A Study of Urgency Vehicle Routing Disruption Management Problem

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Abstract—If a transit vehicle breaks down on a schedule trip, there are some vehicles in the system need to serve this trip and the former plan must be changed. For solving the urgency vehicle routing problem with disruption that may be vehicle breakdowns or traffic accidents in the logistics distribution system, through the analysis of the problem and the disruption measurement, the mathematics model is given based on the thought of disruption management. For the characteristics of the problem, a Lagrangian relaxation is given to simplify the model, and decompose the problem into two parts. The Lagrangian multiplier is given by subgradient method and the subproblems are solved by saving approach to gain the initial solution. A fast insertion algorithm is given to obtain a feasible solution for the primal problem. The results show that the algorithm designed in this paper performs very well for solving the urgency vehicle routing disruption management problem.

Index Terms—urgency vehicle scheduling problem; disruption management; Lagrangian heuristic; saving approach; insertion algorithm

I. INTRODUCTION

With the swift and dynamitic development of information transmission and internet, the convenient e-commerce mode is being accepted gradually. Compared with the information flow and cash flow which are rapid and efficient, the efficiency of the logistics in e-commerce mode becomes the key of the high speed cycling of trading and the convenience for the customers. And the logistics distribution is always disrupted by some events. When such disruption happens, the allocation of the vehicles within relative limited transport capacity becomes a problem to be solved. Vehicle routing problem is well-known as a NP hard problem. And the problem in this study which is VRP with vehicles breaks down is even. So it is necessary to give a solution which will cover benefits of all costumers, goods providers and logistics companies.

The idea[1-2] of disruption management is such an optimization thought which benefits of all costumers, goods providers and logistics companies will be considered synthetically. This theory has been adopted in the domain of flight plan [3], railway scheduling [4] and supply chain coordination [5] and so on. Especially, this idea has been got several fruitful productions, such as Carlson [6], Rosenberger [7], and Yan [8] and so on. Comparing to the flight plan, the disruption management problem in the distribution is more complex and defiant. In some previous studies, the authors have given the vehicle routing disruption management problem and some solution approaches based on the thought of disruption management. A disruption management model and an improved genetic algorithm were given to solve the VRP with the changes of time windows and delivery weight of customers [9]. An improved tabu search algorithm was given to solve the VRP with backhaul and time window [10].

The multi-depot urgent vehicle routing problem with vehicles break down has not yet been found in the papers at domestic or abroad. This study based on the multi-depot vehicle routing problem which is more complicated and practical than the single-depot vehicle routing problem. Some scholars have been doing researched in this field. And many methods have been used to solve MDVRP, such as two-phase heuristic method [11], variable neighborhood search [12], tabu heuristic [13] and genetic arithmetic [14]. From different point of view, these methods have obtained good results.
But these methods were all designed for the common MDVRP, not for the MDVRP with vehicles break down.

After the problem analysis and the basic of mathematics model, a mathematics model is given for the urgent vehicle routing problem with vehicles break down based on the theory of disruption management. Based on the measurement of disruption, A Lagrangian relaxation method is given to divide and simplify the problem and the Lagrangian multiplier is given by the subgradient. And then, an inserted heuristic is designed to make the solver feasible. An example is given to prove the validity of the algorithm which is introduced in this study.

II. PROBLEM ANALYSIS AND THE MODEL BASIC

When disruption events happens, the command and control center need to know the timely information for all the system, including the information of breakdown vehicles and other vehicles. In this study, all the information can be obtained timely. GPS, GIS and vehicle condition monitoring system etc may be used to get this information.

A. Problem Analysis

Firstly, we give the problem in this paper a brief analysis and abstraction. Before a vehicle in a transit system completes all the tasks in its schedule, this vehicle can be defined to be a disabled vehicle if it can not complete the tasks for some reasons and can not repair in an acceptable time. The place and time of the vehicle breaks down will be referred as the breakdown point DP and breakdown time Dt respectively. The tasks which are not completed will be referred to as task set D. All the transit vehicles { k0+k } have the respective start location; start service time s0i+0 and the distribution plan themselves. After they finish all the tasks, they will get back to their own depot at the time s0i+0. The demand and service time of customer are i and s0i, respectively.

When the disruption happens, all the transit vehicles k∈Vm in the system are the backup vehicles except the disabled vehicles. All the vehicles start form the depot m can be referred to as Vm. The start and get back time of vehicle k can be referred as s0m=0 and s0m+1,k respectively. And the service time of customer i by the vehicle k can be referred as s0m after the disruption event happens. The total time of the service time of customer i and the driving time can be referred as t0i.

