Kernel Nyström Method for Light Transport

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The Kernel Nyström Method for Light Transport

Reconstruct light transport matrix from small number of acquired images

Exploits linear and nonlinear data coherence

Adaptive measurement techniques

Kernel Nyström Relighting

Effectively reconstructs light transport matrix to render new lighting conditions.

OUTLINE

Mathematical Definition

$$\begin{bmatrix} \mathbf{V} \end{bmatrix} = \begin{bmatrix} & \mathbf{T} & \end{bmatrix} \cdot \begin{bmatrix} \mathbf{L} \end{bmatrix}$$

V : outgoing radiance seen by camera, m pixels
T : $m \times n$ light transport matrix
L : vector of incident radiance from n light sources

(Ng et al. 03, Peers et al. 09)

MATHEMATICAL DEFINITION

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(Ng et al. 03, Peers et al. 09)

Previous Methods

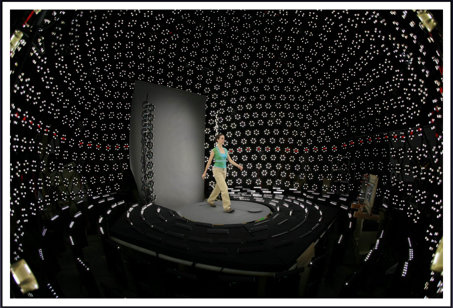
Three main categories:

- Brute Force
- Sparsity Based
- Coherence Based

PREVIOUS METHOD: BRUTE-FORCE

Debevec et al., Light Stage

- Massive array of lights
- Capturing thousands of images

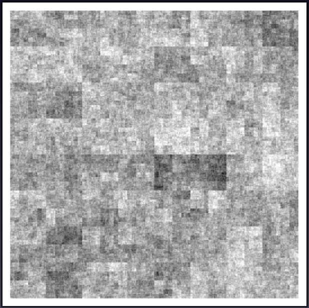


(Relighting Human Locomotion. Debevec et al., 06)

PREVIOUS METHOD: SPARSITY-BASED

Peers et al.

- Project Haar wavelet noise to infer reflectance matrix
- Does not handle complex occlusions

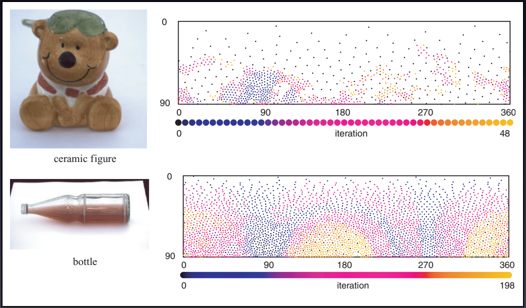


(Inferring reflectance functions from wavelet noise. Peers et al., 05)

PREVIOUS METHOD: COHERENCE-BASED

Fuchs et al.

- Adaptive sampling scheme for reflectance field
- Exploits coherence in either rows or columns



(Adaptive sampling of reflectance fields. Fuchs et al., 07)

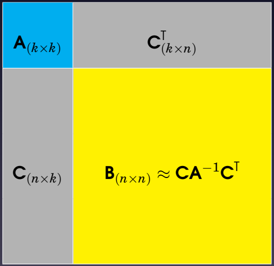
Outline

1. Introduction
   - Mathematical Definition
   - Related Work
2. The Kernel Nyström Method
   - Asymmetric Generalization
   - The Kernel Extension
   - Adaptive Light Transport Measurement
3. Conclusion
   - Results
   - Future Work

Inspiration: the Symmetric Nyström Method

Reconstructs low-rank symmetric matrix from sparsely sampled columns; accurate when $\operatorname{rank}(\mathbf{T}) \leq k$

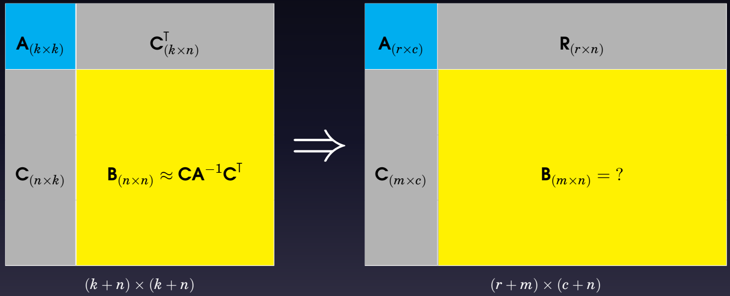
$$\mathbf{T} = \begin{bmatrix} \mathbf{A} & \mathbf{C}^\mathsf{T} \\ \mathbf{C} & \mathbf{B} \end{bmatrix} \approx \begin{bmatrix} \mathbf{A} & \mathbf{C}^\mathsf{T} \\ \mathbf{C} & \mathbf{C}\mathbf{A}^{-1}\mathbf{C}^\mathsf{T} \end{bmatrix}$$



$(k+n)\times(k+n)$

(Williams and Seegor 00)

Asymmetric Light Transport Matrix



Asymmetric light transport matrices with image pixels as columns and light sources as rows

The Asymmetric Nyström Method

If $\operatorname{rank}(\mathbf{T}) = \operatorname{rank}(\mathbf{A})$, we can expect the following relationships:

$$\begin{bmatrix} \mathbf{C} & \mathbf{B} \end{bmatrix} = \mathbf{P} \begin{bmatrix} \mathbf{A} & \mathbf{R} \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} \mathbf{R} \\ \mathbf{B} \end{bmatrix} = \begin{bmatrix} \mathbf{A} \\ \mathbf{C} \end{bmatrix} \mathbf{Q}$$

