Optimal Resource Allocation in Satellite Networks: Certainty Equivalent Approach versus Sensitivity Estimation Algorithms

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Abstract: in this paper we consider a resource allocation problem for a satellite network, where variations of fading conditions are added to those of traffic load. Since the capacity of the system is finite and divided in finite portions, the resource allocation problem reveals to be a discrete stochastic programming problem one, which is typically NP-Hard. We propose a new approach based on the minimization over a discrete constraint set using an estimation of the gradient, obtained through a “relaxed continuous extension” of the performance measure. The computation of the gradient estimation is based on the Infinitesimal Perturbation Analysis technique, applied on a Stochastic Fluid Model of the network. No closed forms of the performance measures, nor additional feedbacks concerning the state of the system and very mild assumptions on the probabilistic properties about the statistical processes involved in the problem are requested. Such optimization approach is compared with a dynamic programming algorithm that maintains a perfect knowledge about the state of the satellite network. Such comparison shows that the sensitivity estimation capability of the proposed algorithm allows to maintain the optimal resource allocation even in dynamic conditions and it is able to provide performance even better than the one reached by employing the dynamic programming approach.

Key words: Satellite networks, Resource Allocation, Optimization, Sensitivity Estimation, Dynamic Programming.

1. Introduction

In computer networks extending over large geographical areas and in multiservice packet switching communication networks, in the presence of limited resources (buffers, bandwidth, or processing capacity), several forms of control are exerted to maintain a desired level of performance for all users and traffic types. Especially nowadays, in the Internet community, several research efforts are still evolving in order to establish optimization mechanisms able to support Quality of Service (QoS) schemes in the Internet ([Chao02, Spect03]).

Optimization problems of telecommunication networks have a discrete stochastic programming nature: in a stochastic resource allocation scenario, the decision variables are non-negative integers and must be modified along time in order to optimize the system performance, for example in terms of blocking probability of the connection requests, packet loss probability, mean delay or delay jitter of the packets (see, e.g., [Wal00, Car84, Cel92, Bolla02, Cass1_02, Cass2_02, Cass01, Marb00]). Such problems are NP-hard (see, [Cass1_02, Cass01] and references therein) and are often solved by means of centralized approaches, see for example [Barnhart95, Barnhart93, Cel03, Baglietto02] for what concerns Call Admission Control (CAC), adaptive bandwidth allocation strategies and pricing issues. The control systems are, in fact, strictly based on closed form expressions for the performance measure. For example, in [Bolla02, Cel03, Bolla01] the Tsybakov-Georganas formula for the cell loss probability in the presence of self-similar traffic ([Tsybakov98]) is used. The main drawback of these approaches is due to the fact that conditions for the applicability of closed form functional costs are difficult to implement in real-life contexts. Such optimization approaches act according to a certainty equivalent optimization technique, namely, a mapping between the current statistical behaviour of the system and the parameters of the functional costs must be periodically performed on line in order to maintain good performance of the resource allocation algorithms (see, e.g., [Bolla02, Cel03, Bolla01]). Moreover, not only closed forms for important performance measures (e.g., mean delay and delay jitter of the packets) are not always available (for example in the presence of self-similar traffic), but also “[...] even under Markovian assumptions for processes of queueing systems, there are only limited cases where closed form expressions can be obtained” ([Cass93]), and, in general, it is very difficult to assure that in a real application scenario some strict hypotheses are verified. All these techniques need also the application of dynamic programming algorithms, whose on-line implementation in a real context is quite unpractical due to the well-known “curse of dimensionality” problem of the dynamic programming algorithms (see, e.g., [Zoppoli02] and references therein).
The application of algorithms, able to estimate the sensitivity of the performance measure could help in providing sub-optimal control decisions without the adoption of closed form functional costs and the application of such, computationally expensive, dynamic programming algorithms. If such sensitivity estimation algorithms are computationally light, they can be employed on line with a small computational effort (see, e.g., [Cass1_02, Cass2_02, Cass01, Davoli03]). The possibility of completely decentralize the sensitivity estimation and the resource allocation strategies constitutes an attractive property, too, since the adoption of a centralized unit that periodically monitors all the components of the system cannot be implemented in a real context ([Vazquez98]).

Such sensitivity estimation algorithms can be based on the so-called Perturbation Analysis (PA) technique. PA is a sensitivity estimation technique for Discrete Event Systems (DESs) ([Cass93, Wardi02, Rub93, Sun02]). It is based on the observation of the sample paths followed by the stochastic processes of a DES and gives an estimation of the derivative of the performance index, allowing the application of a gradient-based algorithm in order to optimize the system performance. Such optimization strategies are known in the literature as “on-line surrogate optimization methodologies” because they act on line, with a gradient-based algorithm, by applying a “surrogate” relaxation of the discrete functional cost ([Cass1_02, Cass01]). It is shown in [Cass2_02, Davoli03] that the application of an on-line surrogate optimization methodology, together with a particular PA technique, succeeds in optimizing the resource allocation of a telecommunication network.

Our first aim, in this work, is to present a novel solution for the bandwidth allocation in a satellite environment based on an on-line surrogate optimization methodology. Without loss in generality concerning the developing of the optimization algorithm, we consider a resource allocation problem currently quite popular in the telecommunication community: the resource allocation in satellite networks. Such optimization problem is even more difficult than the typical problems of the terrestrial broadband networks, simply because channel degradation effects must be taken into account together with the traffic changes. However, as will be clear from the following, the model proposed can be easily generalized for several other telecommunication application scenarios and functional costs.

Moreover, we shall investigate how such optimization approach can ameliorate the performance of a control strategy based on a closed form expression of the performance index. This latter optimization technique, employed in previous works [Bolla02, Davoli03], needs the so-called certainty equivalent assumption, namely a perfect knowledge about the statistical proprieties of the satellite system must be always in effect. In this way, each time a change in the statistical behaviour of the system is detected, a new call to a proper optimization procedure guarantees the maintenance of the optimal resource allocation among all the components of the system. In fact, it is necessary to periodically perform on line a mapping between the current state of the network and the parameters of the employed closed form functional cost. Two drawbacks can severely deteriorate the performance of such optimization strategy. The first one concerns the presence of errors over the measures performed in order to estimate the current state of the network. The second one regards the possibility that the current statistical behaviour of the system does not conform to the hypoteses assumed a-priori to provide a particular closed form expression of the performance index (for instance, a self-similar behaviour of the traffic sources is supposed to be always in effect in [Bolla02, Davoli03]). In [Bolla02, Davoli03], these issues have been left open for future research. In this paper, we shall go deep into this subject by investigating how much the adoption of an on-line surrogate methodology is able to face these drawbacks. We shall discover that it shows a surprising self learning capability that guarantees dynamic reactions to the statistical changes of the system, without any direct feedback over the system’s state and under very mild assumptions concerning the statistical behaviour of the traffic sources.

The paper is organized as follows. In the next Section we formulate the model of the satellite network together with the formulation of the discrete stochastic optimization problem for such environment, then, in Section III, we illustrate our certainty equivalent approach, and, in Section IV, the technique used in order to compute the performance derivative estimation. In Section V, the on-line surrogate optimization algorithm is addressed and, in Section VI, we summarize all the proposed optimization techniques. In Section VII we show the simulation results and in Section VIII some conclusions and future work will be finally proposed.

2. The model of the Satellite System

In satellite networks, dynamically varying fading conditions over the channel can heavily affect the transmission quality, especially when working in Ka band, where the effect of rain over the quality of transmission is more significant. In the literature, it is possible to find optimal policies developed in case of a finite quantity of transmission energy for satellite network devices. [Cruz99, Uysal01, Fut02] show a dynamic programming formulation of the problem that leads, for special cases, a closed form optimal policy, in order to find a tradeoff between the minimization of the energy required to send a fixed amount of data and the maximization of the throughput over a fading channel. Resource allocation for fading multi-user broadcast channels is a popular topic also in information theory (see, e.g., [Gold97, Tsybakov02, Ween98]). In these works, the problem is analyzed and solved at the physical layer: a power
allocation is performed in order to obtain good reactions to variable fading conditions. In [Bolla02, Cel03, Davoli03] and in this work the control system is located at the physical and at the data link (or upper) layers and provides adaptive bandwidth allocation strategies in order to minimize the loss probability of the overall system along a finite time horizon.

We consider a fully meshed satellite network that uses bent-pipe geostationary satellite channels. This means that the satellite performs only the function of a repeater and it does not make any demodulation of data. The system operates in \textit{MF-TDMA} mode. The master station maintains the system synchronization other than performing capacity allocation to the traffic stations. The master station performance is the same as the others, thus the role of master can be assumed by any station in the system. This assures that the master normally operates in pretty good conditions, because when the current master’s attenuation exceeds a given threshold, its role is assumed by another station that is in better conditions. To counteract the signal attenuation the system operates up-link power control, bit and coding rates changing. Traffic stations transmit in temporal slots assigned by the master, each one generally on different TDMA carriers (frequencies). The multi-frequency feature allows us to divide the system capacity into a number of sub-channels, so that the stations can be downsized with respect to a pure TDMA system.

