

Amplitude calculations for 3-D Gaussian beam migration using complex-valued traveltimes

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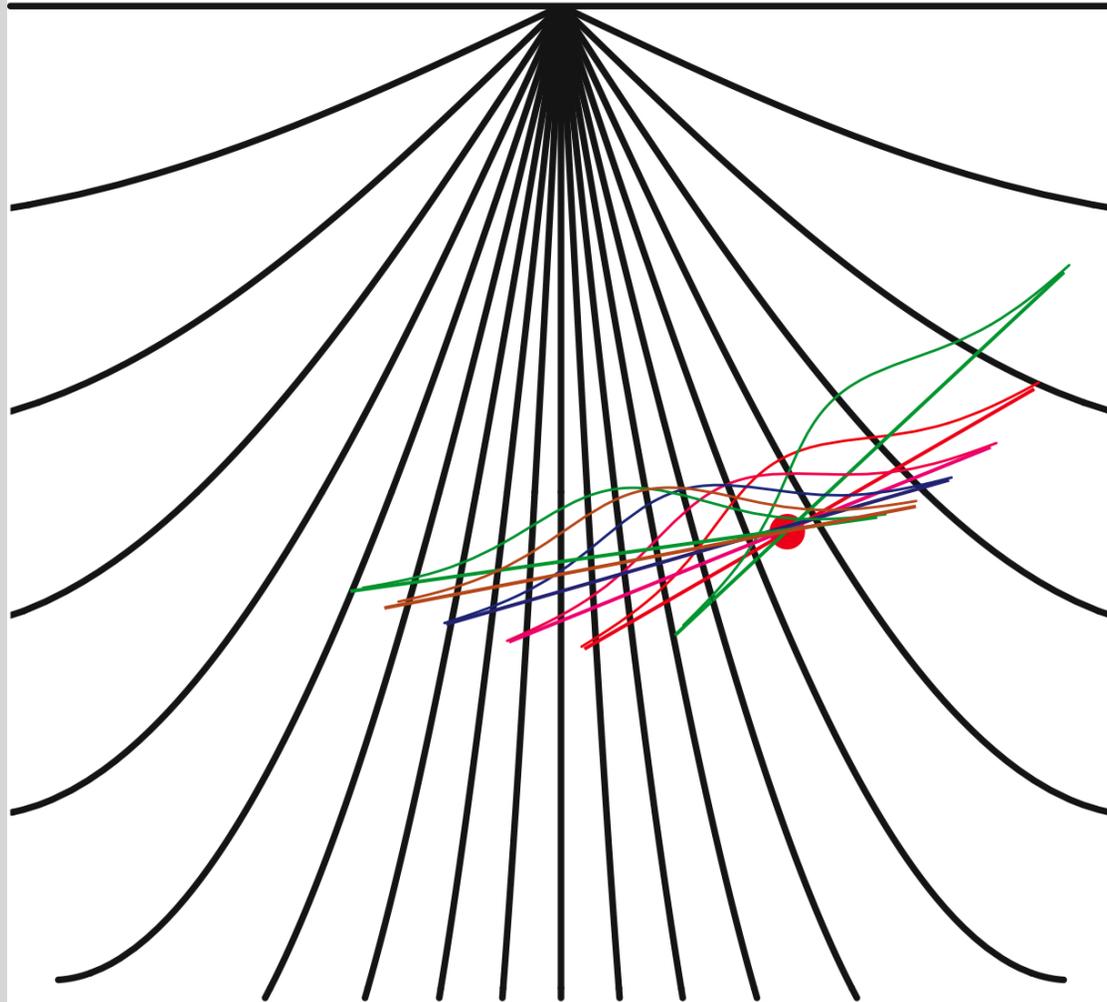
***Presenter**



