Finite-time synchronization of delayed neural networks with Cohen–Grossberg type based on delayed feedback control

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ABSTRACT

This paper is concerned with finite-time synchronization for a class of delayed neural networks with Cohen–Grossberg type. Different from the existing related results, the time-delayed feedback strategy is utilized to investigate finite-time synchronization of delayed Cohen–Grossberg neural networks. By constructing Lyapunov functions and using differential inequalities, several new and effective criteria are derived to realize local and global synchronization in finite time of the addressed neural networks based on two different time-delayed feedback controllers. Besides, the upper bounds of the settling time of synchronization are estimated. Furthermore, as corollaries, some sufficient conditions are given to achieve finite-time synchronization of delayed cellular neural networks. Finally, some numerical examples are provided to verify the theoretical results established in this paper.

**1. Introduction**

Asymptotic stability of nonlinear systems, introduced by Lyapunov at the end of the 19th [1], has been widely proposed and requires that the system trajectories converge to the desired aim over the infinite time. In many applications, however, it is desirable and more valuable that the convergence of a dynamical system is realized in finite time rather than infinite time, this is so-called finite-time stability. In fact, classical optimal control theory provides several examples of systems that exhibit convergence to the equilibrium in finite time [2]. A well-known example is the double integrator with bang-bang time-optimal feedback control [3]. Moreover, such a property is also useful to design higher order sliding controller [4].

In recent years, finite-time stability and stabilization of nonlinear systems has drawn an increasing attention. Initial work on finite-time stability mainly focused on discontinuous dynamic systems which can deteriorate the system transient performance [5,6]. To extend finite-time stability to general dynamical systems, Bhat and Bernstein [7] investigated finite-time stability of multi-dimensional continuous autonomous systems. Nowadays, finite-time stability has been extended to switched systems [8], non-autonomous dynamical systems [9,10], impulsive dynamical systems [11], stochastic nonlinear systems [12,13]. In contrast to the existing results on finite-time stability of nonlinear systems without delay, there are fewer results for nonlinear time-delay systems. This is because the stability analysis of nonlinear time-delay systems is more complicated than that of systems without delay. As the authors in [14] pointed out, it is really difficult to find a Lyapunov functional to propose finite-time stability of delay differential equations.

On the other hand, as one of the most popular and typical neural network models, Cohen–Grossberg neural network has attracted considerable attention due to its potential applications in classification, parallel computing, signal and image processing since its emergence in 1983 by Cohen and Grossberg [15]. It is well known that time delays especially time-varying delays are unavoidably encountered in the signal transmission among the neurons due to the finite switching speed of neurons and amplifiers, which will affect the stability of the neural system and may lead to some complex dynamic behaviors such as instability, chaos, oscillation or other poor performance of the neural networks. Hence, time delays should be introduced into network modelling and neural networks including time delays are more valuable in theory and practice. Nowadays, a large number of results on delayed Cohen–Grossberg neural networks have been derived [16–26]. One of the hot topics in the investigation of neural networks is chaos synchronization due to its important applications in numerous fields of science and engineering such as combinational optimization [27], image processing [28], cognition [29], biology [30,31]. Besides, it was also used to understand self-organization behavior in the brain as well as in ecological systems. Currently, the investigation of synchronization on Cohen–Grossberg neural networks mainly focus on asymptotic synchronization [21–26], which implies that the response chaotic neural network can track the drive neural network over the infinite horizon.
However, it is desirable in practice and more realistic that the networks can be synchronized in finite time rather than merely asymptotically. Based on the idea, finite-time stochastic stabilization was proposed for BAM neural networks with uncertainties in [32]. Nevertheless, there are few results to be published on finite-time synchronization of delayed Cohen–Grossberg neural networks to the best of our knowledge. Therefore, it is interesting to fill the gap.

Motivated by the above analysis, in this paper, we investigate finite-time synchronization of Cohen–Grossberg neural networks with time-varying delays. The main contribution of this paper lies in the following aspects. Firstly, a differential inequality is introduced, which plays a vital role in the investigation of finite-time synchronization. Secondly, two simple and effective controllers are proposed based on time-delayed feedback strategy. Moreover, several new criteria are derived to realize finite-time synchronization of the addressed Cohen–Grossberg neural networks without writing the complex error dynamical system. As a special case, some sufficient conditions are given for the finite-time synchronization of delayed cellular neural networks. Finally, some numerical examples are provided to verify the theoretical results established in this paper.