In the ordinary vehicle scheduling problem, there is no need to consider assigning a specific vehicle to trips, since all the vehicles are identical. And we can assign them arbitrarily after the schedule is determined. But in this paper, each vehicle in the system is different because both the location and the cargo capacity at the breakdown time are different. The cargo capacity in the disabled vehicle is Q0m, the costs and the service cargo capacity will be taken by vehicle k from depot m can be referred as Pm. When the disruption event happens, all the location of service vehicles in the system can be referred as a dummy depot. There is only one vehicle in each of dummy depot. If there are some vehicles in the real depot, they also can be referred as spare service vehicles. The only problem needs to consider is whether the service vehicles have enough time windows and cargo capacity. If the service vehicles are sent from the real depot, the additional costs will be incurred. And the penalty coefficient α is given. The penalty coefficient of loss for canceling some customers, the deviation of the service time and the deviation of the routes are referred as β, δ, γ, respectively.

If we only consider the total cost of transportation in this paper, that can not meet the demands of reality obviously. For example, the service vehicle drivers may not be familiar with the routes which are different with the former routes. After the disruption events happens, new schedule needs less changes with the former one. The optimal objective is that the service time and the transport routes of the new schedule are close to the former one based on the theory of disruption management, and the costs of new schedule are also need to be considered.

B. The Operation of The Former Problem

When disruption events happen, the other vehicles may not right in their own depot and the position of these vehicles are all different. The position recovery operation can solve this problem properly. There are two situations for the transport vehicles when disruption events happen, right in depot or not. For the vehicles which are not in the depot, the positions of them are deemed to dummy customers. The distance between the dummy customers and depot is 0, and the distribution costs and time are also 0. All the transport vehicles must start from the real depots and directly to the dummy customers. When they complete all the tasks, they need get back to their real depots. And that transform can make the vehicle routing disruption management problem to be a multi-depot vehicle routing problem.

When disruption events happen, we assume there is only one vehicle, and this vehicle can be described as a special customer. In the system there is only one customer which is a pickup set and the pickup capacity is equal to the goods in vehicle when the disruption event happens. Because the pickup and delivery operation are existing at the same time, the disruption problem transform into a pickup and delivery vehicle routing problem. If the tasks of the breakdown vehicles are more than one, the correspondence of the pickup and delivery point is one to more. The model for the delivery problem can not satisfy this problem obviously.

We assume the breakdown vehicle to be a dummy depot and the position is the same too. The delivery value for this dummy customer is minus. If there is a transport vehicle to serve this customer, the goods in this vehicle will increase. The attention point is whether the demand can satisfy the capacity of transport. If so, the pickup and delivery vehicle routing problem transform into a delivery vehicle routing problem. And this operation can
not satisfy the problem, in the next chapter there are the other operations used to solve this problem.

III. THE IDENTIFICATION AND MEASUREMENT OF DISRUPTION

Before the mathematics model is built, whether the event can be considered as a disruption event and how to measure the disruption degree must be confirmed.

A. The Identification of Disruption

The disruption events may affect one or more aspects of the logistics distribution system. If the infection does not change the former plan, that event can not be considered to a disruption event. For example, the infection is the changing of service. The former plan does not need to be changed and no customer will be affected. That infection can not be considered to a disruption event.

If the infection make the former plan can not continue, a new plan must be generated. But there is no service quality of customers decreasing. That infection can not be a disruption event. For example, a vehicle breaks down before completing all the tasks. But that vehicle can recovery and complete all its tasks in an acceptable time. No customers are disturbed. That infection can not be considered to a disruption event too.

(1) Vehicle $k_i$ breaks down at $T_2$ for some reasons, and that vehicle can not recovery till the time of logistic distribution system ending. That event can be considered as a disruption event. $s^0_{ik} + I > LT_2$, $I$ is a big enough positive number.

(2) Vehicle $k_i$ breaks down at $T_2$ for some reasons, and that vehicle can not recovery in an acceptable time. That event can be considered as a disruption event. $s^0_{ik} + T_i > LT_2$, $T_i$ is the acceptable time.

B. The Measurement of Disruption

Three points of view are given to the measurement of disruption management problem in this paper. They are the service time of customers, the routes of service vehicles and the costs of the logistics provider, respectively. The main bodies involved in that problem are customers, service vehicles and the logistics provider. The contents involved are service time, routes and costs.

