The Problem with CAT02 and Its Correction

Changjun Li [1], Esther Perales [2], M Ronnier Luo [1] and Francisco Martínez-Verdú [2]

1: Department of Colour Science, University of Leeds, Leeds LS2 9JT, UK
2: Department of Optics, Pharmacology and Anatomy, University of Alicante, Alicante, Spain

Abstract

It was reported that the CAT02 imbedded in the CIECAM02 suffers from predicting the corresponding colours with negative tristimulus values. To overcome this problem, a mathematical approach is proposed for modifying the CAT02. This approach combines the non-negativity constraint for the corresponding colours' tristimulus values with the minimisation of the colour differences between the tristimulus values of the corresponding colours obtained by visual observations and tristimulus values of the corresponding colours predicted by the model, which resulted in a constrained non-linear optimisation problem. A revised matrix is established using the MATLAB routine " fmincon ".

The performances of the CAT02 with various matrices including the original CAT02 matrix and the new matrix are tested using the visual data sets and the optimum colours. Test results show that the CAT02 with the new matrix successfully predicted corresponding colours' tristimulus vales without negative values for all optimum colours and colour matching functions of two standard observers under the test illuminants considered. However, accuracy with the new matrix for predicting the visual data becomes about 1 CIELAB colour difference unit worse compared with the original CAT02. It seems that the accuracy has to be sacrificed in order to ensure the non-negativity constraint for the tristimulus values of the corresponding colours.

Introduction

Chromatic adaptation can be considered as the most important colour appearance phenomena and has long been extensively studied. A Chromatic Adaptation Transform (CAT) is capable of predicting corresponding colours, which are defined as pairs of colours that have the same colour appearance when one is viewed under one illuminant (for example D65) and the other is under a different illuminant (for example A). The CAT02 was embedded in the CIECAM02 [1] and its full procedure is given in reference [2].

Recently, Brill and Süsstrunk [3, 4, 5] reported that CIECAM02 has problem with predicting lightness perceptual attribute for certain colour samples. It was also reported that the major problem with CIECAM02 is that its domain is smaller than the ICC (International Color Consortium) PCS (Profile Connection Space) [6] which is the common colour space for colour management. During the CIE meeting in Beijing, July 2007, a technical committee: CIE TC8-11 on *CIECAM02-Mathematics* was formed to improve the CIECAM02.

It was found that all colours which fail the CIECAM02 occur on or near the CIE chromaticity locus. Besides, it was widely thought that the main problem comes from the chromatic adaptation transform, which predicts the corresponding colours with negative tristimulus values. Hence, modifications towards the CAT02 were carried out. One of the suggestions by Brill and Süsstrunk [5] is to replace the CAT02 matrix

$$M_{02} = \begin{pmatrix} 0.7328 & 0.4296 & -0.1624 \\ -0.7036 & 1.6975 & 0.0061 \\ 0.003 & 0.0136 & 0.9834 \end{pmatrix} \tag{1}$$

by

$$M_{BS} = \begin{pmatrix} 0.7328 & 0.4296 & -0.1624 \\ -0.7036 & 1.6975 & 0.0061 \\ 0. & 0. & 1 \end{pmatrix} \tag{2}$$

Note that first two rows of the two matrices are the same. The difference occurs at the last row. It can be found that for some colours the CAT02 with the original matrix (equation (1)) predicts corresponding colours with negative tristimulus values, but the CAT02 with the BS (Brill and Süsstrunk) matrix (equation (2)) works fine.

Verdú et al [7] in 2007 computed and visualised the gamut boundary of the optimum colours under multi-light sources and CIE standard colorimetric observer. In order to compare the gamut boundaries under different illuminants, the CAT02 was employed to transform all boundaries under a single reference illuminant. It was found that CAT02 failed to predict many of corresponding colours from the boundaries. In other words, for many colours from the boundaries, the CAT02 predicted their corresponding colours with negative tristimulus values. The CAT02 with the BS matrix given by Brill and Süsstrunk was also used to predict the corresponding colours. It also failed for many of them. Some colours (in terms of chromaticity coordinates shown by circles) which failed the CAT02 under different illuminants were shown in Figure 1, where the full curves are the CIE chromaticity loci (1931 and 1964). It can be seen that all colours are located on the locus of CIE chromaticity diagram. Hence searching a new matrix for the CAT02 is necessary.