\subsection{A Stochastic Fluid Model of the Satellite Network}

We base our problem formulation on a \textit{Stochastic Fluid Model} (SFM) of the telecommunication network. SFMs have been proposed for modelling the workload flow in substitution of traditional packet-based queueing models. SFMs adopt a fluid-flow point of view rather than the transaction-flow point of view of traditional queueing models (see e.g., [Ren92] for an overview concerning this topic). In the last years SFMs have been recognized as suitable models for performance analysis of queueing networks. Such networks are designed to transport fixed-size data units, over high-speed transmission links on the order of gigabits per second [Wardi94, Ren92] (e.g., ATM or DVB). Moreover, as is shown in ([Wardi02, Cass93, Wardi01, Sun02, Wardi00, Wardi94]), the IPA techniques for the optimization of performance parameters at the packet level are strictly based on a SFM of the system.

With a notation that slightly differs from [Wardi02], we adopt a SFM for each single satellite station. Each station has a finite-capacity buffer of fixed size $Q$ and a single server (Fig. 2.1.1):

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure.png}
\caption{Model of the single satellite station}
\end{figure}

This buffer is aimed at receiving a variable bit rate traffic from different sources. The scheme above can be referred to as the \textit{Basic} SFM: it consists of a “fluidized queue”, with a single class fluid source (see, e.g., [Wardi02] or [Wardi00]). The stochastic processes associated with this model and useful for our optimization algorithm are:

- $\beta(t)$ : the service rate process, namely, the maximal fluid discharge rate from the server;
- $\alpha(t)$ : the input flow rate (inflow) process into the SFM;
- $\gamma(t)$ : the buffer workload process, namely, the fluid volume in the buffer;
- $\gamma(t)$ : the loss rate (overflow) process due to a full buffer.

All of these stochastic processes evolve over a time horizon $[0, T]$, with $T>0$.

Such SFM can be viewed as a dynamical system, whose evolution is determined by the inflow and service rate processes (the so called “defining processes” $\alpha(t), \beta(t)$), while the other two processes (called “derived processes” $\gamma(t)$) can be derived as follows. At time $t$ if $x(t)=0$ and $\alpha(t) - \beta(t) \leq 0$, or $x(t)=C$ and $\alpha(t)-\beta(t) \geq 0$, then $\frac{dx(t)}{dt} = 0$, otherwise $\frac{dx(t)}{dt} = \alpha(t) - \beta(t)$. For the overflow rate process $\gamma(t)$, $\frac{d\gamma(t)}{dt} = \alpha(t) - \beta(t)$ if $x(t)=C$, otherwise $\frac{d\gamma(t)}{dt} = 0$. In the next Section, we shall put in evidence that, with only such “ingredients”, it is possible to establish a simple way to determine an estimate of the gradient of the cost function in order to optimize the system performance.
2.2 The traffic model

Even if the on-line surrogate optimization methodology we are going to formulate is not related to a specific behaviour of the traffic sources, we shall adopt, in the simulation results, a specific traffic model in order to dispose of a closed form functional cost of the performance index. Hence, we now introduce the traffic model adopted for each station’s input rate (inflow) process $\alpha_i(t), i = 1, \ldots, N$.

In the last years, analyses of packet-based traffic have demonstrated that its main statistical characteristics have good affinity to Self-Similar processes. Intuitively, self-similar traffic is supposed to present the same statistical behaviour over large time intervals. W.E. Leland, M.S. Taqqu, W. Willinger and D.V. Wilson analyzed Ethernet traffic ([Leland94]), highlighting its self-similar nature, as V. Paxson and S. Floyd [Floyd95] extended these observations to the TCP protocol over WANs. Besides, M.W. Garrett and W. Willinger [Garret94] proposed this model also for Variable Bit Rate (VBR) traffic. We shall adopt, in the following, such statistical model for the traffic sources. Here we present a brief description of the main features of such type of traffic and how it has been implemented in our simulations.

Suppose to have $M$ independent sources, each one generating a traffic flow at constant cell rate $R$ [cells/s] for a random time period $\tau$, at the end of which the source does not send any packet for a random time interval $\sigma$. Such types of sources are called in the literature “on-off sources” and they are aimed at modelling VBR traffic (e.g., real time or streaming audio-video applications) [Pitts00]. As in [Cel03, Bolla02, Bolla01, Tsybakov98, Kim01], we suppose that the cell dimension is fixed; the on and off time intervals can be expressed as multiples of $T$, where $T$ is the time necessary, for a source, to complete the generation of a new cell, $T = \frac{L}{B_p}$, where $L$ is the payload length of the cell (e.g., for the ATM cell: 48 bytes of payload plus 5 bytes of overhead) and $B_p$ is the peak bit rate, i.e., the source’s bit rate during its active period. The succession of several periods, as mentioned above, creates a traffic path as depicted in Figure 2.2.1:

\[ \tau' \]

\[ \sigma' \]

\[ T \]

Figure 2.2.1: Typical sample path of an on-off source

where $\tau'$ and $\sigma'$ are two possible realizations of the random variables $\tau$ and $\sigma$ that describe the statistical behaviour of the single on-off traffic source. Let us suppose that $\tau$ follows a heavy-tailed distribution, for example the Pareto one:

\[ \Pr(\tau = t) = c \cdot t^{-(\alpha+1)} \]  (2.2.2)

where $c$ is a normalization constant such that:

\[ c = \frac{1}{\sum_{t=1}^{\infty} t^{-(\alpha+1)}} \]  (2.2.3)

From these equations it follows that, for $1 < \alpha < 2$, $\tau$ has a finite average value:

\[ \bar{\tau} = E\{\tau\} = \sum_{t=1}^{\infty} c \cdot t^{-\alpha} \]  (2.2.4)

but infinite variance $E\{\tau^2\}$ ([Pitts00]). It is shown in [14] that the aggregation of $M$ independent sources (the $Y$ process depicted in Fig. 2.2.2) with such probability distribution over $\tau$ determines an aggregated flow with self-similar properties and this has a dramatic impact over the resources that must be reserved to such flow in order to guarantee cell-level QoS constraints (see, e.g., [Tsybakov98, Pitts00]).
We suppose that each input flow rate process $\alpha_i(t)$ is composed by such self similar stochastic processes. The statistical parameters that describe such process are, for each station $i$, the peak bit rate $B^i_p$ and the burst arrival rate $\lambda^i_{\text{burst}} = \frac{M^i}{\tau^i + \sigma^i}$, where $\tau^i$ and $\sigma^i$ are the mean time duration of the burst and of the silence periods, respectively ([1, 4, 13, 14]); i.e., the buffer of each station sees a mean number of active bursts of $\lambda^i_{\text{burst}}$ and, in each burst of mean time duration $\tau^i$, a cell arrival rate of $\frac{B^i_p}{L}$.

2.3 The performance measure

The performance measure of interest is the loss volume $L_F(\cdot)$ over the interval $[0,T]$:

$$L_F(\cdot) = \int_0^T \gamma(\theta,t) \, dt$$

(2.3.1)

Following [Wardi02, Cass2_03, Sun02], we note that the SFM of the satellite system depends on a real-valued parameter $\theta \in \mathbb{R}$, the service capacity of the buffer. We now indicate this dependence by writing $\beta(\theta,t), x(\theta,t)$ and $\gamma(\theta,t)$, and this modification leads to the conclusion that also the loss volume depends on $\theta$:

$$L_F(\theta) = \int_0^T \gamma(\theta,t) \, dt$$

(2.3.2)

The satellite system is composed by $N$ stations, each of which is provided with a single buffer with service capacity $\theta_i(t)$ and its specific inflow rate process $\alpha_i(t)$. The $N$ stations are supposed to dispose of the same buffer capacity $c$, and the total loss volume becomes the sum of the contributions of each station:

$$L_F(\theta_1,\ldots,\theta_N) = \sum_{i=1}^N \int_0^T \gamma_i(\theta_i,t) \, dt$$

(2.3.3)

The total capacity of the satellite must be divided between the $N$ stations. Let $\theta^d(t) = [\theta^d_1(t),\ldots,\theta^d_N(t)]$ be the vector of the service capacities allocated to each station at time $t$. $\theta^d(t)$ must belong to the following constraint set:

$$\Theta_d = \left\{ \theta^d(t) \in \mathbb{N}^N : \theta^d_i(t) = h_i \cdot \text{MAU}, \ h_i \in \mathbb{N}, \ \sum_{i=1}^N \theta^d_i(t) = K \right\}$$

(2.3.3)
\( \theta_i^d(t) \) is a discrete parameter, in the sense that the allocated service rate for each station is a discrete number of Minimum Allocation Units (MAUs), namely, the smallest portion of bandwidth that can be allocated to a station. \( K \) is the total service capacity available for the satellite system.