(1) The service time of customer. After the disruption events happen, two species service time of customers which are not served will change their routes. One of them is the remaining customer of the disabled vehicles, and the other one is the customers of the vehicles which will change their routes. Some service time of customers will change. The customers may be critical of that and the satisfaction of there customer may decline. So the service time of customers need to be changed to a lesser extent and the satisfaction of the customer may be affected lesser.

After the disruption happens, the deviation between new plan and former plan can be performed by the service time to the customers. The new service time may earlier or later than the former plan. The difference of service time between new plan and the former one can be describe as $|d|$. For the logistics distribution system, the whole time departure can be described as $\sum_{k \in V} \sum_{m \in \mathbb{M} \cap R} \sum_{j \in \mathbb{R}} \sum_{t \in \mathbb{T}} t^n_{ijk} |d|_{ijk}$ and the time departure for the depots are 0.

(2) The routes of service vehicles. After the disruption events happen, if the service vehicles serve the breakdown point, the routes of the service vehicles will change immediately. The deviation of routes will be caused both the vehicles sending from the real depots and the vehicles on the way. Considering the infection of traffic and drivers, too much change in the new schedule may let the feasibility down. Based on the theory of disruption management, the deviations of routes need to be changed to a lesser extent and that conflict will be solved.

When the disruption events happen, some new routes must be build for saving the breakdown vehicles. The routes existent in the former plan can be described as $R(p)$ and the routes in the new plan can be described as $R(\overline{p})$. The whole route departure can be described as $\sum_{k \in V} \sum_{m \in \mathbb{M} \cap R} \sum_{j \in \mathbb{R}} \sum_{t \in \mathbb{T}} t^n_{ijk}$. The new route $(i, j, k, m)$ is $R(\overline{p}) / R(p)$, $t^n_{ijkm} = 1$. The whole mileage is $\sum_{k \in V} \sum_{m \in \mathbb{M} \cap R} \sum_{j \in \mathbb{R}} \sum_{t \in \mathbb{T}} t^n_{ijk}$. No changing routes $R(\overline{p}) \cap R(p)$ can be described as $t^n_{ijkm} = 0$.

(3) The costs of logistics provider. After the disruption events happen, there will be more costs from the deviation of routes, sending new vehicles and canceling some customers to the logistics provider. The costs of sending new vehicles and canceling customer $i$ can be referred as $c_{i}$ and $c_i$. Although the costs are not the most important thing to consider based on the theory of disruption management, too much cost will fall short of the benefits of customers obviously.

The new plan may generate new costs, including the cost of changing routes, sending new vehicles and canceling the customers. The cost coefficient of sending vehicle $k_0$ is $c_{ij}$ and the cost coefficient of canceling customer $i$ is $c_i$. The whole costs of changing routes can be described as $\sum_{k \in V} \sum_{m \in \mathbb{M} \cap R} \sum_{j \in \mathbb{R}} \sum_{t \in \mathbb{T}} t^n_{ijk} c_{ij}$, the whole cost of sending new vehicles can be described as $\sum_{k \in V} \sum_{m \in \mathbb{M} \cap R} \sum_{j \in \mathbb{R}} \sum_{t \in \mathbb{T}} t^n_{ijk} c_{ij}$, and the whole cost of canceling customers can be described as $\sum_{k \in V} \sum_{m \in \mathbb{M} \cap R} \sum_{j \in \mathbb{R}} \sum_{t \in \mathbb{T}} \beta c_i (1 - x^n_{ijk})$.

IV. MATHEMATICAL MODELS

The problem description in this study can be described as follow: these are $M$ depots, $N$ customers, and $k_n$ vehicles for each depot. There are $R$ tasks need to serve including the special customer $DP$ and $R^*$ means
There are different service time, demand and location for each customer. The disruption events will be found by dispatching center. The dispatching center can generate a new schedule immediately by considering the deviation of service time, the deviation of routes and costs integrated. And the disruption events will be solved and the delivery system will run naturally. The mathematical model is described as following based on the analysis:

\[ x_{ijk}^m = \begin{cases} 1, & \text{route}(i, j) = 1 \\ 0, & \text{else} \end{cases} \]

\[ \forall i, j \in R, \forall k \in V_m, i \neq j, i \neq M + 1, j \neq 0, m \in \{0\} \cup D \]

\[
\text{min} \quad \text{Lex}:
\]