The paper is organized as follows. After problem description and preliminaries in Section 2, Section 3 presents two controllers and several criteria for finite-time synchronization of delayed Cohen–Grossberg neural networks. In Section 4, the effectiveness and feasibility of the developed methods are shown by several numerical examples.

2. Problem description and preliminaries

In this paper, we consider a class of Cohen–Grossberg neural networks with variable delays described by

\[
\dot{x_i}(t) = -d_i(x_i(t)) \left[ a_i(x_i(t)) - \sum_{j=1}^{n} b_{ij}f_j(x_j(t)) - \sum_{j=1}^{n} c_{ij}g_j(x_j(t - \tau_j(t))) + l_i(t) \right],
\]

where \(i \in \mathcal{I} = \{1, 2, \ldots, n\}, n \geq 2\), corresponds to the number of units in a neural network, \(x_i(t)\) denotes the state variable of the \(i\)th neuron at time \(t\), \(d_i(\cdot)\) represents an amplification function, \(a_i(\cdot)\) is an appropriately behaved function, \(b_{ij}, c_{ij}\) denote the connection strengths of the \(j\)th neuron on the \(i\)th neuron, respectively, \(f_j(\cdot)\) and \(g_j(\cdot)\) denote the outputs of the \(j\)th neuron at time \(t\) and \(t - \tau_j(t)\), where \(\tau_j(t)\) is the time delay of \(j\)th neuron and corresponds to finite speed of axonal signal transmission at time \(t\), \(l_i(t)\) is the input from outside of the networks.

System (1) is supplemented with initial values given by

\[
x_i(s) = \phi_i(s), \quad s \in [t_0 - \tau, t_0], \quad i \in \mathcal{I},
\]

where \(t_0 \geq 0\), \(\tau = \max_{i \in \mathcal{I}} \{\sup_{s \in \mathbb{R}} \tau_i(s)\}\). \(\phi_i(\cdot) = (\phi_i(1), \ldots, \phi_i(s))\) denotes the Banach space of all continuous functions mapping \([t_0 - \tau, t_0]\) into \(\mathbb{R}^n\) with norm defined by

\[
\|\phi\| = \sup_{s \in [t_0 - \tau, t_0]} \max_{i \in \mathcal{I}} |\phi_i(s)|.
\]

Throughout this paper, for system (1), we always assume that \((H_1)\) amplification functions \(d_i(\cdot)\) are continuous and there exist positive numbers \(\delta_i\) and \(\delta_{ij}\) such that \(0 < \delta_i \leq d_i(t) \leq \delta_{ij}\) for \(t \geq 0\) and \(i \in \mathcal{I}\).

\((H_2)\) Functions \(f_j\) and \(g_j\) satisfy Lipschitzian condition, that is, there exist constants \(C_i > 0\) and \(C_{ij} > 0\) such that

\[
|f_j(x) - f_j(y)| \leq C_i|x - y|, \quad |g_j(x) - g_j(y)| \leq C_{ij}|x - y|
\]

for all \(x, y \in \mathbb{R}\) and \(i \in \mathcal{I}\).

(H2) For any \(i \in \mathcal{I}\), there exists a constant \(\alpha_i\) such that

\[
(y - x)(a_i(y) - a_i(x)) \geq \alpha_i(y - x)^2
\]

for \(x, y \in \mathbb{R}\).

In this paper, we will make drive-response chaotic neural networks with time-varying delays achieve synchronization in finite time by designing some effective controllers. For simplicity, we refer to model (1) as the drive system, and consider a response chaotic network described as follows:

\[
\dot{y}_i(t) = -d_i(y_i(t)) \left[ a_i(y_i(t)) - \sum_{j=1}^{n} b_{ij}f_j(y_j(t)) - \sum_{j=1}^{n} c_{ij}g_j(y_j(t - \tau_j(t))) + l_i(t) \right],
\]

where \(i \in \mathcal{I}, y_i\) denote the states of the response system, \(u_i(t)\) is an external control input to realize synchronization in finite time, the rest of parameters are the same as system (1).