Figure 2.3.1 The model of the satellite system.

2.4 The fading effect and problem formulation

The effect of fading, supposed to be unique for each transmitting station (i.e., each station is supposed to transmit to destinations affected by the same fading levels [Bolla02, Cel03]), is modeled as a reduction in the bandwidth actually “seen” by a traffic station ([Bolla02, Cel03]). The fading effect is represented by a variable \( \phi_i \), independent of \( \theta_i \), that shows how the bandwidth is reduced. For each station \( i \), at time \( t \), the “real” \( \theta_i(t) \) is:

\[
\theta_i(t) = \phi_i(t) \cdot \theta_i^d(t); \quad \phi_i(t) \in [0,1]; \quad i = 1,\ldots,N
\]

\( \phi_i(t) = 1 \) corresponds to no fading effect over the links of station \( i \) at time \( t \), while \( \phi_i(t) = 0 \) corresponds to the so-called “outage” situation for station \( i \), i.e., at time \( t \) the station \( i \) sees a service rate process \( \beta_i(t) = 0 \), in spite of any possible allocation of the service capacity \( \theta_i^d(t) \).

In our model the fading effect involves, as mentioned, a reduction of the bandwidth actually “seen” by the station, or, equivalently, an increase in the bandwidth required by the traffic sources to maintain the same Bit Error Rate (BER). We suppose the presence of fade countermeasures located at the physical layer, totally managed by the single earth station. It is expected to provide the desired BER through FEC codes ([Cel03, Bolla02, Cel92, Car84, Alagoz01]). Whenever the fading effect causes errors over the packets, an adaptive control can monitor the C/N (Carrier/Noise Power) factor and, on the basis of this measure, increase the redundancy of the packets sent introduced by the FEC. In this way, for each station, the available bandwidth is reduced: since more bits are necessary to transmit a single packet (because of FEC coding), the outflow cell rate can be considered as modified by the fading effect. Clearly, heavier fading conditions will involve a more consistent decrease of the allocated bandwidth, because more coding protection of data will be necessary, and vice versa.

So, we are going to discuss the combination of a BER-related fade countermeasure technique with a resource allocation problem, where a master control station is supposed to be responsible of the reallocation of bandwidth, but not necessarily supposed to know the fading level of each station, as on the contrary, it is done in [Bolla02, Cel03]: in this way we reduce the centralization of the system, because the master only has to know the IPA’s sensitivity estimation of each station, ignoring the information about the fading level.
The optimization problem can now be stated, it consists in finding out the optimal bandwidth allocation, over the finite horizon \([0, T]\), \(\text{Opt} \theta_d(t)\), in a such a way that:

\[
\text{Opt} \theta_d(t) = \arg \min_{\theta(t) \in \Theta; \tau(t) \in [0, T]} J[\theta_d(t)]
\]

\[
J[\theta_d(t)] = E_{\omega_1, \ldots, \omega_N} \left[ \sum_{i=1}^{N} L_f(\phi(t) \cdot \theta_d^i(t)) \right] = \sum_{i=1}^{N} \int_{0}^{T} p(\phi(t) \cdot \theta_d^i(t), t) \, dt
\]

where \(\omega_i\) is the generic sample path for station \(i\), namely, a realization of the stochastic processes that characterize the temporal evolution of station \(i: \alpha_i(t), \beta_i(t), \phi_i(t), i = 1, \ldots, N\).

The expectation \(E_{\omega_1, \ldots, \omega_N} \left[ \right]\) is over all the feasible sample paths \(\omega_i \in \Omega_i\) for each station \(i\). Eq. (2.4.2) expresses a hard discrete stochastic programming problem. Even when the setting is deterministic and the expectation is not requested, this class of problems is \(NP\)-hard (see e.g., [Cass01, Cass1_02] and references therein). In some cases, depending upon the form of the objective function \(J(\theta^d)\) (e.g., separability, convexity), efficient algorithms based on finite-stage dynamic programming or generalized Lagrange relaxation methods are known (see, e.g., [Ibaraki88]). Alternatively, if no a priori information is known about the structure of the problem, some forms of search algorithms is employed (e.g. Simulated Annealing [40], or Genetic Algorithms [41] techniques). When the system operates in a stochastic environment and no closed-form expression of \(L_f(\phi(t) \cdot \theta_d^i(t))\) is possible, the situation is further complicated by the need of estimating \(E_{\omega_1, \ldots, \omega_N} \left[ \sum_{i=1}^{N} L_f(\phi(t) \cdot \theta_d^i(t)) \right]\). This generally requires Montecarlo simulation approaches or direct measurements made on the system that are far to be realistic for real on-line optimization strategies.

In order to solve such heavy drawbacks, following [Cass01, Cass1_02, Cass2_02, Davoli03], we shall formulate a new optimization algorithm based on a sensitivity estimation procedure and we shall compare it with the optimization technique employed in [Bolla02, Davoli03] based on a closed form expression of the performance index and the adoption of a dynamic programming algorithm.

### 3. The Optimization algorithm based on a Certainty Equivalent approach

Also in the presence of a self-similar behaviour of the traffic sources, it is possible to dispose of analytical models for the computation of the loss probability performance ([Tsymbakov98, Pitts00, Kim01]). Such closed form expressions could be used in the aforementioned resource allocation framework in order to optimize the system performance (e.g., [Bolla01, Bolla02, Cel03]). Anyway, they need to assume a perfect knowledge of the system’s state and a strong consumption of computing power, due to the continuous on-line minimization of a global cost through the adoption of a proper dynamic programming algorithm.

We now describe in details such optimization strategy. Following the model employed in [Bolla02, Cel03, Bolla01], we adopt the Tsymbakov-Georganas formula for the cell loss probability \(P_{\text{Loss}_i}\) of each station \(i:\)

\[
P_{\text{Loss}_i}(\theta_d^i) = \begin{cases} 
\min \left\{ \frac{c \cdot \lambda^i \cdot R^a}{(\alpha - (\alpha - 1)) \cdot (X_i - \lambda^i \cdot R \cdot T)} \cdot (Q_i)^{-\alpha_1} \cdot 1, \right. & \text{if } X_i > \lambda^i \cdot R \cdot T \\
1 & \text{otherwise} \end{cases} 
\]

(3.1)

Some of the parameters appearing in (3.1) have been already defined in the subsection 2.2 (\(\alpha\) and \(c\) belonging to the Pareto distrubution over the burst and silence periods of the sources, denoted with \(\tau\) and \(\sigma\)). The others are explained in the following. Let \(T\) be a reference time interval (slot), to which we shall refer all the relevant parameters of the cell queue of each station \(i\). The slot also represents the minimum duration of a burst, and the burst length \(\tau\) is expressed as an integer nuber of slots. Let \(B_p\) be the peak generation rate [bits/s] of each sources, supposed for simplicity, and without loss in generality, to be equal for each transmitting station, and \(L\) the number of bits in a cell. Then,
\[ R = \left\lceil \frac{\hat{T} \cdot B_p}{L} \right\rceil \] is the number of cells generated by an active burst in a slot (\( \lceil w \rceil \) being the smallest integer greater than or equal to \( w \)). Suppose that the number of new sources becoming active in each slot are i.i.d. Poissonian with parameter \( \lambda_i = \lambda_{\text{burst}} \cdot \hat{T} \). If \( H \) is the cell’s header length in bits, then \( X_i = \left[ \frac{\theta_i^d \cdot \theta_i}{L + H} \cdot \hat{T} \right] \) represents the bandwidth \( \theta_i^d \), assigned to station \( i \) degraded according to the current value of fading \( \phi_i \), expressed in cells per slot (\( \lfloor w \rfloor \) being the largest integer less than or equal to \( w \)).

Once that a closed form expression is available, and supposing to know perfectly, for each active station \( i \), all the traffic parameters and the current fading levels necessary to correctly update Eq. (3.1), it is possible to employ a proper dynamic programming algorithm in order to optimally distribute the available channel capacity among the stations. The optimization problem formulated in Eqs. (2.4.2) and (2.4.3) has to be slightly modified by taking into account Eq. (3.1), thus stating the problem of the minimization of the overall loss probability at each time instant \( k = 1, 2, \ldots \). A new bandwidth reallocation is performed:

\[
\text{Opt } \theta^d(k) = \arg \min_{\theta^d(k) \in \Theta} J(\theta^d(k)) \tag{3.2}
\]

\[
J(\theta^d(k)) = \sum_{i=1}^{N} P_{\text{Loss}}(\theta_i^d), \quad k = 1, 2, \ldots \tag{3.3}
\]

The index \( k \) denotes the reallocation time instants at which a new solution of (3.2) is computed according to the current state of the network. The expectation operator \( E \) is now useless, because, in order to face the problem formulated by Eqs. (2.4.2) and (2.4.3), a first hard assumption has been taken over the statistical behavior of the traffic sources, thus leading to Eq. (3.1) and then, an optimization procedure is employed to find the solution of (3.2) and (3.3) at the beginning of each reallocation time instant \( k = 1, 2, \ldots \). On the other hand, in the on-line optimization methodology that we are going to investigate, the adoption of an on-line gradient descent technique will allow us to spread over time the solution of (2.4.2) and (2.4.3), instead of concentrating it at the beginning of the reallocation time instants.

