Selection of stocks using constrained fuzzy AHP and PROMETHEE

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Abstract

In a portfolio selection problem, the first step is to choose suitable stocks at the right time. Many approaches and principles have been used to help investor solve the problem. This paper documents a new approach for the portfolio selection problem based on the combined method of Constrained Fuzzy Analytic Hierarchy Process (CFAHP) and the Preference Ranking Organization Method for Enrichment Evaluations (PROMETHEE). The proposed approach is structured around two stages: the first stage, the CFAHP is used to evaluate the attribute weights; the second stage, the alternatives are then ranked by PROMETHEE. The integrated approach exhibits advantages in that more objective attribute weights can be obtained; and furthermore, it reduces the huge computation in CFAHP and gets more reasonable ranking results. This approach has been applied in Chinese stocks as a real case and the results show its usefulness in facilitating the final stock selection decisions for the investor.

Keywords: Stock selection, Multiple criteria decision making, Constrained Fuzzy Analysis Hierarchy Process (CFAHP), Preference Ranking Organization Method for Enrichment Evaluations (PROMETHEE).

1. Introduction

The key issue for the portfolio selection problem is to choose suitable stocks at the right time. Many quantitative methods are applied to solve the portfolio selection problem after Markowitz [1] using the mean-variance (MV) method in mathematics. For example, the portfolio selection problem was solved by using the CFAHP [2]. Chen [3] used dynamic portfolio theory model based on minimum semi-absolute deviations criterion with an application in the Chinese stock market.

The Analysis Hierarchy Process (AHP) was proposed by Satty in 1980 [4]. After its appearance, it has been used in many areas [5][6]. Nevertheless, it requires pair-wise comparisons between attributes and alternatives in order to set up decision matrices, which may result in huge computation and low accuracy. The Preference Ranking Organization Method for Enrichment Evaluations (PROMETHEE) was developed by Brans [7]. It is a simple method in conception and application. In spite of its popularity, the attribute weights of alternatives are subjectively determined which may lead to invalid results. To overcome such disadvantages, an integrated approach has been proposed. It consists of two steps: Firstly, determine the attribute weights by AHP and then rank the alternatives by PROMETHEE. This way, the huge computation in AHP can be greatly reduced, while the attribute weights in PROMETHEE can be obtained objectively.

In fact, the effective integrated approach that incorporate the AHP and PROMETHEE has been used in many areas. For example, a combined AHP and PROMETHEE approach was used to select the most appropriate policy by encouraging people to choose a more sustainable vehicle [8]. In fact, people prefer to express their attitudes in linguistic terms rather than real numbers. The fuzzy set theory proposed by Zadeh [9] provides an effective method by translating the linguistic terms into triangular numbers. The combination of AHP and fuzzy set theory has been received more and more attention. Ahari [10] used the fuzzy AHP (FAHP) in a solution to the portfolio selection problem.

This paper addresses the problem of how to rank a finite number of stocks in a stock market. As a further improvement to existing methods, the combination of CFAHP and PROMETHEE is used to select the suitable stocks. With the application of CFAHP, the uncertain level of fuzzy information can be reduced and objective weights can be obtained. Then in ranking the alternatives by PROMETHEE, the huge computation can be reduced and the results turn out to be more reasonable. With the advantages of the integrated approach, an efficient solution to the stock selection problem can be developed.

The rest of the paper is organized as follows: Section 2 introduces some basic definitions of fuzzy set theory and the second method of constrained fuzzy AHP. Section 3 describes the method of the PROMETHEE. Section 4 applies the combined approach to produce an efficient solution to the stock selection problem. Section 5 draws the conclusions.
2. The method of Constrained fuzzy AHP

2.1. Basic concepts

To facilitate the precise description of our proposed approach, let us first introduce a special type of fuzzy numbers, the triangular fuzzy number, which can be used to replace the linguistic variable under vagueness environment. In this section, some basic definitions of triangular fuzzy number are stated [11][12] as below:

Definition 2.1. A fuzzy number is a special set \( F = \{ (x, \mu_F(x), x \in R) \} \), where \( x \) takes its value on the real line \( R : -\infty < x < +\infty \) and \( \mu_F(x) \) is a continuous mapping form \( R \) into the closed interval \([0 1]\).