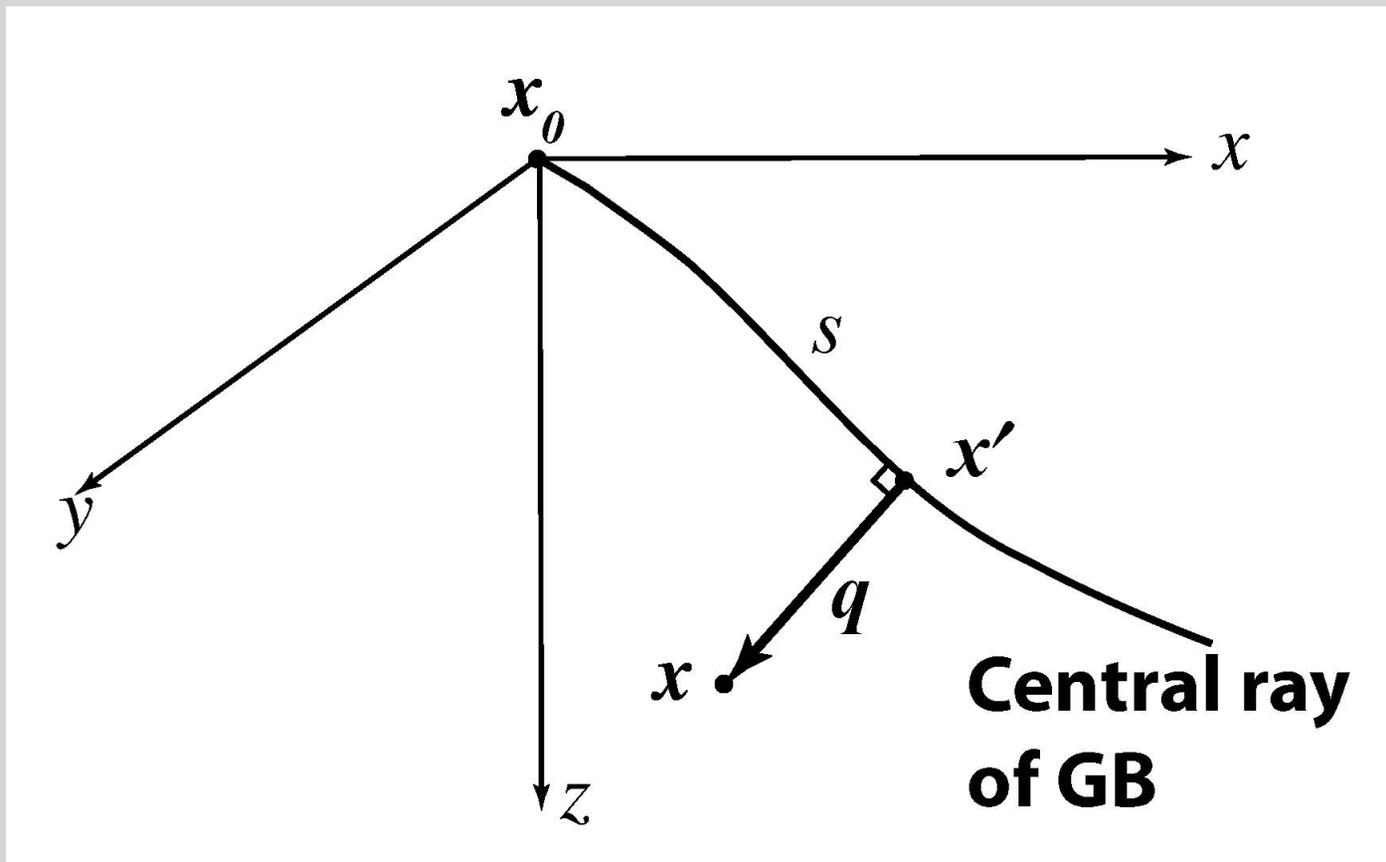
Gaussian beams

- **Extend ray theoretic modeling to complex exponents**
- **Waves: sum over beams**

Sum over beams for Green's function



One beam



Kirchhoff migration with GB's

- **Requires two Green's functions**
Each requires integral/sum over beams
 - **2D: two additional integrals**
 - **3D: four additional integrals**

Ross Hill's method

- **Methods of steepest descent for integrals with complex exponents**
 - **2D: two integrals \longrightarrow one**
 - **3D: four integrals \longrightarrow two**