**Definition 1.** Drive-response systems (1) and (3) are said to be locally synchronized in finite time if there exists a constant \(\kappa > 0\) such that for any solutions of systems (1) and (3) denoted by \(x(t) = (x_1(t), \ldots, x_n(t))^T \) and \(y(t) = (y_1(t), \ldots, y_n(t))^T\) with different initial values \(\phi, \varphi \in \Omega = \{\psi \in \mathbb{C}([t_0 - \tau, t_0], \mathbb{R}^n) : \|\psi\| < \kappa\}\) there is a real number \(0 \leq T < \infty\) which depends on the initial values \(\phi\) and \(\varphi\) such that

\[
\lim_{{t \rightarrow t_0 + T}} \|y(t) - x(t)\| = 0.
\]

and \(\|y(t) - x(t)\| \neq 0\) for \(t \geq t_0 + T\). Furthermore, if \(\Omega = \mathbb{C}([t_0 - \tau, t_0], \mathbb{R}^n)\), drive-response systems (1) and (3) are said to be globally synchronized in finite time. Here, \(\|\cdot\|\) is the Euclidean norm defined by \(\|\phi\| = \sum_{i=1}^{n} |\phi_i|\) for any \(\phi = (\phi_1, \ldots, \phi_n)^T \in \mathbb{R}^n\).

**Definition 2.** Let \(V(x) : \mathbb{R}^n \rightarrow \mathbb{R}^+\) be a continuous function. \(V\) is said to be positive definite if \(V(0) = 0\) and \(V(x) > 0\) for \(x \neq 0\).

The following lemma can be found in [33].

**Lemma 1 (Comparison lemma).** Let \(J\) be a segment of \(R\), if the scalar differential equation

\[
\dot{y} = h(y), \quad y \in J
\]

has a global semi-flow \(\mu : R_+ \times J \rightarrow R_+\), and \(g : [a, b] \rightarrow J\) is a continuous function such that

\[
g(t) \leq h(g(t)), \quad t \in [a, b],
\]

then \(g(t) \leq \mu(t, g(a))\) for all \(t \in [a, b]\).

**Lemma 2** (see [34]). Assume that a continuous and positive-definite \(V(t)\) satisfies the following differential inequality:

\[
\dot{V}(t) \leq -CV(t), \quad t \geq t_0, \quad V(t_0) \geq 0,
\]

where \(c > 0\) and \(0 < \eta < 1\) are constants. Then, for any given \(t_0\), \(V(t)\) satisfies the following inequality:

\[
V^{1-\eta}(t) \leq V^{1-\eta}(t_0) - C(1 - \eta)(t - t_0), \quad t_0 \leq t \leq t_1,
\]

and

\[
V(t) = 0, \quad t \geq t_1
\]

with \(t_1\) given by

\[
t_1 = t_0 + \frac{V^{1-\eta}(t_0)}{C(1 - \eta)}.
\]

The following differential inequality will play an important role in later analysis, its initial idea comes from [35].
Lemma 3. Assume that a continuous and positive-definite $V(t)$ satisfies the following differential inequality:

$$V(t) \leq V(t) - kV^\eta(t), \quad t \geq t_0, \quad V(t_0) \geq 0,$$

where $I > 0$, $k > 0$ and $0 < \eta < 1$ are constants. Then, when $V^{1-\eta}(t_0) < k/I$, the following results are true:

$$V(t) = e^{k(t-t_0)}\left[V^{1-\eta}(t_0) - \frac{k}{I} + \frac{k}{T} - k^{1-\eta}(t-t_0)\right]^{1/(1-\eta)}, \quad t_0 \leq t \leq t_1,$$

and

$$V(t) = 0, \quad t \geq t_1$$

with $t_1$ given by

$$t_1 = t_0 + \frac{1}{k\eta-1}\ln\left(1 - \frac{1}{k}V^{1-\eta}(t_0)\right).$$

Proof. Consider the following differential equation:

$$\dot{y}(t) = ly(t) - k\eta y(t), \quad t \geq t_0$$

with initial value $y(t_0) = V(t_0)$.

