Coarse-to-Fine Multiscale Affine Invariant Shape Matching and Classification

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Abstract

In this paper, a multiscale algorithm for matching and classifying 2-D shapes is developed. The algorithm uses the 1-D Dyadic Wavelet Transform (DWT) to decompose a shape’s boundary into multiscale levels. Then the coarse to fine matching and classification are achieved in two stages. In the first stage, the global features are extracted by calculating the curve moment invariants of the approximation coefficients. By calculating the normalized cross correlation of the 1-D triangle area representation of the detail coefficients, the local similarity is achieved by the second stage. The proposed algorithm is invariant to the affine transformation and to the boundary starting point variation. In addition, the results demonstrate that the new algorithm is not sensitive to small boundary deformations.

1. Introduction

In the last decade, the matching and classifying of shapes that undergo geometrical transformations has attracted a great deal of attention in shape analysis and recognition systems. After a shape in an image is located and segmented, representation and description techniques are used to efficiently characterize the shape. Then, a similarity/dissimilarity measure is employed to find the closest match to the shape(s) from a dataset. This measure facilitates the unsupervised classification (clustering) of the shapes into meaningful or useful clusters.

Typically, there are two main types of unsupervised classification (clustering) algorithms, namely partitional and hierarchical [3]. The hierarchical clustering procedures are among the most popular ones due to their conceptual simplicity, and can be represented by a tree, called a dendrogram, by using the linkage rules. The later determine how the clusters are connected in order to form new ones, and what the new cluster distances will be.

The selection and combination of a shape’s features is crucial to shape matching and classification. The multiscale decomposition of the shapes using Wavelet Transform (WT) is adopted in this research because the transform captures both local and global shape features. Also, the multiscale decomposition techniques are considered to be the most promising ones in shape analysis. The reason for this is that such techniques have the capability to such methods to take into account the evolution of a shape, as it is subjected to more and more smoothing.

1.1. Related Work

Many researchers have used the Wavelet Transform (WT) for matching and classification. Some have attempted to apply the WT in 2-D domain (region-based techniques); others have chosen to apply the transform to a 1-D shape boundary (contour-based technique). Two-dimensional geometric moments have been used with the WT by [8] and [6]. Other techniques have consisted of combinations of the WT and Fourier Transform (FT) [2], the wavelet multiscale features and the Hopfield neural networks [5], or the line moment, WT, and FT. [1]. Multiscale moment-based techniques have been employed in [4] for object matching and recognition, where the moments were computed for different boundary signature scales.

In [4], a 1-D Continuous Wavelet Transform (CWT) was adopted to represent the shape boundary (this representation is called the w-representation). For classification, the Fourier descriptors has been combined as global features and the w-representation as local features. Wavelet moment invariants has been derived in [8] to distinguish between seemingly similar shapes. For the classification application, these invariants have been used with a min-distance classifier. In [9], the authors have proposed a wavelet-based rotational invariant target classification system. The first three central moments have been computed for only the third coarse scale of the 2-D wavelet transform of images.
Differences classifiers, including the min-distance classifier, k-nearest neighbor, and the feed-forward back propagation computational NN has been tested.

In this paper, a new algorithm, based on the multiscale decomposition of the wavelet representation, is introduced. The algorithm uses both the approximation and the detail coefficients to match and classify shapes.

2. Shape Matching and Classification Algorithm

The proposed algorithm in Figure 1 extracts the outer boundary of the shape by using one of the known boundary extractor algorithms (in this study, the bug following technique is used [7]). The extracted 2-D boundary is then converted into two 1-D sequences ($x(k)$ and $y(k)$).

If the shape is subjected to a geometric transformation such as the affine transformation, then the boundary is distorted by the same transformation.

2.1. Feature extraction

After the shape boundary has been extracted and parameterized, a 1-D Dyadic Wavelet Transform (DWT) is applied to the boundary sequences $x(k)$ and $y(k)$. Also, the 1-D DWT is applied to $x(k)$ and $y(k)$ to obtain the different approximation and detail coefficients. The boundary sequences ($x(k)$ and $y(k)$) are decomposed to a certain wavelet scale level $L$,

$$
\begin{bmatrix}
x(k) \\
y(k)
\end{bmatrix} = \begin{bmatrix}
\sum_{n} a_{x,l,n} \tilde{\phi}_{L,n}(k) \\
\sum_{n} a_{y,l,n} \tilde{\phi}_{L,n}(k)
\end{bmatrix} + \sum_{l=1}^{L} \sum_{n} d_{x,l,n} \tilde{\psi}_{l,n}(k) + \sum_{l=1}^{L} \sum_{n} d_{y,l,n} \tilde{\psi}_{l,n}(k),
$$

(1)

where $a_{x,l,n}$ are the approximation coefficients for $x(k)$ at scale $L$, $a_{y,l,n}$ are the approximation coefficients for $y(k)$ at scale $L$, $d_{x,l,n}$ are the detail coefficients for $x(k)$ at scale $l$, $d_{y,l,n}$ are the detail coefficients for $y(k)$ at scale $l$, $\tilde{\phi}_{L,n}(k)$ are the dual scaling functions at scale $L$, and $\tilde{\psi}_{l,n}(k)$ are the dual wavelet functions at scale $l$. Figure 2 illustrates the approximation and the detail coefficients plotted as 2-D contours.