The minimization of (3.3) can be performed through a dynamic programming approach. [Ross95] shows how to employ a dynamic programming algorithm in order to solve the optimization of the overall blocking probability of a multiservice system at the call level in the presence of a limited set of bandwidth resources. For our specific optimization problem, we have modified such an algorithm in order to take into account the presence of the Loss Probability formula (3.1). The modified version is reported in the Appendix A.

Such algorithm is polynomial with respect to the number of stations and to the total number of available MAUs in the system. Hence, its computational burden seems to be not so heavy as expected. Unfortunately, the total number of available MAUs can be very large in presence of a satellite link with a high capacity and with the adoption of small values in the MAU parameter (from 100 Kbps down to lower values). So, if also the number of active stations is high, such optimization procedure could need several seconds to terminate successfully. This drawback can severely degrade its performance. As we shall also show in the simulation results, since the adoption of high MAU values is not recommendable (as it leads to poor performance of the optimization algorithm, too), if such dynamic programming algorithm has to be employed, a proper trade-off must be found out in order to limit its computational burden without adopting too high values of MAUs.

Even if our on-line surrogate optimization methodology will reveal to be computationally lighter, we do not insist, in this paper, about its suitability due to its lower computational complexity, but we shall try to highlight how it is able, in virtue of its sensitivity estimation capability, to achieve better solutions than the ones obtained employing the aforementioned dynamic programming algorithm.
4. Performance derivative estimation

In this section, we develop our on-line surrogate optimization technique. In order to generate a gradient descent of bandwidth reallocations, it is necessary to dispose of a derivative estimation of the performance index. Since our challenging task is to build an optimization technique able to manage any possible statistical behaviour of the traffic sources, we employ a derivative estimation technique which assumes very mild a-priori hypotheses concerning the stochastic processes involved in the system.

4.1 The unbiasedness condition

As far as the derivative estimate of the performance metric, \( L(\cdot) \), is concerned (denoted in the following with \( L(\cdot) \) to limit the notational burden), we note that the IPA technique proposed in [Wardi94, Cass93, Wardi00, Wardi01, Wardi02, Cass2_03] for SFMs of a telecommunication network, can be efficiently employed to satisfy our needs. In order to obtain gradients of performance metrics, IPA derives the effect on the system of a small (infinitesimal) perturbation on parameters that influence the evolution of the system. One of the main advantages of IPA is that no a priori information on the form of \( L(\cdot) \) is required, since the gradient estimates are computed directly from the current sample path \( \omega \) of the system ([Wardi94, Cass93, Wardi00, Wardi01, Wardi02, Cass2_03]).

It has been demonstrated that IPA would yield an “unbiased” estimator for a large class of networks in a SFM setting ([Wardi94, Cass93, Wardi00, Wardi01, Wardi02, Cass2_03]). The concept of unbiasedness is an essential property for the use of IPA in an optimization framework and it can be explained as follows. Let \( L(\theta) \) be a generic performance measure observed on a generic sample path (e.g., loss volume, cumulative workload, etc.), which is a function of the parameter of interest \( \theta \) (e.g., service rate, buffer dimension). If \( \Omega \) is the set of all feasible sample paths and \( \omega \) a generic one, an IPA estimator is defined to be unbiased if the derivative operator can be replaced with the expectation operator and vice versa:

\[
\frac{d}{d\theta} E[L(\theta)] = E\left[\frac{dL}{d\theta}\right]
\]

More formally, we can explain this concept showing Eq. (3.1.1) through:

\[
E\left[\lim_{\Delta\theta \to 0} \frac{L(\theta + \Delta\theta) - L(\theta)}{\Delta\theta}\right] = \lim_{\Delta\theta \to 0} E\left[\frac{L(\theta + \Delta\theta) - L(\theta)}{\Delta\theta}\right]
\]

To build an estimator of the performance derivative using only the current sample path \( \omega \), it is necessary to allow the interchange of the expectation with the limit. This issue requires that \( L(\theta) \) satisfy some particular characteristics ([4]). Much work has been conducted in the last few years in the operations research community to study which are the conditions for the employment of such “Perturbation Analysis” technique (often called “Sensitivity Estimation”, too) in a Discrete Event System ([Wardi94, Cass93, Wardi00, Wardi01, Wardi02, Cass2_03, Sun02, Kesidis96, Kumaran98, Miyoshi98, Wardi99]). The three conditions that ensure the unbiasedness of IPA derivatives (as shown in [Cass93], [Fu02] Lemma A2, and more specifically in [Wardi94]) are:

**Condition 1:** for every \( \theta \in \Theta \), \( \frac{\partial L(\cdot)}{\partial \theta} \) exists, with probability 1 (w.p.1).

**Condition 2:** \( L(\cdot) \) is Lipschitz continuous through \( \Theta \) w.p.1.

**Condition 3:**

\[
\frac{\partial L(\cdot)}{\partial \theta} \text{ satisfies}
\]

\[
E[\sup_{\theta \in \Theta} \left| \frac{\partial L(\cdot)}{\partial \theta} \right|] < +\infty
\]

Where \( \Theta(\frac{\partial L(\cdot)}{\partial \theta}) \) consists of all elements \( \theta \in \Theta \) for which the derivative \( \frac{\partial L(\cdot)}{\partial \theta} \) exists.

Our choice of modelling the satellite system with a SFM leads to the applicability of an unbiased IPA estimator. In general, due to the discontinuities of \( L(\theta) \) over \( \Theta \) for several DES models, traditional queueing models give biased
derivative estimators that do not satisfy Eq. (4.1.1) (see [Cass93, Wardi00, Wardi02] for an overview concerning this topic). On the contrary, adopting the SFM of Section 2.1, it is proved (e.g., in [Wardi02]) that our performance function \( L() \) satisfies conditions 1, 2 and 3 and a guaranteed unbiased IPA estimator can be obtained.

### 4.2 Gradient estimation

Once we have found out the right Perturbation Analysis technique for gradient estimation applicable to our context, namely, the IPA applied to a SFM of the network, the next step is to show the IPA formulas adopted. To this aim, we make the following assumptions (also adopted in [Wardi02]):

**Assumption 1**: the function \( \alpha(t) - \beta(t, \theta) \) is piecewise continuously-differentiable in the interval \([0, T]\).

**Assumption 2**: w.p.1, no multiple events may occur simultaneously.

**Assumption 3**: the sample derivative always \( \frac{\partial L(\theta)}{\partial \theta} \) exists.

Let these three assumptions be in effect, and let now \( B_k \) be an “active” period of the buffer between two times of bandwidth reallocation, namely, a period of time in which the buffer is non-empty; we denote it with:

\[
B_k(\xi_k(\theta), \eta_k(\theta))
\]  

(4.2.1)

where \( \xi_k \) is the start point and \( \eta_k \) the end point of \( B_k \). We now define the index set \( \Gamma^k(\theta) \) as:

\[
\Gamma^k(\theta) = \{All \ time \ instants \ in \ increasing \ order \ [v_{\theta}^{k}, \ldots, v_{\theta}^{N_k}] \ during \ B_k \ when \ a \ loss \ occurs \ in \ the \ buffer\}
\]

Clearly, \( \Gamma^k(\theta) \) strictly depends on \( \theta \). Let \( v_{\theta}^{N_k} \) be the instant of time when the last loss occurs during \( B_k \). Then, for every \( \theta \in \Theta \) it can be demonstrated that (see Fig. 4.2.1):

\[
\frac{\partial L^k(\theta)}{\partial \theta} = -(v_{\theta}^{N_k} - \xi_k(\theta))
\]  

(4.2.2)

![Figure 4.2.1 Gradient estimation looking only at the sample path of the buffer state](image)

Namely, the contribution to the required derivative of each active period \( B_k \), during which some loss occurred, is the length of the time interval from the start of \( B_k \) until the last time point in \( B_k \) at which the buffer is full. For the proof of Eq. (4.2.5) and its unbiasedness properties see [Wardi02].
At the time of bandwidth reallocation at the station $i$, denoting by $N_{Bi}$ the number of active periods between two consecutive bandwidth reallocation time instants where at least one loss occurs, we dispose of an estimation of the gradient performance:

$$\frac{\partial L_i(\theta)}{\partial \theta} = \sum_{k=1}^{N_{Bi}} \frac{\partial L_i^k(\theta)}{\partial \theta}; i=1,..,N$$  \hspace{1cm} (4.2.3)

The IPA performance derivative obtained by Eqs. (4.2.2) and (4.2.3) is also called "nonparametric", since it is computable directly from an observed sample path $\omega$ without any knowledge concerning the probability distributions of the stochastic processes involved in the system ([Wardi94, Wardi02, Cass93, Wardi01, Cass2_03, Wardi00]). This fact implies that it is applicable not only in off-line simulation settings (aimed, for example, at the planning of the telecommunication network), but also in real on-line scenarios for practical network management and control ([Wardi94, Wardi02, Cass93, Wardi01, Cass2_03, Wardi00, Wardi99, Sun02, Kesidis96, Kumaran98, Miyoshi98]). In the next Section we shall proceed according to this last direction.

