sum_{s=k_A+k_B+1}^{k_A+N_B} (s - k_A - k_B)P(L = s) + (N_B -$   
 $k_B)P(L > N_B + k_A)).$  On the other hand, if  $p_{A,2} > p_{B,2} + (v_{A,2} - v_{B,2}), \Pi_A^{CS,OS} =$   
 $\bar{\Pi}''_A(p_{A,2}, p_{B,2}) = v_{A,1}(N_A - k_A) + p_{A,2}(\sum_{s=N_B+1}^{N_B+k_A} (s - N_B)P(L = s) + k_A P(L > N_B + k_A)),$   
 $\Pi_B(CS, OS) = \bar{\Pi}''_B(p_{A,2}, p_{B,2}) = v_{B,1}(N_B - k_B) + p_{B,2}(\sum_{s=1}^{k_B} sP(L = s) + k_B P(L >$   
 $k_B)) + (p_{B,2} - v_{B,1} - R_B)(\sum_{s=k_B+1}^{N_B} (s - k_B)P(L = s) + (N_B - k_B)P(L > N_B)).$  Let  
 $\psi_A, \psi_B$  denote the mixed strategy equilibrium of Firms  $A$  and  $B$  where  $\psi_A(a)$  denote  
the probability that Firm  $A$  chooses a price  $p_{A,2}$  less than or equals to  $a$ ,  $\psi_B(b)$   
denote the probability that Firm  $B$  chooses a price  $p_{B,2}$  less than or equals to  $b$ ,  
 $a \in [\bar{a}_0, v_{A,2}], b \in [\bar{b}_0, v_{B,2}].$  Consider Firm  $B$ . For  $p_{B,2} = b, b \in [\bar{b}_0, v_{B,2}],$  the expected  
payoff of Firm  $B$  is give by  $\Pi_B^{CS,OS} = Prob(p_{A,2} < b + (v_{A,2} - v_{B,2}))\bar{\Pi}'_B(p_{A,2}, b) +$   
 $Prob(p_{A,2} > b + (v_{A,2} - v_{B,2}))\bar{\Pi}''_B(p_{A,2}, b),$  which simplifies to

$$\Pi_B^{CS,OS} = \bar{\Pi}''_B(p_{A,2}, b) + Prob(p_{A,2} < a)(\bar{\Pi}'_B(p_{A,2}, b) - \bar{\Pi}''_B(p_{A,2}, b)), \quad (11)$$

$a = b + (v_{A,2} - v_{B,2}).$  When  $p_{B,2} = b = v_{B,2},$  the expected payoff of Firm  $B$  is

$$\Pi_B^{CS,OS}(v_{A,2}, v_{B,2}) = \bar{\Pi}'_B(v_{A,2}, v_{B,2}). \quad (12)$$

Equating (11) and (12), we obtain

$$\psi_A(a) = Prob(p_{A,2} < a) = \frac{\bar{\Pi}'_B(v_{A,2}, v_{B,2}) - \bar{\Pi}''_B(p_{A,2}, b)}{\bar{\Pi}'_B(p_{A,2}, b) - \bar{\Pi}''_B(p_{A,2}, b)}. \quad (13)$$

By equating  $Prob(p_{A,2} < \bar{a}_0) = 0,$  we find that  $\bar{b}_0$  is such that

$$\bar{\Pi}'_B(v_{A,2}, v_{B,2}) = \bar{\Pi}''_B(p_{A,2}, \bar{b}_0). \quad (14)$$