![Figure 2. The Multiscale representation of a shape using wavelet decomposition.](image)

The affine invariant curve moments in [11] are computed for the approximation coefficients at all the scale levels. The Moment invariants are used for matching of two different curves. There are definite advantages of using the moment invariants in the novel algorithm: the representation becomes invariant to the starting point of the shape boundary, the algorithm is more robust to the affine transformation, and the moments reduce the size of the extracted feature vectors from the shapes. To compute the similarity by using the wavelet detail coefficients, a preprocessing is conducted to ensure the affine invariance of the coefficients. The details for $x(k)$ and $y(k)$ are represented by computing the area of the triangle that is formed by any three points on the detail contour.

For the affine transformation, the area is multiplied by a constant value which is equal to the determinant of the transformation. If the contour is formed by the detail coefficients $d_{x(n)}$ and $d_{y(n)}$ at a specific scale level, then the area equation of the distorted detail coefficients will be

$$
A = \frac{1}{2} \begin{bmatrix}
d_{x1} & d_{y1} & 1 \\
d_{x2} & d_{y2} & 1 \\
d_{x3} & d_{y3} & 1
\end{bmatrix},
$$

(2)

where $d_{xn}$ and $d_{yn}$ are any detail coefficients points on the details contour at a specific scale level. The distance between these points can be adjusted according to the variations of the curve. The absolute affine invariant area is calculated by dividing equation (2) by the maximum triangle...
area at each level. The normalized cross-correlation function is adapted to measure the similarity between any two 1-D area representation functions.

2.2. Matching stage

The computation of the dissimilarity between two shapes is based on the Euclidian distances between the moment invariants of the wavelet approximation coefficients at all the scale levels. If the dissimilarity between two shapes is low (i.e., a high similarity), then the shapes are globally similar. To investigate the local similarity, the maximum values of the normalized cross-correlations between the detail coefficients that are represented by the area of the triangles are computed.

For the shape clustering, the hierarchical clustering algorithm is applied by using Ward’s linkage rules. Ward’s method is considered one of the best hierarchical methods [3]. The hierarchical clustering is represented by a tree called a dendrogram. The classes are obtained by horizontally cutting the dendrogram at different distance values.

3. Experimental Results

The shapes in the experiment described in this paper, are exhibited in Figure 3. They are categorized into 14 groups, and each group contains four similar shapes with small boundary deformations and different starting point positions. All the shapes are resampled so that they have 256 points. As a result, the DWT decomposes the sequences \( x(k) \) and \( y(k) \) into eight different scale levels.

The majority voting according to the results of the first five scale levels determines the shape clustering. The last few scale levels are discarded from the clustering, because these levels do not contain much information, compared to that of the first few levels. Figure 4 illustrates the dendrogram for the approximation coefficients at a scale level \( L = 3 \). Figure 5 portrays the clustering results by cutting the dendrogram to obtain seven classes. The results of the clustering indicate that the shapes are grouped according to their global features. Figure 6 demonstrates the fine clustering results of seven classes. It is clearly evident that the detail coefficients are not reliable for grouping globally similar shapes. In other words, approximations are good for constructing clusters and details are good for splitting and breaking these clusters into smaller ones. Figure 7 illustrates an example of the fine clustering of two clusters from Figure 5, where each coarse clustering is split into two clusters.

To test the affine invariance of the algorithm, 14 shapes, each representing a group from Figure 3, are selected and distorted. A sample of the affine distorted shapes is provided in Figure 8. These distorted shapes are obtained by applying the following equation:
Figure 8. A sample of the affine distorted shapes.

Figure 9. Affine distorted shapes clustered into 14 classes.

\[ T_{\text{affine}} = \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix} \begin{bmatrix} 1 & sk \\ 0 & 1 \end{bmatrix}, \]  

(3)

where \( sc \) is the scale, \( \theta \) is the rotation angle, and \( sk \) is the skew parameter. The affine transformation parameters used in the experiment are \( sc \in \{1, 0.7\} \), \( \theta \in \{0, 60^\circ, 120^\circ\} \), and \( sk \in \{0, 0.4, 0.9\} \).

Figure 9 exhibits the clustering results for the affine distorted shapes by cutting the dendrogram to obtain 14 clusters. It is clear that the affine distorted shapes are successfully grouped together.

4. Conclusions

In this paper, a new multiscale algorithm is presented for matching and clustering shapes according to their wavelet coefficients. The algorithm is easy to implement and uses simple and efficient techniques. Also, the algorithm is invariant to the affine transformation as well as to the starting point variation of the shape’s boundary. The results indicate that both the global and local features are important to distinguish between shapes. The coarse clustering is achieved by considering the dissimilarities computed from the approximation coefficients. The similarities computed from the details coefficients are used to split the selected coarse clusters into new fine clusters.

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References