5. The on-line surrogate optimization methodology

Since our aim is to apply the estimator for the derivative of the performance parameter $L(\cdot)$ formulated in the previous Section, it is necessary to "relax" the discrete constraint set $\Theta_d$ into a continuous one $\Theta_c$. Namely, the $L(\cdot)$ is used to optimize a continuous bandwidth allocation $\theta^c$ with the following constraints:

$$\theta^c \in \Theta_c, \Theta_c = \left\{ \theta^c \in \mathbb{R}^c; i=1,..,N; \sum_{i=1}^{N} \theta^c_i = K \right\}$$  \hspace{1cm} (5.1)

As proposed in [Cass01, Cass1_02] the discrete functional cost defined over $\Theta_d$ is transformed into a "surrogate" one that works over $\Theta_c$. We construct by Eqs. (4.2.2) and (4.2.3) an estimation of the gradient, according to the current measured sample path, and we apply a sequence of minimization steps until the optimum is reached.

The following scheme illustrates each step of the optimization algorithm, whose computation can be decentralized. Initially, the bandwidth resources are equally distributed among the stations and, during the system evolution, each station $i$ must:

1. **observe** its buffer temporal evolution according to the current sample path $\omega_i$ and bandwidth allocation $\theta_i^d(k) \in \Theta_d$;

2. **compute** the gradient estimation $\frac{\partial L_i(\theta_i(k))}{\partial \theta_i(k)}$ according to Eqs. (4.2.2) (4.2.3);

3. **adjust** the value of its bandwidth allocation using the gradient method:

$$\theta^c_i(k+1) = \theta^d_i(k) - \eta k \frac{\partial L_i(\theta_i(k))}{\partial \theta_i(k)};$$

4. **convert** $\theta^c_i(k+1)$ in the nearest discrete feasible neighbour $\theta_i^d(k+1)$ in such a way that $\theta_i^d(k+1) \in \Theta_d$ or **transfer** to the master station, which will perform this conversion, a new bandwidth allocation request of $\theta_i^c(k+1)$;

Fig. 5.1 The algorithm adopted in each station in order to achieve an optimal resource allocation

In step 4, as is shown in [Cass01], the nearest feasible neighbour $\theta_i^d(k+1) \in \Theta_d$ of $\theta_i^c(k+1)$ can be determined using an algorithm based on the Simplex Method. However, it is possible to apply a simpler algorithm (see e.g.
[Cass1_02 for further details), based on the individuation of $N+1$ discrete neighbours of $\theta^i_c(k+1) \in \Theta_c$, not necessarily all feasible, and on the selection of one of them, which satisfies the discrete constraint set $\Theta_d$. This algorithm is briefly explained in the Appendix B.

The projection of each $\theta^i_c(k+1)$ to the constraint discrete set $\Theta_d$ can be performed by the master station in the following way. At the reallocation time instant $k+1$ the master station receives from each station all $\theta^i_c(k+1)$, $i = 1,\ldots,N$; then, it can map them to the discrete feasible bandwidth allocations $\theta^d_i(k+1) \in \Theta_d$ applying the aforementioned algorithm. In this way the allocation strategy is partially decentralized: each station formulates its local sensitivity estimation of the gradient and formulates a new bandwidth request $\theta^i_c(k+1)$ to the master station; finally, the master station determines the new discrete optimal allocation $\theta^d_i(k+1) \in \Theta_d$. To completely decentralize the reallocation algorithm, namely, to release the necessity of a master station in the system, it could be possible to establish something similar to the Link State (LS) information exchange of the QoS routing in a MPLS environment ([Crawley98, Apo98]). In LS routing, network nodes should be aware of the state of the links, possibly located several hops away. This calls for a periodic flooding exchange of LS information, which contributes extra traffic to the network. Our model needs a flooded and periodic (but very light) exchange of Node State (NS) information concerning the local sensitivity estimation of the gradient of each station actually present in the satellite network. In fact, if each station knows the new bandwidth requests $\theta^i_c(k+1)$ of the others stations, it can calculate its new bandwidth allocation $\theta^d_i(k+1)$ by itself. In this latter situation, such completely decentralized optimization algorithm looks like a "cooperative" strategy based on a team theory framework where each agent, located in each station, calculates, on the basis of its local information and on the information received from other agents, the new optimal control aimed at reaching a global optimum for the system performance ([Ho72, Baglietto01]).

It has been demonstrated in [Cass01, Cass1_02] that this surrogate optimization approach guarantees the convergence to the optimal resource allocation. In [Cass2_02], a similar approach is applied for the optimization of the Call Admission Control in a circuit switched network, and, at the best of our knowledge, this is the first time that such technique is adopted to optimize the performance of a telecommunication network at the packet level. The computational effort required to both the gradient estimation procedures and the algorithm adopted in Step 4 are polynomial in the state space, namely the computational complexity of such optimization approach grows polynomially with respect to the dimension of the network. For this reason, and in spite of the mild assumptions requested for the applicability of the adopted IPA technique, we could claim that the proposed optimization algorithm can be efficiently applied in real online scenarios, according to different statistical behaviours of the traffic sources.

6. Bandwidth allocation strategies

We now summarize all the bandwidth allocation strategies employed in the following simulation results. At the end of each simulation, the final loss volume is computed in terms of the overall Loss Probability among the stations of the satellite system.

**CF&DP (Closed Form functional cost with Dynamic Programming optimization approach):** the certainty equivalent approach formulated in Section 3 is employed. For all stations in the satellite system, a perfect knowledge on both the state of traffic sources and the fading levels is supposed to be always in effect for each time of bandwidth reallocation. However, the sensitivity of the solution obtained with the adoption of this technique will be investigated, with respect to possible estimation errors over the real traffic sources’ state.

**SE&GD (Sensitivity Estimation and Gradient Descent optimization approach):** the IPA technique described in the previous section is adopted, and derivative estimations are computed. After that, the gradient algorithm illustrated in Section 5 is applied using a gradient stepsize $\eta_k$:

$$\theta^i_c(k+1) = \theta^d_i(k) - \eta_k \frac{\partial L_i(\theta^i_c(k))}{\partial \theta^i_c(k)} ; i = 1,\ldots,N$$  \hspace{1cm} (6.1)

where $\eta_k$ is modified at each step according to the following relationship:

$$\eta_k = \{(a \cdot 10^6) - k \cdot (b \cdot 10^3)\} \hspace{1cm} (6.2)$$

$$a = 12; b = 200$$
The rationale of this choice stems from the fact that, in stochastic optimization problems, the convergence of (6.1) is related to the decreasing behaviour for the stepsizes $\eta_k$ (see e.g. [Rob51, Tsy71, Ermo80, Benv90]). Of course, the function specifying how $\eta_k$ has to decrease as the step $k$ increases must be empirically evaluated in order to reach a “good” level of optimality for the DES under investigation. The $a=12, b=200$ setting was found out by means of simulation analysis and the best combination of the parameters $a, b$ was obtained.

It is important to point out the fact that no feedback about the state of the system (i.e., fading levels, traffic sources’ state) is necessary to apply the optimization descent (6.1).

**Optimal (Optimal bandwidth allocation solution):** since the SE&GD technique is guaranteed to achieve the optimal resource allocation after the sub-optimal transient period introduced by the employed gradient descent (6.1), it is possible to calculate the optimum system performance by observing the optimal allocations reached in steady state. Then, in a second stage of simulation, it is possible to re-apply such optimal allocations and to measure the optimum system performance.

7. **Simulation results**

We now illustrate a comparison among the aforementioned optimization techniques. Different traffic scenarios of variable traffic load and fading levels are taken into account. An investigation about the sensitivity of the obtained solutions with respect to the system parameters (e.g., MAU dimension, reallocation time intervals) will be also taken into account.