This paper describes a general method for generating matrix for the chromatic adaptation transform to overcome this problem.

Mathematical Approach for Searching a New Matrix for the CAT02

CAT02 [2] can be compactly expressed by

$$s_c = M^{-1}\Lambda M s \tag{3}$$

Here $s = (X \quad Y \quad Z)^T$ is the tristimulus values of the object colour under the test illuminant $s_w = (X_w \quad Y_w \quad Z_w)^T$, $s_c = (X_c \quad Y_c \quad Z_c)^T$ is the corresponding colour under the reference illuminant $s_{wr} = (X_{wr} \quad Y_{wr} \quad Z_{wr})^T$. M is the CAT02 matrix given by equation (1). Superscript T denotes the transpose of a vector or a matrix. Λ is a diagonal matrix depending on the test and reference illuminants and the degree of adaptation and is given by

$$\Lambda(D)=\begin{pmatrix} D\beta\frac{R_{wr}}{R_w}+1-D & & \\ & D\beta\frac{G_{wr}}{G_w}+1-D & \\ & & D\beta\frac{B_{wr}}{B_w}+1-D \end{pmatrix}, \text{ with } \beta=\frac{Y_w}{Y_{wr}} \tag{4}$$

Note that D is the degree of adaptation between 0 and 1. It is a full adaptation if $D=1$ and it has no adaptation if $D=0$. Note also that

$$\Lambda(0)=\begin{pmatrix} 1 & & \\ & 1 & \\ & & 1 \end{pmatrix},\quad \Lambda(1)=\begin{pmatrix} \beta\frac{R_{wr}}{R_w} & & \\ & \beta\frac{G_{wr}}{G_w} & \\ & & \beta\frac{B_{wr}}{B_w} \end{pmatrix}, \tag{5}$$

$$\Lambda(D)=D\Lambda(1)+(1-D)\Lambda(0) \tag{6}$$

Thus, it follows from equations (3-6) that

$$s_c(D)=M^{-1}\Lambda(D)Ms=M^{-1}[D\Lambda(1)+(1-D)\Lambda(0)]Ms=DM^{-1}\Lambda(1)Ms+(1-D)s \tag{7}$$

Since $(1-D)s\geq 0$ for any $s\geq 0$, $s_c(D)\geq 0$ if $M^{-1}\Lambda(1)Ms\geq 0$. Thus, the matrix M should be chosen so that

$$s_c(1)=M^{-1}\Lambda(1)Ms\geq 0 \tag{8}$$

Next note that for any positive number μ, $M^{-1}\Lambda(1)Ms\geq 0$ is equivalent to $M^{-1}\Lambda(1)M(\mu s)\geq 0$. Thus we only need consider s defined in terms of chromaticity coordinates, i.e.

$$s=\begin{pmatrix} x \\ y \\ z \end{pmatrix}, \text{ with } x+y+z=1. \tag{9}$$

Let s_1 and s_2 be the two samples, their chromaticity coordinates (x,y) are shown in Figure 1 as crosses. Thus, any point s on the dotted line between s_1 and s_2 can be expressed as:

$$s=\sigma s_1+\omega s_2, \text{ with } \sigma\geq 0,\ \omega\geq 0, \text{ and } \sigma+\omega=1 \tag{10}$$

Now $M^{-1}\Lambda(1)Ms\geq 0$ for any s on the dotted line in Figure 1 is equivalent to $M^{-1}\Lambda(1)Ms_1\geq 0$ and $M^{-1}\Lambda(1)Ms_2\geq 0$. Therefore, the matrix M should be chosen so that for any point s on the CIE chromaticity locus inequality (8) holds.