\[
P_1: \sum_{i \leq j, m \in \{0\} \cup D} \sum_{j \in R} \sum_{k \in V_m} x_{ijk}^m (t_{ij}^m - t_{ijk}^m + \delta(1-x_{ijk}^m))
\]

\[
P_2: \alpha \sum_{i \leq j, m \in \{0\} \cup D} \sum_{j \in R} \sum_{k \in V_m} \sum_{c_{ij}}^m (c_{ij} - t_{ijk}^m + \gamma)^m (1-x_{ijk}^m)
\]

\[
P_3: \alpha \sum_{i \leq j, m \in \{0\} \cup D} \sum_{j \in R} \sum_{k \in V_m} \sum_{c_{ij}}^m (c_{ij} t_{ijk}^m + \beta c_{ij} (1-x_{ijk}^m))
\]

Constraints:

\[
\sum_{i \leq j, m \in \{0\} \cup D} \sum_{j \in R} x_{ijk}^m \geq 1
\]

\[
\sum_{i \leq j, m \in \{0\} \cup D} x_{ijk}^m \leq 1 \quad \forall k \in V_m
\]

\[
\sum_{i \leq j, m \in \{0\} \cup D} x_{ijk}^m = 1 \quad \forall k \in V_m
\]

\[
\sum_{m \in \{0\} \cup D} x_{ijk}^m \leq 1 \quad \forall k \in V_m
\]

\[
\sum_{m \in \{0\} \cup D} x_{ijk}^m = 0 \quad \forall k \in V_m
\]

\[
\sum_{m \in \{0\} \cup D} \sum_{j \in R} x_{ijk}^m \leq Q^m \quad \forall k \in V_m
\]

\[
\sum_{m \in \{0\} \cup D} \sum_{k \in V_m} P^m_{ij} = Q^m_{ij} \quad \forall k \in V_m
\]

\[
S_{ijk}^m + I(1-x_{ijk}^m) \leq S_{jk}^m \quad \forall i, j \in R, i \neq M + 1, j \neq 0, k \in V_m
\]

\[
ET_{ij} \leq S_{ijk}^m \leq LT_{ij} \quad i \in R', k \in V_m
\]

There are three parts in the objective function. The purpose of Function (1) is that while servicing as many customers as possible, keep the deviation of starting servicing time lowest. The Function (2) expresses the deviation of routes between the original plan and the adjustment plan. The purpose of Function (3) is to keep the cost of deviation is lowest.

Constraint (4) states that the disabled vehicles must be served at least once. Constraint (5) states that each customer is serviced once at most, which is that each customer is serviced once, or not be serviced at all because of the canceling strategy. Constraint (6) and (7) state that each vehicle leaves the depot, after arriving at a customer the vehicle leaves again, and finally arrives at the corresponding depot. Constraint (8) states that each rescue vehicle must pickup goods first then delivery goods. Constraint (9) states that the vehicles must not travel between depots. Constraint (10) states that the need of each route must not exceed the sum of goods from disable vehicle and rescue vehicles. Constraint (11) states that all of the goods has been taken away by the backup vehicles are equal to the goods in the disabled vehicle. Constraint (12) states that the time of traveling from the customer i to the customer j for vehicle k is less than s_{ik}^m + l_{ij}. I is a number which large enough. Constraint (13) is the time window constraints. Constraint (14) is the definition of routes deviation, which is increased, decreased or unchanged routes, by compared with the original optimal plan.

V. LAGRANGIAN HEURISTIC

Based on the characteristics of the disruption management and the complexity of the mathematics model, the Lagrangian heuristic is given to solve the urgent vehicle routing problem. The flow of the algorithm is as follow:

Step1: Relax the hard constraint and decompose the problem into two parts.

Step2: An improved saving algorithm is given to solve the subproblems.

Step3: The Lagrangian multiplier is revised again and again till we get a satisfactory one.

Step4: Based on the steps as above, an insertion heuristic is given to get feasible solution.

Step5: Repeat Step2 to Step4, till a most satisfactory solution we get or a given time.

A. Lagrangian Relaxation

The hard constraint (5) is slacked [8] and the former problem is divided into two subproblems which named as Lag(S1) and Lag(S2) respectively. One part aims at the vehicles which must pass through DP and pickup goods belong to S1. The other part aims at the vehicles that do not need to pass through DP and pickup goods belong to S2. Two constraints of the subproblems are all broaden.
For Lag(S₁), the missing constraints are mainly the constraints of time windows. The results may not accord with the time windows. For Lag(S₂), the missing constraints are mainly the constraints of transport capacity. The results may not accord with the transport capacity. And when the vehicles get back to the depot, there may be some goods still in the vehicles. For obtaining a better solution, an objective function is adopted to both of the two subproblems. The residual goods are given a large penalty coefficient \( \varphi \). The purpose of this function is to reduce the residual goods in the vehicles as less as possible. Once the results are obtained, \( \varphi \) is set to 0.