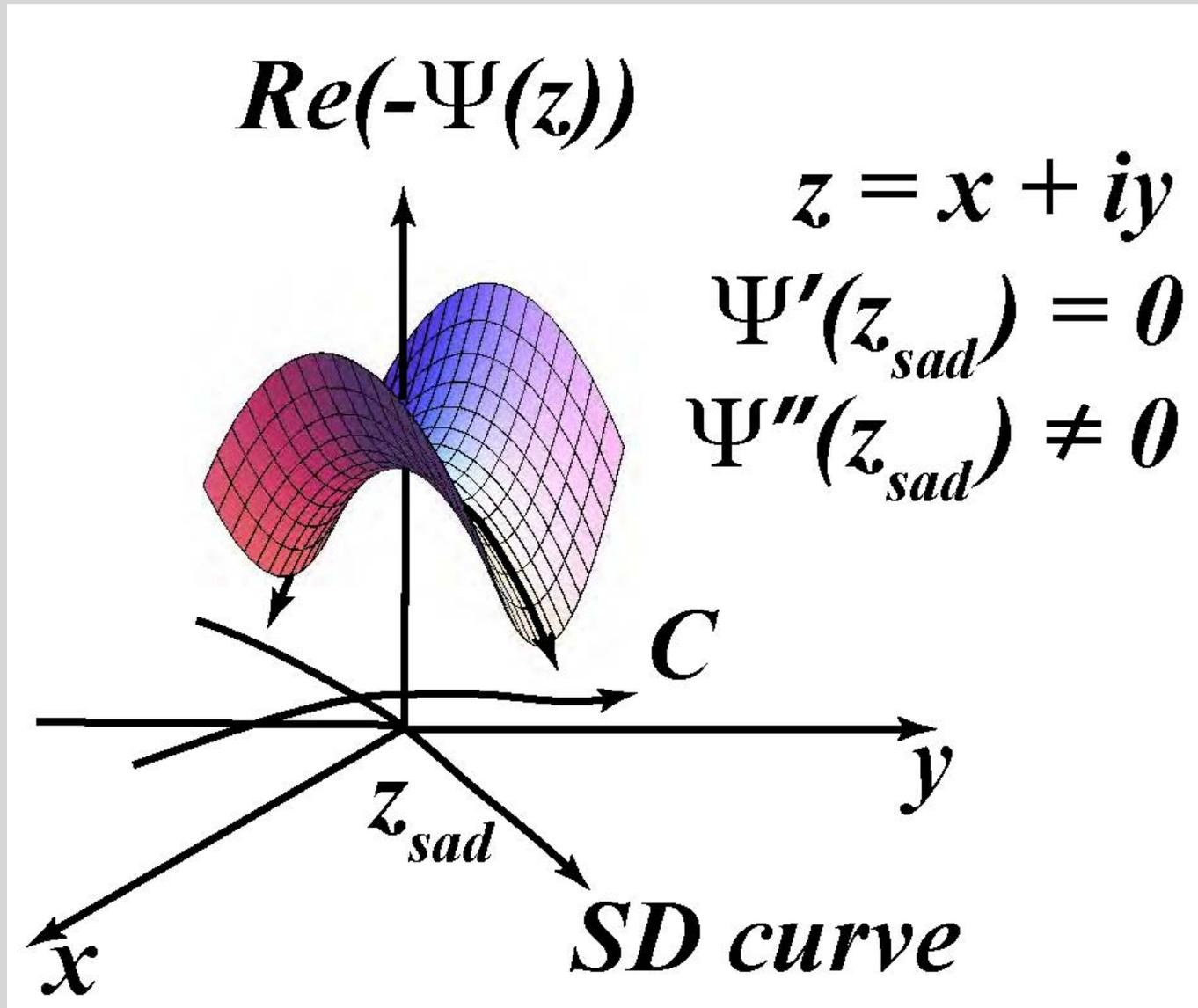
Prototype integral in 1D

$$I(\omega) = \int f(z) \exp\{-\omega\Psi(z)\} dz$$

$$\frac{d\Psi}{dz} = 0, \quad z = z_{sad}$$

$$\Psi_{zz} = \left. \frac{d^2\Psi}{dz^2} \right|_{z=z_{sad}} \neq 0$$

Method of steepest descent



Objective

- Deform contour C on to steepest descent path
- $Im(\Psi)$ constant on steepest descent path \longrightarrow
- Only $Re(\Psi)$ varies on SD path