Using a similar argument, we deduce that  $\Pi_A^{CS,OS}(p_{A,2}, p_{B,2}) = Prob(p_{B,2} < a + (v_{B,2} -$   
 $v_{A,2}))\bar{\Pi}''_A(p_{A,2}, p_{B,2}) + Prob(p_{B,2} > a + (v_{B,2} - v_{A,2}))\bar{\Pi}'_A(p_{A,2}, p_{B,2})$  can be written as

$$\Pi_A^{CS,OS} = \bar{\Pi}'_A(a, p_{B,2}) + Prob(p_{B,2} < b)(\bar{\Pi}''_A(a, p_{B,2}) - \bar{\Pi}'_A(a, p_{B,2})), \quad (15)$$

where  $b = a + (v_{B,2} - v_{A,2})$ . When  $p_{B,2} = b = v_{B,2}$ ,

$$\Pi_A^{CS,OS}(v_{A,2}, v_{B,2}) = \bar{\Pi}_A(v_{A,2}, v_{B,2}). \quad (16)$$

As before, we equate (15) and (16) to obtain

$$\psi_B(b) = Prob(p_{B,2} < b) = \frac{\bar{\Pi}_A''(v_{A,2}, v_{B,2}) - \bar{\Pi}_A'(a, p_{B,2})}{\bar{\Pi}_A''(a, p_{B,2}) - \bar{\Pi}_A'(a, p_{B,2})}. \quad (17)$$

Finally,  $Prob(p_{B,2} < \bar{b}_0) = 0$  implies that  $\bar{a}_0$  solves

$$\bar{\Pi}_A''(v_{A,2}, v_{B,2}) = \bar{\Pi}_A'(\bar{a}_0, p_{B,2}). \quad (18)$$

Thus, the equilibrium pricing strategies for the firms are characterized by (13), (14), (17) and (18).

The expected payoffs for Firms  $A$  and  $B$  are given by

$$\begin{aligned} \Pi_A^{CS,OS} &= v_{A,1}(N_A - k_A) + v_{A,2} \left( \sum_{s=N_B+1}^{N_B+k_A} (s - N_B)P(L = s) + k_A P(L > N_B + k_A) \right), \\ \Pi_B^{CS,OS} &= v_{B,1}(N_B - k_B) + v_{B,2} \left( \sum_{s=k_A+1}^{k_A+k_B} (s - k_A)P(L = s) + k_B P(L > k_A + k_B) \right) + \\ &\quad (v_{B,2} - v_{B,1} - R_B) \left( \sum_{s=k_A+k_B+1}^{k_A+N_B} (s - k_A - k_B)P(L = s) + \right. \\ &\quad \left. (N_B - k_B)P(L > N_B + k_A) \right). \end{aligned}$$

To determine the number of units reserved by Firms  $A$  and  $B$  at the equilibrium, let  $\Pi_A(k, k_B)$  be the expected payoff of Firm  $A$  if it reserves  $k$  units, given that Firm  $B$  reserves  $k_B$  units. We omit the notation  $(CS, OS)$  here. Then  $\Pi_A(k, k_B) - \Pi_A(k - 1, k_B)$  simplifies to  $-v_{A,1} + v_{A,2}P(L \geq N_B + k)$ , which is greater than zero if and only if  $P(L \geq N_B + k) > \frac{v_{A,1}}{v_{A,2}}$ . Hence,  $k_A^*(CS, OS)$  is the largest integer such that

$$P(L \geq N_B + k_A^*(CS, OS)) > \frac{v_{A,1}}{v_{A,2}}. \quad (19)$$

Similarly, let  $\Pi_B(k_A, k)$  denote the expected payoff of Firm  $B$  if it reserves  $k$  units, given that Firm  $A$  reserves  $k_A$  units. Upon simplification,  $\Pi_B(k_A, k) - \Pi_B(k_A, k-1) = -v_{b,1} + (v_{B,1} + R_B)P(L \geq k_A + k) > 0$  if and only if

$$P(L \geq k_A + k) > \frac{v_{B,1}}{v_{B,1} + R_B}. \quad (20)$$

Combining (19), (20), we deduce that at the equilibrium,  $k_A^*(CS, OS), k_B^*(CS, OS)$  are such that  $P(L \geq N_B + k_A^*(CS, OS)) > \frac{v_{A,1}}{v_{A,2}}$ ,  $P(L \geq k_A^*(CS, OS) + k_B^*(CS, OS)) > \frac{v_{B,1}}{v_{B,1} + R_B}$ . More specifically,  $k_A^*(CS, OS) = \min(\max(0, \kappa_A - N_B), N_A)$ ,  $k_B^*(CS, OS) = \min(\max(\kappa_B^R - k_A^*(CS, OS), 0), N_B)$ .  $\square$