Multiplying $y^{-\eta}(t)$ both sides of Eq. (9), we have

$$\frac{1}{1-\eta}\dot{y}(t) = ly(t) - k, \quad t \geq t_0$$

where $z(t) = y^{-\eta}(t)$. By using the constant variation formula and noting that $y(t) \geq 0$ for all $t \leq t_0$, it follows from (10) that

$$y(t) = e^{k(t-t_0)}\left[y^{1-\eta}(t_0) - \frac{k}{I} + \frac{k}{T} - k^{1-\eta}(t-t_0)\right]^{1/(1-\eta)}, \quad t_0 \leq t \leq t_1,$$

and

$$y(t) = 0, \quad t \geq t_1.$$

From Lemma 1, one obtains

$$V(t) \leq e^{k(t-t_0)}\left[V^{1-\eta}(t_0) - \frac{k}{I} + \frac{k}{T} - k^{1-\eta}(t-t_0)\right]^{1/(1-\eta)}, \quad t_0 \leq t \leq t_1,$$

and

$$V(t) = 0, \quad t \geq t_1.$$

The proof is completed. □

The following inequality is called Jensen inequality, which can be found in [36].

Lemma 4. If $a_1, a_2, ..., a_n$ are positive numbers and $0 < r < p$, then

$$\left(\sum_{i=1}^{n} a_i^p\right)^{1/p} \leq \left(\sum_{i=1}^{n} a_i^r\right)^{1/r}.$$

3. Main results

In this section, two different cases are considered to design the controllers for response system (3) to realize finite-time synchronization between system (1) and system (3).

For convenience, denote

$$\beta_i = \sum_{j=1}^{n} |b_{ji}|F_i - a_i, \quad i \in I.$$

Firstly, we consider the first case, that is, $\beta_i \leq 0$ for all $i \in I$. In this case, to achieve finite-time synchronization, the controller in response system (3) is designed as the following form:

$$u_i(t) = -\alpha \text{sign}(e_i(t))|e_i(t)|^\eta - \delta_i \text{sign}(e_i(t))|e_i(t-t_1(t))|,$$

where $e_i(t) = y_i(t) - x_i(t)$ denotes the synchronization error, $\delta_i > 0$ for $i \in I$ and $\alpha > 0$, $0 < \eta < 1$.

Based on the controller, the following results are derived.

Theorem 1. Assume that $(H_1)-(H_3)$ hold. If $\beta_i \leq 0$ for all $i \in I$, then drive system (1) and response system (3) are globally synchronized in finite time under the controller (11) with

$$\delta_i \geq \sum_{j=1}^{n} |c_{ij}|G_i, \quad i \in I.$$ (12)

Moreover, the settling time of synchronization $T_0$ satisfies

$$T_0 \leq \frac{\|e(t_0)\|^{1-\eta}}{\alpha d(1-\eta)}$$

where $d = \min\{|d_i|, i = 1, 2, ..., n\}$.

Proof. Construct the following Lyapunov function:

$$V(t) = \sum_{i=1}^{n} \text{sign}(e_i(t)) \int_{t_0}^{e_i(t)} \frac{1}{d(s)} ds.$$ (14)

Evidently,

$$\frac{1}{d} \sum_{i=1}^{n} |e_i(t)| \leq V(t) \leq \frac{1}{d} \sum_{i=1}^{n} |e_i(t)|,$$

where $d = \min\{|d_i|, i \in I\}$ and $\overline{d} = \min\{|\overline{d}_i|, i \in I\}$.

Calculate the derivative of $V(t)$ and apply conditions $(H_1)-(H_3)$, we have

$$V(t) = \sum_{i=1}^{n} \text{sign}(e_i(t)) \left[-(a_i(y_i(t) - a_i(x_i(t))) + \sum_{j=1}^{n} b_{ji}(y_j(t) - f_j(x_j(t))))

+ \sum_{j=1}^{n} c_{ij}(y_j(t-t_1(t)) - g_j(x_j(t-t_1(t))))

- \alpha \text{sign}(e_i(t))|e_i(t)|^\eta - \delta_i \text{sign}(e_i(t))|e_i(t-t_1(t))|\right]

\leq - \sum_{i=1}^{n} \left(a_i - \sum_{j=1}^{n} |b_{ji}| F_i\right) |e_i(t)| - \alpha \sum_{i=1}^{n} |e_i(t)|^\eta