Through the analysis of above, the two subproblems can be described as follow:

\[
\text{Lag}(S₁) : \\
\min \left( 0 + \sum_{\mathcal{E} \in \mathcal{D}} \sum_{\mathcal{R} \in \mathcal{D}} \sum_{\mathcal{K} \in \mathcal{D}} x_{\mathcal{I} \mathcal{J} \mathcal{M} \mathcal{K}} - 1 \right) \\
+ \varphi \sum_{\mathcal{K} \in \mathcal{D}} \sum_{\mathcal{R} \in \mathcal{D}} \sum_{\mathcal{M} \in \mathcal{D}} \sum_{\mathcal{K} \in \mathcal{D}} \left( Q^m_i + P^m_i - r_i x_{\mathcal{I} \mathcal{J} \mathcal{M} \mathcal{K}} \right) \\
\sum_{\mathcal{K} \in \mathcal{D}} \sum_{\mathcal{R} \in \mathcal{D}} \sum_{\mathcal{M} \in \mathcal{D}} x_{\mathcal{I} \mathcal{J} \mathcal{M} \mathcal{K}} \geq 1
\]

\[
\sum_{\mathcal{K} \in \mathcal{D}} \sum_{\mathcal{R} \in \mathcal{D}} \sum_{\mathcal{M} \in \mathcal{D}} x_{\mathcal{I} \mathcal{J} \mathcal{M} \mathcal{K}} \geq Q^m_i \quad \forall k \in V_w
\]

\[
\sum_{\mathcal{M} \in \mathcal{D}} P^m_i = Q^m_i \quad \forall k \in V_w
\]

\[
\sum_{\mathcal{K} \in \mathcal{D}} x_{\mathcal{I} \mathcal{J} \mathcal{M} \mathcal{K}} = 1 \quad \forall k \in V_w
\]

\[
\sum_{\mathcal{M} \in \mathcal{D}} x_{\mathcal{I} \mathcal{J} \mathcal{M} \mathcal{K}} = 1 \quad \forall k \in V_w
\]

\[
\sum_{\mathcal{K} \in \mathcal{D}} x_{\mathcal{I} \mathcal{J} \mathcal{M} \mathcal{K}} = 0 \quad \forall k \in V_w
\]

\[
\sum_{\mathcal{K} \in \mathcal{D}} x_{\mathcal{I} \mathcal{J} \mathcal{M} \mathcal{K}} = 0 \quad \forall i = m \in \mathcal{D}, k \in V_w
\]

\[
x_{\mathcal{I} \mathcal{J} \mathcal{M} \mathcal{K}} = \{0,1\} \quad \forall i, j \in \mathcal{D}, k \in V_w
\]

\[
x_{\mathcal{I} \mathcal{J} \mathcal{M} \mathcal{K}} = \begin{cases} 
1 & (i, j, m, k) \in E(\overline{p}) / E(p) \\
0 & (i, j, m, k) \in E(\overline{p}) \cap E(p) \\
-1 & (i, j, m, k) \in E(\overline{p}) \cap E(p)
\end{cases}
\]

\[
x_{\mathcal{I} \mathcal{J} \mathcal{M} \mathcal{K}} = \{0,1\} \quad \forall i, j \in \mathcal{D}, k \in V_w
\]

Through the analysis of above, the two subproblems can be described as follow:

\[
\text{Lag}(S₂) : \\
\min \left( 0 + \sum_{\mathcal{E} \in \mathcal{D}} \sum_{\mathcal{R} \in \mathcal{D}} \sum_{\mathcal{K} \in \mathcal{D}} x_{\mathcal{I} \mathcal{J} \mathcal{M} \mathcal{K}} - 1 \right) \\
+ \varphi \sum_{\mathcal{K} \in \mathcal{D}} \sum_{\mathcal{R} \in \mathcal{D}} \sum_{\mathcal{M} \in \mathcal{D}} \sum_{\mathcal{K} \in \mathcal{D}} \left( Q^m_i + P^m_i - r_i x_{\mathcal{I} \mathcal{J} \mathcal{M} \mathcal{K}} \right) \\
\sum_{\mathcal{K} \in \mathcal{D}} \sum_{\mathcal{R} \in \mathcal{D}} \sum_{\mathcal{M} \in \mathcal{D}} \sum_{\mathcal{K} \in \mathcal{D}} \left( Q^m_i + P^m_i - r_i x_{\mathcal{I} \mathcal{J} \mathcal{M} \mathcal{K}} \right)
\]