Formula

$$I(\omega) = \int f(z) \exp\{-\omega\Psi(z)\} dz$$

$$I(\omega) \sim \sqrt{\frac{2\pi}{\omega\Psi_{zz}}} f(z_{sad}) \exp\{-\omega\Psi(z_{sad})\}$$

Formula

$$I(\omega) \sim \sqrt{\frac{2\pi}{\omega \Psi_{zz}}} f(z_{sad}) \exp \{ -\omega \Psi(z_{sad}) \}$$

Phase comes from

$$- \arg(\Psi_{zz})/2$$

Iterated Method of Steepest Descent

$$\int f(\mathbf{z}) \exp\{-\omega\Psi(\mathbf{z})\} dz_1 dz_2, \quad \mathbf{z} = (z_1, z_2)$$

$$\nabla_{\mathbf{z}}\Psi(\mathbf{z}) = \mathbf{0}, \quad \mathbf{z} = \mathbf{z}_{sad} = (z_{1sad}, z_{2sad})$$

$$\det[\Psi] \neq 0. \quad \Psi = \left[\frac{\partial^2 \Psi}{\partial z_i \partial z_j} \right]$$

Iterated Method of Steepest Descent

$$\int f(\mathbf{z}) \exp\{-\omega \Psi(\mathbf{z})\} dz_1 dz_2, \quad \mathbf{z} = (z_1, z_2)$$

$$I(\omega) \sim \frac{2\pi}{\omega} \frac{f(\mathbf{z}_{sad})}{\sqrt{\det[\Psi(\mathbf{z}_{sad})]}} \exp\{-\omega \Psi(\mathbf{z}_{sad})\}$$

Prototype integral over beams

$$I(\mathbf{x}, \mathbf{x}_s, \mathbf{x}_r, \omega) = \int_{D_{x_s}} \frac{dp'_{s1} dp'_{s2}}{p'_{s3}} \int_{D_{x_r}} \frac{dp'_{r1} dp'_{r2}}{p'_{r3}}$$

$$\cdot A_{GB}^*(\mathbf{x}'_s, \mathbf{x}_s) A_{GB}^*(\mathbf{x}'_r, \mathbf{x}_r) \exp\{-\omega \Psi(\mathbf{x}'_s, \mathbf{x}_s, \mathbf{x}'_r, \mathbf{x}_r)\}$$

$$\Psi(\mathbf{x}'_s, \mathbf{x}_s, \mathbf{x}'_r, \mathbf{x}_r) = i[T(\mathbf{x}'_s, \mathbf{x}_s) + T(\mathbf{x}'_r, \mathbf{x}_r)]^*$$