Finally, the matrix M should be chosen to fit the experimental data well. Eight data sets were used for deriving the CMCCAT2000 [8]. They are the data sets: the Colour Science Association of Japan (CSAJ), Kuo & Luo, Lam and Rigg, Helson et al., LUTCHI, Breneman, Braun & Fairchild, and McCann. For details about each of the data sets, see reference [8] and references there. However, there is a strong objection to using the McCann data because its viewing conditions, mainly using chromatic illuminants under low luminance level, are largely different from the other data sets. The CAT02 matrix given by equation (2) was obtained by excluding the McCann data

[9]. Here the McCann data set is also excluded. For measuring the fit to the experimental data, CIELAB colour difference [10] is used.

Thus, the above discussions lead to a constraint and non-linear optimization problem. MATLAB routine **fmincon** was used for solving the problem. 82 illuminants were used for the optimisation. Illuminants details including tristimulus values and colour temperatures are listed in Table 1. The chromaticity coordinates for all illuminants are shown in Figure 2. The dotted curves in Figure 2 are the CIE 1931 (black) and 1964 (blue) chromaticity loci. The full curve is the convex boundary of the two loci. As the analysis above, it is sufficient to use the points on the convex boundary for the constraints (equation (8)). The matrix finally found is given as follows:

$$M_{new}\begin{pmatrix} 1.007245 & 0.011136 & -0.018381 \\ -0.318061 & 1.314589 & 0.003471 \\ 0 & 0 & 1 \end{pmatrix} \tag{11}$$

Note that the last row of the matrix is the same as the last row of the BS matrix (equation (2)).

Performance Evaluation with New Matrix

The CAT02 [1, 2] with four different matrices were used for the testing. The four matrices are the original CAT02 matrix defined by equation (1), the new matrix defined by equation (11), the BS matrix defined by equation (2), and the HPE matrix [1] given by equation (12) below:

$$M_{HPE} = \begin{pmatrix} 0.38971 & 0.68898 & -0.07868 \\ -0.22981 & 1.1834 & 0.04641 \\ 0 & 0 & 1 \end{pmatrix} \tag{12}$$

Firstly, the visual experimental data sets were used for testing the CAT02 performance with different matrices. The average CIELAB colour differences are listed in Table 2. There are 21 data sets from seven groups of data. The last row lists the mean colour difference values of all 21 data sets. As expected, the CAT02 with its original matrix performs the best since the matrix was derived based on the 21 data sets via minimising the overall mean colour difference without any further constraints. The CAT02 with BS matrix becomes slight worse compared with the CAT02 with its original matrix. The CAT02 with the HPE matrix ranks the third. The CAT02 with the new matrix performs the worst, which can also be expected since more constraints were added when the matrix was optimised.

Next, the CAT02 with the four matrices were tested using the two CIE colorimetric standard observers as samples to see the corresponding colour's non-negativity. These testing colours correspond to those loci colours in Figure 1. They provide a severe test for checking robustness of the CATs. In total, there are 942 samples. 81 illuminants were used as test illuminants and D65 was used as reference illuminant. The test results are listed in Table 3, where NOI means the number of illuminants under which the CAT02 with one of the four matrices: Original, BS, HPE, and New predicted corresponding colours with negative tristimulus values and NOT means the number of times under all illuminants the CAT02 predicted corresponding colours with negative tristimulus values. It can be seen that the CAT02 with original matrix predicted corresponding colours with negative tristimulus values under 65 out of 81 illuminants, and the corresponding colours with tristimulus values occurred 28861 times under the

65 illuminants. When the BS matrix was used, the NOI=63 and NOT=6710. When the HPE matrix was used, the NOI=39 and NOT=1517. But when the new matrix was used, NOI=0, which means that CAT02 with the new matrix can successfully predict corresponding colours under all illuminants considered. Thus, it can be expected that the CAT02 with the new matrix can predict corresponding colours for all samples on or inside the chromaticity locus.