We have developed a C++ simulator for the network of queues that models the satellite system (see Fig. 2.3.1). Such satellite system is made by 2 earth stations. An ATM structure of the packets is employed. The simulations performed fall in the category of the so-called “finite time horizon” or “terminating” simulations [Paw02]. The Independent Replications technique for the analysis of stochastic simulation systems [Paw02] (i.e., the repetition of the same simulation with different pseudorandom number generators until a confidence interval is reached for the performance parameter) was applied. For all the results presented, the width of the confidence interval over the loss volume at the end of $T$ is less then 1% for 95% of the cases.

7.1 **Traffic load changes**

The time horizon of the following simulation scenario $T$ is fixed to 3.0 minutes and the channel capacity $K$ is fixed at 80.0 Mbps. We suppose no fading attenuations acting over the system (i.e., $\phi_i(t) = \lambda_i, \forall t \in [0, T]$). According to the self-similar traffic model introduced in Section 2, 100 on-off sources, with Pareto distributed burst periods of activity, generate a traffic stream which is aggregated in a unique traffic flow. Such flow constitutes the inflow process of each buffer of the satellite system. Each source is supposed to transmit at a peak bit rate $p_B$ of 1.0 Mbps. During the simulation, the average values of active and silence periods (respectively $\bar{\tau}$ and $\bar{\sigma}$) are changed following the scheme in the chart below:

<table>
<thead>
<tr>
<th>Time Interval</th>
<th>0.0 – 60.0 s</th>
<th>60.0 – 120.0 s</th>
<th>120.0 – 240.0 s</th>
</tr>
</thead>
<tbody>
<tr>
<td>Activity Period (sec)</td>
<td>$\bar{\tau}$</td>
<td>$\bar{\tau}$</td>
<td>$\bar{\tau}$</td>
</tr>
<tr>
<td>Station 1</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td>Station 2</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
</tr>
</tbody>
</table>

Since the number of on-off sources for each station is fixed at $M^i = 100$ for $i = 1, 2$, the station in high traffic conditions sees a burst arrival rate of $\lambda^i_{\text{burst}} = \frac{M^i}{\bar{\tau}^i + \bar{\sigma}^i} = 50$ bursts/s, $i = 1, 2$, while, for the one in low traffic conditions, the burst arrival rate is $\lambda^i_{\text{burst}} = 25$ bursts/s, $i = 1, 2$. Both stations are provided with a finite buffer of 100 cells, thus guaranteeing a reasonable bound for the mean delay and delay jitter. For instance, with an allocation of 25.0 Mbps (that is a lower bound of the following bandwidth allocations), such bound is around 1.7 ms for each ATM cell. The MAU of each station is 100 Kbps.

In the chart above it is shown that, every minute, traffic statistics have been changed, inverting the role of the heavy-traffic station and the low-traffic one. The reallocation period has been kept fixed at 1.0 second. 1.0 second between every bandwidth reallocation is a realistic value for a satellite network ([Bolla02, Cel03]). In fact, the master
control station has to get the sensitivity estimation of each earth station to compute the next allocation of bandwidth, and, particularly with geostationary satellites, which are supposed to be located at a distance of about 36,000 km from earth with a Round Trip Time near to 500-600 ms, it is necessary to take into account the relevance of the propagation time for this information. Fixing the reallocation time period at 1.0 second, we suppose to adopt a feasible lapse of time necessary for the bandwidth reallocation signal to become available to each station in the system.

A sample path in the SE&GD’s bandwidth allocations is depicted in Figure 7.1.1, where, for each station, the fraction of the total system’s capacity assigned by the SE&GD technique is visualized. It is clear how the algorithm is able to react to traffic variations: in the first minute, Station 2 suffers of heavier traffic load, and a larger quantity of MAUs has been allocated to it. The situation is inverted in the second minute, and it is evident in this case how SE&GD provides more resources to Station 1. Finally, in the last minute, the situation is brought back to the one of the beginning. The optimal steady states in the bandwidth allocations are founded by looking at the allocations performed by the SE&GD technique at the end of the transient periods of the employed gradient descent (6.1).

Figure 7.1.1. Variable traffic load scenario, Optimal vs. SE&GD's bandwidth allocations.
In Fig. 7.1.2, the CF&DP’s bandwidth allocations are depicted together with the ones obtained through the SE&GD technique. The CF&DP reveals to be a good heuristic for the bandwidth allocation, because it is able to follow the variable traffic conditions. However, it is not able to maintain the best resource allocation: the bandwidth allocation to the station in heavier traffic load is lower than the one obtained by applying the SE&GD technique (around 47.0 Mbps using the CF&DP technique in front of the 52.0 Mbps obtained with the SE&GD algorithm). This, clearly, has an impact on the system performance. In Fig. 7.1.3, the overall loss probability of the system is shown. With the “CF&DP ErrX” notation, we denote the application of the CF&DP technique with a percentual error over the traffic load foreseen at station 1, e.g., with “CF&DP ErrX” we highlight the performance of the CF&DP technique in which the feedback over the state of the first station underestimates the real traffic load with a percentual error that amounts to \( X \times 100\% \) over the real value. For example, with \( X=10\% \), a \( \lambda_{burst} \) of 45 burst/s is employed in the functional cost (3.3) of the CF&DP technique instead of the real value of 50 burst/s. Such estimation errors can severely decrease the CF&DP’s performance, especially if they affect the feedback with a percentual error around 30\%. On the other hand, with a perfect feedback on the system’s state, the CF&DP technique reaches good performance, but only the application of the SE&GD technique guarantees the best approximation of the optimal solution.

It seems that the SE&GD technique is able to learn the optimal equilibrium in the bandwidth allocation, thus guaranteeing better performance than the one reached by the certainty equivalent approach, even in presence of a traffic load whose statistical behaviour is very close to the assumptions adopted to provide a closed form expression of the loss performance metric. The intuition behind such efficiency stems from the fact that, by means of its gradient descent, the SE&GD technique tries to follow the resource allocation that guarantees that all the components of the gradient of the functional cost achieve the same values. In this way, the equilibrium point that corresponds to the minimum value of performance index can be obtained. Moreover, since the gradient estimation procedure is continuously performed during the system evolution, such optimal equilibrium reveals to be adaptive to the current realization of the stochastic processes involved in the system. As a consequence, without any direct feedback over the system’s state, such technique is able to learn the best resource allocation according to the chosen performance index and with respect to the current realization of the stochastic processes. On the other hand, the CF&DP technique, even if it disposes of a perfect knowledge over the system’s state, is not able to catch the optimal resource allocation. The employed closed form expression of the performance index can be seen as only an heuristic indication about the performance achieved by the system under the current state of the network. The application of the closed form expression (3.1) is not able to provide
the “sensitivity measures” performed by the SE&GD technique. For this reason, it should be expected that only the SE&GD technique is able to learn the bandwidth allocation that exactly solves the resource allocation problem under the current realization of the stochastic processes involved in the system.

7.2 A priori assumptions over the traffic sources versus their real statistical behaviour.

There is one deeper motivation behind the inability of the CF&DP approach in providing the optimal solution of the resource allocation problem. Such motivation is related to the possible differences between the statistical behaviour of the traffic processes assumed a-priori and the real one. The a-priori assumptions made over the traffic sources can reveal to be insufficient to catch their real statistical behaviour. In order to highlight this fact, we show an example related to the self-similarity assumption made in this paper, in which, even if such assumptions seem to be always in effect, the parameters of the closed form functional cost reveal to be insufficient in accurately describing the real statistical behaviour of the traffic processes.

In fact, in order to employ the closed form loss probability formula (3.1), the assumption of a self-similar behaviour of the inflow processes has been made a-priori. As we have already mentioned in Section 2.2, a self-similar behavior of a traffic flow means that it maintains a high variability in the rate produced by its inflow process. The Tsybakov-Georganas formula (3.1) assumes an exact self-similar behaviour, namely, such high variability can be always observed at every time scale we measure the bit rate produced by the traffic source. A traffic process composed by a group of on-off sources shows an exact self-similar behaviour only when the number of the sources tends to infinity (the so-called asymptotical self-similarity; see, e.g., [Tsybakov98] for further details). In a real scenario, this hypothesis reveals to be quite unrealistic, since the number of traffic sources is always finite and, in practice, different asymptotical self-similar behaviours can be obtained by varying the number $M$ of on-off sources in the aggregate flow.

In Fig. 7.2.4, the temporal evolution of a self similar flow is depicted with different values of $M$. With $M = 100$, the $\tau$ and $\sigma$ parameters are both fixed at 1.0 s and with $M = 300$, they are fixed at 1.0 s and 5.0 s, respectively. In this way, the obtained $\lambda_{burst}$ is always 50 burst/s and the impact of the $M$ parameter on the traffic traces can be highlighted.