**p' 's: initial transverse slownesses
in Cartesian coordinates**

Hill's transformation of coordinates

Offset slownesses

$$p'_{h1} = p'_{r1} - p'_{s1}, \quad p'_{h2} = p'_{r2} - p'_{s2}$$

Midpoint slownesses

$$p'_{m1} = p'_{r1} + p'_{s1}, \quad p'_{m2} = p'_{r2} + p'_{s2}$$

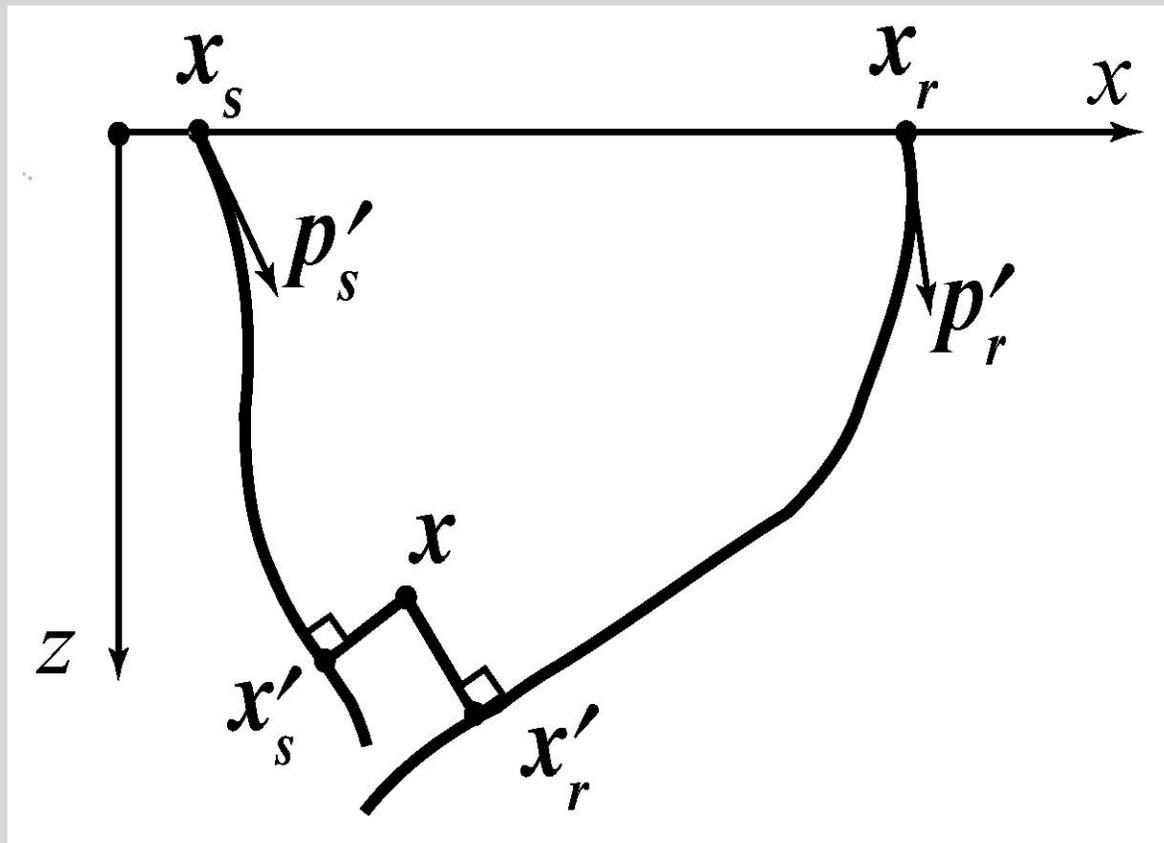
- **Apply iterated steepest descent to integrals over offset slownesses**
- **Integrate numerically in midpoint slownesses**

After iterated steepest descent calculation

$$I \sim \frac{\pi}{2\omega} \int_{D_m} \frac{dp'_{m1} dp'_{m2}}{\sqrt{\det[\Psi]}} A_{GB}^*(\mathbf{x}'_s, \mathbf{x}_s) A_{GB}^*(\mathbf{x}'_r, \mathbf{x}_r) \cdot \exp\{-i\omega[T(\mathbf{x}'_s, \mathbf{x}_s) + T(\mathbf{x}'_r, \mathbf{x}_r)]^*\}, \mathbf{p}_h = \mathbf{p}'_{h \text{ sad}}(\mathbf{p}'_m)$$

$$\Psi = \left[\frac{\partial^2 \Psi}{\partial p_{hI} \partial p_{hJ}} \right] \quad \det[\Psi] \neq 0$$

Typical p_m



$$\frac{\partial^2 \Psi}{\partial p'_{hj} \partial p'_{hk}} = i \left[\frac{\partial^2 T(\mathbf{x}'_r, \mathbf{x}_r)}{\partial p'_{rj} \partial p'_{rk}} + \frac{\partial^2 T(\mathbf{x}'_s, \mathbf{x}_s)}{\partial p'_{sj} \partial p'_{sk}} \right]^*, \quad j, k = 1, 2$$

??????