Finally, the CAT02 with each of the four matrices was tested using the generated optimum colours [7] under the 81 test illuminants shown in Table 1 to check the non-negativity of the corresponding colour's tristimulus values. CIE illuminant D65 was used as the reference illuminant. For each illuminant, 99 lightness planes starting from L*=1 to L*=99 at 1 L* unit length interval. Under the 81 illuminants, the total number of optimum colours (TNOC) generated is 20,252,868. The test results are listed in Table 4, where NOI means the number of illuminants under which the CAT02 predicted corresponding colours with negative tristimulus values, and where the NOCF means the number of optimum colours which failed the CAT02 under certain illuminants. For example, for CAT02 with the original matrix, NOI=62, and NOCF=2,708,615. Thus, CAT02 with the original matrix failed to predict corresponding colours for about 13 percent of the optimum colours. For the CAT02 with BS matrix, NOI=61, NOCF= 697,945. Thus, the CAT02 with the BS matrix has a problem for over 3 percent of the optimum colours. For the CAT02 with HPE matrix, NOI=38, NOCF=41,239. In this case, the CAT02 predicted corresponding colours with negative tristimulus values for about 0.2 percent optimum colours. While for CAT02 with the new matrix, NOI=0, and NOCF=0. Thus, it successfully predicted corresponding colours for all the optimum colours. According to the none-negativity test for the tristimulus values of the corresponding colours, the best is the CAT02 with the new matrix, followed by the CAT02 with the HPE matrix, the CAT02 with the BS matrix third, and the CAT02 with the original matrix the worst. The results further confirmed that the present mathematical approach to modify the chromatic adaptation matrix for the CAT02 is correct.

Conclusions

A mathematical approach has been proposed for searching the chromatic adaptation matrix for the CAT02. This approach combines the non-negativity constraint for the corresponding colours' tristimulus values with the minimisation of the colour differences between the tristimulus values of the corresponding colours obtained by visual observations and tristimulus values of the corresponding colours predicted by the model, which resulted in a constrained non-linear optimisation problem. For satisfying the non-negativity constraints, it has been shown that it is sufficient if samples on the convex boundary of the two CIE standard colorimetric observers satisfy the constraints, which largely reduces the complexity of the problem. For fitting the experimental results, all the data sets used to derive the original matrix have been used again here. A new matrix has been found using the MATLAB routine " fmincon ".

The performances of the CAT02 with the Original, BS, HPE, and New matrices have been tested. According to the test for fitting the visual results, the ranking order from the best to the worst is the CAT02 with the Original, BS, HPE, and New matrices respectively. The accuracy of the CAT02 with the new matrix is about 1 CILELAB colour difference unit larger than that of the original CAT02. While according to the non-negativity test using the CIE standard colorimetric observers and the optimum colours under 81 illuminants, the ranking order is exactly the reverse order of the fitting visual result test. The CAT02 with the new matrix is the only one combination which successfully predicted corresponding colours for all optimum colours and the CIE standard colorimetric observers. Thus it seems that the accuracy has to be sacrificed in order to ensure the non-negativity constraint for the corresponding colours. However, 1 CIELAB colour difference unit seems to be small since observer variation in the chromatic adaptation experiments might be much higher than that.

It was thought that the CIECAM02 problem is also solved by replacing the original CAT02 matrix by the new one. Unfortunately, it was found that some of the optimum colours also failed the CIECAM02. It was also found that the failure of the CIECAM02 using the optimum colours has a relation with the correlated colour temperature (CCT) of the test illuminant. Further work toward repairing the CIECAM02 is under way.