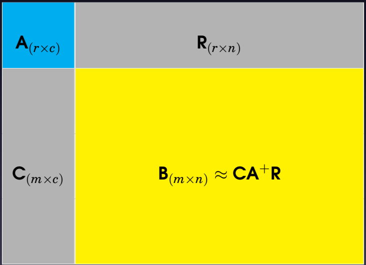
implying $\mathbf{C} = \mathbf{PA}$, $\mathbf{R} = \mathbf{AQ}$, and $\mathbf{B} = \mathbf{PR} = \mathbf{CQ}$. Thus,

$$\mathbf{B} = \mathbf{PAQ} = \mathbf{PAA}^{+}\mathbf{AQ} = \mathbf{CA}^{+}\mathbf{R}$$

$\mathbf{A}^{+}$ denotes the Moore-Penrose pseudoinverse of $\mathbf{A}$ with property $\mathbf{AA}^{+}\mathbf{A} = \mathbf{A}$ and is found by applying SVD to $\mathbf{A}$.

The Asymmetric Nyström Method

$$\mathbf{T} = \begin{bmatrix} \mathbf{A} & \mathbf{R} \\ \mathbf{C} & \mathbf{B} \end{bmatrix} \approx \begin{bmatrix} \mathbf{A} & \mathbf{R} \\ \mathbf{C} & \mathbf{C}\mathbf{A}^{+}\mathbf{R} \end{bmatrix}$$



$(r + m) \times (c + n)$

Works under assumption: $\mathrm{rank}(\mathbf{T}) = \mathrm{rank}(\mathbf{A})$

The "Kernel Trick"

Standard approach for enhancing performance of machine learning algorithms based on nonlinear transformations of input.

Map vectors in the data space to a higher dimension feature space.

(Cristianini and Shawe-Taylor 00)

The Kernel Method

$$f(\mathbf{T}) \approx \mathbf{K} = \begin{bmatrix} f(\mathbf{A}) & f(\mathbf{R}) \\ f(\mathbf{C}) & f(\mathbf{C})(f(\mathbf{A}))^{+} f(\mathbf{R}) \end{bmatrix}$$

$$\mathbf{T} \approx f^{-1}(\mathbf{K})$$

See paper for more details on how f is a kernel method.

The Light Transport Kernel

Use the nonlinear power function

$$f(x) = x^{\gamma}$$

- One parameter, optimal function easy to find
- Produces enhanced reconstructions

Estimating the Light Transport Kernel

We use golden section search to find optimal $\gamma \in_{\log} [0.001, 1000]$ that minimizes

$$g(\gamma) = \frac{||f(\mathbf{A})||_*}{||f(\mathbf{A})||_2} \int_0^1 \frac{1}{f'(x)} p(x)\, \mathrm{d}x \ ,$$

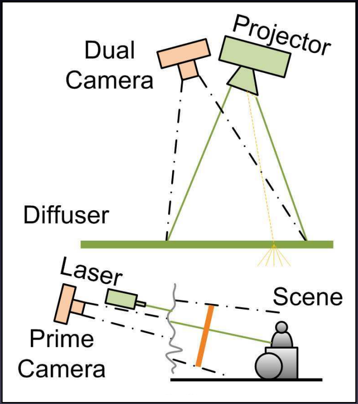
$$||\mathbf{X}||_* = \sum_i \delta_i \ , \quad ||\mathbf{X}||_2 = \max_i \{\delta_i\}$$

where the δ_i's are singular values of $\mathbf{X}$ and p is the distribution density of values in $\mathbf{A}$ (assumed to be identical to that in $\mathbf{T}$).

Optimization takes only few seconds.

(Press et al. 92).
See paper for more details.

ACQUISITION EQUIPMENT SETUP



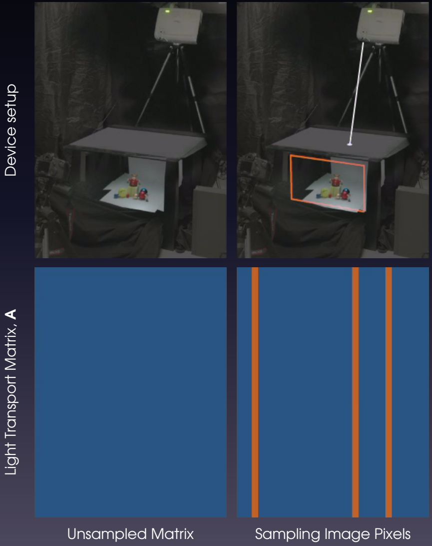
Light Transport Measurement

Device setup

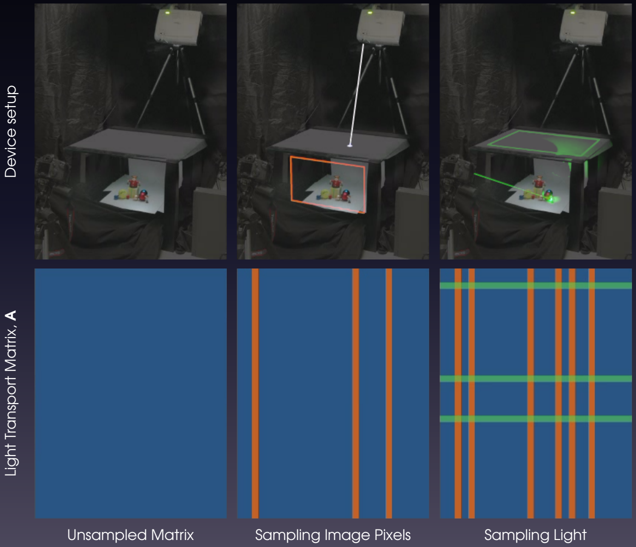
Light Transport Matrix, **A**

Unsampled Matrix

Light Transport Measurement



Light Transport Measurement



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Results

Future Work

Thank you!

Any questions?