Definition 2.2. A triangular fuzzy number is denoted by \( \tilde{A} = (l, m, u) \), where \( l < m < u \). The membership function of \( \tilde{A} \) is defined as

\[
\mu_{\tilde{A}}(x) = \begin{cases} \frac{x-l}{m-l}, & l \leq x \leq m, \\ \frac{u-x}{u-m}, & m \leq x \leq u, \\ 0, & \text{otherwise}. \end{cases}
\]  

If \( l = m = u \), the triangular fuzzy number degenerates to a real number.

Definition 2.3. The distance of the two triangular fuzzy numbers \( \tilde{A}_1 = (l_1, m_1, u_1) \) and \( \tilde{A}_2 = (l_2, m_2, u_2) \) is defined as:

\[
d(\tilde{A}_1, \tilde{A}_2) = \sqrt{\frac{1}{3}[(l_1 - l_2)^2 + (m_1 - m_2)^2 + (u_1 - u_2)^2]}. \]  

2.2. The second method of Constrained fuzzy AHP

CFAHP contains two methods in application. In this paper, we introduce the second method of the CFAHP which using a simple mathematical programming model as the constrained function.

Assume that there are \( n \) attributes and \( m \) alternatives. By using the triangular fuzzy numbers, the fuzzy pair-wise comparison matrix \( \tilde{S} \) of attributes is represents by \( \tilde{s}_{ij} = (l_{ij}, m_{ij}, u_{ij}), \ 1 \leq i, j \leq n \), To keep the symmetry, we set \( (\tilde{s}_{ji}) = (1/ u_{ij}, 1/ m_{ij}, 1/ l_{ij}), \forall i \neq j \) and \( (\tilde{s}_{jj}) = (1, 1, 1), \forall i = j \).

Denote \( S \) as the fuzzy synthetic extent, whose elements are \( s_{li} = (l_{li}, m_{li}, u_{li}), 1 \leq i, j \leq n \) where the indices \( l, m \) and \( u \) represent the lower, medium and upper respectively. \( s_{mi} \) can be evaluated using a crisp formula below:

\[
s_{mi} = \left( \prod_{j=1}^{n} m_{kj} \right)^{1/n} / \sum_{k=1}^{n} \left( \prod_{j=1}^{n} m_{kj} \right)^{1/n} \quad (i, j, k = 1, \ldots, n), \tag{3}
\]

and \( s_{li} \) can be estimated by the simple mathematical programming model

\[
s_{li} = \min \left[ \left( \prod_{j=1}^{n} a_{ij} \right)^{1/n} / \sum_{k=1}^{n} \left( \prod_{j=1}^{n} a_{kj} \right)^{1/n} \right] \quad (i, j, k = 1, \ldots, n), \tag{4}
\]

subject to

\[
a_{kj}l_{kj}, \forall j > k; a_{kj} = 1/a_{kj}, \forall j < k; a_{jj} = 1. \tag{5}
\]

and \( s_{ui} \) can be calculated by the crisp mathematical programming model

\[
s_{ui} = \max \left[ \left( \prod_{j=1}^{n} a_{ij} \right)^{1/n} / \sum_{k=1}^{n} \left( \prod_{j=1}^{n} a_{kj} \right)^{1/n} \right] \quad (i, j, k = 1, \ldots, n), \tag{6}
\]

subject to

\[
a_{kj}l_{kj}, \forall j > k; a_{kj} = 1/a_{kj}, \forall j < k; a_{jj} = 1. \tag{7}
\]

The \( \tilde{s}_i = (s_{li}, s_{mi}, s_{ui}), i = 1, 2, \ldots, n \), can be used to evaluate the relative importance of attributes also.

Different from the existing papers of other authors, we can further convert them into deterministic weight vector with extent analysis technique [13]. The concrete steps are as follows:

Step 1: Compute the possibility of \( \tilde{s}_2 = (s_{2i}, s_{2m}, s_{2u}) \geq \tilde{s}_1 = (s_{1i}, s_{1m}, s_{1u}) \). It is defined as:
The concrete formula is:

\[ v(\tilde{s}_2 \geq \tilde{s}_1) = \sup_{y \geq x} \min(u_{\tilde{s}_1}(x), u_{\tilde{s}_2}(y)) \].