+ \sum_{i=1}^{n} \left(\sum_{j=1}^{n} |c_{ij}| G_i - \delta_i\right) |e_i(t-t_1(t))|

\leq - \alpha \sum_{i=1}^{n} (|e_i(t)|)^\eta.$$ (15)

It follows from Lemma 4 that

$$\sum_{i=1}^{n} (|e_i(t)|)^\eta \geq \left(\sum_{i=1}^{n} |e_i(t)|\right)^\eta,$$

which together with (15) and (16),

$$\dot{V}(t) \leq -\alpha \left(\sum_{i=1}^{n} |e_i(t)|\right)^\eta \leq -\alpha \overline{d}^\eta V(t).$$

According to Lemma 2, we obtain

$$V(t) \leq V(t_0) - \alpha \overline{d}^\eta (1-\eta)t^{1/(1-\eta)}, \quad t_0 \leq t \leq t_0 + T,$$

and

$$V(t) = 0, \quad t \geq t_0 + T,$$

where

$$T = \frac{V^{1-\eta}(t_0) - \alpha \overline{d}^\eta (1-\eta)t_0^{1/(1-\eta)}}{\alpha \overline{d}^\eta (1-\eta)}.$$ (17)

Hence, $V(t)$ converges to zero in finite time $t_0 + T$, and it follows from (15) that system (1) and system (3) are finite-timely synchronized and the settling time of synchronization is estimated by $T$. The proof of Theorem 1 is completed. □

In the following, we consider another case that there exists a nonempty set $C \subset I$ such that

$$\sum_{j=1}^{n} \overline{|b_{ji}|} > a_i, \quad i \in C.$$ (17)
and
\[ \sum_{j=1}^{n} |b_j|F_i \leq a_i, \quad i \in \mathcal{I} \cap \mathcal{C}. \] (18)

In this case, the following criteria are established to ensure synchronization in finite time.

**Theorem 2.** Assume that \((H_1)-(H_3)\) hold. If there exists a nonempty set \(C \subset \mathcal{I} \) such that \((17)\) and \((18)\) are satisfied, then under the controller \((11)\), drive system \((1)\) and response system \((3)\) are locally synchronized with the finite time
\[ T_0 \leq \frac{1}{\beta d \eta (\eta - 1)} \ln \left( 1 - \frac{\beta d}{\alpha d} \|e(t_0)\|^{1-\eta} \right), \] (19)
where \(\beta = \max_{i \in \mathcal{I}} |\sigma_i| \beta_i > 0\), that is to say, drive system \((1)\) and response system \((3)\) are finitely synchronized if \(\|e(t_0)\|^{1-\eta} < \alpha d / \beta d\).

**Proof.** Similar to the proof of Theorem 1, we can easily derive that
\[ V(t) \leq \sum_{i=1}^{n} \left( \frac{1}{\beta d \eta} \|e(t_0)\|^{1-\eta} \right). \] In view of \(\|e(t_0)\|^{1-\eta} < \alpha d / \beta d\), then
\[ V(t) < \alpha d / \beta d. \]
According to Lemma 3, we have
\[ V(t) \leq \alpha d / \beta d \left[ V(1)^{(1-\eta)} - \frac{\alpha d}{\beta d} \sum_{i=1}^{n} \|e(t)\|^{1-\eta} \right] \leq \alpha d / \beta d, \]
and
\[ V(t) = 0 \quad \text{for} \quad t \geq t_0 + T \] with \(T\) given by
\[ T = \frac{1}{\beta d \eta (\eta - 1)} \ln \left( 1 - \frac{\beta d}{\alpha d} \|e(t_0)\|^{1-\eta} \right). \]
Hence, \(V(t)\) converges to zero within finite time \(T\), and it follows from \((15)\) that system \((1)\) and system \((3)\) are finite-time synchronized in finite time \(T\). The proof of Theorem 2 is completed. \(\square\)

**Theorem 2** gives some sufficient conditions to ensure the local finite-time synchronization between systems \((1)\) and \((3)\). To realize global synchronization in finite-time, the following results are proposed.