\[
x_{\mathcal{I} \mathcal{J} \mathcal{M} \mathcal{K}} + t_y - I(1 - x_{\mathcal{I} \mathcal{J} \mathcal{M} \mathcal{K}}) \leq \sigma_{\mathcal{I} \mathcal{J} \mathcal{M} \mathcal{K}} \\
\forall i, j \in \mathcal{D}, i \neq M + 1, j \neq 0, k \in V_w
\]

\[
E_{\mathcal{I} \mathcal{J} \mathcal{M} \mathcal{K}} \leq \sigma_{\mathcal{I} \mathcal{J} \mathcal{M} \mathcal{K}} \leq M_{\mathcal{I} \mathcal{J} \mathcal{M} \mathcal{K}} \\
\forall i \in \mathcal{D}, k \in V_w
\]

\[
x_{\mathcal{I} \mathcal{J} \mathcal{M} \mathcal{K}} + d_i = \sigma_{\mathcal{I} \mathcal{J} \mathcal{M} \mathcal{K}} \\
\forall i \in \mathcal{D}, k \in V_w
\]

\[
\sum_{j \in \mathcal{D}} \sum_{\mathcal{R} \in \mathcal{D}} \sum_{\mathcal{M} \in \mathcal{D}} x_{\mathcal{I} \mathcal{J} \mathcal{M} \mathcal{K}} = 1 \\
\forall k \in V_w
\]

\[
\sum_{j \in \mathcal{D}} \sum_{\mathcal{R} \in \mathcal{D}} \sum_{\mathcal{M} \in \mathcal{D}} x_{\mathcal{I} \mathcal{J} \mathcal{M} \mathcal{K}} = 1 \\
\forall k \in V_w
\]

\[
\sum_{j \in \mathcal{D}} \sum_{\mathcal{R} \in \mathcal{D}} x_{\mathcal{I} \mathcal{J} \mathcal{M} \mathcal{K}} = 0 \\
\forall i = m \in \mathcal{D}, k \in V_w
\]

\[
x_{\mathcal{I} \mathcal{J} \mathcal{M} \mathcal{K}} = \{0,1\} \\
\forall i, j \in \mathcal{D}, k \in V_w
\]

\[
x_{\mathcal{I} \mathcal{J} \mathcal{M} \mathcal{K}} = \begin{cases} 
1 & (i, j, m, k) \in E(\overline{p}) / E(p) \\
0 & (i, j, m, k) \in E(\overline{p}) \cap E(p) \\
-1 & (i, j, m, k) \in E(\overline{p}) \cap E(p)
\end{cases}
\]

\[
x_{\mathcal{I} \mathcal{J} \mathcal{M} \mathcal{K}} = \{0,1\} \\
\forall i, j \in \mathcal{D}, k \in V_w
\]

\[
\text{B. The Solving of the Subproblems}
\]

For Lag(S₁), the customers must include SD and the corresponding customers where goods picked up at SD. Thus, the problem could be regard as a vehicle routing problem with picking up and delivery. An improved saving arithmetic is introduced to solve this part [9]. First, link i, SD and the customer where goods picked up and the final depot in one route. And then, the thought of saving arithmetic is adopted up to insert each customer into the route and calculate the saving value until satisfied. And for Lag(S₂), all of the vehicles do not need to serve SD, but directly to other customers. For \( \forall \mathcal{I} \in \mathcal{S₂} \), it is necessary to find one route back to the depot and the objective function is minimized. Thus, for each vehicle, the problem is translated into a shortest path problem with minimum cost. Considering the characteristics of this problem, this subproblem can use solving algorithm too.

An improved saving algorithm is given in this study based on the thought of disruption management. The basic of the saving algorithm is the disruption value, but not the saving value. With the disruption value decreasing, the new plan is close to the reality. The steps of the improved saving algorithm are as follow:

(1) After connect these three points, if the total demand of goods does not exceed the loading amount of vehicle, this route is formed. Other customers which are not connected to the depot can be inserted into this route till the first customer who make the total demand of goods exceed the loading amount of vehicle. And then, remove part of the goods which make vehicle overload. Or there is no customer can satisfy the demand. The route form completely.