Step 2: Define the possibility degree of a triangular fuzzy number \( \tilde{s} \) which is greater than other triangular fuzzy numbers \( \tilde{s}_i, i = 1, 2, \ldots, n \), as:

\[ v(\tilde{s} \geq \tilde{s}_1, \tilde{s}_2, \ldots, \tilde{s}_n) = v((\tilde{s} \geq \tilde{s}_1) \land (\tilde{s} \geq \tilde{s}_2) \land \ldots \land (\tilde{s} \geq \tilde{s}_n)) = \min_i v(\tilde{s} \geq \tilde{s}_i), \] (9)

let \( d'(\tilde{s}_i) = \min_k d'(\tilde{s}_i \geq \tilde{s}_k), k = 1, 2, \ldots, n, k \neq i \). Determine the deterministic weight vector by

\[ W' = (d'(\tilde{s}_1), d'(\tilde{s}_2), \ldots, d'(\tilde{s}_n)). \] (10)

Step 3: Normalize the vector, thus we get,

\[ W = (d(\tilde{s}_1), d(\tilde{s}_2), \ldots, d(\tilde{s}_n)). \] (11)

Obviously, \( W \) is a non-fuzzy vector.

The CFAHP not only reduces the vagueness level, but also it obtains the deterministic vector. But it needs huge computation to rank the alternatives. To overcome the disadvantages, the PROMETHEE is used to rank the alternatives.

3. The method of PROMETHEE II

This section introduces PROMETHEE II, which provides a complete ranking of alternatives from the best to the worst one. In order to ensure the objectivity of the attribute weights, we use the CFAHP mentioned above to calculate the weight of each criterion \( w_j \).

The implementing procedure for PROMETHEE II is stated as follows:

Step 1: Obtain the pair-wise comparison matrix between the attributes and alternatives.

Step 2: Calculate the deviation between the evaluations of \( a \) and \( b \) on a particular criterion \( g_j \)

\[ d_j(a, b) = g_j(a) - g_j(b) \] (12)

Step 3: Application of the preference function

\[ p_j(a, b) = F_j[d_j(a, b)] \] (13)

Where \( p_j(a, b) \) translates the \( d_j(a, b) \) into a preference degree ranging from 0 to 1. There are six possible types of preference function: (1) usual shape, (2) U-shape function, (3) V-shape function, (4) level function, (5) linear function, (6) Gaussian function.

Step 4: Calculate global preference index:

\[ \pi(a, b) = \sum_{j=1}^{n} p_j(a, b) w_j, \forall a, b \in A \] (14)

Step 5: Evaluate outranking flows:

\[ \Phi^+(a) = \frac{1}{n-1} \sum_{x \in A} \pi(a, x) \] (15)

\[ \Phi^-(a) = \frac{1}{n-1} \sum_{x \in A} \pi(x, a) \] (16)

where \( \Phi^+(a) \) denotes the positive outranking flow and \( \Phi^-(a) \) means the negative outranking flow.

Step 6: Computer net outranking flow:

\[ \Phi(a) = \Phi^+(a) - \Phi^-(a) \] (17)

The alternative with the higher net flow is superior. Usually, we use DECISION LAB software to solve the PROMETHEE II.
4. A numerical example based on CFAHP and PROMETHEE II

Choosing the suitable stocks from Shanghai and Shenzhen stock markets in China is a useful work before the portfolio selection. As an illustration, only four stocks have been chosen, A₁ (stock code: 000002), A₂ (stock code: 000069), A₃ (stock code: 000157), A₄ (stock code: 000411). The combined method contains two stages. The first stage is calculating the weight of attribute by CFAHP. The second stage is ranking the alternatives by PROMETHEE II.

Stage I: Use the CFAHP to calculate the attribute weight.

Step 1: Built the hierarchy structure of decision making problem. According to the company financial report on www.hexun.com, there are five attributes: \( C_1 \)- growth ability, \( C_2 \)-debt paying ability, \( C_3 \)-cash flow, \( C_4 \)-business capacity, \( C_5 \)-profit ability. Fig 1 shows the hierarchy structure.