We do know about

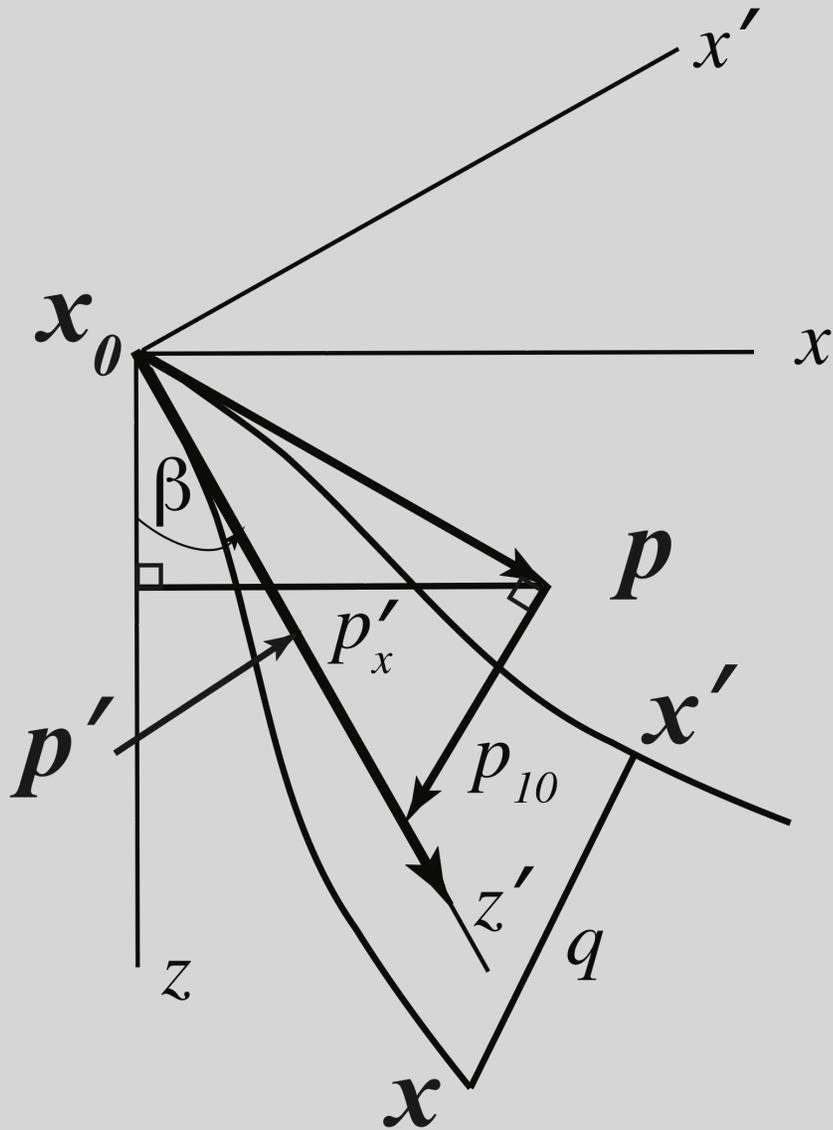
$$\mathbf{M} = \mathbf{T}_q = \left[\frac{\partial^2 T}{\partial q_\mu \partial q_\nu} \right]_{\mathbf{q}=\mathbf{0}}$$

Objective

Transform derivatives in p'_r and p'_s
to derivatives in q_r and q_s

Two steps

2D case



- $x_0 = x_s$ or x_r

- Rotate $\longrightarrow \beta$

- $p'_x \longrightarrow p_{10}$

- $p_{10} \longrightarrow q$

3D Case

- **Two angles, β_1, β_2**
- **Rotation matrix, Γ**

3D Case

- Two angles, β_1, β_2
- Rotation matrix, Γ
- Defines ray-centered p_0 in terms of Cartesian p'
- Ray-centered ray theory:

$$p_0 \longrightarrow q$$

Matrix Ψ

$$\Psi = -i\omega_r w_0^2 \left\{ \frac{1}{v_{0r}(0)} \Gamma_r^T Q_{GBr}^{-1} Q_{2r} \Gamma_r + \frac{1}{v_{0s}(0)} \Gamma_s^T Q_{GBs}^{-1} Q_{2s} \Gamma_s \right\}^*$$