(2) If connect these three points, the total demand of goods are equal to the loading amount of vehicle and they all satisfy their time windows. The route form completely.
(3) If connect these three points, the total demand of good exceed the loading amount of vehicle. Remove part of the goods which make vehicle overload. If they all satisfy their time windows, the route form completely.

(4) Repeat the operations above, till all the customers are connected into the routes and the result can be seemed to be a initial solution to the relation problem.

C. Lagrangian Multiplier

Because the Lagrangian function is a concave function and piecewise continuous, the subgradient optimization is used to solve the Lagrangian multiplier. For each given \( \lambda_i \), a subgradient \( s(\lambda_i) \) is chosen from the subgradient vector \( \partial g(\lambda) \). And let \( \lambda_{s, i} = \max \{ \lambda_i + \theta_i s(\lambda_i), 0 \} \), where \( \theta_i = \frac{z_{cv}(i) - z_{lb}(i)}{\| s(\lambda_i) \|} \). The parameter \( \rho \) determines the step of the gradient direction. Through trial and error, it is found that if \( LR(\lambda) \) ascend, \( \rho = 2 \); if the value of \( LR(\lambda) \) steadiness in some steps, \( \rho = 1 \). \( z_{cv}(i) \) are the upper bound of the optimal objective of IP, and the value of that change with \( i \) to get better. \( z_{lb}(i) \) is the lower bound of \( z_{lb}(\lambda_i) \). In order to solve quickly, it is taken as a fixed value in this paper.

D. Making Solution Feasible

The Lagrangian relaxation approach can be used to get an initial solution. But this solution may not consistent with the problem because the hard constraint (2) is relaxed. So it is necessary to find a way to make the initial solution feasible.

It is necessary to find a way to make the initial solution feasible because of the relation. For considering the demand of rapid solving, an insertion algorithm is given to settle this problem quickly. For the situation of that one trip is covered by multiple paths, remove all paths except one path which has minimum deviation with original plan. And by doing so, more paths could be provided for the trips which are not covered. For the situation of there are trips which are uncovered, an insertion algorithm is given to insert the trips into the neighbor pair of trips. And try to decrease the difference from the original plan. If the trip inserted can not satisfied the time windows, sending new vehicles to serve this trip; if there are no more vehicles in the depot, cancel this trip.

The steps of insertion heuristic are as follow:

Step1: Remove redundant covering from the partial paths.

Step2: For each uncovered trip \( i \), choose feasible insertion positions. If the position satisfies the constraints, make the trip insert into the corresponding position and computing the cost of increment. If there is no feasible position for this trip, send the new vehicle to serve that trip. Otherwise, cancel it.

<table>
<thead>
<tr>
<th>TABLE I. DATA OF DEPOT AND CUSTOMER</th>
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Step3: Compare the costs of increment for each trip inserted; find the situation of minimum deviation.

Step4: Repeat Step2 and Step3 until all the uncovered are inserted into the routes or cancelled.

VI. EXPERIMENTAL RESULTS AND ANALYSIS

A. Initial Parameters

A program based on the algorithm in this study is compiled by C++. The hardware environment is Intel Pentium 4 CPU 3.06GHz and the ram is 512MB. It is assumed that the speed of all of the vehicles is 1 and discharged time is 0. The cost of each distance unit is 2.5 and the cost of sending new car is 100. The penalty of early arrive is 0.2 and delay arrive is 2. And 31, 32, 33, 34 states depot and each depot have four 450 vehicles. Position coordinate, transport goods and time windows can be seen from table 1.

B. Route Optimization

For getting the optimal plan without disruption and the feasibility of the algorithm in this paper, the mathematics model needs to be transformed. The transform process can be described as follow:

The set of former routes \( P \) is set an empty set and all the customers belong to set \( R \). All the vehicles belong to set \( V \). The total cargo volume of each vehicle must not exceed the cargo capacity of this vehicle. The weighting factors are designed as follow: \( \alpha = 0 \), \( \gamma = 0 \), \( \delta = 0 \), \( \eta \) is a large enough number. The time between the service time in the new schedule and the original service time in the former plan is referred as 0. If the sending strategy is adopted in the mathematics model, the cost of sending a new vehicle is \( c_i \). For the problem of MDVRPTW, \( c_i = 0 \).