To show the self-similarity of a traffic flow, a sequence of measures (about the bit rate produced by the inflow process during a finite time observation period) must be performed over the inflow process (see, e.g., [Pitts00]). An exact self-similar behaviour guarantees that, in spite of every possible dimension of such observation window, high values in the input rate are observed. On the other hand, in the presence of a more regular traffic process, such measures tend quite rapidly to the mean value of the process’s input rate as the dimension of the observation window increases. We denote with $W$ the dimension of such observation window. Three different values of $W$ are employed: 0.1 s, 1.0 s and 10.0 s. As expected, with 100 sources in the aggregated flow, an observation window of $W = 10.0$ s always returns measures very close to the mean rate of the input process (around 48.0 Mbps). On the other hand, with 300 sources in the aggregated flow the measures performed every 10.0 seconds show a sensible variance over the the mean rate of the inflow process. To better emphasize this difference, in Fig. 7.2.5, the variance above the measures performed in the $W = 10.0$ case are shown. Clearly, in the $M = 100$ case, the variance is always around zero, while in the $M = 300$ case, an higher variability over the performed measures arises.
As we have briefly shown, even with equal values of the $\lambda_{\text{burst}}$ parameter, an inflow process composed by a group of on-off sources, whose mean periods of activity $\tau$ are Pareto distributed, can show, in practice, different self-similar behaviours. As a result, we could claim that the a-priori assumptions about the traffic sources, even if necessary for the employment of the closed form formula (3.1), are only an approximation of the real behaviour of the inflow processes. The CF&DP technique cannot react to such sensible differences. In fact, if the $\lambda_{\text{burst}}$ and the $\tau$ parameters are maintained constant in updating eq. (3.1), it remains insensitive to any change in the number of sources in the aggregated flow and, as a consequence, to any possible different self-similar realization of the inflow processes. So, it can only guarantees an approximation of the optimal resource allocation, because it is not able to distinguish among traffic sources that can follow, in practice, sensible differences in their statistical behaviour.

Clearly, in order to take into account the real statistical behaviour of the traffic sources, it should be necessary to apply proper estimation algorithms over the inflow processes and this can require the adoption of complex estimation techniques. However, the SE&GD algorithm developed in this paper, in virtue of its sensitivity estimation capability directly applied over the gradient of the chosen performance metric, reveals to be a very effective optimization...
technique, as it is able to learn the real impact of statistical behaviour of the traffic sources over the system performance.

7.3 Fading changes

We consider now the effect of the fading phenomenon. The time horizon of the simulation scenario $T$ has been increased to 20.0 minutes and the channel capacity $K$ is fixed at 50.0 Mbps. Again, a group of on-off sources with Pareto distributed burst periods of activity constitutes the inflow process for each satellite station. The peak bit rate $B_p$, the mean burst $\tau$ and silence period $\sigma$ of such on-off sources are fixed to 1.0 Mbps, 1.0 s and 1.0 s, respectively. The number of on-off sources for each station is fixed at $M^i = 10$, $i = 1, 2$. All of the other system’s parameters (buffer dimensions, MAU values and reallocation time interval’s length) are maintained the same as in the latter simulation scenario. This time, no traffic changes take place, namely, each inflow process generates a $\lambda^i_{burst} = 5$ bursts/s for each station $i$ of the satellite system, $i = 1, 2$.

The employed fading processes come from [Bolla02, Cel03], where real-life fading attenuation samples are taken from a data set chosen from the results of experiments, in Ka band, carried out on the Olympus satellite by the CSTS (Centro Studi sulle Telecomunicazioni Spaziali) Institute, on behalf of the Italian Space Agency. The up-link (30 Ghz) and down-link (20 Ghz) samples considered were 1-second averages, expressed in dB, of the signal power attenuation with respect to clear sky conditions. The Carrier/Noise Power $(C/N_0)$ factor is monitored at each station and, on the basis of its values, different convolutional data coding rates are applied in order to limit the BER below a chosen threshold of $10^{-7}$. Six different fading classes are defined with the corresponding redundancy factors $\delta^i_{level}$ of the coding rate ($\delta^i_{level} \geq 1.0$):

$$\phi(i) \in \{0.0, 0.15625, 0.3125, 0.625, 0.8333, 1.0\}, \ i = 1, 2$$

and, consequently, the bandwidth reduction can be computed as $\theta_i = \phi_i \cdot \delta^i_{level}; \phi = \frac{1}{\delta^i_{level}}$. As is shown in Fig. 7.3.1, the employed fading processes determine strong peaks of channel degradation, especially for the first station.

![Fading levels at station 1 & 2](image)

**Figure 7.3.1.** Fading changes scenario, fading levels.

In Figs. 7.3.2 and 7.3.3, the bandwidth allocations of the SE&GD and the CF&DP techniques are depicted for each satellite station. This time, the application of the CF&DP technique guarantees more sensible differences in the bandwidth allocations, since the SE&GD technique reacts to fading variations only when the first strong attenuation at station 1 arises (after around 500 seconds of simulation). This is due to the fact that, in the first 500 seconds, the bandwidth attenuation at station 2 does not imply high loss probability values and, as a consequence, also the gradient component of station 2 does not reveal high differences with respect to the gradient values of station 1. On the other hand, the CF&DP technique, in spite of its perfect feedback over the fading states, it is always able to perform little changes in the bandwidth allocation, as it is clear by looking at its bandwidth allocations in the first 500 seconds of simulation. Anyway, this fact does not allow the CF&DP technique to outperform the SE&GD’s performance. If we look at the bandwidth allocations during the strong peaks of fading attenuations at station 1 (after the first 500 seconds of simulation), we note that the CF&DP technique, as in the previous simulation scenario, is not able to reach the optimal resource allocation, particularly when station 1 is affected by the lowest fading level $\phi = 0.15625$. On the other hand, since the SE&GD technique continuously updates the gradient estimation values, it is able to find the new equilibrium point at a higher value in the bandwidth allocation for station 1. Such difference is emphasized in Fig. 7.3.4, where the
aforementioned techniques are compared with respect to the bandwidth allocation at station 1. Finally, the obtained loss probability performance is shown in Fig. 7.3.5, where the presence of estimation errors over the state of the traffic load is also taken into account. As in the latter simulation scenario, “CF&DP ErrX” means that the feedback over the state of the first station underestimates the real traffic load with a percentual error of X⋅100%. Looking at the obtained results, it is quite clear how the SE&GD technique is able to reach the best performance. Even though, this time, the CF&DP technique reveals a performance more insensitive with respect to estimation errors over the traffic state (because the major impact over the system performance is due to the effect of the fading changes), it is far away to maintain the optimal solution of the resource allocation problem.

Figure 7.3.2. Fading changes scenario, SE&GD’s bandwidth allocation.

Figure 7.3.3. Fading changes scenario, CF&DP’s bandwidth allocation.

Figure 7.3.4. Fading changes scenario, CF&DP vs. SE&GD’s bandwidth allocation at station 1.
One final remark is necessary concerning the SE&GD’s bandwidth allocations in this simulation scenario. If we look at the SE&GD’s bandwidth allocations in the last 240 seconds of simulation, we can observe that the equilibrium point reached by the SE&GD technique with respect to the last fading values ($\phi_1 = 1.0$, $\phi_2 = 0.8333$) is far away from the one expected. In fact, the optimal solution for the last 240 seconds should be quite close to an equally distributed resource allocation as happens in the CF&DP’s bandwidth allocation case. On the contrary, the SE&GD’s bandwidth allocation remains quite close to the previous one that corresponds to the fading values $\phi_1 = 0.15625$, $\phi_2 = 1.0$. This is due to the fact that, even if a strong change in the fading levels of the time intervals [725;960] [960;1200] arises, the gradient values computed during the time interval [960;1200] are very low and, as a consequence, they are not sufficient to carry on the bandwidth allocation to a new equilibrium point closer to an equally distributed bandwidth allocation. The rationale behind such inefficiency comes from the fact that, for the particular sample paths shown in Figs. 7.3.2-7.3.4, using the SE&GD technique, very low losses values are observed for both stations in the time interval [960;1200] and, as a consequence, also the loss derivatives achieve low values, thus leading to a suboptimal bandwidth allocation for the last time interval [960;1200]. However, as also shown in the SE&GD’s results of Fig. 7.3.5 (averaged over 25 independent replications of the proposed simulation scenario in order to meet the confidence interval requirement), in the presence of higher loss values during the time interval [960;1200], the SE&GD technique is always able to react and move to a much more different equilibrium point in the bandwidth allocations.

Anyway, the aforementioned SE&GD’s behaviour in the last time interval [960;1200] would induce a deeper investigation concerning the possibility of reaching only a sub-optimal resource allocation with respect to particular realizations of the stochastic processes involved in the system. This is subject of ongoing reasearch taking into account not only further simulation scenarios for the resource allocation problem addressed in this paper, but also for other resource allocation frameworks (for example in the context of terrestrial wireless networks) and for other performance measures (e.g., delay and delay jitter).