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Illuminants	X	Y	Z	T_c (K)
P20	127.43	100	14.52	2,000
P30	108.13	100	39.34	3,000
P40	100.98	100	64.43	4,000
P50	98.14	100	86.23	5,000
P60	97.08	100	104.30	6,000
P70	96.78	100	119.09	7,000
P80	96.84	100	131.19	8,000
P90	97.05	100	141.16	9,000
P100	97.33	100	149.44	10,000
P110	97.61	100	156.38	11,000
P120	97.89	100	162.25	12,000
P130	98.16	100	167.27	13,000
P140	98.41	100	171.59	14,000
P150	98.64	100	175.35	15,000
P160	98.84	100	178.64	16,000
P170	99.03	100	181.53	17,000
P180	99.21	100	184.11	18,000
P190	99.37	100	186.40	19,000
P200	99.51	100	188.46	20,000
P400	100.98	100	207.43	40,000
P600	101.48	100	213.4	60,000
P800	101.72	100	216.3	80,000
P1000	101.87	100	218	100,000
D40	99.66	100	60.97	4,000
D50	96.42	100	82.51	5,000
D55	95.68	100	92.13	5,500
D60	95.26	100	100.85	6,000
D65	95.04	100	108.87	6,500
D70	94.96	100	116.01	7,000
D75	94.97	100	122.60	7,500
D80	95.02	100	128.44	8,000
D90	95.24	100	138.62	9,000
D100	96.08	100	148.34	10,000
D110	95.78	100	154.12	11,000
D120	96.05	100	160.08	12,000
D130	96.30	100	165.17	13,000
D140	96.53	100	169.56	14,000
D150	96.74	100	173.36	15,000
D160	96.93	100	176.69	16,000
D170	97.10	100	179.63	17,000
D180	97.26	100	182.23	18,000
D190	97.41	100	184.55	19,000
D200	97.54	100	186.62	20,000
D400	98.85	100	205.66	40,000
D600	99.28	100	211.55	60,000

D800	99.5	100	214.38	80,000
D1000	99.62	100	216.03	100,000
A	109.85	100	35.58	2,856
C	98.07	100	118.22	6,776
E	100	100	100	5,455
HP1	128.44	100	12.54	1,959
HP2	114.90	100	25.58	2,506
HP3	105.57	100	39.82	3,144
HP4	100.38	100	62.97	4,002
HP5	101.68	100	67.61	4,039
F1	92.87	100	103.77	6,430
F2	99.18	100	67.39	4,225
F3	103.80	100	49.93	3,446
F4	109.20	100	38.88	2,938
F5	90.9	100	98.82	6,346
F6	97.34	100	60.26	4,148
F7	95.04	100	108.74	6,496
F8	96.43	100	82.42	4,998
F9	100.38	100	67.94	4,149
F10	96.38	100	82.35	4,999
F11	100.96	100	64.35	3,999
F12	108.12	100	39.28	3,000
FL3.1	109.27	100	38.69	2,932
FL3.2	101.99	100	65.18	3,965
FL3.3	91.69	100	99.12	6,280
FL3.4	109.54	100	37.78	2,904
FL3.5	102.11	100	70.25	4,086
FL3.6	96.89	100	80.88	4,894
FL3.7	108.38	100	38.82	2,979
FL3.8	99.69	100	61.29	4,006
FL3.9	97.43	100	81.05	4,853
FL3.10	97.06	100	83.87	5,000
FL3.11	94.50	100	96.72	5,854
FL3.12	108.40	100	39.31	2,984
FL3.13	102.85	100	65.65	3,896
FL3.14	95.51	100	81.55	5,045
FL3.15	95.10	100	109.06	6,509

Table 1: The tristimulus values for *82* illuminants and their correlated colour temperatures.

	Matrix for the CAT02			
Data Sets	Original	B&S	HPE	New
csaj.da.dat	5.36	5.57	6.33	6.59
kuo.da.dat	6.94	6.83	7.94	7.36
kuo.dt.dat	3.57	3.68	4.57	3.89
lam.da.dat	5.61	6.06	7.18	8.06
helson.da.dat	7.12	7.45	7.99	9.39
lutchi.da.dat	7.92	7.93	7.92	10.34
lutchi.dd.dat	6.88	6.88	6.53	7.76
lutchi.dw.dat	7.46	7.85	9.14	7.63
Brene.p1.dat	7.09	7.4	6.98	10.11
Brene.p2.dat	5.3	5.95	4.97	8.23
Brene.p3.dat	7.85	8.53	9.29	9.63
Brene.p4.dat	9.99	10.83	11.74	12.6
Brene.p6.dat	8.95	9.31	8.36	10.78
Brene.p8.dat	6.41	7.6	8.86	10.29
Brene.p9.dat	15.51	16.19	17.44	17.79
Brene.p11.dat	5.29	5	3.68	7.25
Brene.p12.dat	5.64	5.76	5.04	6.43
RIT1.dat	2.92	2.93	3.48	3.8
RIT2.dat	5.05	5.04	5.46	5.18
RIT3.dat	4.19	4.39	6.01	5
RIT4.dat	3.38	3.26	3.69	4.15
Mean	**6.59**	**6.88**	**7.23**	**8.20**