![Figure 1. The hierarchy structure](image-url)

Step 2: Evaluate each attribute by using linguistic terms from Table 1, which is adopted from Chen [14].

<table>
<thead>
<tr>
<th>Definition</th>
<th>Triangular fuzzy number</th>
</tr>
</thead>
<tbody>
<tr>
<td>Just equal (JE)</td>
<td>(1, 1, 1)</td>
</tr>
<tr>
<td>Equally important (E)</td>
<td>(1/2, 1, 3/2)</td>
</tr>
<tr>
<td>Weakly important (W)</td>
<td>(1, 3/2, 2)</td>
</tr>
<tr>
<td>Moderately important (M)</td>
<td>(3/2, 2, 5/2)</td>
</tr>
<tr>
<td>Strongly important (S)</td>
<td>(2, 5/2, 3)</td>
</tr>
<tr>
<td>Absolutely important (A)</td>
<td>(5/2, 3, 7/2)</td>
</tr>
</tbody>
</table>

To avoid the subjective attribute weights, two experts are invited to give their own decision matrices \( S^1 \) and \( S^2 \). Diagonal elements are defined as (1, 1, 1). It only needs to consider the elements above the diagonal: \( \tilde{s}_{ij} = (l_{ij}, m_{ij}, u_{ij}), 1 \leq i < j \leq n \). Thus, the elements below the diagonal is \( \tilde{s}_{ij} = (1/u_{ij}, 1/m_{ij}, 1/l_{ij}), i > j \).

\[
S^1 = \begin{pmatrix}
(1, 1, 1) & (2/3, 1, 2) & (1, 1, 1) & (1/3, 2/5, 1/2) & (1/3, 2/5, 1/2) \\
(1/2, 1, 3/2) & (1, 1, 1) & (1/2, 1, 3/2) & (1/2, 2, 3/1) & (2/5, 1/2, 2/3) \\
(1, 1, 1) & (2/3, 1, 2) & (1, 1, 1) & (2/5, 1/2, 2/3) & (2/5, 2, 3) \\
(2, 5/2, 3) & (1, 3/2, 2) & (3/2, 2, 5/2) & (1, 1, 1) & (1, 1, 1) \\
(3/2, 5/2) & (1, 1, 1) & (2/5, 1/2, 2/3) & (1/3, 2, 2) & (2/3, 1, 2) \\
(1, 3/2, 2) & (1, 3/2, 2) & (5/2, 3, 7/2) & (1, 3, 2) & (1, 1, 1) \\
(1/2, 2/3, 1) & (2, 5/2, 3) & (1, 1, 1) & (2/3, 3, 1) & (1, 1, 1) \\
(2, 7/1, 3/2, 5) & (3/2, 5, 1/2) & (1, 1, 1) & (2, 5, 1/2, 2) & (1, 1, 1) \\
(1/2, 2, 3/2) & (1/2, 2, 3) & (2/3, 1, 2) & (3/2, 2, 5/2) & (1, 1, 1) \\
(1, 1, 1) & (1, 1, 1) & (1, 1, 1) & (1, 1, 1) & (1, 1, 1)
\end{pmatrix}
\]

\[
S^2 = \begin{pmatrix}
(1, 1, 1) & (2/5, 1/2, 2/3) & (1, 2/3, 1) & (1, 3/2, 2) & (2/3, 1, 2) \\
(3/2, 5/2) & (1, 1, 1) & (1, 1, 1) & (1, 3/2, 2) & (1, 1, 1) \\
(1, 3/2, 2) & (1/2, 2, 3/1) & (1, 1, 1) & (2, 5, 2, 3) & (1/2, 1, 3/2) \\
(1, 2, 2/3, 1) & (2, 7, 1/3, 2/5) & (3/2, 5, 1/2) & (1, 1, 1) & (2, 5, 1/2, 2/3) \\
(1/2, 1, 3/2) & (1/2, 2, 3/1) & (2/3, 1, 2) & (3/2, 2, 5/2) & (1, 1, 1)
\end{pmatrix}
\]

Step 3: To keep the symmetry, the geometric mean method is applied to evaluate the decision matrices. It can be expressed in mathematics as:

\[
S = (S^1 \otimes S^2)^{1/2}.
\]