**Theorem 3.** Assume that \((H_1)-(H_3)\) hold. If there exists a nonempty set \(C \subset \mathcal{I} \) such that \((17)\) and \((18)\) are satisfied, then drive system \((1)\) and response system \((3)\) are globally synchronized within finite time under the following controller:
\[ u_i(t) = -k_i e_i(t) - \alpha \Gamma_i e_i(t) \] (12)
where \(k_i \geq 0\) for \(i \in \mathcal{I} \cap \mathcal{C}\) and \(k_i \geq \beta_i \) for \(i \in \mathcal{C}\). Furthermore, the settling time of synchronization \(T_0\) satisfies
\[ T_0 \leq \frac{\|e(0)\|^{1-\eta}}{\alpha (1-\eta)} \] (21)

**Proof.** Similar to the proof of Theorem 1, we can easily derive that
\[ V(t) \leq \sum_{i=1}^{n} \left( k_i + \alpha \sum_{j=1}^{n} |b_j|F_i \right) |e_i(t)| - \alpha \sum_{i=1}^{n} |e_i(t)|^{\eta}. \]

According to Lemma 2,
\[ V(t) \leq \frac{\|e(0)\|^{1-\eta}}{\alpha (1-\eta)} \] (19)
Hence, \(V(t)\) converges to zero in finite time \(T\) and it follows from \((15)\) that system \((1)\) and system \((3)\) are finite-time synchronized in finite time \(T\). The proof of Theorem 3 is completed. \(\square\)

**Corollary 1.** Assume that \((H_2)-(H_3)\) hold. If \(\beta_i \leq 0\) for all \(i \in \mathcal{I}\), model \((1)\) is degenerated to the following cellular neural networks:
\[ x_i(t) = -a_i x_i(t) + \sum_{j=1}^{n} b_{ij} f_j(x_j(t)) + \sum_{j=1}^{n} g_{ij} e_i(t - \tau_j(t)) + I_i. \] (23)
Correspondingly, the response chaotic network \((3)\) is replaced by
\[ y_i(t) = -a_i y_i(t) + \sum_{j=1}^{n} b_{ij} f_j(y_j(t)) + \sum_{j=1}^{n} g_{ij} e_i(t - \tau_j(t)) + I_i + u_i(t). \] (24)
In this case, assumption \((H_2)\) is true with \(d_i = \widetilde{d}_i = 1\) for \(i \in \mathcal{I}\). The following results are directly derived from Theorems 1–3.

**Corollary 2.** Assume that \((H_2)-(H_3)\) hold. If there exists a nonempty set \(C \subset \mathcal{I} \) such that \((17)\) and \((18)\) are satisfied, then under the controller \((11)\), drive system \((22)\) and response system \((23)\) are globally synchronized in finite time under the controller \((11)\) with
\[ \delta_i \geq \sum_{j=1}^{n} |c_{ji}| G_i, \quad i \in \mathcal{I}. \] Moreover, the settling time of synchronization \(T_0\) satisfies
\[ T_0 \leq \frac{\|e(0)\|^{1-\eta}}{\alpha (1-\eta)} \]

**Corollary 3.** Assume that \((H_2)-(H_3)\) hold. If there exists a nonempty set \(C \subset \mathcal{I} \) such that \((17)\) and \((18)\) are satisfied, then drive system \((22)\) and response system \((23)\) are globally synchronized in finite time under the controller \((20)\) and the settling time of synchronization \(T_0\) satisfies
\[ T_0 \leq \frac{\|e(0)\|^{1-\eta}}{\alpha (1-\eta)} \]

**Remark 1.** In \([21,22]\), the synchronization of Cohen–Grossberg neural networks with constant amplification function has been studied. To investigate asymptotical synchronization for the general Cohen–Grossberg neural networks with amplification function \(d_i(t)\), some interesting results were derived by different control strategies \([23–26]\) and their main step to propose
synchronization is write the error dynamical system, but the error dynamical system is very complex because of the amplification functions and the results are quite complex. To avoid writing the explicit complex error system and reduce the possible conservatism, a different Lyapunov function in this paper is constructed to study synchronization.