Table 2: Average CIELAB colour difference values under each data set for the CAT02 with CAT02, Brill & Süsstrunk, HPE, and New matrices.

	CAT02 with different matrix			
	Original	BS	HPE	New
NOI	65	63	39	0
NOT	28861	6710	1517	0

Table 3: Number of illuminants (NOI) under which the CAT02 predicted corresponding colours with negative tristimulus values and number of times (NOT) under all illuminants the CAT02 predicted corresponding colours with negative tristimulus values

	CAT02 with different matrix			
	Original	BS	HPE	New
NOI	62	61	38	0
TNOC	2,708,615	697,945	41,239	0

Table 4: Number of illuminants (NOI) under which the CAT02 predicted corresponding colours with negative tristimulus values and number of optimum colours (NOCF) (from total number of optimum colours (TNOC) of 20,252,868) which failed the CAT02.

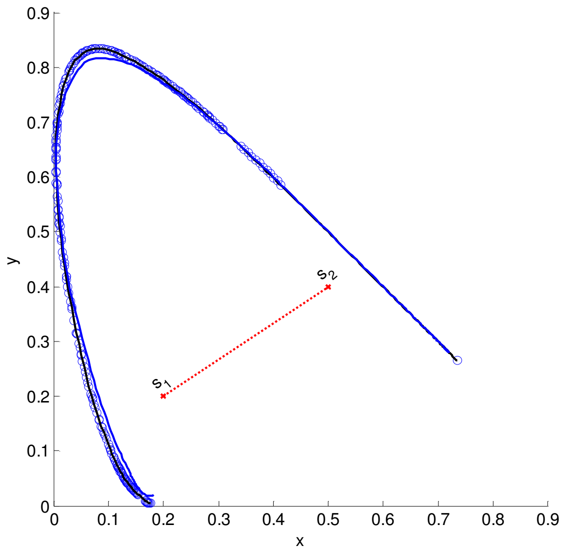

Figure 1: Chromaticity coordinates (circles) for the colours which failed the CAT02. The full curves are the CIE chromaticity loci (1931 and 1964).

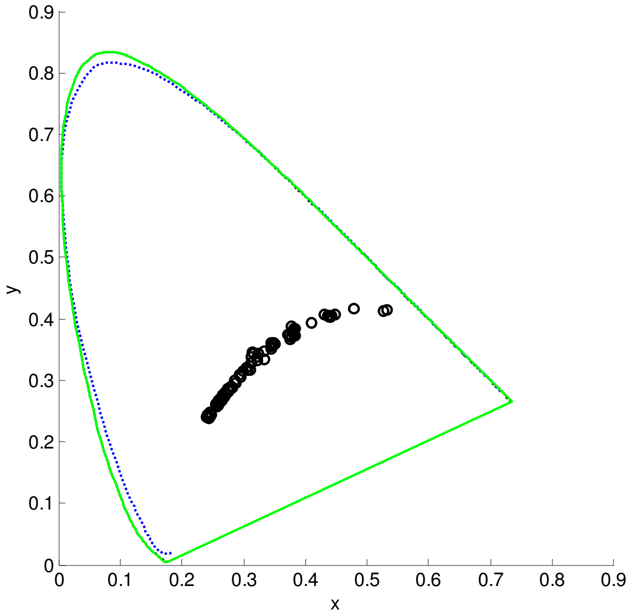

Figure 2: Chromaticity coordinates (circles) for 82 illuminants. The dotted lines are the CIE1931 (black) and 1964 chromaticity loci. The full curve is the convex boundary of the two standard observers.