The optimization route is 1167.50, the optimization cost is 3029.33 and the number of vehicles is 10. The routes are depot 31: (1) 31-22-26-7-31, (2) 31-23-29-14-31, (3) 31-18-3-31; depot 32: (4) 32-12-17-8-32, (5) 32-25-24-17-32, (6) 32-13-1-2-32; depot 33: (7) 33-10-5-6-21-33, (8) 33-28-11-15-33, (9) 33-4-9-20-33; depot 34: (10) 34-19-16-30-34. The results are better than paper [10].

C. Disturbance Generation

For validating the difference between disruption management, the common optimal method and rescheduling method are shown as can be seen from table 2, table 3 and table 4.

D. The Comparison Between Three Methods

The results of the disruption management, common optimal method and rescheduling method are shown as can be seen from table 2, table 3 and table 4.

| TABLE II. THE DISRUPTION SITUATIONS FROM THE CONVENTIONAL METHOD |
|-------------------------|----------------|----------------|-------------|
| Disruption events | Time departure | Route departure | Costs       |
| 1                     | 17.20          | 83.54          | 308.85      |
| 2                     | 148.95         | 131.92         | 429.8       |
| 3                     | 56.74          | 145.51         | 563.78      |
| 4                     | 82.05          | 207.56         | 681.13      |

| TABLE III. THE DISRUPTION SITUATIONS FROM RESCHEDULING |
|-------------------------|----------------|----------------|-------------|
| Disruption events | Time departure | Route departure | Costs       |
| 1                     | 190.4          | 85.51          | 245.6       |
| 2                     | 237.6          | 126.11         | 361.80      |
| 3                     | 33.34          | 70.45          | 179.71      |
| 4                     | 164.39         | 221.28         | 656.01      |

| TABLE IV. THE DISRUPTION SITUATIONS FROM RESCHEDULING |
|-------------------------|----------------|----------------|-------------|
| Disruption events | Time departure | Route departure | Costs       |
| 1                     | 14.80          | 76.54          | 191.35      |
| 2                     | 69.27          | 105.01         | 262.53      |
| 3                     | 28.58          | 52.66          | 244.23      |
| 4                     | 77.86          | 224.89         | 624.45      |

| TABLE V. COMPARATIVE RESULTS OF DISRUPTION MANAGEMENT, SCHEDULING ACCORDING TO THE ORIGINAL PLAN |
|-------------------------|----------------|----------------|-------------|
| Comparative results | Time departure | Route departure | Costs       |
| One vehicle  | DBD and DBC | 49.40% | 15.73% | 38.56% |
| DBD and DBC | 49.40% | 15.73% | 38.56% |
| More vehicle | DBD and DBC | 23.31% | 21.39% | 30.22% |
| DBD and DBC | 46.17% | 4.86% | -3.94% |

E. The analysis of results

(1) As shown in table 1, the deviation of serving times, the deviation of routes and the costs of adjusting schedule are all reduced significantly comparing the
common method. So it can be concluded that the method designed in this study has a better solution. 
(2) It is hard to avoid the fluctuation of results since the choice of subgradient and the insertion of customers are adopted in the process of urgency vehicle routing problem. Through times and times experiment, we found that the stability of algorithm in this study is still better than a simple heuristic algorithm. 
(3) Because the problem studied in this paper is NP hard problem. The results shown in table 1 are hard to contrast with the optimal solution that is gotten by exhaustion approach and it is difficult to find comparable data from the correlative literatures. But the algorithm proposed has a good convergences (at current hardware environment, below 1 minute on average), which is crucial for an algorithm to work efficiently with the urgency vehicle routing problem with vehicles breakdown.

VII. CONCLUSIONS

The urgency vehicle routing problem aims at proposing an approach to solve the disruption events for the delivery system. Although there are many studies in the domain of VRP, the situation in the real world were not considered in the existing solution approaches, such as vehicle breakdowns or traffic accidents. In this paper, the urgency vehicle routing problem is extended to consider these aspects, and the method can be used in more real situation in this world.

In order to get an objective with minimize of the deviation of service times, deviation of routes and total costs, a disruption management mathematical model and the corresponding lagrangian heuristic are given. On the basis of the available model of VRP, the mathematical model is established the customer, the goods provider and the logistics facilitator comprehensively. The initial solution is given by the lagrangian relaxation. An insertion procedure is used in the primal heuristic to obtain a feasible solution or the primal problem.

There are many relative domain need to be considered in the future study, such as the variety of time windows and the changing of the place of receipt, the urgency vehicle routing problem with mixed disturbed events.

ACKNOWLEDGEMENTS

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