5.3 Variable MAU’s dimensions and reallocation time interval’s length.

As mentioned before, the reallocation period becomes a critical parameter in a satellite environment under different traffic conditions. We now go deep into this topic. Suppose that no fading degradations affect the satellite system. Fig. 5.3.1 depicts, with respect to different traffic load conditions at the two stations of the satellite system, the advantage of adopting an “ideal” period of 0.1 seconds in place of the one adopted, 1.0 second. We show in this way that, in our case, the ideal period does not involve a strong enhancement in the system performance, while fixing the period to 10.0 seconds induces a much more significant detriment.

Also the size of the MAU is a very important parameter in our discrete optimisation problem, because its excessive granularity would involve a too high computational burden in the dynamic programming optimization approach. On the other hand, if the MAU is sized to a large value, the accuracy of the solution could be strongly deteriorated. Figure 5.2.7 depicts exactly this problem at various traffic conditions, and shows that the case of MAU=1 Mbps does not achieve largely worse performance than the case of MAU=100 Kbps, while the situation is strongly different for MAU=6 Mbps, particularly at high traffic loads.
6. Conclusions and Future Work

A novel optimization algorithm, called SE&GD (since it follows a Sensitivity Estimation and a Gradient Descent approach), based on the gradient estimation of the Infinitesimal Perturbation Analysis technique has been applied to react to fading effects and traffic load changes over a satellite network. Such optimization algorithm, due to its sensitivity estimation capability, does not need any closed form expression of the performance measure. The SE&GD optimization approach has been compared with an optimization technique based on a closed form expression of a performance measure and on the application of a dynamic programming algorithm (CF&DP).

The results have shows how the SE&GD technique allows strong performance improvements both in variable fading and traffic scenarios. More in particular, SE&GD reveals to be a very effective technique in on-line operating conditions, since it permits to compute sensitivity estimations based only on sample paths of the real system, and to catch the main features of the stochastic system ("active learning" [Cass93]), in order to optimally react when variations on the environment take place. This last conclusion is enforced from the fact that no feedback about fading level or traffic load is necessary for the application of the IPA sensitivity estimation procedure. Moreover, its suitability in real on-line optimization scenarios is due to the fact that the SE&GD technique requires a lighter computation effort with respect to the CF&DP algorithm.

Future work can include the application of the proposed SE&GD approach in order to solve other resource allocation problems for other important QoS parameters, such as delay and delay jitter. Different application scenarios will be taken into consideration, for example for wireless, or QoS networks.
Appendix A. Dynamic programming algorithm for the minimization of the overall loss probability by means of the Tsybakov-Georganas formula (3.1).

The algorithm employed by the CF&DP approach is based on the adoption of the loss probability formula (3.1). Then, in order to find out the optimal resource allocation, a dynamic programming algorithm must be used at each reallocation time instant.

We briefly report the C pseudocode for such algorithm. It is important to underline the fact that the required computational complexity is $O(bandmax^2 \cdot N)$, where $bandmax$ is the maximum number of available Minimum Allocation Units (MAUs) to the satellite system and $N$ is the number of active stations in the system. The proof regarding such computational complexity can be easily obtained by observing that the initialization of the dynamic programming cost-to-go requires a number of assignments equal to $bandmax^2 \cdot N$ (look at the triple “for” cycle at the beginning of the pseudocode of the next page).

```c
int bandmax = Maximum number of available MAUs in the satellite system;
int N = number of active stations in the system;
int allocations[N] = bandwidth allocations among the stations according to the current state of the network;

/* return the value of the closed form functional cost according to the current state of station j that receives a bandwidth allocation of i MAUs */
double FunctionalCost(\lambda_j^{burst}, \tau_j; \phi_j , i){
    // apply the loss probability formula (3.1) according to the traffic and fading parameters \lambda_j^{burst}, \tau_j; \phi_j and bandwidth allocation of i MAUs */
}

//return the minimum value of a vector addressed by the “*pointer” variable and with “numb” elements
double minimum(double *pointer, int numb){
}

//return the position of the element in the vector addressed by the “*pointer” variable and with “numb” elements equal to the value of the variable “valuecomparison”
int position(double* pointer, int numb, double valuecomparison){
}

double h[N][bandmax+1];  //dynamic programming cost-to-go. Given an amount of K MAUs available for station k and k-1, different allocation choices are available: allocations[k]=i, allocations[k-1]=K-i, i=1,...,K. The best bandwidth allocation \hat{r} satisfies:

\hat{r} = \arg \min_i \{ FunctionalCost(\lambda_i^{burst}, \tau_i; \phi_i , i) + FunctionalCost(\lambda_{i-1}^{burst}, \tau_{i-1}; \phi_{i-1} , K-i) \}

and h[k][K] contains the cost value of such an assignment.
```
// For all active stations j = 1,...,N compute the functional cost with i MAUs available
for (j=0, j<=N, j++)
  for (i=0, i<= bandmax, i++)
    cost[j][i]=FunctionalCost( λburst, μj, φj , i);

for (i=0, i<= bandmax, i++)
  h[0][i]=cost[0][i];

for (k=1, k<=N-1, k++)
  for (i=0, i<= bandmax, i++)
    for (j=0, j<= i, j++)
      // j MAUs to station k, i-j MAUs to station k-1 imply a cost value equal to temp[j]
      temp[j]=cost[k][j]+h[k-1][i-j];
    // h[k][i]: minimum cost corresponding to i MAUs available for stations k and k-1
    h[k][i]=minimum(&temp[0],i);

int capacity = bandmax;
double lastmin;
for(j=0, j<= bandmax, j++)
  h[N-1][j]=cost[N-1][j]+h[N-2][capacity-j];
  // lastmin: minimum cost corresponding to bandmax MAUs available for stations N and N-1
  lastmin=minimum(&h[N-1][0], bandmax);
}

// allocate to station N a number of MAUs with respect to the minimum value of h[N-1][0]
allocation[N-1]=position(&h[N-1][0], bandmax,lastmin);

// The available bandwidth is now bandmax - allocation[N-1]
capacity= capacity - allocations[N-1];

for (k=N-2, k>0, k--)
  for (j=0 to capacity)
    // cost corresponding to j MAUs to station k and capacity-j MAUs to station k-1
    z[j]=cost[k][j]+h[k-1][capacity-j];
  /* allocate to station k the number of MAUs which corresponds to h[k][capacity] ,
  namely, the minimum cost corresponding to capacity MAUs available for stations k and k-1 */
  allocations[k]=position(&z[0],capacity,h[k][capacity];
  capacity= capacity - allocations[k];
}

// for station 1 the current value of capacity is finally available
allocations[0]=capacity;
Appendix B. Mapping algorithm for the conversion of the surrogate continuous allocation to the discrete bandwidth allocation with respect to the channel constraint.

The algorithm that allows the conversion from $\theta^c$ to the nearest discrete feasible neighbour $\theta^d$ is described as follows:

- Apply a normalization in order to guarantee $\theta^c \in \Theta^c$, $\Theta^c = \left\{ \theta^c_i \in \mathbb{R}^+ | i = 1, \ldots, N; \sum_{i=1}^{N} \theta^c_i = K \right\}$;
- Initialize the index set $I = \{1, \ldots, N\}$ and define a temporary vector $v = \theta^c - \lfloor \theta^c \rfloor$;
- While $I \neq \emptyset$ do:
  1. $\bar{\theta}(n) = \sum_{j=1}^{N} e_j$, where $n = \text{arg min} \{v_j, j \in I\}$, and $e_j$ is a vector with all 0 components, except for the $j^{th}$ one, which is 1. $\bar{\theta}(n)$ is the $n^{th}$ vector of a set of $N+1$;
  2. $\alpha_{n} = v_n$;
  3. $v \leftarrow v - \alpha_{n} \bar{\theta}(n)$;
  4. $I \leftarrow I \setminus \{n\}$;
- $\bar{\theta}(0) = 0$;
- $\alpha_0 = 1 - \sum_{n=1}^{N} \alpha_n$;
- Define a set of $N+1$ discrete feasible neighbours $S(\theta^d)$ as follows:
  
  $S(\theta^d) = \left\{ \theta^d(n) : \theta^d(n) = \theta^d(n) + \lfloor \theta^c \rfloor \text{ for } n = 1, \ldots, N \right\}$

Fig. 4.1.2 The algorithm adopted to map the bandwidth allocations in the constraint discrete set $\Theta_d$

Once we have found this last set $S(\theta^d)$, which is composed of vectors whose components are between $\lfloor \theta^c(n) \rfloor$ and $\lfloor \theta^c(n) \rfloor + 1$, it is possible to notice that not all of them are feasible, and only one respects the constraint $\theta^d \in \Theta_d$; this one is the nearest feasible neighbour of $\theta^c(k+1)$ we were looking for. For further details about the demonstration of the algorithm see [34].
References