Remark 2. Although there are some results on asymptotical synchronization for Cohen–Grossberg neural networks [23–26], there is no results to be published on finite-time synchronization for the networks because of the theoretical lack of finite-time stability for delayed dynamical systems. In this paper, the time-delayed feedback control is applied to propose finite-time synchronization of delayed Cohen–Grossberg neural networks and the proposed methods as well as the derived criteria are new. Obviously, the methods used in this paper can be extended to investigate finite-time synchronization of delayed complex networks and delayed chaotic systems.

4. Numerical simulations

Consider the following Cohen–Grossberg neural networks with variable delays:

\[
\dot{x}_i(t) = -d_i(x_i(t)) \left[ a_i(x_i(t)) - \sum_{j=1}^{2} b_{ij} f_j(x_j(t)) - \sum_{j=1}^{2} c_{ij} f_j(x_j(t-\tau_j(t))) \right]
\]

(24)

for \( i = 1, 2 \), where \( f_j(x) = \tanh x \), \( a_1(x_1(t)) = 1.4x_1(t) \), \( a_2(x_2(t)) = 1.6x_2(t) \), \( b_{11} = 1.8, b_{12} = -0.1, b_{21} = -2, b_{22} = 0.4, c_{11} = -1.7, c_{12} = -0.6, c_{21} = 0.5, c_{22} = -2.5 \) and \( \tau_1(t) = \tau_2(t) = \frac{e^t}{1+e^t}, \quad d_1(x) = d_2(x) = 0.7 + 0.1 \frac{1}{1+x^2} \).

Evidently, \( 0.7 \leq d_i(x) \leq 0.8, F_i = G_i = 1 \) for \( i = 1, 2 \) and \( a_1 = 1.4, a_2 = 1.6 \). Hence, assumptions \((H_1)–(H_2)\) are satisfied. The numerical simulation of system (24) is represented in Fig. 1.

Consider the finite-time synchronization of driving system (24) and driven system described by

\[
\dot{y}_i(t) = -d_i(y_i(t)) \left[ a_i(y_i(t)) - \sum_{j=1}^{2} b_{ij} f_j(y_j(t)) - \sum_{j=1}^{2} c_{ij} f_j(y_j(t-\tau_j(t))) - u_i(t) \right],
\]

(25)

where \( a_i, d_i, b_{ij}, c_{ij}, f_j, \tau_j \) are defined in system (24).

By simple computation, \( \beta_1 = 2.4 \) and \( \beta_2 = -1.1 \). Firstly, we consider the finite-time synchronization under controller (11). Chosen \( \eta = 0.8, \alpha = 2.5, \delta_1 = 2.2, \delta_2 = 3.1 \), then by Theorem 2, system (24) and system (25) are finite-time synchronized when \( \|e(0)\| \leq 0.6290 \). To verify the result, Figs. 2–4 are provided with \( \|e(0)\| \leq 0.5 \), in this case \( T_0 \leq 8.0826 \) by (19).

In the following, we consider the finite-time synchronization under controller (20). Parameters \( \eta, \alpha, \delta_1, \delta_2 \) are the same as the above controller and \( k_1 = -2.4, k_2 = 0 \) are chosen. Evidently, all conditions of Theorem 3 are satisfied, therefore system (24) and system (25) are globally finite-time synchronized, which are verified by Figs. 5–7 with \( \|e(0)\| \leq 3 \), in this case \( T_0 \leq 3.5592 \) by (21).

![Fig. 1. The phase diagram of system (24) with the initial values \( x_1(0) = 0.2 \) and \( x_2(0) = 0.2 \) for \( \theta \in [-1, 0] \).](image)

![Fig. 2. Synchronization curves of \( x_1 \) and \( y_1 \).](image)

![Fig. 3. Synchronization curves of \( x_2 \) and \( y_2 \).](image)

![Fig. 4. Synchronization error \( \|e(t)\| \).](image)
5. Conclusion

In this paper, the local and global synchronization was investigated for a class of delayed Cohen–Grossberg neural networks. Two different time-delayed feedback controllers were designed and some criteria are derived to ensure the realization of synchronization in finite time for the addressed neural networks. Moreover, as some special cases, some sufficient conditions were given for the finite-time synchronization of delayed cellular neural networks. Finally, some numerical examples are provided to verify the theoretical results. It is believed that those proposed controllers and the derived results are new and effective. The future work will focus on the investigation of finite-time synchronization of delayed complex networks.

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References


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