## More Numerical Results.

Table A1: Numerical Results ( $\frac{v_{i,1}}{v_{i,2}} = 0.333$ )<sup>3</sup>

$\frac{v_{i,1}}{v_{i,2}}$	$\frac{v_{i,1}}{v_{i,1}+R_i}$	$N_i$	$ 1 - \frac{N_i+N_j}{\bar{s}} $	$(\rho^D, \rho^U)$
0.333	0.5	10	0.5	$(-0.099, 0.000)^*$
			0.25	$(-0.068, 0.042)^*$
			0.1	$(-0.085, 0.020)^*$
		20	0.5	$(-0.134, 0.000)$
			0.25	$(-0.105, 0.034)^*$
			0.1	$(-0.089, -0.017)^*$
0.333	0.667	10	0.5	$(-0.008, 0.046)^*$
			0.25	$(-0.017, 0.171)$
			0.1	$(-0.012, 0.142)$
		20	0.5	$(-0.118, 0.045)$
			0.25	$(-0.058, 0.160)$
			0.1	$(-0.019, 0.099)$
0.333	0.833	10	0.5	$(-0.069, 0.136)$
			0.25	$(0.014, 0.262)$
			0.1	$(0.032, 0.217)$
		20	0.5	$(-0.108, 0.135)$
			0.25	$(-0.030, 0.250)$
			0.1	$(0.024, 0.170)$

<sup>3</sup>If  $1 - \frac{N_i+N_j}{\bar{s}} > 0$ , there is under capacity while if  $1 - \frac{N_i+N_j}{\bar{s}} < 0$ , there is over capacity.  $\rho^D$  ( $\rho^U$ ) denotes the downside (upside) ratio  $\frac{\Pi_i^{OS,OS} - \Pi_i^{CS,CS}}{\Pi_i^{CS,CS}}$  when  $1 - \frac{N_i+N_j}{\bar{s}} < 0$  ( $> 0$ ). In these examples in Tables A1 and A2, we assume that firms are symmetric and  $L$  follows a uniform distribution on  $[0, \bar{s}]$ .

**Table A2: Numerical Results** ( $\frac{v_{i,1}}{v_{i,2}} = 0.4$ )

$\frac{v_{i,1}}{v_{i,2}}$	$\frac{v_{i,1}}{v_{i,1}+R_i}$	$N_i$	$ 1 - \frac{N_i+N_i}{\bar{s}} $	$(\rho^D, \rho^U)$
0.4	0.5	10	0.5	$(-0.101, 0.000)^*$
			0.25	$(-0.086, -0.022)^*$
			0.1	$(-0.055, -0.039)^*$
		20	0.5	$(-0.103, 0.000)^*$
			0.25	$(-0.090, -0.028)^*$
			0.1	$(-0.078, -0.042)$
0.4	0.667	10	0.5	$(-0.082, 0.055)^*$
			0.25	$(-0.033, 0.120)$
			0.1	$(0.027, 0.091)$
		20	0.5	$(-0.086, 0.054)^*$
			0.25	$(-0.040, 0.110)$
			0.1	$(-0.008, 0.087)$
0.4	0.833	10	0.5	$(-0.070, 0.163)$
			0.25	$(-0.001, 0.219)$
			0.1	$(0.076, 0.172)$
		20	0.5	$(-0.076, 0.162)$
			0.25	$(-0.009, 0.211)$
			0.1	$(0.038, 0.166)$