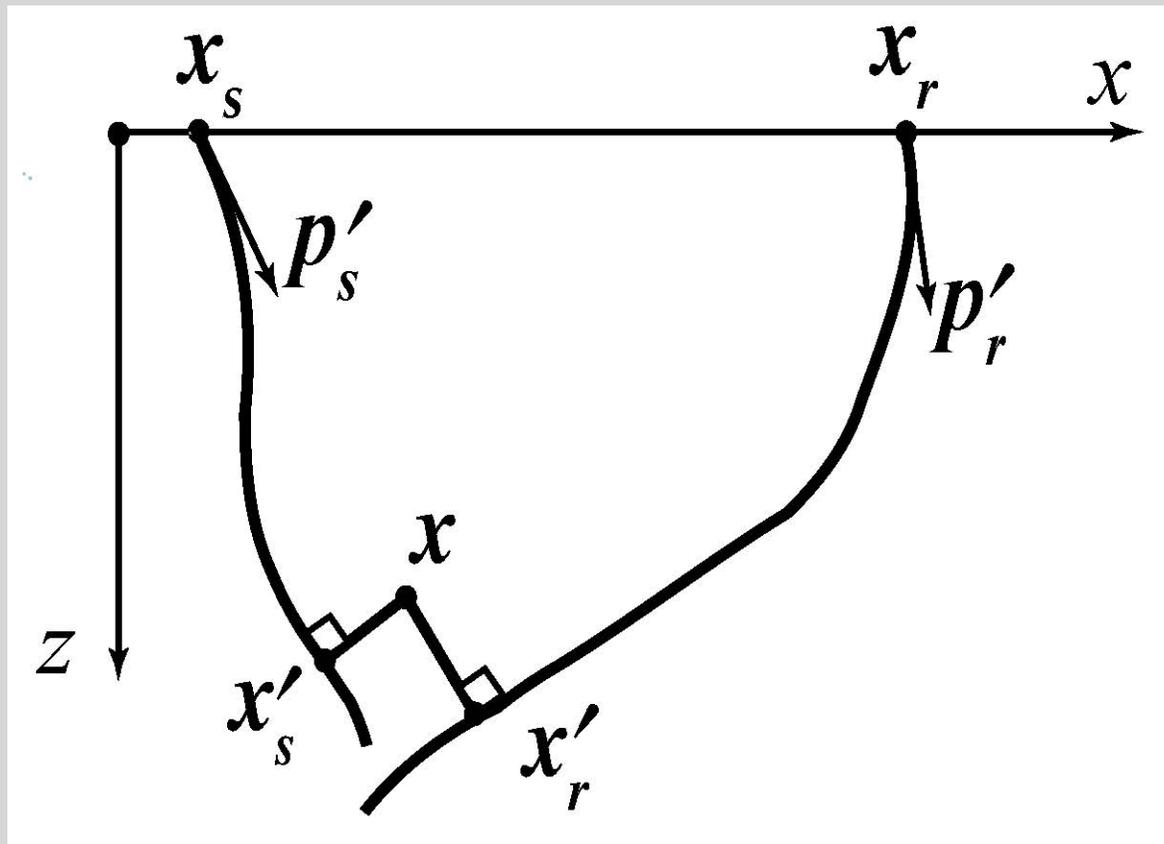
- Γ 's: Rotation matrices
- Q_2 's: Červený's fundamental solutions for point sources

Matrix Ψ

$$\Psi = -i\omega_r w_0^2 \left\{ \frac{1}{v_{0r}(0)} \Gamma_r^T Q_{GBr}^{-1} Q_{2r} \Gamma_r + \frac{1}{v_{0s}(0)} \Gamma_s^T Q_{GBs}^{-1} Q_{2s} \Gamma_s \right\}^*$$

**Compute Γ 's on central rays,
not on ray through image
point!**

Typical p_m



Summary and Conclusions

- Start from double integrals in p'_s and in p'_r
- Transform to double integrals in p'_h and p'_m
- Apply steepest descent to double integral in p'_h

Summary and Conclusions

- Need determinant of matrix Ψ
- Transform derivatives in p'_h to derivatives in q for rays from source and receiver
- Obtain the Hessian matrix $[\Psi]$