The concrete results are follows:

\[
S = (S^1 \otimes S^2)^{1/2}.
\]
Step 4: Compute the fuzzy synthetic extents by using the second method of the CFAHP. The results are as follows:

\[ S^5 = \begin{pmatrix} 
(1, 1, 1) & (0.52, 0.71, 1.16) & (0.71, 0.82, 1) & (0.58, 0.78, 1) & (0.47, 0.63, 1) \\
(0.87, 1.41, 1.94) & (1, 1, 1) & (0.71, 1.23, 1.73) & (1.12, 1.41, 1.87) & (1.63, 0.87, 1.16) \\
(1, 1.23, 1.41) & (0.58, 0.82, 1.41) & (1, 1, 1) & (0.89, 1.12, 1.41) & (1.15, 0.82, 2.12) \\
(1, 1.29, 1.73) & (0.54, 0.71, 0.89) & (0.71, 0.89, 1.12) & (1, 1, 1) & (0.63, 0.71, 0.82) \\
(1, 1.58, 2.12) & (0.87, 1.16, 1.58) & (0.47, 0.63, 1) & (1.26, 1.41, 1.58) & (1, 1, 1) 
\end{pmatrix} \]

Step 5: Evaluate the weight vector through the extent analysis method and normalize it. The attribute weight of alternatives is: \( W = (0.1061, 0.2708, 0.2497, 0.1323, 0.2411) \).

Stage II: Rank the alternatives by PROMETHEE II.

Step 6: The pair-wise comparisons between the attributes and alternatives are shown in Table 2.

<table>
<thead>
<tr>
<th>( A_1 )</th>
<th>( A_2 )</th>
<th>( A_3 )</th>
<th>( A_4 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>71.82</td>
<td>61.84</td>
<td>54.81</td>
<td>95.17</td>
</tr>
<tr>
<td>1.46</td>
<td>11.04</td>
<td>53.72</td>
<td>38.21</td>
</tr>
<tr>
<td>57.94</td>
<td>92.16</td>
<td>47.62</td>
<td>43.48</td>
</tr>
<tr>
<td>11.9</td>
<td>15.67</td>
<td>3.94</td>
<td>71.38</td>
</tr>
<tr>
<td>11.9</td>
<td>70.85</td>
<td>63.95</td>
<td>56.64</td>
</tr>
</tbody>
</table>

Step 7: Evaluate the deviation between each evaluation \( A_i \) on criterion \( C_p \). The concrete results are shown in Table 3.

<table>
<thead>
<tr>
<th>( d(A_1, A_2) )</th>
<th>( d(A_1, A_3) )</th>
<th>( d(A_1, A_4) )</th>
<th>( d(A_2, A_3) )</th>
<th>( d(A_2, A_4) )</th>
<th>( d(A_3, A_4) )</th>
<th>( d(A_4, A_4) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>9.98</td>
<td>-9.58</td>
<td>-34.22</td>
<td>-3.77</td>
<td>-58.95</td>
<td></td>
<td></td>
</tr>
<tr>
<td>17.01</td>
<td>-52.62</td>
<td>10.32</td>
<td>7.96</td>
<td>-52.05</td>
<td></td>
<td></td>
</tr>
<tr>
<td>-23.35</td>
<td>-36.75</td>
<td>14.46</td>
<td>-59.48</td>
<td>-44.74</td>
<td></td>
<td></td>
</tr>
<tr>
<td>-9.98</td>
<td>9.58</td>
<td>34.22</td>
<td>3.77</td>
<td>58.95</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7.03</td>
<td>-42.68</td>
<td>44.54</td>
<td>11.73</td>
<td>6.90</td>
<td></td>
<td></td>
</tr>
<tr>
<td>-33.33</td>
<td>-27.17</td>
<td>48.68</td>
<td>-55.71</td>
<td>14.21</td>
<td></td>
<td></td>
</tr>
<tr>
<td>-17.01</td>
<td>52.26</td>
<td>-10.32</td>
<td>-7.96</td>
<td>52.05</td>
<td></td>
<td></td>
</tr>
<tr>
<td>-7.03</td>
<td>42.68</td>
<td>-44.54</td>
<td>-11.73</td>
<td>-6.89</td>
<td></td>
<td></td>
</tr>
<tr>
<td>-40.36</td>
<td>15.51</td>
<td>4.14</td>
<td>-67.44</td>
<td>7.31</td>
<td></td>
<td></td>
</tr>
<tr>
<td>23.35</td>
<td>36.75</td>
<td>-14.46</td>
<td>59.48</td>
<td>44.74</td>
<td></td>
<td></td>
</tr>
<tr>
<td>33.33</td>
<td>27.17</td>
<td>-48.68</td>
<td>55.71</td>
<td>-14.21</td>
<td></td>
<td></td>
</tr>
<tr>
<td>40.36</td>
<td>-15.51</td>
<td>-4.14</td>
<td>67.44</td>
<td>-7.31</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Step 8: Application of the preference function, the usual criterion:

\[ p_j(a, b) = \begin{cases} 
1, & d_j(a, b) > 0; \\
0, & d_j(a, b) \leq 0.
\end{cases} \] \( \forall a, b \in A_i, i = 1 \ldots 4 \)

Then the following is the value of \( p(a, b) \):

<table>
<thead>
<tr>
<th>( p(A_1, A_2) )</th>
<th>( p(A_1, A_3) )</th>
<th>( p(A_1, A_4) )</th>
<th>( p(A_2, A_1) )</th>
<th>( p(A_2, A_2) )</th>
<th>( p(A_2, A_3) )</th>
<th>( p(A_2, A_4) )</th>
<th>( p(A_3, A_1) )</th>
<th>( p(A_3, A_2) )</th>
<th>( p(A_3, A_3) )</th>
<th>( p(A_3, A_4) )</th>
<th>( p(A_4, A_1) )</th>
<th>( p(A_4, A_2) )</th>
<th>( p(A_4, A_3) )</th>
<th>( p(A_4, A_4) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

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Step 9: Calculate global preference index:  
\[ \pi(a, b) = \sum_{j=1}^{k} p_j(a, b)w_j, \forall a, b \in A, i = 1...4. \]  
The value of \( \pi(a, b) \) is shown in Table 5.

<table>
<thead>
<tr>
<th>( p(A_i, A_j) )</th>
<th>( A_1 )</th>
<th>( A_2 )</th>
<th>( A_3 )</th>
<th>( A_4 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>

Step 10: Calculate the positive outranking flow \( \Phi^+(A_i) \) and the negative outranking flow \( \Phi^-(A_i) \). The net outranking flow \( \Phi(A_i) \) is also obtained. The score of the value is shown in Table 6.

<table>
<thead>
<tr>
<th>( \pi(A_i, A_j) )</th>
<th>( A_1 )</th>
<th>( A_2 )</th>
<th>( A_3 )</th>
<th>( A_4 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Phi^+(A_1) )</td>
<td>0.2813</td>
<td>0.7046</td>
<td>0.5148</td>
<td>0.4993</td>
</tr>
<tr>
<td>( \Phi^-(A_1) )</td>
<td>0.7187</td>
<td>0.2954</td>
<td>0.4852</td>
<td>0.5007</td>
</tr>
<tr>
<td>( \Phi(A_1) )</td>
<td>-0.4374</td>
<td>0.4093</td>
<td>0.0295</td>
<td>-0.0014</td>
</tr>
<tr>
<td>Rank</td>
<td>4</td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
</tbody>
</table>

Actually, the alternative with the higher net flow is superior. The results in the Table 6 show that ranking of the alternatives is \( A_2 > A_3 > A_1 > A_4 \). The investor can choose \( A_2 \) and \( A_3 \) as the investment stocks in this case. The final result gives a guide to investor how to choose suitable stocks at the right time.

5. Conclusions

In this paper, an integrated approach is proposed for ranking stocks in exchange market. Compared with existing models, the proposed approach has its own advantages. Investors prefer to express their attitudes by linguistic terms rather than real numbers. In evaluation of attributes, it utilizes the CFAHP to calculate the weight vector through translating the linguistic terms into triangular numbers, which avoid the subjective weight required by PROMETHEE. In ranking of alternatives, the PROMETHEE is used, which also reduces the huge computation in CFAHP. The application of the combined method is convenient. The numerical example has showed the usefulness of this approach to the stock portfolio selection problem.

6. References